

1. 9/10

$$A1=5*10$$

$$A2=10*3$$

$$A3=3*12$$

$$A4=12*5$$

$$A5=5*50$$

$$A6=50*6$$

what method are you using ? how does it work ? what is the recursive definition that it gives ? What do the tables below represent ?

BUILD THE TABLE

m	1	2	3	4	5	6
1	0	150				
2		0	360			
3			0	180		
4				0	3000	
5					0	1500
6						0

$m(1,1)=0$ because it is just $A1$ and no need for calculation

$m(1,2)=150=5*10*3$ from $A1, A2$ the rest are for the same reason



s	1	2	3	4	5	6
1		1				
2			2			
3				3		
4					4	
5						5
6						

$s(1,2)$ is 1 because the multiplication of $A1*A2$ is based on $A1$

$$\begin{aligned}
 m(4,3) &= A_1 \cdot (A_2 \cdot A_3) \quad \text{or} \quad (A_1 \cdot A_2) \cdot A_3 \\
 &= m[1,1] + m[2,3] + 5 \times 6 \times 2 \\
 &= 960 \\
 \text{so } m(1,3) &= 330, \quad s(1,3) = 2.
 \end{aligned}$$

$$\begin{array}{cccc}
 A_1 & A_2 & A_3 & A_4 \\
 5 \times 6 & 6 \times 3 & 3 \times 2 & 2 \times 5 \\
 A_5 & A_6 & & \\
 6 \times 5 & 5 \times 6 & &
 \end{array}$$

$$\begin{aligned}
 m(2,4) &= A_2 \cdot (A_3 \cdot A_4) \quad \text{or} \quad (A_2 \cdot A_3) \cdot A_4 \\
 &= m[2,2] + m[3,4] + 10 \times 3 \times 5 \quad \text{or} \quad m[2,3] + m[4,4] + 10 \times 2 \times 5 \\
 &= 180 + 150 = 330 \quad \text{or} \quad 360 + 600 = 960.
 \end{aligned}$$

$$\text{so } m(2,4) = 330, \quad s(2,4) = 2.$$

$$\begin{aligned}
 m(3,5) &= A_3 \cdot (A_4 \cdot A_5) \quad \text{or} \quad (A_3 \cdot A_4) \cdot A_5 \\
 &= m[3,3] + m[4,5] + 3 \times 2 \times 5 \quad \text{or} \quad m[3,4] + m[5,5] + 3 \times 5 \times 5 \\
 &= 300 + 1800 = 2100 \quad \text{or} \quad 180 + 750 = 930 \\
 \text{so } m(3,5) &= 930, \quad s(3,5) = 4.
 \end{aligned}$$

$$\begin{aligned}
 m(4,6) &= A_4 \cdot (A_5 \cdot A_6) \quad \text{or} \quad (A_4 \cdot A_5) \cdot A_6 \\
 &= m[4,4] + m[5,6] + 12 \times 5 \times 6 \quad \text{or} \quad m[4,5] + m[6,6] + 12 \times 5 \times 6 \\
 &= 1800 + 3600 = 5400 \quad \text{or} \quad 3000 + 3600 = 6600 \\
 \text{so } m(4,6) &= 5400, \quad s(4,6) = 4.
 \end{aligned}$$

$$\begin{aligned}
 m(1,4) &= A_1 \cdot (A_2 \cdot A_3) \cdot A_4 \quad \text{or} \quad (A_1 \cdot (A_2 \cdot A_3)) \cdot A_4 \quad \text{or} \quad m_1 \cdot (m_2 \cdot m_3 \cdot m_4) \\
 &= m[1,3] + m[4,4] + 5 \times 12 \times 5 \quad \text{or} \quad m[1,2] + m[3,4] + 5 \times 3 \times 5 \quad \text{or} \quad m[1,1] + m[2,4] + 5 \times 1 \times 5 \\
 &= 630 \quad \text{or} \quad 150 + 180 + 75 = 405 \quad \text{or} \quad 330 + 150 = 480 \\
 \text{so } m(1,4) &= 405, \quad s(1,4) = 2.
 \end{aligned}$$

