

Q1

Algorithm MultipleX(A, n,X)

Input array A of n integers and Integer X we want to check

Output the string that show which index is multiple of X

for $i \leftarrow 0$ to $n - 1$ do

 if $A[i] \bmod X = 0$ then

 display " Index i with value A[i]"

 end for

return

- a) use for loop,if mod X=0,then its is multiple of X,then can System.out.println ("Index "+i+" with value "+A[i]+"")
- b) $\geq n$, the for loop is from $i=0$ to $i=n-1$,and there is no nested loop in the for loop,other sentence are just $O(1)$
- c) $\leq n$, $\Theta(n)$ is n ,then $O(n)$ is \geq , $\Omega(n)$ is \leq
- d) $O(1)$,there is no recursion so we don't need ADT like stack to hold other method

(2)

Algorithm MultipleX(A, n,X)

Input array A of n integers and Integer X we want to check

Output the string that show which index is multiple of X

T<-an empty array with length $n+1$ // queue with length n can contain only $n-1$ element

$f \leftarrow 0, r \leftarrow 0$

for $i \leftarrow 0$ to $n-1$ do

 ($T[i] \leftarrow A[i]$

$r \leftarrow r+1$)

end for

while $f \neq r$ do

 if $T[f] \bmod X = 0$ then

 display "Index f with value T[f]"

$f \leftarrow f+1$ //deq()

 end if

end while

return

- a) create a new array with $n+1$ length, see the new array as queue, $f=r=0$,then copy the old array into new array,with $r++$ n times. the rest thing is just deq until the queue is empty
- b) $\geq n$, the for loop is from $i=0$ to $i=n-1$,and there is no nested loop in the for loop the while loop will repeat n times and there is no nested loop in the while loop
- c) $\leq n$, $\Theta(n)$ is n ,then $O(n)$ is \geq , $\Omega(n)$ is \leq
- d) $O(n)$,we use the new array as queue ADT to store the data in old array

2.

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1. for i=0 to n-1 do                                n
2. Res[i]=0                                           n
3. end for
4. for i=0 to n-2 do                                n-1
5. for j=i+1 to n-1 do                              n-1+n-2+....1
6. if A[i]≤A[j] then                                n-1+n-2+....1
7. Res [j]= Res [j]+1
8. else
9. Res[i]= Res[i]+1                                step7+9= n-1+n-2+....1
10. end if
11. end for
12. end for
13. for i=0 to n-1 do                                n
14. B[Res [i]]= A[i]                                n
15. end for
16. Return B                                         1

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a) $O(n^2)$, $\Omega(n^2)$,the biggest step is the for loop from 5 to 11, $n-1+\dots+1=n*(n-1)/2=O(n^2)$

b) From 1-3,wil create a array **Res (0,0,0,0,0,0,0)**

when i=0, for j=1 to 6 ,if $A[0] \leq A[j]$,res[j]++,else res[i]++

Res[i]++,res[j]++,res[i]++,res[i]++.res[i] ++,res[i]++, res[0]=5,res[2]=1

I=1,FOR J= 2 TO6, res[1]++,res[2]++,res[3]++,res[5]++,res[6]++.

Res[1]=1,res[2]=2,res[3]=1,res[5]=1,res[6]=1

I=2. For j=3 to 6, res[2]++*4,res[2]=6

I=3,for j=4 to6, res[3]++,res[5]++,res[6]++. Res[3]=2,res[5]=2,res[6]=2

I=4 ,for j= 5 to 6, res[5]++,res[6]++,res[5]=3,res[6]=3

I=5, ,for j=6, res[6]++, res[6]=4

For 4-12, **res(5,1,6,2,0,3,4)**

For 13-15,

$B[RES[0]]=A[0] \rightarrow B[5]=88,B[1]=12,B[6]=94,B[2]=17,B[0]=2,B[3]=36.B[4]=69$

$B(2,12,17,36,69,88,94)$

FOR 16, RETURN $B(2,12,17,36,69,88,94)$

c) It sort the array from small to big.

With the nested for loop , the element reverse[i] shows the rank in size of $A[i]$ in whole array A. Why? It compares 2 element with each other once, which $A[i]$ is bigger, reverse[i]++, why $j=i+1$ in second for loop? Because we only want to compare once, if it start with $j=0$, then we will repeat.

At the last for loop, we just input the element of A into B with the order with reverse[i]

d) Yes, now time complexity is n^2 , we can just use heap sort, which time complexity is $n \log n$

e) Yes, heapsort just need $O(1)$ for exchange, while the old algothium need an array of n size to exchange

3.

1., $\Theta(n) = (\log n)^3 < g(n)$, fn is $O(g(n))$

2. $\Theta(n) = n^{1.5} > g(n)$, fn is $\Omega(g(n))$

3. $\Theta(n) = n > g(n)$ fn is $\Omega(g(n))$

4. $\Theta(n) = n^{0.5} > g(n)$ fn is $\Omega(g(n))$

5. $\Theta(n) = 2^n > g(n)$ fn is $\Omega(g(n))$

6. $\Theta(n) = 2^{10n} < g(n)$ fn is $O(g(n))$

7. $\Theta(n) = (n^n)^5 < g(n)$ fn is $O(g(n))$