

1 **From Low-Quality to High-Quality: Generating Structure-Preserved**

2 **Atomic-Scale HRTEM images via an Enhanced CycleGAN**

3 **Abstract** High-quality high-resolution transmission electron microscopy (HRTEM)
4 images are essential for linking materials structure–property relationships at the atomic
5 scale. However, capturing dynamic processes with high temporal resolution inevitably
6 leads to severely degraded HRTEM images under low-dose imaging conditions, which
7 substantially limits accurate structural analysis. In this paper, we develop a structure-
8 preserving HRTEM restoration framework that enhances low-quality HRTEM images
9 with blurred or incomplete atomic arrangements using a generative deep learning
10 approach while maintaining physical fidelity. Specifically, we propose HRTEM-GAN,
11 a cycle-consistent generative framework that operates under unpaired training
12 conditions and performs patch-level distribution modeling between high- and low-
13 quality image domains, while explicitly incorporating frequency-domain constraints to
14 preserve atomic-scale structural fidelity. This design enables effective restoration of
15 low-dose HRTEM images, yielding structurally coherent atomic lattices that are fully
16 suitable for subsequent recognition and quantitative analysis. The proposed method is
17 validated on a real experimental dataset acquired during *in situ* imaging of Au catalysts
18 under CO oxidation conditions. Compared with representative methods, HRTEM GAN
19 achieves substantial improvements in image restoration quality and consistently
20 enhances downstream atomic column recognition performance. These results
21 demonstrate the potential of the proposed framework to facilitate reliable atomic-scale
22 analysis in HRTEM studies.

23 **Keywords** HRTEM, Atomic-resolution image restoration, Unpaired image-to-image
24 translation, Generative adversarial networks.

25 **1 Introduction**

26 Understanding and controlling material properties through atomic-scale structure
27 has long been a central pursuit in materials science, as atomic arrangements

28 fundamentally govern material functionality and performance [1–4]错误!未找到引用
29 源。. High-resolution transmission electron microscopy (HRTEM), therefore, plays a
30 pivotal role in reliable materials characterization [5–7]错误!未找到引用源。; however,
31 realistic experimental constraints often result in severely degraded images, which in
32 turn hinder accurate identification of atomic sites and subsequent structural analysis
33 [4,8]. For instance, in catalytic systems, chemical reactions are often accompanied by
34 rapid atomic-scale structural rearrangements occurring on timescales of tens of
35 milliseconds [9–11]错误!未找到引用源。. Capturing image sequences at such high
36 temporal resolution inevitably yields images that are strongly affected by shot noise,
37 further complicating atomic-scale feature extraction and statistical analysis. It is
38 generally impractical to suppress noise by increasing the incident electron beam
39 intensity, as high-dose exposure to high-energy electrons can induce damage to the
40 material. This motivates the need to recover high-quality (LQ) atomic-resolution
41 information from degraded low-quality (LQ) HRTEM images while preserving
42 physical fidelity [12,13].

43 In recent years, a variety of computational methods, including conventional image
44 processing algorithms [14–16]错误!未找到引用源。错误!未找到引用源。 and deep
45 learning techniques [17–19]错误!未找到引用源。, have been applied to mitigate the
46 inherent limitations of microscopy imaging and improve image quality. Conventional
47 rule-based methods, such as Wiener filtering, bilateral filtering, and BM3D, have been
48 widely used for noise suppression and basic image enhancement [16,20]. While
49 effective in attenuating random noise, these methods rely on fixed assumptions and
50 limited local statistics, making it difficult to robustly handle complex backgrounds and
51 severe image degradation. Recently, deep learning (DL)-based methods, leveraging
52 their strong non-linear modeling capacity, have been extensively explored for electron
53 microscopy image processing, primarily targeting low-level image enhancement tasks
54 such as denoising, super-resolution, and generation [17].

Representative DL-based denoising approaches for TEM typically rely on supervised learning with synthetic or simulated training data [18,19,21]. For example, Lin et al. [18] introduced the AtomSegNet framework, in which convolutional encoder–decoder networks are trained on physics-informed simulated images. Mohan et al. [12] further proposed a simulation-based denoising (SBD) framework, demonstrating that CNNs trained on carefully designed forward models can effectively denoise low-SNR TEM images and generalize to real experimental data, particularly when large receptive fields are used to capture non-local atomic periodicities. More recently, frequency-aware enhancement strategies have been explored, incorporating spatial–frequency interactions to better exploit the periodic nature of atomic arrangements. Li et al. [22] proposed a framework that integrates a spatial-frequency interaction network with noise calibration-based data synthesis to model frequency-domain information. Nevertheless, most existing methods still struggle under severely degraded imaging conditions. Importantly, most methods rely on synthetic images to construct paired samples for supervision, while overlooking the rich and realistic texture characteristics present in real high-quality experimental images.

To alleviate the dependence on paired supervision, unpaired image-to-image translation frameworks have been actively studied [23–26]**错误!未找到引用源。错误!**
未找到引用源。. In this context, approaches based on the Cycle-Consistent Generative Adversarial Network (CycleGAN) [26–28] have attracted increasing attention as a pioneering framework capable of translating images between two domains without requiring paired training samples. Building upon this paradigm, Quan et al. [29] proposed an asymmetrically cyclic adversarial network for unpaired denoising in electron microscopy, which extends the CycleGAN framework to learn forward and inverse noise mappings for artifact suppression without paired clean targets. However, the method still relies primarily on cycle-consistency for structural preservation and lacks explicit atomic-scale geometric or physical constraints; their whole-image-level modeling with weak implicit constraints limits controllability at the atomic scale,

83 frequently resulting in inaccurate atomic reconstruction and inconsistent atomic
84 positions.

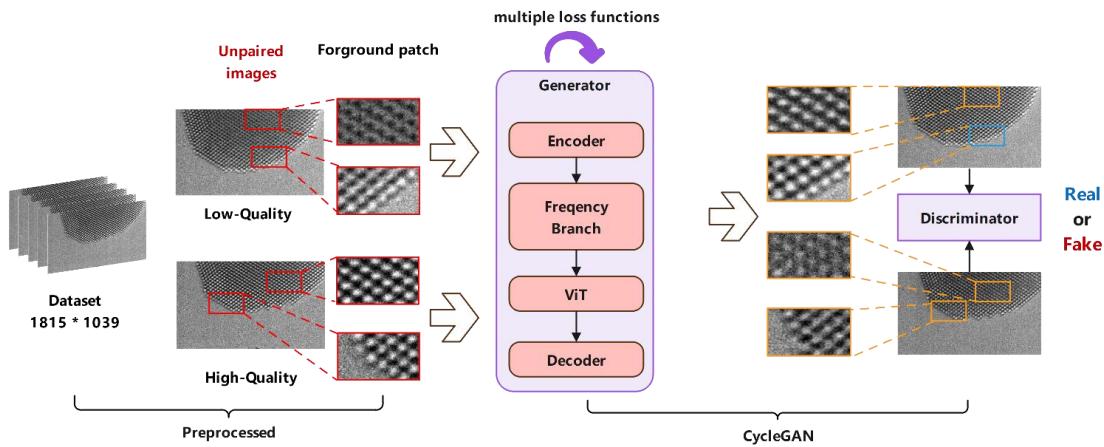
85 In this paper, we propose HRTEM-GAN, a CycleGAN-based framework designed
86 for structure-preserved image restoration in atomic-resolution HRTEM imaging.
87 Instead of performing whole-image translation, HRTEM-GAN adopts patch-level
88 modeling, which reduces the effective receptive scope of generation and allows the
89 network to focus on fine-grained atomic structures, alleviating incomplete atomic
90 recovery commonly observed in full-image generation. To further improve the
91 modeling of long-range atomic correlations, a Vision Transformer (ViT) [30] module is
92 incorporated at the bottleneck of the generator, enabling global dependency modeling
93 beyond local convolutional neighborhoods. In addition, we explicitly introduce
94 frequency-domain modeling and constraints to exploit the intrinsic spectral
95 characteristics of HRTEM images. By enforcing spatial-frequency consistency via a
96 dedicated frequency branch and frequency-aware losses, the proposed framework
97 balances image quality enhancement and atomic-scale structural fidelity. We have
98 performed extensive quantitative and qualitative evaluations on a real experimental
99 dataset, demonstrating that HRTEM-GAN outperforms existing methods of TEM
100 image quality enhancement and atomic structure preservation. Evaluations using
101 AtomSegNet further show improved atomic recognition accuracy and stability,
102 underscoring the value of the proposed framework for reliable quantitative atomic-scale
103 analysis.