$$\begin{aligned}
 m(2,5) &= (A_2 \cdot A_3) \cdot A_4 \cdot A_5 \quad \text{or} \quad (A_2 \cdot A_3) \cdot (A_4 \cdot A_5) \quad \text{or} \quad m_2 \cdot (m_3 \cdot m_4 \cdot m_5) \\
 &= m[2,4] + m[5,5] + 6 \times 5 \times 5 \quad \text{or} \quad m[2,3] + m[4,5] + 10 \times 2 \times 5 \quad \text{or} \quad m[2,2] \times m[3,5] + 10 \times 3 \times 5 \\
 &= 330 + 1500 = 1830 \quad \text{or} \quad 360 + 3000 + 600 = 3960 \quad \text{or} \quad 930 + 1500 = 2430 \\
 \text{so } m(2,5) &= 1830, \quad s(2,5) = 2.
 \end{aligned}$$

$$m(3,6) = m_3 \cdot m_4 \cdot m_5 \cdot m_6 \\ (A_3 \cdot A_4 \cdot A_5) A_6 \\ = m[3,5] + m[6,6] + 3 \times 5 \times 6 \\ = 180 + 180 \\ = 360$$

$$s(3,6) = 1770$$

$$m_3 = m_4 \cdot m_5 \cdot m_6 \\ (A_3 \cdot A_4) A_5 A_6 \\ = m[3,4] + m[5,6] + 3 \times 5 \times 6 \\ = 180 + 180 + 90 \\ = 450$$

$$s(3,6) = 4$$

$$m_3 = A_3 \cdot (A_4 \cdot A_5 \cdot A_6) \\ = m[3,3] + m[4,6] + 3 \times 12 \times 6 \\ = 180 + 240 \\ = 420$$

$$m(1,5) = (A_1 \cdot A_2 \cdot A_3 \cdot A_4) A_5 \\ = m[1,4] + m[5,5] + 4 \times 5 \times 5 \\ = 40 + 125 \\ = 165$$

$$m_1 = A_1 \cdot (A_2 \cdot A_3 \cdot A_4 \cdot A_5) \\ = m[1,1] + m[2,5] + 5 \times 6 \times 5 \\ = 243 + 225 \\ = 468$$

$$m(1,5) = (A_1 \cdot A_2) \cdot (A_3 \cdot A_4 \cdot A_5) \\ = m[1,2] + m[3,5] + 5 \times 3 \times 5 \\ = 15 + 135 + 75 \\ = 225$$

$$m_1 = (A_1 \cdot A_2 \cdot A_3) \cdot (A_4 \cdot A_5) \\ = m[1,3] + m[4,5] + 5 \times 12 \times 5 \\ = 330 + 300 + 300 \\ = 930$$

$$s(1,5) = 1655, \quad s(1,5) = 4$$

$$m(2,6) = A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \\ = m[2,5] + m[6,6] + 10 \times 5 \times 6 \\ = 240 + 300 + 300 \\ = 840$$

$$m_2 = A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6 \\ = m[2,2] + m[3,6] + 10 \times 5 \times 6 \\ = 177 + 180 = 357$$

$$m(2,6) = (A_2 \cdot A_3) \cdot (A_4 \cdot A_5 \cdot A_6) \\ = m[2,3] + m[4,6] + 10 \times 12 \times 6 \\ = 360 + 180 + 720 \\ = 1110$$

$$m_2 = (A_2 \cdot A_3 \cdot A_4) \cdot (A_5 \cdot A_6) \\ = m[2,4] + m[5,6] + 10 \times 5 \times 6 \\ = 330 + 150 + 300 \\ = 780$$

$$s(2,6) = 1150$$

$$s(2,6) = 2$$

$$m(1,0) = A_0(A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6) \quad \text{or} \quad (A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5) \cdot A_6$$

$$= m(1,1) + m(2,6) + 5 \times 6 \times 6 = 160 + 360 + 180 = 2010$$

$$= 160J + 180 = 315J$$

$$\text{or } (A_1 \cdot A_2 \cdot A_3 \cdot A_4 \cdot A_5 \cdot A_6) \quad \text{or} \quad (A_1 \cdot A_2 \cdot A_3 \cdot A_4) \cdot (A_5 \cdot A_6)$$

