Appendix for: Large-Signal Stability of Power Systems with Mixtures of GFL, GFM and GSP Inverters

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APPENDIX A
DERIVATION OF MODEL FOR TWO-INVERTER-INFINITE-BUS SYSTEM

$$\begin{aligned}
X_{\Delta 12} &= \tfrac{X_1X_2+X_1X_g+X_2X_g}{X_g}\\
X_{\Delta g1} &= \tfrac{X_1X_2+X_1X_g+X_2X_g}{X_2}\\
X_{\Delta g2} &= \tfrac{X_1X_2+X_1X_g+X_2X_g}{X_1}\\
X_{\Sigma 1} &= X_1+X_g, X_{\Sigma 2} = X_2+X_g\\
X_{1+2//g} &= X_1+\tfrac{X_gX_2}{X_{\Sigma 2}}, X_{2+1//g} = X_1+\tfrac{X_gX_1}{X_{\Sigma 1}}
\end{aligned} \tag{A1}$$

A. The expression of Both IBRs are GFM inverters

$$\begin{cases}\dot{\delta}_1 = k_{\text{droop1}}[P_{1ref} - \frac{V_1V_2\sin(\delta_1-\delta_2)}{X_{\Delta 12}} - \frac{V_1U_g}{X_{\Delta g1}}\sin\delta_1]\\ \dot{\delta}_2 = k_{\text{droop2}}[P_{2ref} - \frac{V_1V_2\sin(\delta_2-\delta_1)}{X_{\Delta 12}} - \frac{V_2U_g}{X_{\Delta g2}}\sin\delta_2]\end{cases} \tag{A2}$$

where δ_1, δ_2, and P_{1ref}, P_{2ref} represent the power angles, and the power references of the two GFM inverters respectively. V_1 and V_2 are the voltages of two GFM inverters.

B. The expression of Both IBRs are GFL inverters

$$\begin{cases}\dot{\delta}_1 = k_{\text{PLL1}}\left[X_{\Sigma 1}I_{d1} + X_gI_{d2}\cos(\delta_2-\delta_1) - U_g\sin\delta_1\right]\\ \dot{\delta}_2 = k_{\text{PLL1}}\left[X_{\Sigma 2}I_{d2} + X_gI_{d1}\cos(\delta_1-\delta_2) - U_g\sin\delta_2\right]\end{cases} \tag{A3}$$

where δ_1, δ_2, and I_{d1}, I_{d2} represent the angles of PLL, and the current references on d-axis of the two GFL inverters respectively.

C. The expression of IBR1 is GFL inverter and IBR2 is GFM inverter

$$\begin{cases}\dot{\delta}_1 = k_{\text{PLL}}\left[X_{1+2//g}I_{d1} - \dfrac{X_gV_2}{X_{\Sigma 2}}\sin(\delta_1-\delta_2)\right.\\ \qquad\left. - \dfrac{X_2U_g}{X_{\Sigma 2}}\sin\delta_1\right]\\ \dot{\delta}_2 = k_{\text{droop}}\left[P_{ref} + \dfrac{X_gI_{d1}V_2}{X_{\Sigma 2}}\cos(\delta_1-\delta_2)\right.\\ \qquad\left. - \dfrac{U_gV_2}{X_{\Sigma 2}}\sin\delta_2\right]\end{cases} \tag{A4}$$

D. The expression of IBR1 is GFL inverter and IBR2 is GSP inverter

$$\begin{cases}\dot{\delta}_1 = k_{\text{PLL1}}\left[X_{1+2//g}I_{d1} - \dfrac{X_gV_{ref}}{X_{\Sigma 2}}\sin(\delta_1-\delta_2)\right.\\ \qquad\left. - \dfrac{X_2U_g}{X_{\Sigma 2}}\sin\delta_1 + \varepsilon_{lp}(k_v, k_{\text{PLL2}}, (\delta_1-\delta_2))\right]\\ \dot{\delta}_2 = k_{\text{PLL2}}\left[X_{\Sigma 2}I_{d2} + X_gI_{d1}\cos(\delta_1-\delta_2) - U_g\sin\delta_2\right]\end{cases} \tag{A5}$$

where

$$\begin{aligned}
\varepsilon_{lp} &= \epsilon_v g_1(\delta_1-\delta_2) + \epsilon_{\text{PLL}}h_1(\delta_1-\delta_2)\\
g_1(\delta_1-\delta_2) &= \frac{X_g}{X_{\Sigma 2}}V_{ref}\sin(\delta_1-\delta_2)\\
&\quad + \frac{X_g}{X_{\Sigma 2}}U_g\cos\delta_2\sin(\delta_2-\delta_1)\\
&\quad + \frac{{X_g}^2}{X_{\Sigma 2}}I_{d1}\sin(\delta_2-\delta_1)^2\\
h_1(\delta_1-\delta_2) &= \frac{X_g}{X_{\Sigma 2}}\cos(\delta_1-\delta_2)
\end{aligned} \tag{A6}$$