104 **2 Experimental**

105 **2.1 Pipeline overview and dataset preparation**

106 Our framework comprises two stages arranged in a sequential pipeline (see Fig.
107 1). Firstly, the acquired HRTEM images are divided into high- and low-quality groups
108 (unpaired) and cropped into patches that are then classified into foreground (nanocrystal
109 area) and background (support area). Further details of this preprocessing are provided
110 in Supplementary Note I (online). Secondly, the proposed HRTEM-GAN model is

111 trained exclusively on unpaired high- and low-quality foreground patches, enabling the
 112 network to focus on learning atomic patterns. We employ CycleGAN to establish
 113 bidirectional mappings between the high- and low-quality domains, using two
 114 generators and two discriminators constrained by cycle consistency. We next detail our
 115 HRTEM-GAN (Section 2.2), frequency-enhanced feature interaction (Section 2.3), and
 116 implementation details (Section 2.4).



117

118 **Figure 1. Overall pipeline.**

119 Our dataset was obtained from *in situ* aberration-corrected transmission electron
 120 microscopy experiments, capturing surface step-site dynamics of Au catalysts during
 121 CO oxidation at room temperature. Due to low electron dose and high temporal
 122 resolution, the acquired HRTEM images exhibit significant quality variations. The
 123 images were therefore screened based on spatial resolution, atomic-column clarity, and
 124 lattice integrity, and subsequently divided into high-quality and low-quality subsets that
 125 are intrinsically unpaired and non-aligned, forming an unpaired training dataset.

126 **2.2 HRTEM-GAN**

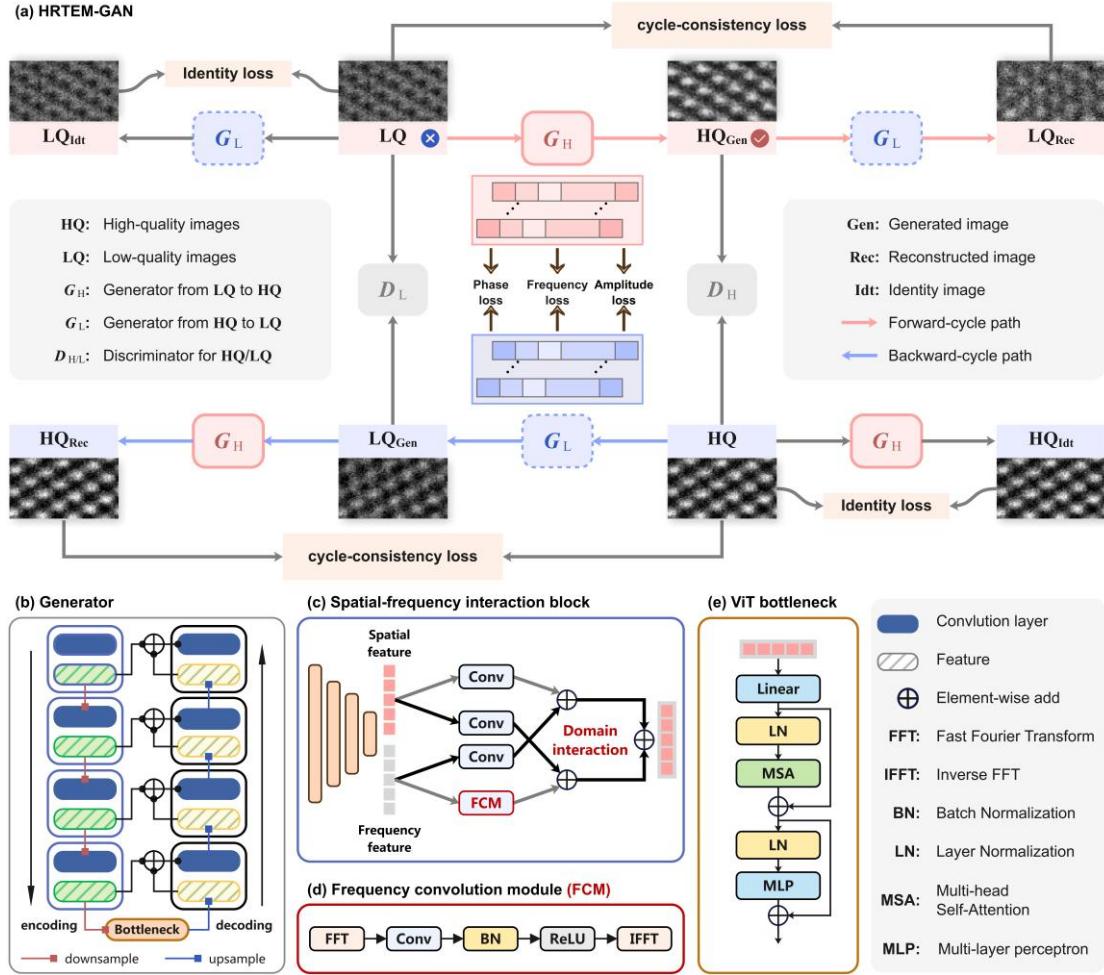
127 To address the challenge of obtaining perfectly aligned pairs of noisy experimental
 128 images and clean ground-truth images in HRTEM experiments — which renders
 129 standard supervised learning methods (e.g., Pix2Pix [31]) inapplicable—we propose
 130 HRTEM-GAN (see Fig. 2(a)). It is based on CycleGAN and learns robust cross-domain
 131 mappings using unpaired data. In particular, the cycle-consistency constraint

132 encourages an image translated from the source domain to the target domain and back
133 to remain faithful to the input, which is essential for preserving the geometric
134 arrangement of atomic columns during enhancement. Here, we consider unpaired
135 translation between the low-quality and high-quality HRTEM domains, denoted as LQ
136 and HQ, respectively. We train two generators $G_H \equiv G_{LQ \rightarrow HQ}$ and $G_L \equiv G_{HQ \rightarrow LQ}$,
137 together with discriminators D_H and D_L . The base objective is formulated as

$$\begin{aligned}\mathcal{L}_{\text{base}} = & \mathcal{L}_{\text{GAN}}(G_H, D_H) + \mathcal{L}_{\text{GAN}}(G_L, D_L) + \lambda_{\text{cyc}} \mathcal{L}_{\text{cyc}}(G_H, G_L) \\ & + \lambda_{\text{idt}} \mathcal{L}_{\text{idt}}(G_H, G_L),\end{aligned}\tag{9}$$

138 where \mathcal{L}_{GAN} , \mathcal{L}_{cyc} , and \mathcal{L}_{idt} are defined in [Supplementary Note III](#).

139 Nevertheless, standard CycleGAN architectures often struggle to fully resolve the
140 high-frequency periodic patterns intrinsic to atomic lattices. To address this limitation,
141 HRTEM-GAN introduces two major modifications.



142

143 **Figure 2. Schematic diagram of the proposed HRTEM-GAN.** (a) Overall HRTEM-
 144 GAN framework. (b) Enhanced UNet-ViT generator. (c) Spatial-frequency interaction
 145 block. (d) Frequency convolution module (FCM). (e) ViT bottleneck.

