Planning of electricity production and transmission

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Abstract

A country's electricity grid is a large-scale and exceptionally complex system, and there are many factors affecting power production and transmission, such as the voltage amplitudes and phase angles between the nodes in the network, etc., and generally speaking, the maximum capacity of the generators in the nodes is limited. This paper addresses the possible influencing factors and constraints on the generator production modeling, so as to achieve to minimize the cost of production of active power under the premise of meeting the consumption demand of consumers in the electricity network, which in turn will provide references and suggestions for the corresponding planning of power production and transmission.

1 Introduction

Electricity is the bloodline of economic and social development, and the benign economic growth is intimately related to reasonable electricity planning. Electricity planning is oriented to provide power generation resources within the main grid and meet the demand for power consumers, through the balance control of the the generation of active power and reactive power, analysis and adjustment, so that the different power production bodies the within the electricity grid coordinate the operation to achieve the overall optimization of the economic benefits of the grid.

Therefore, to plan for electricity production and transmission, the key lies in the construction and rational planning of the power system, and the security and balance of the electricity grid should be guaranteed while continuously increasing the supply of electricity, then to optimize economic efficiency while maximizing the possibility of meeting the demand for electricity production.

In an alternating current used electric grid, the instantaneous power at a point in the circuit is the rate of energy flow through that point at a given time instance. However, not all instantaneous power provides net energy transfer over an AC cycle. This is due to that energy is stored in components such as inductors and capacitors in the grid itself. As defined, the active power is known as the net energy transfer, the average over the length of one cycle, which is the power that a receiving unit on the other side of the transmission grid can actually use when a generator transmits power in a transmission grid. The remaining part of the energy in the system, which does not result in a net energy transfer, is known as reactive power. Also, when two nodes in the grid are connected for power transmission, there is an associated voltage amplitude and a voltage phase angle that determine the corresponding active and reactive power. When planning electric power production and transmission, we need to consider both the active and the reactive power. It is assumed that each generator can produce or absorb reactive power, and that the generators are the only objects in the network that can absorb the reactive power. Each generator can generate a non-negative amount of active power, but cannot exceed some upper limit related to maximum capacity, and the price per generated unit power varies among the generators. This report models the production and price of generators in the current network with respect to these constraints, with the aim of minimizing the cost of production of active power while meeting consumer demand for active power, and builds on the results, in particular how to anticipate the impact of the maximum capacity of a generator on cost if it changes in actual manufacturing, and to quickly arrive at the most advantageous option accordingly, while alleviating the computing effort.

2 Mathematical Formulation

2.1 Modelling Preparation

- **Figure about power flows in the network**

  The core of the optimization problem is the generation, utilization and flow of power, the main body of power generation and utilization is the generator and the consumer, while the generator and the consumer are distributed in different nodes, the power flow occurs between the two nodes. Therefore, the process of power flow is closely related to the connection of each node. The following figure shows the process of power flow, which also contains the location information of generator and consumer.

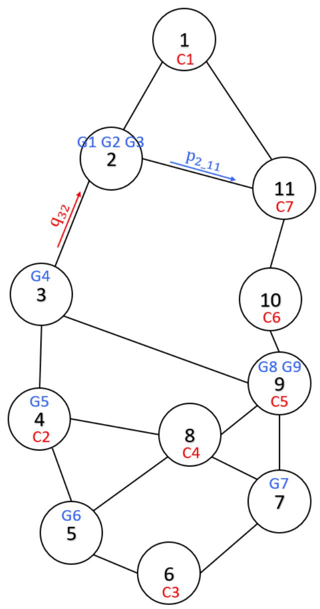

Figure 1: Diagram depicting node, generator, and consumer networks

  $\boldsymbol{Gi}$ and $\boldsymbol{Cj}$ represent the $\boldsymbol{i}$-th generator and $\boldsymbol{j}$-th consumer, respectively, located in different nodes, depicted as circles in the diagram. $\boldsymbol{p_{lm}}$ denotes the active power flow from node $\boldsymbol{l}$ to node $\boldsymbol{m}$ (e.g., $\boldsymbol{p_{2\text{-}11}}$), while $\boldsymbol{q_{ml}}$ represents the reactive power flow from node $\boldsymbol{m}$ to node $\boldsymbol{l}$ (e.g., $\boldsymbol{q_{32}}$).

