

Final Project

Due dates:

Presentation: December 09, 2025
Report: December 12, 2025

Submit report on Canvas: <https://canvas.illinois.edu/courses/61487/assignments/1356826>

Groups · This project may be completed in **groups of up to three students**. All members will receive the **same grade** and are expected to contribute equally. Any concerns about unequal contributions should be communicated directly to the instructor.

Project content · This final project assignment is designed to introduce you to graduate-level multiphase flow research. You will study and critically review the scientific literature on a specific multiphase flow topic, and verify theoretical concepts and/or models through small numerical simulations and/or experiments. This assignment supports the following educational objectives:

- Research and select relevant high-quality scientific literature.
- Understand, synthesize, and explain theories, results, and models.
- Organize complex information into a structured, rigorous, and critical document.
- Present your work clearly, concisely, and professionally in both oral and written formats.
- Enrich the course content by exploring phenomena or models not covered in class so far.

The main steps of the project are:

1. Choose a model or theoretical concept related to multiphase flows (**before November 25**).
2. Study and review the existing scientific literature related to this topic.
3. Verify the model or theoretical concept using small experiments and/or numerical simulations.
4. Assess the validity and limitations of the selected model or theoretical concept.

Evaluation · You will present your work in class on **December 9**, followed by a written report due on **December 12**. Your presentation must fit within **10 minutes** (this will be strictly enforced) and contain the following key items: (i) literature review, (ii) description of your choice of model/theory, (iii) verification results, and (iv) assessment of validity and limits. The written report must fit within **4 to 6 pages**, excluding references, and must use the **Journal of Fluid Mechanics L^AT_EX template** (also directly available on **Overleaf**). Your literature review should reference ~ 20 – 30 journal articles, including foundational papers of the research field, relevant review articles, and recent work from the last decade (i.e., the state of the art). Whenever possible, you are encouraged to produce synthesis tables or sketches that summarize and support your statements. Code used for simulations or data processing does not count toward the page limit and may be submitted as supplementary material.

The presentation and report each contribute 50% of the final project grade. Grading rubrics are provided at the end of this document.

Research topic suggestions · You may choose any of the suggested topics below or propose your own. If you wish to propose your own topic, it must meet the following requirements:

- It must be different from any previous or current work you have done for your MSc or PhD thesis.
- It must receive instructor approval by **November 25, 2025**.

In other words, you may propose a topic that is related to your graduate research as long as you can clearly show that it is a new direction and not something you have previously worked on.

List of suggested final project topics:

Project 1: Nonlinear stability analysis of a cylindrical liquid jet · Linear stability theory predicts the breakup of a cylindrical jet into droplets separated by the most unstable wavelength, $\lambda_c \simeq 9R$, but cannot predict the formation of satellite droplets, which appear both in experiments (see Fig. 1) and simulations. In the early 1970's, Rutland and Jameson [7] argued that the nonlinear stability of Yuen [9] explains the formation of these satellite droplets.

Suggested workflow for your final project:

1. Review the literature on the nonlinear stability of a cylindrical liquid jet and the prediction of satellite-droplet formation as a result of the Plateau-Rayleigh instability (this includes but is not limited to the references [9, 7]).
2. Use the Yuen/Rutland model (or any other model you may have found) to predict satellite-droplet sizes, and validate these predictions using small experiments and/or numerical simulations, e.g., using the open-source multiphase flow solver **Basilisk** and its **axisymmetric Plateau-Rayleigh example**.
3. Assess the accuracy and limitations of the model.

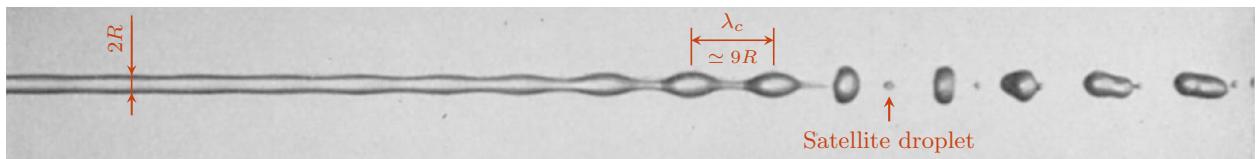


Figure 1: Illustration of the Plateau-Rayleigh instability of a millimeter-size water jet, from the experiments of Donnelly and Glaberson [3].