146 Firstly, we impose feature-level alignment on the fused bottleneck representations
 147 of the two generators G_H and G_L to encourage the generated samples to approximate
 148 the feature characteristics of the target domain. Let z_H and z_L denote the
 149 corresponding bottleneck features. Feature consistency is enforced from two
 150 complementary aspects: frequency-domain alignment and distributional alignment.

151 **Frequency-domain alignment.** We apply the fast Fourier transform (FFT) to the
 152 bottleneck features and penalize the discrepancy between their magnitude spectrum,
 153 thereby encouraging the translated results to preserve consistent spectral statistics and
 154 reduce unrealistic high-frequency artifacts. Formally, let $\mathcal{F}(\cdot)$ denote the FFT and $|\cdot|$
 155 its magnitude spectrum. The frequency-domain alignment loss is defined as

$$\mathcal{L}_{\text{FFT}} = \mathbb{E}_{z_H, z_L} [\| |\mathcal{F}(z_H(x))| - |\mathcal{F}(z_L(y))| \|_1], \quad (9)$$

156 where $\|\cdot\|_1$ denotes the element-wise ℓ_1 norm.

157 **Distributional alignment via the Characteristic Function (CF).** In addition, we
 158 introduce the Characteristic Function (CF) [32] as a robust measure to match feature-
 159 space distributions between the two domains. The CF yields complex-valued responses
 160 whose real and imaginary parts encode cosine and sine components, naturally
 161 supporting an amplitude–phase decomposition. The amplitude characterizes how
 162 strongly and broadly the feature distribution responds to each probing direction, which
 163 correlates with the coverage of fine-grained textures and local contrast variations. The
 164 phase, by contrast, is more sensitive to structural shifts and thus penalizes misalignment
 165 that would manifest as lattice distortion or atomic-position drift. Accordingly, we
 166 enforce joint amplitude and phase consistency between the bottleneck features of G_H
 167 and G_L . This motivates our design to enforce joint amplitude and phase consistency
 168 between the two domains:

$$\mathcal{L}_{\text{CF}} = \| |\text{CF}(z_H)| - |\text{CF}(z_L)| \|_1 + \lambda_\phi (1 - \cos(\angle \text{CF}(z_H) - \angle \text{CF}(z_L))), \quad (9)$$

169 where $|\cdot|$ and $\angle(\cdot)$ denote the magnitude and phase of the complex-valued CF output,
 170 respectively, and λ_ϕ balances the phase term. Details of the CF construction are
 171 deferred to [Supplementary Note V](#).

172 In summary, the total loss of our training is:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{base}} + \lambda_{\text{CF}} \mathcal{L}_{\text{CF}} + \lambda_{\text{FFT}} \mathcal{L}_{\text{FFT}}, \quad (9)$$

173 where λ_{CF} and λ_{FFT} are weighting hyperparameters that balance the contributions of
 174 the CF constraint and the frequency-domain consistency constraint to the overall
 175 training objective.

176 Secondly, our HRTEM-GAN moves beyond CycleGAN whose conventional U-
 177 Net [33] generator by integrating a dual-branch architecture in each encoder layer (see
 178 Fig. 2(c)). Specifically, considering that spectral features naturally encode global

179 periodicity, we introduce an independent frequency branch alongside the spatial branch
180 to preserve the intrinsic long-range order and fine-grained details of crystal lattices
181 (detailed in the following section). To further enhance the generator’s ability to recover
182 lattice-level structural regularity, a ViT [30] module is incorporated at the bottleneck of
183 the dual-branch encoder–decoder architecture. While convolutional operations
184 effectively capture local atomic textures, their limited receptive field constrains the
185 modeling of long-range periodicity. Addressing this, the fused spatial-frequency
186 features at the bottleneck are flattened into a token sequence, and two-dimensional
187 positional encodings are added to preserve the original lattice geometry. The token
188 sequence is then processed by several stacked Multi-head Self-Attention (MSA) layers,
189 enabling the network to learn global dependencies between arbitrary spatial locations
190 and to represent crystal lattices with coherent long-range ordering. The globally
191 contextualized features are subsequently reshaped back into a two-dimensional feature
192 map and propagated into the decoder through skip connections. By embedding global
193 self-attention at the structural bottleneck, this hybrid design enforces global lattice
194 consistency while retaining high-fidelity local atomic details provided by the
195 convolutional and frequency branches.

196 **2.3 Frequency-enhanced feature interaction**

197 Complementing the independent frequency branch in each layer of the encoder,
198 the input HRTEM image is processed by a standard convolutional layer to obtain
199 primary features, which are then split along the channel dimension to initialize parallel
200 spatial and frequency streams. After that, we employ the spatial-frequency interaction
201 block [22] to progressively enhance both feature streams. As illustrated in Fig. 2(d),
202 convolutional modules are used to capture local geometric structures and atomic
203 neighborhoods, while frequency convolution modules (FCM) emphasize periodic
204 lattice patterns and high-frequency structural details by leveraging Fourier transform.

205 To prevent semantic drift and ensure that both representations evolve in a mutually
206 consistent manner, a cross-branch information exchange mechanism is incorporated
207 within each interaction block. Specifically, the updated features from one branch are

208 passed through an additional convolutional layer and added to the counterpart branch,
209 enabling reciprocal refinement. This design ensures that spatial features are
210 continuously informed by global periodic cues, while frequency-domain features
211 remain anchored to localized structural geometry.

212 After the interaction stages, the enhanced spatial–frequency features are fused into
213 a unified representation and propagated through the decoder to reconstruct the output
214 image. This integrated design significantly improves the generator’s capacity to restore
215 atomic lattice periodicity, preserve sharp high-frequency details, and maintain local
216 structural fidelity under unpaired training conditions. The restored image patches are
217 finally reassembled into full-resolution frames, yielding high-quality HRTEM images
218 with globally coherent and physically meaningful atomic arrangements.

219 **2.4 Implementation details**

220 Our model is trained on unpaired data. All experiments were implemented in
221 PyTorch and trained on an NVIDIA 4090 GPU with CUDA acceleration. The networks
222 were optimized using the Adam optimizer with an initial learning rate of 2×10^{-4} and
223 $\beta_1 = 0.5$, where the learning rate was kept constant for the first 150 epochs and then
224 linearly decayed to zero over the following 20 epochs. An unaligned dataset setting with
225 an high-to-low translation direction was adopted, and all input images were resized to
226 286×286 and randomly cropped to 256×256 during training, with a batch size of
227 128. The model followed the standard CycleGAN configuration with 64 base feature
228 channels for both generator and discriminator, employed a PatchGAN discriminator,
229 and was trained using the least-squares GAN objective, together with an image buffer
230 of size 50 to stabilize training. Note that although HRTEM-GAN is trained with
231 bidirectional mappings between LQ and HQ domains, practical deployment primarily
232 uses the LQ→HQ branch.

233 **3 Results and discussion**

234 **3.1 Evaluation protocols**

235 To comprehensively evaluate the proposed method, we conducted both image
236 generation quality assessment and recognition performance evaluation. For image
237 generation, quantitative metrics were adopted to measure the distribution consistency
238 between generated images and real images, complemented by qualitative feature-space
239 visualization. Specifically, Fréchet Inception Distance (FID) [34] and Kullback-Leibler
240 (KL) divergence [35] were used for quantitative evaluation, while t-SNE visualization
241 [36] was employed for qualitative analysis. For recognition, standard pixel-level
242 metrics were adopted to assess the accuracy and overlap between predicted masks and
243 ground-truth annotations, including Precision, Dice coefficient, and Intersection over
244 Union (IoU). The formulas for computing the evaluation metrics are provided in
245 **Supplementary Note III.**

246 As comparison baselines, we compared HRTEM-GAN with both conventional
247 denoising algorithms and recent learning-based restoration methods. Specifically, we
248 included Wiener filtering and ABSF as representative classical frequency-/filtering-
249 based approaches, and BM3D as a widely used patch-based denoiser. In addition, we
250 evaluated three learning-based alternatives, i.e., AtomSegNet [18], SBD [12], and SFIN
251 [22], which have been adopted in prior work for improving TEM image quality and
252 downstream atomic recognition. To assess recognition performance in a controlled
253 manner, we used the same segmentation network (AtomSegNet) to predict atomic
254 masks from the restored images produced by each restoration method, and then
255 computed pixel-level metrics (Precision, Dice, and IoU) against the ground-truth
256 annotations.