- **Adjacency matrix**

  We utilize the adjacency matrix $\boldsymbol{E}$ to denote the connectivity of nodes as illustrated in Figure 1. A matrix element $\boldsymbol{e_{ij}} = \boldsymbol{0}$ signifies that there is no edge between nodes $\boldsymbol{i}$ and $\boldsymbol{j}$, whereas $\boldsymbol{e_{ij}} = \boldsymbol{1}$ indicates the presence of a connection. $\boldsymbol{E}$ is a symmetric matrix.

$$\boldsymbol{E} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix}$$

- **Location matrix**
We use matrices $\boldsymbol{G}$ and $\boldsymbol{C}$ to represent the locations of generators and consumers in the network, respectively. In reality, column $\boldsymbol{j}$ of these matrices represents the distribution of generators/consumers in node $\boldsymbol{j}$. The row number where an element in column $\boldsymbol{j}$ equals 1 indicates the serial number of the generator/consumer distributed inside that node.

$$\boldsymbol{G} = \begin{vmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{vmatrix}$$

$$\boldsymbol{C} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

2.2 Modelling

- **Objective function**
We need to formulate an optimization problem that minimizes the cost of active power production. Therefore, we assume that the $\boldsymbol{i}$-th generator produces active power $\boldsymbol{x_i}$ (in pu) , reactive power $\boldsymbol{y_j}$ (in pu) and the active power production cost (in SEK per pu) for each generator, denoted as $\boldsymbol{cost_i}$, is known.

$$\boldsymbol{x_i} \;\; i = 1, ..., n \;(n = 9) \qquad \boldsymbol{y_j} \;\; j = 1, ..., 9$$

$$min \sum_{i=1}^{n} \boldsymbol{x_i} * cost_i$$

- **Active power constrains**
In the network, we primarily have two types of constraints. Firstly, the active constraints. When considering node $\boldsymbol{l}$, the active power produced by generators within node $\boldsymbol{l}$ can either be consumed by consumers within the same node (the demand of each consumer is known), or it can flow to a neighboring node $\boldsymbol{k}$ through an edge if it is not entirely consumed internally. The information about the location of generators and consumers within the node network, as well as the connectivity of nodes, is all encapsulated in the $\boldsymbol{G}$ and $\boldsymbol{C}$ matrices we constructed during the model preparation, as well as the adjacency matrix $\boldsymbol{E}$.

$$\sum_{i=1}^{9} \boldsymbol{x_i} * \boldsymbol{G_{il}} = \sum_{j=1}^{7} demand_j * \boldsymbol{C_{jl}} + \sum_{k=1}^{11} \boldsymbol{p_{lk}} * \boldsymbol{E_{lk}} (k \neq l) \qquad l = 1, ..., 11$$

- **Reactive power constrains**
For reactive power, when we are at node $\boldsymbol{m}$, the reactive power received (from neighboring nodes) and generated by the generators located in that node should be balanced. In other words, the sum of these two types of reactive power should be zero.

$$\sum_{k=1}^{11} \boldsymbol{q_{km}} * \boldsymbol{E_{km}} (k \neq m) + \sum_{i=1}^{9} \boldsymbol{y_i} * \boldsymbol{G_{mi}} = 0 \qquad m = 1, ..., 11$$

- **Ground constrains**
  There are also some fundamental constraints on the active and reactive power produced by each generator $\boldsymbol{i}$, which are related to the active power capacity of generator $\boldsymbol{i}$ (known).

$$0 \leq \boldsymbol{x_i} \leq capacity\ of\ generator\boldsymbol{i}$$

$$-0.3\% * capacity\ of\ generator\boldsymbol{i} \leq \boldsymbol{y_i} \leq 0.3\% * capacity\ of\ generator\boldsymbol{i}$$

- **The computation of $\boldsymbol{p}$ and $\boldsymbol{q}$**
  The expressions for the flow of active power and reactive power between two nodes $\boldsymbol{l}$ and $\boldsymbol{m}$ (with $\boldsymbol{l}$ as the starting point and $\boldsymbol{m}$ as the endpoint) are known and given as follows, they vary with the changes in voltage amplitude $\boldsymbol{v_l}$, $\boldsymbol{v_m}$ (in vu) and voltage phase angle θ_l, θ_m:

$$\boldsymbol{p_{lm}} = \boldsymbol{v_l^2} * \boldsymbol{g_{lm}} - \ \boldsymbol{v_l} * \boldsymbol{v_m} * \boldsymbol{g_{lm}} * \cos(\theta_l - \theta_m) - \boldsymbol{v_l} * \boldsymbol{v_m} * \boldsymbol{b_{lm}} * \sin(\theta_l - \theta_m)$$

$$\boldsymbol{q_{lm}} = -\boldsymbol{v_l^2} * \boldsymbol{b_{lm}} + \ \boldsymbol{v_l} * \boldsymbol{v_m} * \boldsymbol{b_{lm}} * \cos(\theta_l - \theta_m) - \boldsymbol{v_l} * \boldsymbol{v_m} * \boldsymbol{g_{lm}} * \sin(\theta_l - \theta_m)$$

  The magnitude of voltage amplitude and voltage phase angle is also subject to certain constraints:

$$0.98 \leq \boldsymbol{v_{l,m}} \leq 1.02$$

$$-\pi \leq \theta_{l,m} \leq \pi$$

3 Results and analysis

- **Result**

  In our experiment, the overall cost of power production is about 186 SEK. Table 1 shows the amount of active power that each generator generates. In addition, the reactive power that each generator absorbs is also shown in Table 1. Since there is no cost for generating and absorbing the reactive power, we do not need to take them into consideration for the cost.