In (A6), the definition of $\epsilon_v \triangleq 1/(k_vX_{\Sigma 2}+1)$ represents the voltage control error. The $\epsilon_{\text{PLL}} \triangleq \dot{\delta}_2/k_{\text{PLL2}} = v_{q2}$ represents dynamics of the difference between the angle of PLL in IBR2 and its steady state. When k_{PLL2} is large enough, which means that the dynamic of PLL in IBR2 is much faster than PLL1 in IBR1, the perturbed term ϵ_{PLL} can be regarded as zero. Also, when the voltage control gain k_v is large enough, indicating more effective voltage control with smaller error, the term ϵ_v is approximately 0. Under this case ($\epsilon_v \approx 0$, $\epsilon_{\text{PLL}} \approx 0$), the sets of Equations (A4) and (A5) are identical, and they share the same form as shown in Eq. (3) in the paper.

E. The expression of IBR1 is GFM inverter and IBR2 is GSP inverter

$$\begin{cases}\dot{\delta}_1 = k_{\text{droop}}\left[P_{ref} - \dfrac{V_{ref}V_1\sin(\delta_1-\delta_2)}{X_{\Delta 12}}\right.\\ \qquad\left. - \dfrac{V_1U_g\sin\delta_1}{X_{\Delta g1}} + \varepsilon_{mp}(k_v, k_{\text{PLL}}, (\delta_1-\delta_2))\right]\\ \dot{\delta}_2 = k_{\text{PLL}}\left[X_{2+1//g}I_{d2} - \dfrac{X_gV_1}{X_{\Sigma 1}}\sin(\delta_2-\delta_1)\right.\\ \qquad\left. - \dfrac{X_2U_g}{X_{\Sigma 1}}\sin\delta_2\right]\end{cases} \tag{A7}$$

where

$$\begin{aligned}
\varepsilon_{mp} &= \epsilon_v g_2(\delta_1-\delta_2) + \epsilon_{\text{PLL}}h_2(\delta_1-\delta_2)\\
g_2(\delta_1-\delta_2) &= -\frac{X_g}{X_{\Delta 12}X_{\Sigma 1}}{V_1}^2\cos(\delta_1-\delta_2)\sin(\delta_1-\delta_2)\\
&\quad - \frac{X_1}{X_{\Delta 12}X_{\Sigma 1}}V_gV_1\cos\delta_2\sin(\delta_1-\delta_2)\\
&\quad + \frac{V_1V_{ref}}{X_{\Delta 12}}\sin(\delta_1-\delta_2)\\
h_2(\delta_1-\delta_2) &= \frac{V_1\cos(\delta_1-\delta_2)}{X_{\Delta 12}}
\end{aligned} \tag{A8}$$

In (A8), the definition of $\epsilon_v \triangleq 1/\left(k_vX_{2+1//g}+1\right)$ represents the voltage control error. The $\epsilon_{\text{PLL}} \triangleq \dot{\delta}_2/k_{\text{PLL}} = v_{q2}$ represents dynamics of the difference between the angle of PLL in IBR1 and its steady state. When k_{PLL} is large enough, which means that the dynamic of PLL is much faster than GFM, the perturbed term ϵ_{PLL} can be regarded as zero. Also, when the voltage control gain k_v is large enough, indicating more effective voltage control with smaller error, the term ϵ_v is approximately 0. Under this case ($\epsilon_v \approx 0$, $\epsilon_{\text{PLL}} \approx 0$), the sets of Equations (A7) and (A2) are identical, and they share the same form as shown in Eq. (3) in the paper.