Project 2: Drag on nonspherical particles using the Method of Regularized Stokeslets · In particle-laden flows found in Nature or industrial applications, particles are rarely spherical and, as a result, require shape-dependent drag laws. The Method of Regularized Stokeslets (MRS) of Cortez [2] provides a flexible numerical approach for computing the forces acting on arbitrarily shaped bodies in the Stokes flow regime, so long as their surface can be populated by a cloud of (approximately) equidistant marker points. This method can be implemented in a few lines of Python code (typically less than 20) using standard functions of the Numpy library.

Suggested workflow for your final project:

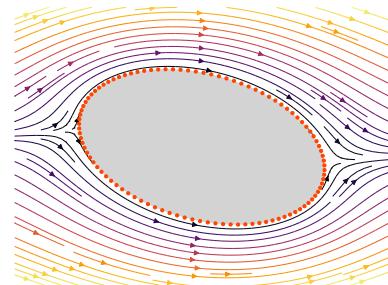


Figure 2: Streamlines of the Stokes flow solution around an ellipsoid produced with the MRS

1. Review the literature on the MRS (including [2]) and on drag laws for nonspherical particles (starting with ellipsoidal particles).
2. Implement the MRS to compute drag on a chosen nonspherical particle in uniform and nonuniform flows. Validate against analytical drag laws, experimental results, and/or other numerical results.
3. Assess the strengths and limitations of the MRS for predicting drag on nonspherical particles.

Project 3: Preferential orientation of ellipsoids · In HW3, you studied the settling of spherical particles in homogeneous isotropic turbulence (HIT). Replacing the spheres by ellipsoids introduces orientation dynamics that typically lead to preferential orientation (e.g., oblate *disk-like* ellipsoids align their symmetry axis with gravity).

Suggested workflow for your final project:

1. Review the literature on preferential concentration and preferential orientation of nonspherical particles in turbulence, with and without gravity.
 2. Use the HIT dataset from HW3 to run one-way coupled simulations of ellipsoidal point-particles with and without gravity. This requires tracking particle orientation (solving angular momentum conservation) and implementing orientation-dependent drag laws (e.g., based on the analytical work of Jeffery [5] and Brenner [1]).
 3. Evaluate the applicability and limits of the orientation mechanisms stipulated in the literature.
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Project 4: Numerical simulation of bubble cavitation · In Chapter 4, we derived the Rayleigh-Plesset equation describing the nonlinear shape oscillation of spherical bubbles in incompressible liquids due to pressure variations. Building upon the work of Plesset [8], later models (e.g., by Keller-Miksis [6] or Gilmore [4]) expand the Rayleigh-Plesset equation by incorporating liquid compressibility, albeit each using different building assumptions.

Suggested workflow for your final project:

1. Review the literature on pressure-driven bubble dynamics and bubble-radius evolution equations in incompressible and compressible liquids.
 2. Use the Rayleigh-Plesset equation and at least one compressible extension to predict bubble-radius evolution under pressure forcing. Validate against experimental results and/or numerical simulations, e.g., using the open-source multiphase flow solver [Basilisk](#) and its [spherically-symmetric and axisymmetric bubble collapse example](#).
 3. Compare the predictive range and limitations of the models considered.
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References

- [1] H. Brenner, *The Stokes resistance of an arbitrary particle*. Chem. Eng. Sci., vol. 18 (1963).
- [2] R. Cortez, *The Method of Regularized Stokeslets*. SIAM J. Sci. Comput., vol. 23 (2001).
- [3] R. J. Donnelly and W. Glaberson, *Experiments on the Capillary Instability of a Liquid Jet*. Proc. R. Soc. Lond. A Math. Phys. Sci., vol. 290 (1966).
- [4] F. R. Gilmore, *The growth or collapse of a spherical bubble in a viscous compressible liquid*. ONR Report No. 26-4 (1952).
- [5] G. B. Jeffery, *The motion of ellipsoidal particles immersed in a viscous fluid*. Proc. R. Soc. Lond A Math. Phys. Sci., vol. 102 (1922).
- [6] J. B. Keller and M. Miksis, *Bubble oscillations of large amplitude*. J. Acoust. Soc. Am., vol. 68 (1980).
- [7] D. F. Rutland and G. J. Jameson, *A non-linear effect in the capillary instability of liquid jets*. J. Fluid Mech., vol. 46 (1971).
- [8] M. S. Plesset, *The Dynamics of Cavitation Bubbles*. J. App. Mech., vol. 16 (1949).
- [9] M. Yuen, *Non-linear capillary instability of a liquid jet*. J. Fluid Mech., vol. 33 (1968).