Table 1: Active and reactive power each generator generates or absorbs

generators	active power generated[pu]	reactive power generated/absorbed[pu]
G_1	0.0048	-0.00006
G_2	0.15	-0.00045
G_3	0.08	-0.00024
G_4	0.07	-0.00021
G_5	0.04	-0.00012
G_6	0.1388	-0.00051
G_7	0.003	-0.00051
G_8	0.253	-0.00078
G_9	0.05	-0.00015

Let us take a deeper look at the active and reactive power in every edge. Tables 2 and 3 illustrate the active and reactive power flow from one node to another, some items remain blank because they are not adjacent.

Table 2: Active power flow from i to j node

i \ j	1	2	3	4	5	6	7	8	9	10	11
1		-0.096									-0.0032
2	0.096		0.058								0.0798
3		-0.0581		0.0981					0.0299		
4			-0.09801		-0.05198						
5				0.0520		0.0648		0.0219			
6					-0.0648		-0.0451				
7						0.0451		0.0033	-0.0454		
8					-0.0219		-0.0033		-0.0646		3
9			-0.0299				0.0454	0.0647		0.0134	
10									-0.0134		-0.0365
11	0.0032	-0.0798								0.0365	

As it mentioned in the title, the formula for each active and reactive power flows from node i to j is as follows.

$$p_{kl} = v_k^2 \cdot g_{kl} - v_k \cdot v_l \cdot g_{kl} \cdot \cos(\theta_k - \theta_l) - v_k \cdot v_l \cdot b_{kl} \cdot \sin(\theta_k - \theta_l)$$

$$q_{kl} = -v_k^2 \cdot b_{kl} - v_k \cdot v_l \cdot b_{kl} \cdot \cos(\theta_k - \theta_l) - v_k \cdot v_l \cdot g_{kl} \cdot \sin(\theta_k - \theta_l)$$

Since b_{kl} and g_{kl} are constant parameters, we only care about the voltage amplitudes v and voltage angles θ. Table 4 describes the final values of the amplitudes and angles when we found the optimal solution.

Table 3: Reactive power flow from i to j node

i \ j	1	2	3	4	5	6	7	8	9	10	11
1		0.0015									-0.0015
2	-0.0011		0.0007								0.0011
3		-0.0005		0.0002					0.0004		
4			0.0004		-0.0003						
5				0.0006		-0.0004		0.0003			
6					0.0008		-0.0008				
7						0.0009		-0.0004	-0.00003		
8					-0.00032		0.0004		-0.0000007		
9			-0.0004				0.0002	0.0003		0.0008	
10									-0.0008		0.0008
11	0.0015	-0.0008								-0.0007	

Table 4: Values of voltage amplitudes and voltage angles

nodes	voltage amplitudes	angles
1	1.019	0.0018
2	1.02	0.0062
3	1.019	0.003
4	1.018	-0.0048
5	1.019	-0.0003
6	1.018	-0.0052
7	1.018	-0.0021
8	1.018	-0.0025
9	1.019	0.0015
10	1.019	0.0006
11	1.019	0.0019

- **Analysis of the optimal values**
  By numerically solving this optimization problem using Julia, we obtain **186.289** as the minimum production cost. The Ipopt solver we used is a local solver. In fact, in this optimization problem, the feasible set is not a convex set. Therefore, we cannot ascertain whether this local minimum is a global minimum. However, local optimality still ensures that this solution holds certain value. We can consider using this solution in practice.

- **Analysis of the perturbation of the generator capacity**
  In reality, the power network is dynamic rather than static, and generators often face the need for upgrades and updates. As operators of the power system, it is crucial to approach this issue from the perspectives of network and economic benefit. The decision of which generator's active power capacity to increase (by 0.01 power unit) should be based on a analysis of the optimum problem.
  From the active power point of view (setting the flow of reactive power to remain in balance), we should choose the **5-th generator** for the following reasons:
  The mathematical essence of increasing the generator's capacity is a **perturbation** of the parameter $\boldsymbol{capacity_i}$ in the corresponding constraints ($\boldsymbol{x_i} \leq capacity_i$). It is essential to consider how this perturbation affects the optimum value (minimum cost). According to the **Shadow Price Theorem**, the marginal change in the optimal value is given by the optimal solution μ^* of the dual problem. Therefore, we should select the smallest element in μ^*, and the corresponding variable $\boldsymbol{x_i}$ represents the generator for which we should upgrade the capacity. Based on the results of μ^*, it should be $\boldsymbol{x_5}$.