257 **3.2 Quantitative results of restoration**

258 We first quantify the distribution-level fidelity of restored images to the HQ
259 domain using feature-based metrics, aiming to evaluate whether a restoration method
260 genuinely narrows the gap between LQ inputs and HQ references in the learned

representation space. Specifically, we report the FID and KL divergence computed between restored foreground patches and real HQ patches. The quantitative comparison across different methods is summarized in Table 1.

Table 1. Quantitative comparison of restored results produced by our method and existing approaches. “KL” denotes KL divergence. The best and second-best results are highlighted in **bold** and underlined, respectively.

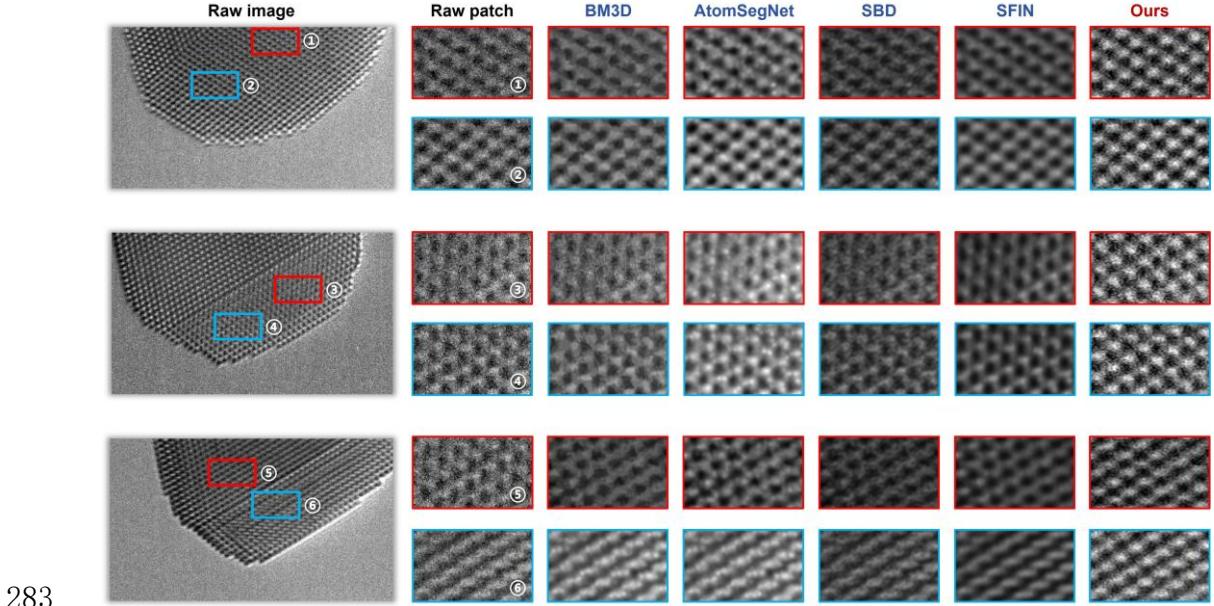
Metric	Conventional methods			Learning-based methods			
	Wiener	ABSF	BM3D	AtomSegNet	SBD	SFIN	Ours
FID ↓	215.825	223.188	132.504	172.224	<u>169.088</u>	211.103	33.622
KL ↓	1.530	1.320	0.424	<u>0.326</u>	0.402	0.348	0.092

Among the conventional denoising methods, BM3D achieves the strongest performance, with an FID of 132.504 and a KL divergence of 0.424, while Wiener and ABSF exhibit substantially larger distribution gaps (FID > 215 and KL ≥ 1.320). The learning-based counterparts (AtomSegNet, SBD, and SFIN) reduce KL divergence to a moderate level (0.326–0.402) but still yield relatively high FID values (169.088–211.103), indicating that the restored outputs remain far from the target HQ distribution in feature space. In contrast, our HRTEM-GAN achieves the lowest FID (33.622) and KL divergence (0.092), outperforming all competing approaches by a large margin and demonstrating substantially improved distribution alignment and restoration fidelity.

3.3 Qualitative results of restoration

3.3.1 Patch-level qualitative comparison

We first inspect atomic-level detail recovery through local magnification. As shown in Fig. 3, two regions are cropped from each HRTEM image restored by BM3D, AtomSegNet, SBD, SFIN, and our proposed method. It is observed that BM3D and other deep learning-based methods are effective in noise reduction but fail to recover the underlying atomic structures.



283

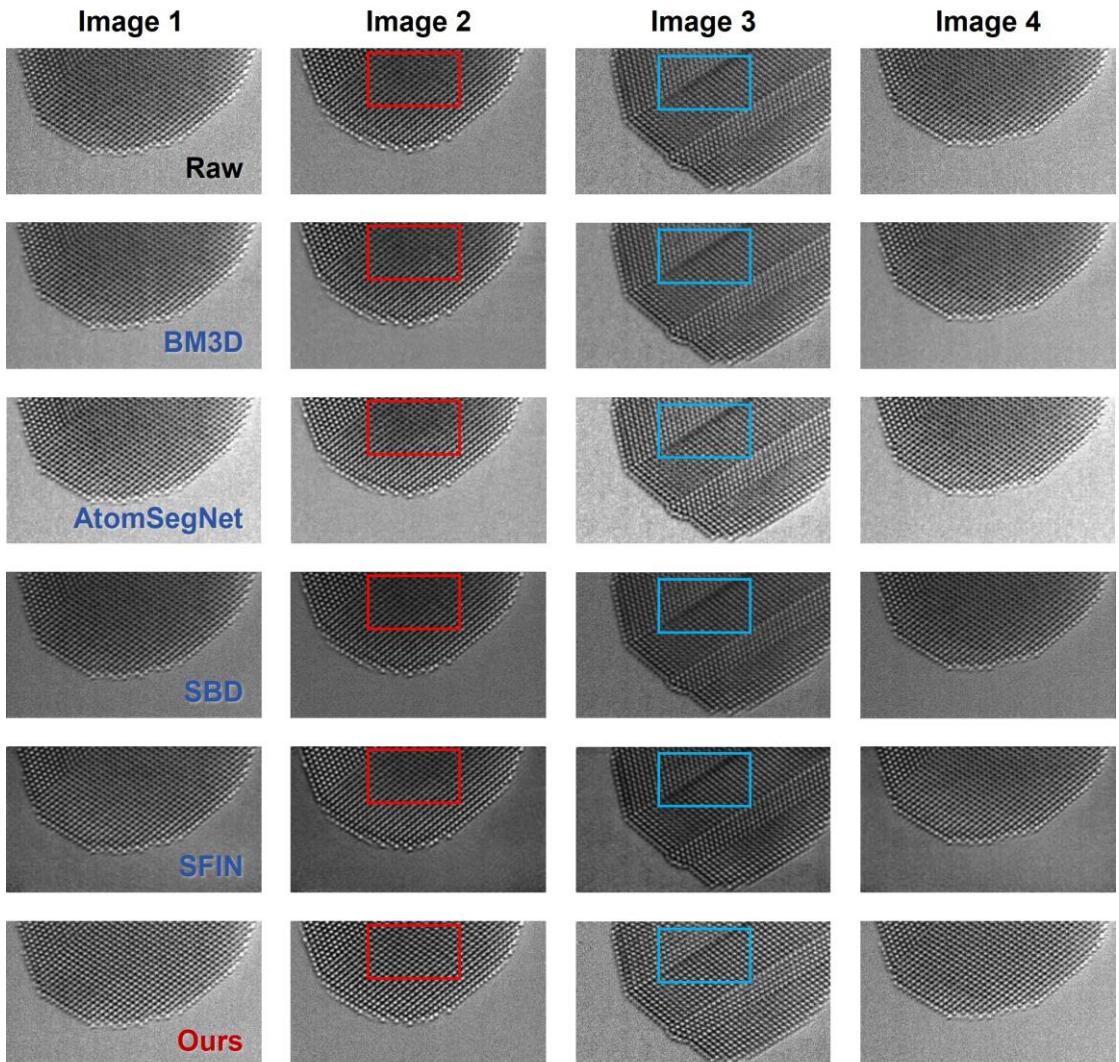
284 **Figure 3. Qualitative comparison of patch-level restored results produced by our**
 285 **method and existing approaches.**

