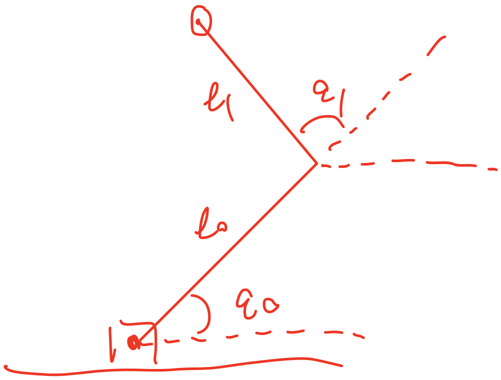


$x_i = l_0 \cos q_0 + l_1 \cos(q_0+q_1)$

$y = l_0 \sin q_0 + l_1 \sin(q_0+q_1)$

$x_i = l_0 \cos q_0 + l_1 \cos(q_0+q_1)$

$$\frac{dx}{d\theta_1} = -l_0 \sin q_0 - l_1 \sin(q_0+q_1)$$

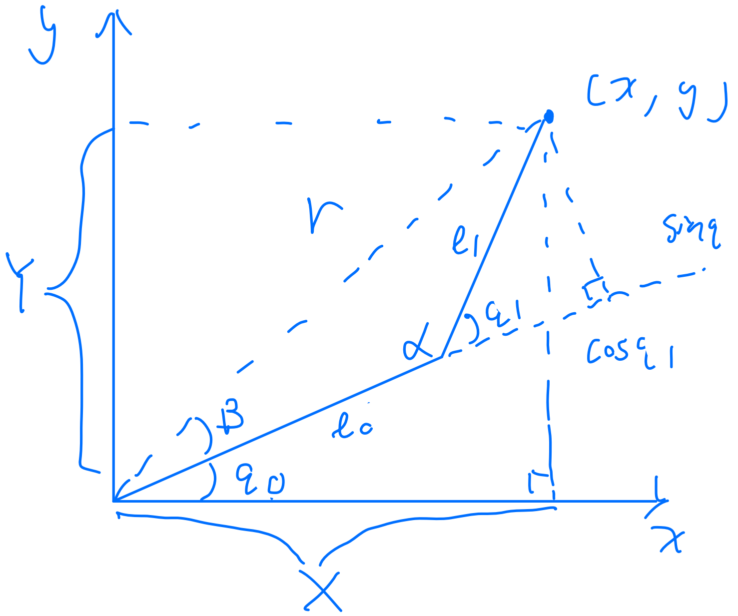
$$\frac{dx}{d\theta_2} = -l_1 \sin(q_0+q_1)$$

$$\frac{dy}{d\theta_1} = l_0 \cos q_0 + l_1 \cos(q_0+q_1).$$

$$\frac{dy}{d\theta_2} = l_1 \cos(q_0+q_1)$$

$$J(\theta) = \begin{bmatrix} \frac{\partial f_1(\theta)}{\partial \theta_1} & \frac{\partial f_1(\theta)}{\partial \theta_2} \\ \frac{\partial f_2(\theta)}{\partial \theta_1} & \frac{\partial f_2(\theta)}{\partial \theta_2} \end{bmatrix}$$

4



$$r^2 = x^2 + y^2$$

$$= l_0 \cos q_0 + l_1 \cos(q_0+q_1) + l_0 \sin q_0 + l_1 \sin(q_0+q_1)$$

$$r^2 = l_0^2 + l_1^2 - 2 l_0 l_1 \cos \alpha = x^2+y^2$$

$$\Rightarrow \cos\alpha = \frac{l_0^2 + l_1^2 - x^2 - y^2}{2 l_0 l_1}$$

$$\alpha = \pi - q_1$$

$$\cos\alpha = -\cos q_1$$

$$\Rightarrow \cos q_1 = \frac{x^2 + y^2 - l_0^2 - l_1^2}{2 l_0 l_1}$$

$$\beta + q_0 = \tan^{-1}\frac{y}{x} \qquad \beta = \tan^{-1}\frac{l_1 \sin q_1}{l_0 + l_1 \cos q_1}$$

$$q_0 = \tan^{-1}\frac{y}{x} - \tan^{-1}\frac{l_1 \sin q_1}{l_0 + l_1 \cos q_1}$$

$$q_1 = \cos^{-1}\left(\frac{x^2 + y^2 - l_0^2 - l_1^2}{2 l_0 l_1}\right)$$

cos is symmetrical about 0, there are two solutions (positive and negative)

since q_1 is also involved in equation of q_0 leading to two different solution for q_0 and q_1

$$\begin{cases} q_0 = \tan^{-1}\frac{y}{x} \mp \tan^{-1}\frac{l_1 \sin q_1}{l_0 + l_1 \cos q_1} \\ q_1 = \pm\cos^{-1}\left(\frac{x^2 + y^2 - l_0^2 - l_1^2}{2 l_0 l_1}\right) \end{cases}$$