$$\mu^* = [0.0, -74.99, -24.99, -111.66, \textbf{-170.66}, -2.59, 0.0, -2.50, -100.00]$$

$$\triangle = -170.66 * 0.01 = -1.7066$$

with $\boldsymbol{x_5}$, if we increase the $\boldsymbol{capacity_5}$ by 0.01 power unit, the optimum solution (minimum cost) with decrease by 1.7066, which is the greatest among all the choices.

4 Conclusion

In conclusion, we have transformed the production and transmission planning problem of electric power into a nonlinear optimization problem. By employing numerical methods, we obtained a local minimum along with its corresponding optimal solution, which serves as the answer to this problem. Additionally, we discussed the choice of upgrading the capacity of generators. In practical applications, both the objective function and constraints may become more complex. However, we can endeavor to mathematically model them and seek to provide optimal solutions through mathematical optimization methods.

5 Appendix

```julia
using JuMP
import Ipopt

# Import data from the data file
include("project_data.jl")

# Create the model object
the_model = Model(Ipopt.Optimizer)

# Create (one set of) variables, and their lower and upper bounds
@variable(the_model, lb[i] <= x[i = 1:n_vars] <= ub[i])
@variable(the_model, theta_lb <= theta[i = 1:n_nodes] <= theta_ub)
@variable(the_model, voltage_lb <= v[i = 1:n_nodes] <= voltage_ub)
@variable(the_model, -0.003 * capacity[i] <= y[i = 1:n_vars] <= 0.003 * capacity[i])
@variable(the_model, p[k = 1:n_nodes, l = 1:n_nodes])
@variable(the_model, q[k = 1:n_nodes, l = 1:n_nodes])


for k in 1:n_nodes
    for l in 1:n_nodes
        if adj_matrix[k, l] == 1
            unregister(the_model, :epr)
            unregister(the_model, :epr2)
            @NLexpression(the_model, epr, v[k]^2 * g[k, l] - v[k]*v[l]*g[k, l]*cos(t
            @NLexpression(the_model, epr2, -1*v[k]^2 * b[k, l] + v[k]*v[l]*b[k, l]*c
            @NLconstraint(the_model, p[k, l] == epr)
            @NLconstraint(the_model, q[k, l] == epr2)


        end
    end
end

# Create the nonlinear objective \sum_i (x_i - 1)^2, which we want to minimize
# @NLobjective(the_model, Min, sum((x[i] - 1)^2 for i in 1:n_vars))
@objective(the_model, Min, sum(cost[i] * x[i] for i in 1:n_vars))

# Active power constraints
for k in 1:n_nodes
    neighbors = [l for l in 1:n_nodes if adj_matrix[k, l] == 1]
    generators = [n for n in 1:n_vars if gl[n] == k]
    consumers = [m for m in 1:n_consumers if cl[m] == k]
    @NLconstraint(the_model, sum(x[n] for n in generators) == sum(p[k, l] for l in n
end

# Reactive power constraints
for k in 1:n_nodes
    neighbors = [l for l in 1:n_nodes if adj_matrix[k, l] == 1]
    generators = [n for n in 1:n_vars if gl[n] == k]
    # generators = [n for n in 1:n_vars if (gl[n] == k) || any(gl[n] == neighbor for
    @NLconstraint(the_model, 0 == sum(q[k, l] for l in neighbors) + sum(y[n] for n i
end
```

```julia
# Print the optimzation problem in the terminal
println(the_model)

# Solve the optimization problem
optimize!(the_model)

# Printing some of the results for further analysis
println("") # Printing white line after solver output, before printing
println("Termination statue: ", JuMP.termination_status(the_model))
println("Optimal(?) objective function value: ", JuMP.objective_value(the_model))
println("Optimal(?) point: ", JuMP.value.(x))
println("Optimal(?) theta: ", JuMP.value.(theta))
println("Optimal(?) voltage: ", JuMP.value.(v))
println("Optimal(?) reactive power: ", JuMP.value.(y))

println("Optimal p:")
for k in 1:n_nodes
    for l in 1:n_nodes
        if adj_matrix[k, l] == 1
            println("p[", k, ",", l, "] = ", JuMP.value(p[k, l]))
        end
    end
end

println("Optimal q:")
for k in 1:n_nodes
    for l in 1:n_nodes
        if adj_matrix[k, l] == 1
            println("q[", k, ",", l, "] = ", JuMP.value(q[k, l]))
        end
    end
end
```