286 Two representative cases further highlight the difference. In region (3), where the
 287 raw observation is severely blurred and atomic contrast is barely discernible, other
 288 methods mainly suppress noise but still fail to reveal stable lattice periodicity. In
 289 contrast, our HRTEM-GAN recovers clearer atomic-point contrast with sharper
 290 boundaries, making the underlying lattice pattern more distinguishable. In region (6),
 291 adjacent atomic columns appear partially merged due to imaging degradations;
 292 competing methods tend to preserve the adhesion after denoising, whereas HRTEM-
 293 GAN better separates the fused atomic points and restores more consistent inter-column
 294 boundaries. Similar trends are observed across the remaining regions.

295 **3.3.2 Image-level qualitative comparison**

296 To evaluate overall restoration quality beyond local patches, we compare full-
 297 image reconstructions produced by HRTEM-GAN and competing approaches.
 298 Following the patch-based inference setting, we adopt an overlapping sliding-window
 299 strategy to aggregate patch-wise predictions into a complete image: each restored patch
 300 is mapped back to its spatial location, and pixel-wise accumulation followed by
 301 averaging is applied within overlapping areas. Afterward, redundant regions introduced

302 by boundary padding are cropped, and the reconstructed image is resized to the original
303 resolution to obtain the final restored output. Figure 4 presents representative image-
304 level results across four test images.



305

306 **Figure 4. Qualitative comparison of image-level restored results produced by our**
307 **method and existing approaches.** Columns correspond to different test images, and
308 rows correspond to different restoration methods (method names are indicated on the
309 left). The red and blue bounding boxes mark representative regions in Image 2 and
310 Image 3.

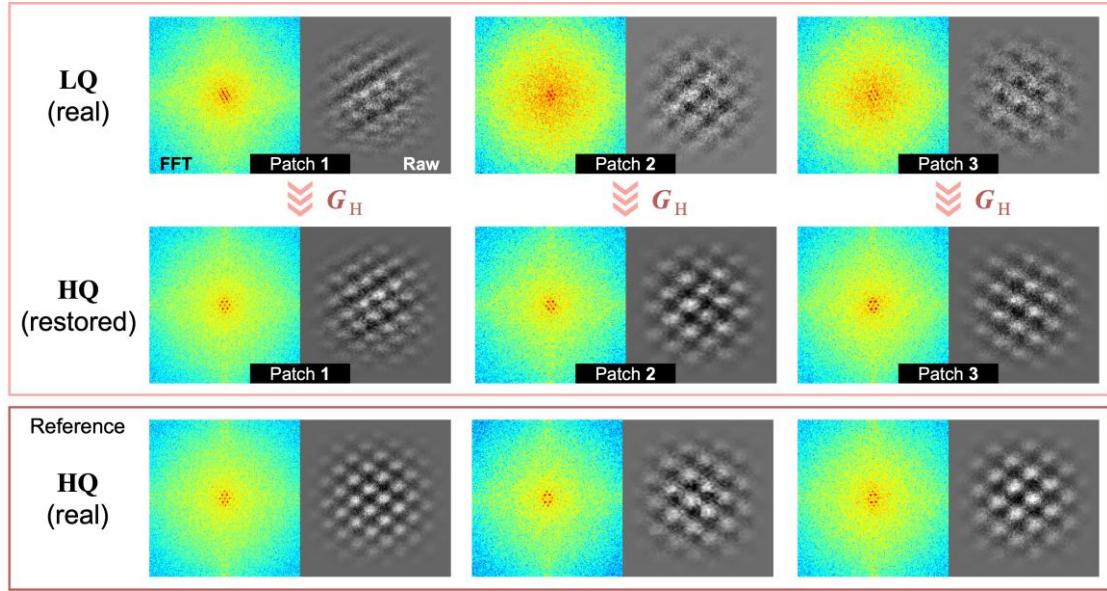
311 As shown in Figure 4, BM3D and the learning-based methods (AtomSegNet, SBD,
312 and SFIN) reduce noise to some extent, but their outputs remain largely smoothing-
313 dominated, with attenuated atomic contrast and locally unstable lattice textures. This

314 limitation is particularly evident in Image 2 and Image 3, where the highlighted regions
315 emphasize challenging structures. In Image 2 (red box), the competing methods tend to
316 over-smooth the lattice fringes, leading to weakened peak–valley contrast and less well-
317 defined atomic columns in the interior lattice region. In contrast, HRTEM-GAN
318 preserves clearer atomic-point contrast and more regular lattice periodicity within the
319 boxed area, maintaining sharper boundaries between adjacent columns while
320 suppressing background noise. In Image 3 (blue box), the specimen exhibits a twin-
321 boundary-like (or stacking-fault-like) interfacial contrast, where the lattice orientation
322 and fringe continuity change across a narrow region. Such defect- or interface-related
323 patterns are particularly challenging under low SNR, as they require preserving not only
324 periodic lattice fringes but also the coherence across the interface. The other methods
325 tend to over-smooth this area and partially wash out the interfacial contrast, resulting in
326 blurred textures and locally disrupted periodicity. Oppositely, HRTEM-GAN better
327 maintains the interface-induced contrast variation while preserving the surrounding
328 lattice order, yielding a more coherent reconstruction with clearer atomic features in the
329 highlighted region. Overall, these observations indicate that the proposed method goes
330 beyond generic denoising and yields restorations that are visually closer to high-quality
331 TEM observations.

332 **3.3.3 Spatial-frequency consistency analysis**

333 Beyond spatial-domain appearance, a key indicator of physically meaningful
334 HRTEM restoration is whether the method recovers the characteristic lattice periodicity
335 in the frequency domain. We compute the 2D FFT log-magnitude after applying a two-
336 dimensional Hann window and visualize it with the zero-frequency component centered.
337 This process suppresses boundary-induced leakage and yields more stable reciprocal-
338 space patterns for assessing lattice-related high-frequency content.