$$= m(1,2) + m(3,6) + 5 \times 3 \times 6 = m(1,4) + m(5,6) + 5 \times 5 \times 6$$

$$= 160 + 770 + 150 = 2010 \quad = 405 + 1500 + 150 = 2010$$

$$\text{or } (A_1 \cdot A_2 \cdot A_3) \cdot (A_4 \cdot A_5 \cdot A_6)$$

$$= m(1,3) + m(4,6) + 5 \times 12 \times 6$$

$$= 330 + 1160 + 360 = 2010$$

So $m(1,6) = 2010$ ✓ $SL(6) = 2$

so

m	1	2	3	4	5	6
1	0	150	330	405	1655	2010
2		0	360	330	2430	1950
3			0	180	930	1770
4				0	3000	1860
5					0	1500
6						0

s	1	2	3	4	5	6
1		1	2	2	4	2
2			2	2	2	2
3				3	4	4
4					4	4
5						5
6						

$s(1,6)=6$

so $(a_1*a_2)(a_3*a_4*a_5*a_6)$

$s(1,2)=1$ only 2 elements, ignore

$s(3,6)=4$

so $(a_1*a_2)(a_3*a_4)(a_5*a_6)$

so the final answer is $(a_1*a_2)((a_3*a_4)(a_5*a_6))$

2. 8/10

s1 | "AGGTAB"
s2 | "GXTXAYB"

		A	G	G	T	A	B
"	"	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0	1	1	1	2	3	3
B	0	1	1	1	2	3	4

The example and table is from <https://www.youtube.com/watch?v=ASoaQq66foQ>

if current column = current row, which means that the current two substrings have the same last char and can eliminate them, and **move to the left-up cell**

		A	G	G	T	A	B
"	"	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0	1	1	1	2	3	3
B	0	1	1	1	2	3	4

✓

(no points removed for this of course)

if current column \neq current row, which means that current two substrings have different last chars, so the length of LCS won't change, **we move to the cell with same number.**

		A	G	G	T	A	B
"	"	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0	1	1	1	2	3	3
B	0	1	1	1	2	3	4

✓

		A	G	G	T	A	B
"	"	0	0	0	0	0	0
G	0	0	1	1	1	1	1
X	0	0	1	1	1	1	1
T	0	0	1	1	2	2	2
X	0	0	1	1	2	2	2
A	0	1	1	1	2	3	3
Y	0	1	1	1	2	3	3
B	0	1	1	1	2	3	4

and use stringbuilder to remember the cells that goes left-up, which is G T A B

pseudo code

```
String reverse_LCS(table t, int row, int col){
    StringBuilder temp = new StringBuilder();
    if (row == 0 || col == 0)
        return temp.toString();
    if (t[row, 0] == t[0, col])
        reverse_LCS(t, row - 1, col - 1);
        temp.append(t[row, 0]);
    else if (t[row - 1, col] == t[row, col - 1])
        reverse_LCS(t, row - 1, col - 1);
    else
        reverse_LCS(t, row, col - 1);
}
```

cause there is no loop, there is at most $m+n$ recursions. so the time complexity is $m+n$

bad explanation. should be : because at most
 call is made each function call, and it lowers either
 row by one, either col by one, or both,
 and base case is at $i=0, j=0$, and the rest is constant time
 → worst case = $O(m+n)$

3. 10/10

Firstly copy these numbers into a new array and sort the array, $O(n \log n)$

Then run LCS algorithm on the sorted array and the original sequence of n numbers, $O(n^2)$

Then the result will must be sorted longest increasing subsequence of the original sequence

4. 4/10

YES,

ek choose the activity that have the latest start time

ek-1 to e1 choose the compatible activity with latest start time at that stage

So it is greedy, because we choose our best choice at every stage

take $e_1', e_2', e_3' \dots e_k'$ as an optimal schedule

if $e_k' \neq e_k$, then we can replace e_k' with e_k , because e_k have later start time than e_k' and get an optimal schedule

The rest are for same reason.