Appendix B
EMT Simulation Parameters

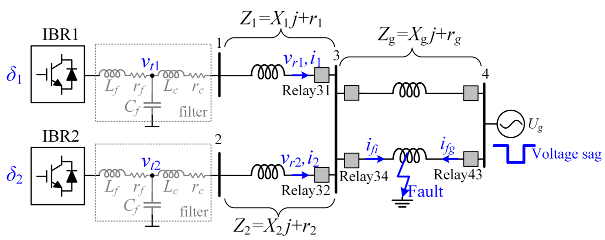

Fig. 2. Two-inverter system and detailed configurations used in EMT simulation.

TABLE B1
System Parameters in Two Paralleled GFL Inverters System

Parameters	Value (p.u.)
Base frequency ω_s	50 Hz
Grid voltage U_g	1
DC voltage U_{dc}	2.5
LCL Filter impedance $L_f j + r_f$	$0.2j + 0.002$
LCL Filter capacitance C_f	0.01
Inner current control loop bandwidth	1 kHz
PLL controller of IBR1 k_{PLL1}	$10 \times 2\pi$
PLL controller of IBR1 k_i	2π
PLL controller of IBR2 k_{PLL2}	$10 \times 2\pi$
PLL controller of IBR2 k_i	2π
Frequency limit of PLL ω_{limit}	± 0.2
Current reference of IBR1 I_{d1}	0.8
Current reference of IBR2 I_{d2}	0.4
Line impedance Z_1	0.2j+0.002
Line impedance Z_2	0.2j+0.002
Line impedance Z_g	0.35j or 0.4j
Fault resistance R_f	0.02

TABLE B2
System Parameters in GFL - GFM(GSP) Inverters System

Parameters	Value (p.u.)
Base frequency ω_s	50 Hz
Grid voltage U_g	1
Impedance Z_1	0.5j+0.025
Impedance (include virtual one) Z_2	0.1j+0.002
Impedance $2Z_g$	0.6j+0.03
Fault resistance R_f	0.001
Fault position (from the infinite bus)	0.8
IBR1 - GFL	
Inner current control loop bandwidth	1 kHz
PLL controller k_{PLL1}	$2.5 \times 2\pi$
PLL controller k_i	$0.25 \times 2\pi$
Frequency limit of PLL ω_{limit}	± 0.2
Current reference I_{d1}	1
DC voltage U_{dc}	2.5
LCL filter impedance $L_f j + r_f$	0.2j+0.02
LCL filter capacitance C_f	0.01
IBR2 - GFM	
$p - \omega$ droop gain k_{droop}	$2.5 \times 2\pi$
Power reference P_{ref}	0.6 in Section IV-A, 0 in Section IV-B
AC Voltage V_2	1
Inner voltage control loop bandwidth	200 Hz
Inner current control loop bandwidth	1 kHz
LCL filter impedance $L_f j + r_f$	0.2j+0.02
LCL filter capacitance C_f	0.1
Current limit I_{limit}	5
$p - \omega$ droop time constant τ_p	$1/(25 \times 2\pi)$
IBR2 - GSP	
Voltage droop k_v	1 or 4 in Section IV-A, 2 in Section IV-B
Voltage reference V_{ref}	1
Current reference I_{d2}	0.6 in Section IV-A, 0 in Section IV-B
Voltage droop filter time scale τ_v	$1/(50 \times 2\pi)$
PLL controller k_{PLL2}	$k_{\text{droop}} \cdot \frac{1}{X_{\Sigma 2}}$

TABLE B3
System Parameters in the GFM - GSP Inverters System

Impedance Z_1	0.5j+0.025
Impedance Z_2	0.1j+0.002
Impedance $2Z_g$	0.6j+0.03
Fault resistance R_f	0.001
Fault position (from the infinite bus)	0.8
IBR1 - GFM	
$p - \omega$ droop gain k_{droop}	$2.5 \times 2\pi$
Power reference P_{ref}	0.8
AC Voltage V_1	1
Inner voltage control loop bandwidth	200 Hz
Inner current control loop bandwidth	1 kHz
Current limit I_{limit}	5
$p - \omega$ droop time constant τ_p	$1/(25 \times 2\pi)$
IBR2 - GSP	
voltage control gain k_v	0, 2, or 4
Voltage reference V_{ref}	1
Current reference I_{d2}	0.2
Voltage control filter time scale τ_v	$1/(50 \times 2\pi)$
PLL controller k_{PLL2}	$6 \times 2\pi$
PLL controller k_i	$0.6 \times 2\pi$