339



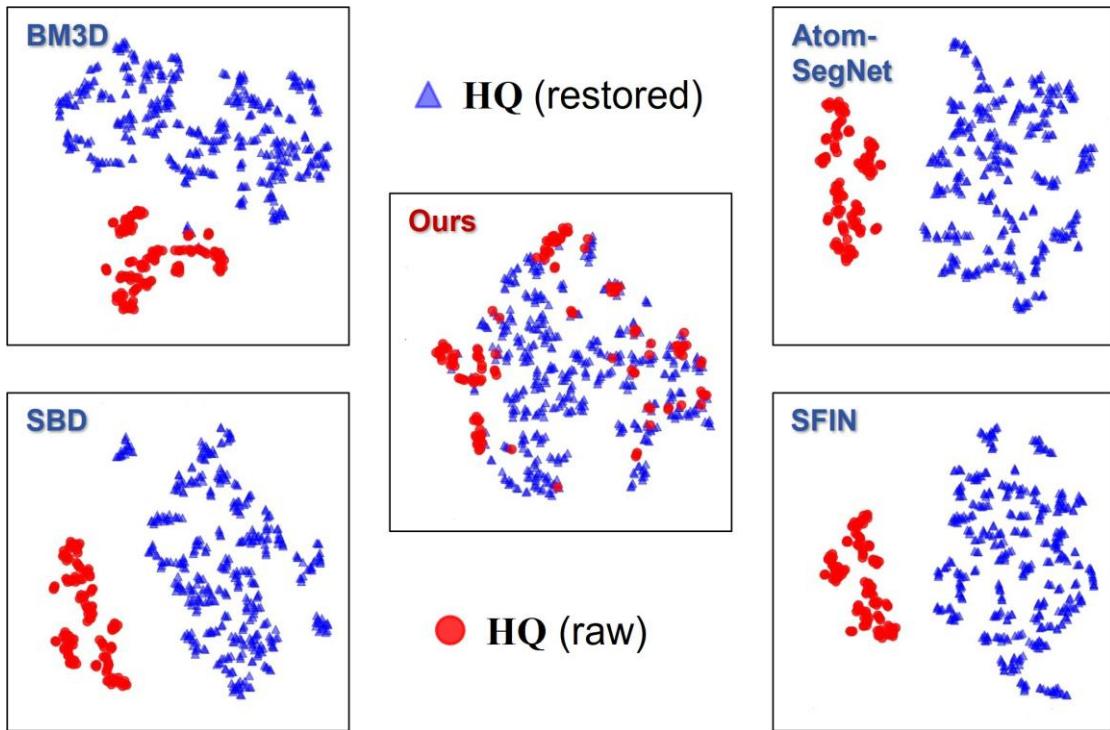
340 **Figure 5. Spatial-frequency comparison of restoration results.** The bottom row
 341 provides representative real HQ patches as a reference for the expected reciprocal-space
 342 patterns (unpaired with the LQ inputs).

343 Under this spatial-frequency inspection (see Fig. 5), LQ experimental patches (the
 344 first row) exhibit attenuated and diffuse high-frequency responses, where reciprocal-
 345 lattice signatures are weak and partially blurred, consistent with diminished fringe
 346 contrast in the spatial domain. In contrast, the outputs restored by G_H (the second row)
 347 show clearer atomic-column modulation and sharper lattice fringes, accompanied by
 348 more discernible Bragg-like peaks (or reciprocal-lattice spots) and reduced spectral
 349 smearing in the log-magnitude representation. These observations suggest that our
 350 HRTEM-GAN improves image quality by reconstructing lattice periodicity rather than
 351 merely amplifying contrast. Representative HQ real patches are additionally provided
 352 as a reference (not paired with the corresponding LQ inputs) to illustrate typical
 353 reciprocal-space patterns of high-quality images, which are qualitatively better matched
 354 by the restored results.

355 **3.3.4 t-SNE visualization analysis**

356 To examine whether restored images are not only visually improved but also
 357 feature-wise aligned with the high-quality (HQ) domain, we visualize the deep-feature

358 embeddings of foreground patches using t-SNE in Fig. 6, where red circles denote real
359 HQ patches and blue triangles denote the corresponding patches obtained after
360 restoration.



361
362 **Figure 6. t-SNE visualization of deep-feature embeddings for atomic patches**
363 **under different restoration methods.** Red circles denote real HQ (raw) patches, and
364 blue triangles denote the corresponding patches obtained after restoration.

365 After applying BM3D and the learning-based methods (AtomSegNet, SBD, and
366 SFIN), the restored features remain noticeably detached from the HQ cluster and often
367 become more scattered, suggesting that these methods mainly perform local
368 denoising/enhancement without effectively mapping the low-quality inputs toward the
369 HQ feature distribution. In contrast, the embeddings produced by HRTEM-GAN show
370 substantially increased overlap with the HQ points and reduced inter-domain
371 discrepancy, indicating that the proposed model better recovers high-frequency
372 structural characteristics and overall statistical properties consistent with the HQ
373 domain, thereby achieving more effective distribution-level alignment.

374 **3.4 Performance of atomic recognition**

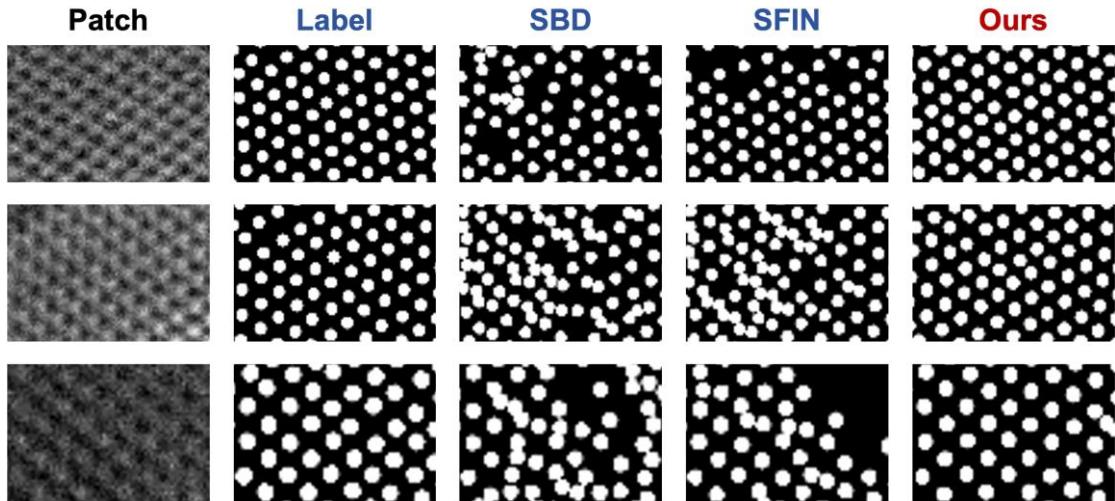
375 To further evaluate how well different restoration models preserve atomic-level
376 structures, we apply the trained AtomSegNet to segment atomic columns on the restored
377 outputs of representative DL method (SBD and SFIN) and quantify structural fidelity
378 using segmentation-based metrics (Precision, Dice, and IoU) against the ground-truth
379 labels, and additionally provide qualitative comparisons of the predicted masks to
380 visually assess atomic-column completeness, separability, and boundary consistency
381 under challenging imaging conditions.

382 **Table 2. Quantitative comparison of segmentation performance under different**
383 **restoration methods.**

Method	Precision	Dice \uparrow	IoU \uparrow
SBD	0.468	0.548	0.386
SFIN	0.692	<u>0.691</u>	<u>0.533</u>
Ours	0.806	0.731	0.578

384 Table 2 summarizes the segmentation accuracy on the restored images. Compared
385 with SBD and SFIN, our method yields consistent gains across all metrics, indicating
386 improved atomic-column structural fidelity after restoration. Specifically, our method
387 achieves a Precision of 0.806, which is higher than SFIN (0.692) and SBD (0.468),
388 suggesting fewer false positives and better separability between adjacent atomic
389 columns. Likewise, the Dice score increases to 0.731 (vs. 0.691 for SFIN and 0.548 for
390 SBD), and IoU improves to 0.578 (vs. 0.533 for SFIN and 0.386 for SBD), reflecting
391 better mask overlap and more complete atomic-column recovery. These quantitative
392 improvements are consistent with the qualitative comparison below.

393



394

Figure 7. Qualitative comparison of atomic-column segmentation results.

