Assignment 1

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Solve the following problems from your textbook (Introduction to Algorithms, second edition, by T. Cormen, C. Leiserson, R. Rivest, C. Stein, McGraw Hill).

Problem 1: Show that the solution of the following recurrence is $\Omega(n \cdot lg\ n)$ for all $n > 1$, $T(2)=3$, $T(3)=9$, and $T(n) = 2T(\lfloor n/2 \rfloor) + n$, where $\lfloor n/2 \rfloor$ means the floor of $n/2$. Also, conclude that the solution is $\Theta(n \cdot lg\ n)$ by proving that $T(n) = O(n \cdot lg\ n)$. Solve for constants and boundary conditions. Do not ignore floor function.

Problem 2: Solve the recurrence $T(n) = T(n/3) + T(2n/3) + n$. In other words prove both Ω, O bounds and conclude Θ bound.

Problem 3: (4.3-2) The recurrence $T(n) = 7T(n/2)+n^2$ describes the running time of an algorithm A. A competing algorithm A' has a running time of $T'(n) = aT'(n/4) + n^2$. What is the largest integer value for a such that A' is asymptotically faster than A?

Problem 4: (7.1-3) Give a brief argument that the running time of PARTITION on a subarray of size n is $\Theta(n)$.

Problem 5: (7.2-2) What is the running time of QUICKSORT when all elements of array A have the same value?

Problem 6: (9.1-1) Show that the second smallest of n elements can be found with $n + \lceil \lg n \rceil - 2$ comparisons in the worst case. (*Hint:* Also find the smallest element.)

Problem 7: (9.3-1) In the algorithm SELECT, the input elements are divided into groups of 5. Will the algorithm work in linear time if they are divided into groups of 7? Argue that SELECT does not run in linear time if groups of 3 are used.

Problem 8: (9.3-8) Let $X[1 .. n]$ and $Y[1 .. n]$ be two arrays, each containing n numbers already in sorted order. Give an $O(\lg n)$-time algorithm to find the median of all $2n$ elements in arrays X and Y.