So it is optimal.

what? seems like

*→ true but write this more formally.
optimal substructure property?*

5. 10/10

item#	1	2	3	4	5
profit	3	6	8	2	3
weight	5	3	2	1	3

pseudo code

`int[][] arr = new int[n][W]; //initialize 2-d array`

`int max_Profit(int[][] arr, int[] weightArr, int[] profitArr, int n, int W){`

`if (arr[n][W] != null) return arr[n][W]; //reuse the buffer arr to save time`

`int result = 0;`

`if (n == 0 || W == 0){`

`result = 0;`

`} //base case`

`else if (weightArr[n] > W) //the weight of current item > current capacity, skip`

`result = maxProfit(arr, weightArr, profitArr, n-1, W);`

`else{`

`result = Math.max(`

`maxProfit(arr, weightArr, profitArr, n-1, W), //skip this item`

`profitArr[n] + maxProfit(arr, weightArr, profitArr, n-1, W-weightArr[n]) //pick this item`

`);`

`arr[n][W] = result;`

very difficult to read, next time just take

a picture of your

programming editor code in your


```
return result;
}
```

we only need to build the $n \times W$ 2D arr (which means we only need to call this function $n \times w$ times including recursion) , and there is no loop in the function, so it is $O(nW)$ time ✓

3 / 10

6.

in ~~Huffman~~ code, the leaf represents the char, path to the leaf represents the code. Assume there is a char 0001, then in full tree, it will have a sibling node 0000, 000 need two nodes to differ these two chars. If we remove 0000, then there is no need to differ. then 000 is enough to represent the original char. Then 0001 isn't the optimal prefix code.

just an algo to build an optimal prefix tree, we are talking about optimal prefix trees in general, which may not be constructed by that algo

technically true. But explain this in a more formal, general manner, not just for one example:

Assume our tree is not full. So, there exists some node x which is not the root, with a

prefix $P = \langle p_1, p_2, \dots, p_{x-1}, p_x \rangle$ which has

no sibling (ie there doesn't exist anyone with a prefix

$\langle p_1, p_2, \dots, p_{x-1}, \text{not}(p_x) \rangle$. Then delete x 's parent and

move x and its subtree in its place. Then all nodes that are x 's descendants can still be differentiated with the differences they have from x 's subtree, and there are no other children of x 's parent that

need any prefix code. All other nodes are unaffected by this move, so they also keep their valid prefix. So our prefix tree is valid for the

original alphabet. Then, it has a more optimal cost because... (continue this proof yourself)

7. 1/10

assume we've already known the minimum spanning tree. The G and G' are just the graphs after adding some new edges. //so that two graphs share the same minimum spanning tree. And we need to prove e is not part of minimum spanning tree. So we can add it and remove it without influencing the minimum spanning tree.

then you need to prove if this is optimal or not: kruskal is optimal but does not guarantee you get to do this.

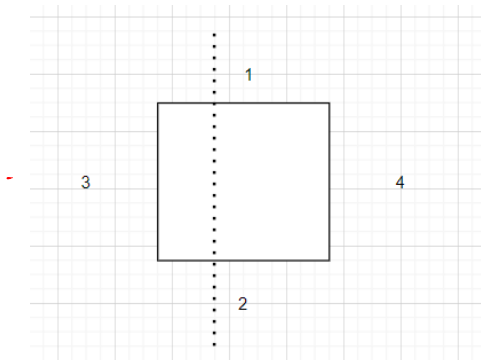
Cause the e is an edge in cycle. We use the Kruskal's method to find the minimum spanning tree. However, we keep all edges of this cycle. And then the final graph will be a minimum spanning tree + the cycles including e . Cause the property of cycle, even remove one edge, the vertices are still connected, so if we want to get the minimum tree. We must remove the edge e . So e is not part of minimum spanning tree

it can be (what) if all edges on cycle cost the same?

10/10

8. No, at this step "**Finally, select the minimum-weight edge in E that crosses the cut (V_1, V_2)** ", we cannot ensure the edges that are removed are the maximum edge in cycle that won't influence the minimum spanning tree.

For example



we will remove 2, then the minimum will be 1,3,4 but in fact the minimum is 1 2 3