395

As shown in Fig. 7, although SBD and SFIN improve the segmentation quality in certain areas, their results still suffer from issues such as atomic region merging, positional shifts, and disruption of lattice structure. In contrast, our method consistently produces more accurate and coherent segmentation results across all test patches. The segmentation results generated by our method exhibit uniformly distributed atomic regions and well-preserved lattice periodicity, maintaining complete and regular atomic structures.

402

Overall, the segmentation results demonstrates that HRTEM-GAN outperforms other learning-based restoration methods. The images restored by HRTEM-GAN enable the atomic segmentation network to achieve more accurate and stable atomic identification, indicating superior structural fidelity in atomic-resolution images. This advantage provides a more reliable image foundation for subsequent atomic modeling and materials analysis.

408

4 Conclusion

409

In this paper, we have developed an HRTEM-image restoration method (HRTEM-GAN) built on the generative method, CycleGAN. Targeted at the atomic-resolution scenario, HRTEM-GAN restores low-quality HRTEM observations into high-quality counterparts while preserving critical structure information, without requiring strictly

413 paired low-/high-quality training data. We combine spatial restoration with frequency-
414 consistent supervision and align the restored features with real high-quality features to
415 enhance the reality. The ViT-based design at the bottleneck helps capture long-range
416 lattice regularity and improves structural faithfulness in complex regions. Extensive
417 experiments demonstrate that the restored outputs exhibit improved visual fidelity and
418 closer distributional alignment with real high-quality images in both pixel space and
419 feature space. Importantly, the enhanced image quality consistently improves
420 downstream atomic-column segmentation performance, yielding higher Precision, Dice,
421 and IoU compared with recent learning-based restoration baselines.

422 In future work, we will explore more condition-aware and physics-guided
423 constraints [37,38] (e.g., incorporating imaging parameter priors or forward models) as
424 well as broader multi-domain training to improve robustness. We also plan to
425 investigate lightweight deployment strategies to facilitate practical integration into
426 routine HRTEM workflows. Overall, HRTEM-GAN provides a practical and effective
427 pathway for restoring atomic-resolution HRTEM images and enhancing subsequent
428 quantitative analysis.

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432 **Compliance with ethics guidelines**

433 **Conflict of interest**

434 All authors declare that they have no conflict of interest.

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Supplementary Material for

540 **From Low-Quality to High-Quality: Generating Structure-Preserved
541 Atomic-Scale HRTEM images via an Enhanced CycleGAN**542 **Supplementary Note I: Image quality classification and reconstruction**

543 **Image quality classification and reconstruction pipeline.** Atomic-scale microscopy
544 video streams were acquired using transmission electron microscopy (TEM), from
545 which individual frames were manually screened according to image quality. Twenty
546 high-quality images and twenty low-quality images were selected to construct an
547 unpaired dataset. The original images, with a resolution of 1936×1112 , were uniformly
548 resized to 256×256 prior to training and subsequently divided into non-overlapping
549 patches of size 32×32 . For each patch, foreground/background pseudo-labels were
550 automatically generated using multiple unsupervised clustering strategies, including
551 clustering based on grayscale distribution features, statistical features such as mean
552 intensity and intensity standard deviation, as well as frequency-domain and structural
553 statistical features. The results from different clustering methods were integrated via a
554 voting mechanism to obtain robust foreground/background labels. In addition, each
555 patch inherited a high-quality or low-quality domain label from its source image. Patch
556 samples annotated with both patch-level and image-level labels were then used to
557 jointly learn reconstruction and classification tasks at the patch level. During training,
558 a U-Net based encoder-decoder architecture was employed as the backbone network,
559 in which the encoder consists of five hierarchical convolutional blocks with
560 downsampling operations to progressively extract multi-scale features, while the
561 decoder symmetrically restores spatial resolution through upsampling combined with
562 skip connections, ultimately producing a single-channel reconstructed output.

563 **Training objectives and optimization.** The proposed framework was trained in a
564 multi-task manner, jointly optimizing image reconstruction and classification
565 objectives at the patch level. For reconstruction, the network was supervised to recover

566 the grayscale intensity of the input patch, where the RGB input was converted to a
567 single-channel grayscale image using a linear combination of color channels. To
568 emphasize the restoration of structurally meaningful regions, the reconstruction loss
569 was computed only on patches labeled as foreground, while background patches were
570 excluded from reconstruction supervision. The reconstruction objective was formulated
571 using the mean squared error (MSE) loss between the reconstructed output and the
572 corresponding grayscale input. In addition, two classification objectives were
573 introduced: a patch-level foreground/background classification loss, which encourages
574 the network to distinguish structural regions from background regions, and an image-
575 level quality classification loss, which predicts whether a patch originates from a high-
576 quality or low-quality image domain. These objectives were combined into a weighted
577 loss function, where the reconstruction loss was assigned a higher weight to prioritize
578 structural fidelity, and the classification losses served as auxiliary constraints to guide
579 feature learning. The network was optimized using the Adam optimizer with an initial
580 learning rate of 1×10^{-3} . Training was performed with a batch size of 16 for 50 epochs.
581 During optimization, the total loss was minimized as a weighted sum of the
582 reconstruction loss, the patch-level classification loss, and the image-level classification
583 loss. Model selection was based on the validation loss, and the network parameters
584 yielding the lowest validation loss were retained for inference.

585 **Supplementary Note II: Atomic region segmentation**

586 **Training Pipeline.** The foreground segmentation model was trained using a fully
587 supervised learning pipeline based on conventional U-Net architecture. The training
588 dataset consists of 105 atomic-resolution images, for which the corresponding
589 foreground masks were manually annotated using LabelMe to delineate atomic regions
590 of interest. Input images and their associated masks were paired on a per-file basis to
591 ensure strict one-to-one correspondence between images and labels. During training,
592 identical random geometric augmentations were synchronously applied to each image-
593 mask pair to preserve spatial alignment. Specifically, center cropping and random
594 rotations were performed on both the input images and their corresponding masks,

595 while appearance-based intensity augmentations were applied exclusively to the input
596 images. After preprocessing and normalization, the input images were fed into the U-
597 Net model, which outputs a single-channel prediction map representing pixel-wise
598 foreground likelihood. For stable model training and evaluation, the dataset was
599 randomly divided into training and validation subsets, and mini-batch optimization was
600 carried out using a standard data-loading strategy. During validation, the predicted
601 probability maps were resized back to the original image resolution for visualization
602 and further analysis.

603 **Training Objectives and Optimization.** The model was trained to perform binary
604 foreground segmentation using a composite loss function that jointly enforces pixel-
605 wise classification accuracy and region-level overlap consistency. Specifically, the
606 training objective combines a binary cross-entropy loss with logits and a Dice loss
607 computed on sigmoid-activated predictions, thereby balancing local discrimination
608 capability with global foreground shape consistency. Model parameters were optimized
609 using the Adam optimizer, with learning rate scheduling applied throughout training to
610 regulate the optimization process. Training was conducted for a fixed number of epochs,
611 with model performance evaluated on a held-out validation set after each epoch.
612 Quantitative evaluation on the validation set demonstrates that the proposed
613 segmentation model achieves a Dice coefficient of 0.9837, indicating highly accurate
614 delineation of atomic regions.

615 **Supplementary Note III: CycleGAN loss formulation**

616 In CycleGAN, the discriminators are updated by back-propagating the loss
617 corresponding to failures in distinguishing real and translated images, commonly
618 referred to as the generative adversarial loss (GAN loss). In this work, we define two
619 image domains, where domain A corresponds to the low-quality HRTEM images and
620 domain B represents the high-quality images:

	$\mathcal{L}_{disc,A} = \mathbb{E}_{x \sim B} \ell_{GAN}(\mathcal{D}_A(\mathcal{G}_{B \rightarrow A}(x)), 0) + \mathbb{E}_{x \sim A} \ell_{GAN}(\mathcal{D}_A(x), 1), \quad (1)$	
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here, ℓ_{GAN} denotes the adversarial loss used in CycleGAN, which can take different forms depending on the GAN variant, such as binary cross-entropy, least-squares loss, or the Wasserstein objective. The labels 0 and 1 indicate fake and real images, respectively, when applicable. The generators are updated by back-propagating loss from three sources: GAN loss, cycle-consistency loss, and identity-consistency loss.