Rubric 1: Oral presentation (50% of the final project grade)

Criteria	Minimal credit ≤ 5	Below expectations 5 – 7	Satisfactory/good 7 – 9	Exemplary 9 – 10	Grade /10
Technical content (25 %) <ul style="list-style-type: none">• Literature review• Theory/model description• Validation and critical assessment	Technical content missing, incorrect, or incoherent. Major conceptual gaps, little to no synthesis, significant inaccuracies.	Covers the main elements but with limited synthesis or conceptual clarity. Some inaccuracies, missing or superficial discussion.	Technical content is accurate and mostly complete. Synthesis is present but may lack depth or integration in some areas.	Demonstrates a deep understanding of the topic. Accurately presents relevant literature and clearly synthesizes theories, models, results, and limitations. Provides a coherent narrative connecting theory, methods, results, and critical evaluation.	
Organization and clarity of the presentation (25 %) <ul style="list-style-type: none">• Logical structure• Time management• Organization within the group	Poorly structured, unclear, unprofessional, or significantly under/over time.	Adequate structure but noticeable issues in pacing or clarity.	Overall clear and well organized; minor timing or clarity issues.	Presentation is exceptionally clear, well-structured, and professional; slides support the narrative very well; timing (10 minutes) is respected.	
Visual quality of slides (25 %) <ul style="list-style-type: none">• Readability• Quality of figures• Conciseness	Difficult to follow or read; visuals distract from content.	Slides are cluttered, filled with dense text, or with low-quality figures.	Mostly clear but with minor readability or layout issues.	Slide design is clear, readable, visually balanced, and professional. Figures/tables are of high quality, appropriately labeled, and enhance understanding.	
Answers to audience questions (25 %) <ul style="list-style-type: none">• Clarity of answers• Ability to defend arguments	Cannot respond meaningfully to any question.	Responses partially correct or incomplete.	Answers most questions well, with minor gaps.	Responds accurately, concisely, and confidently; demonstrates mastery of topic.	

Rubric 2: Written report (50% of the final project grade)

Criteria	Minimal credit ≤ 5	Below expectations 5 – 7	Satisfactory/good 7 – 9	Exemplary 9 – 10	Grade /10
Literature review (20 %) <ul style="list-style-type: none">• Choice of references• Quality of synthesis• Critical evaluation	Minimal, inaccurate, or poorly researched literature.	Incomplete or insufficiently analyzed literature.	Good coverage with moderate synthesis but mostly descriptive.	Comprehensive review with 20 to 30 relevant articles; demonstrates synthesis and critical evaluation; identifies gaps or open questions.	
Explanation of the chosen theory/model (20 %) <ul style="list-style-type: none">• Rigorous description• Clear assumptions• Sound physical understanding	Incorrect, unclear, or absent explanation.	Basic explanation lacking depth or rigor.	Reasonably clear and mostly correct; minor gaps.	Rigorous and precise description of the chosen theory/-model; equations, assumptions, and physical interpretation are summarized effectively.	
Verification methodology and results (20 %) <ul style="list-style-type: none">• Description of the verification technique(s)• Results are clearly presented• Results are reproducible	Insufficient or incorrect verification.	Reasonable but shallow or partially flawed verification.	Solid methodology and results but with limited discussion or clarity.	Verification method is scientifically sound; simulations/-experiments are well designed; results are clearly presented, reproducible, and thoroughly analyzed.	
Critical assessment and insight (20 %) <ul style="list-style-type: none">• Evidence-based criticism• Original insight	Missing or superficial assessment.	Basic criticism without strong support.	Good assessment but lacking depth or nuance.	Thorough evidence-based evaluation of the theory/model's validity, assumptions, and limitations; demonstrates original insight.	
Structure and writing quality (20 %) <ul style="list-style-type: none">• Well structured and balanced report• Proper template use• Quality of supporting figures	Poorly written, disorganized, or improperly formatted.	Adequate structure but unclear writing or formatting issues.	Well organized with minor issues in clarity or formatting.	Exceptionally clear, coherent, rigorous writing; logically organized sections; JFM template used correctly; figures/tables labeled and discussed; free of grammatical and typographical errors.	