Using $G_{A \rightarrow B}$ as an example:

	$\mathcal{L}_{GAN,A} = \mathbb{E}_{x \sim A} \ell_{GAN}(\mathcal{D}_A(G_{A \rightarrow B}(x)), 1),$	(2)
	$\mathcal{L}_{cyc,A} = \mathbb{E}_{x \sim A} \ell_{reg}(\mathcal{G}_{B \rightarrow A}(G_{A \rightarrow B}(x)), x),$	(3)
	$\mathcal{L}_{idt,A} = \mathbb{E}_{x \sim A} \ell_{reg}(\mathcal{G}_{B \rightarrow A}(x), x).$	(4)

And,

	$\mathcal{L}_{gen,A \rightarrow B} = \mathcal{L}_{GAN,A} + \lambda_{cyc} \mathcal{L}_{cyc,A} + \lambda_{idt} \mathcal{L}_{idt,A},$	(5)
	$\mathcal{L}_{gen,B \rightarrow A} = \mathcal{L}_{GAN,B} + \lambda_{cyc} \mathcal{L}_{cyc,B} + \lambda_{idt} \mathcal{L}_{idt,B},$	(6)
	$\mathcal{L}_{CycleGAN} = \mathcal{L}_{disc,A} + \mathcal{L}_{disc,B} + \mathcal{L}_{gen,A \rightarrow B} + \mathcal{L}_{gen,B \rightarrow A},$	(7)

here, ℓ_{reg} can be any regression loss function, and λ_{cyc} and λ_{idt} are combination coefficients.

Supplementary Note IV: Characteristic-function construction

Let $z_H \in \mathbb{R}^{C \times H \times W}$ and $z_L \in \mathbb{R}^{C \times H \times W}$ denote the fused bottleneck feature maps produced by G_H and G_L , respectively. We interpret each spatial location as a C -dimensional feature vector and form two empirical feature sets:

$$\mathcal{U}_H = \{\mathbf{u}_n^H\}_{n=1}^N, \mathcal{U}_L = \{\mathbf{u}_n^L\}_{n=1}^N, N = H \cdot W,$$

where $\mathbf{u}_n^H \in \mathbb{R}^C$ (resp. \mathbf{u}_n^L) is obtained by flattening z_H (resp. z_L) over spatial dimensions. In practice, to reduce computation and improve robustness, we optionally subsample N' vectors uniformly from the N locations (or from multiple images within a mini-batch) and apply channel-wise normalization to stabilize the scale of features.

639 Given a set of probing directions $\{\mathbf{t}_m\}_{m=1}^M \subset \mathbb{R}^C$ (shared by both domains), the
 640 empirical characteristic function of a feature set \mathcal{U} is computed as

641
$$\hat{\phi}(\mathbf{t}_m; \mathcal{U}) = \frac{1}{|\mathcal{U}|} \sum_{\mathbf{u} \in \mathcal{U}} \exp(i \mathbf{t}_m^\top \mathbf{u}), m = 1, \dots, M,$$

642 which yields complex-valued responses. We use the same $\{\mathbf{t}_m\}$ for both \mathcal{U}_H and \mathcal{U}_L to
 643 ensure a consistent comparison. In our implementation, $\{\mathbf{t}_m\}$ can be sampled once and
 644 kept fixed (e.g., i.i.d. from a zero-mean Gaussian with a controlled scale, or uniformly
 645 on the unit sphere followed by a fixed radius), which provides a stable set of “probes”
 646 for distribution matching.

647 We adopt an amplitude–phase decomposition of the CF response:

648
$$A(\mathbf{t}_m) = |\hat{\phi}(\mathbf{t}_m)|, \Phi(\mathbf{t}_m) = \angle \hat{\phi}(\mathbf{t}_m),$$

649 where $|\cdot|$ and $\angle(\cdot)$ denote the magnitude and phase (computed via $\text{atan2}(\Im(\cdot), \Re(\cdot))$),
 650 respectively. The CF loss enforces both magnitude (coverage) and phase (alignment)
 651 consistency between domains:

652
$$\mathcal{L}_{\text{CF}} = \frac{1}{M} \sum_{m=1}^M (|A_H(\mathbf{t}_m) - A_L(\mathbf{t}_m)| + \lambda_\phi (1 - \cos(\Phi_H(\mathbf{t}_m) - \Phi_L(\mathbf{t}_m)))),$$

653 where $A_H(\mathbf{t}_m) = |\hat{\phi}(\mathbf{t}_m; \mathcal{U}_H)|$ and $A_L(\mathbf{t}_m) = |\hat{\phi}(\mathbf{t}_m; \mathcal{U}_L)|$ (similarly for Φ_H, Φ_L).
 654 The phase term uses the circular distance $1 - \cos(\Delta\Phi)$ to respect the 2π -periodicity
 655 of angles and to avoid discontinuities. Unless otherwise stated, both terms are averaged
 656 over the M probing directions and over mini-batches.

657 Practical note: we set small constants (e.g., ϵ) where needed for numerical
 658 stability when computing phases, and we compute \mathcal{L}_{CF} on mini-batch features so that
 659 it serves as a stochastic approximation of dataset-level distribution matching.

660 Formally, the characteristic function of the feature distribution is defined as
 661 follows:

$$\Phi_x(t) = \mathbb{E}[\cos(\langle t, x \rangle)] + j\mathbb{E}[\sin(\langle t, x \rangle)], \quad (8)$$

662 where x denotes the feature representation, t is the frequency argument, and the
663 cosine term represents the amplitude component while the sine term captures the phase
664 component of the feature distribution.

665 **Supplementary Note V: Evaluation metrics**

666 FID computes the distance between two multivariate Gaussian distributions fitted
667 to deep feature representations of real and generated images, defined as:

$$\text{FID} = \|\mu_r - \mu_g\|_2^2 + \text{Tr}(\Sigma_r + \Sigma_g - 2(\Sigma_r \Sigma_g)^{1/2}), \quad (10)$$

668 where (μ_r, Σ_r) and (μ_g, Σ_g) denote the mean and covariance of features extracted
669 from real and generated images, respectively. A lower FID indicates closer feature
670 distributions.

671 KL divergence is employed to further quantify the difference between the two
672 distributions, expressed as:

$$D_{\text{KL}}(P\|Q) = \sum_x P(x) \log \frac{P(x)}{Q(x)}, \quad (11)$$

673 where P and Q represent the feature distributions of real and generated samples.
674 Smaller KL values imply higher distributional consistency.

675 t-SNE visualization for projecting high-dimensional features into a low-
676 dimensional space, enabling an intuitive comparison of feature clustering between real
677 and generated images.

678 To assess recognition accuracy, we employed Precision, Dice coefficient, and
679 Intersection over Union (IoU) as evaluation metrics. Precision measures the proportion
680 of correctly predicted foreground pixels and is defined as:

$$\text{Precision} = \frac{\text{TP}}{\text{TP} + \text{FP}}, \quad (12)$$

681 where TP and FP denote the numbers of true positive and false positive pixels,
682 respectively. The Dice coefficient evaluates the overlap between predicted masks and
683 ground truth, formulated as:

$$\text{Dice} = \frac{2\text{TP}}{2\text{TP} + \text{FP} + \text{FN}}, \quad (13)$$

684 while IoU is defined as:

$$\text{IoU} = \frac{\text{TP}}{\text{TP} + \text{FP} + \text{FN}}, \quad (14)$$

685 with FN representing false negatives. Higher values of Precision, Dice, and IoU indicate
686 better recognition performance.

687