Problem 1: Assume P(t-1) = 1000, sigma = 0.1 (square root of 0.01)

Classic Brownian Motion:

Mean: P(t-1) = 1000

SD: sigma = 0.1

Arithmetic Returns:

Mean: P(t-1) * (1+ mean (r)) = P(t-1) = 1000

SD: P(t-1) * sigma = P(t-1) * 0.1 = 100

Logarithmic Returns:

Mean: P(t-1) * 1.005 = 1005

SD: P(t-1) * 0.1003 = 100.3

Result from simulations:

```
 Mean for classic brownian:  999.9995764869564
Standard Deviation for classic brownian:  0.10028488883679941
```

```
 Mean for arithemtic:  999.5764869562208
Standard Deviation for arithemtic:  100.28488883679954
```

```
 Mean for logrithemtic:  1004.6051557678768
Standard Deviation for logbrithemtic:  100.68913003976503
```

From the results, we can see that the answers we got from simulations are approximately the same as that of the number I manually calculated.

Problem 2:

```
Value at Risk (VaR) for META using different methods:
Normal VaR: 0.038250
EWMA VaR: 0.029952
T-distribution VaR: 0.032426
AR(1) Model VaR: 0.002442
Historical Simulation VaR: 0.028843
```

VaR from the normal distribution is the highest. VaR for T-distribution is less than normal distribution. This might indicate the distribution of returns for META does not has a fat tail. The AR (1) model only tries to predict the relationship between t-1 and t. As a result, it might not show an accurate number. EWMA VaR and VaR historical simulation show fair predictions.

Problem 3:

Result for EWMA VaR:

```
exclude: ['ELV', 'MMC', 'VRTX', 'REGN', 'CB', 'CI', 'ETN', 'SLB', 'PGR', 'BSX']
Portfolio A VaR (EWMA, $): $543878.58
Portfolio B VaR (EWMA, $): $338768.19
Portfolio C VaR (EWMA, $): $437773.32
exclude: ['ELV', 'MMC', 'VRTX', 'REGN', 'CB', 'CI', 'ETN', 'SLB', 'PGR', 'BSX']
Total Portfolio VaR (EWMA, $): $3902861.54
```

This method puts more weight on the recent data. If we add all VaR values for each individual Portfolio, we will get approximately 1.3 million dollars which is significantly lower than that of the total VaR. This means for this model, the VaR is not subadditive, which makes VaR non-coherent as a risk measure.

Result for historical simulation:

```
excluded: ['ELV', 'MMC', 'VRTX', 'REGN', 'CB', 'CI', 'ETN', 'SLB', 'PGR', 'BSX']
Portfolio A VaR (Historical, $): $17250.47
Portfolio B VaR (Historical, $): $11093.34
Portfolio C VaR (Historical, $): $21052.49
excluded: ['ELV', 'MMC', 'VRTX', 'REGN', 'CB', 'CI', 'ETN', 'SLB', 'PGR', 'BSX']
Total Portfolio VaR (Historical, $): $45598.76
```

For the alternative method, I choose historical simulation VaR. On the contrary of EWMA, historical method cast the same importance on every element in the dataset. By calculating historical VaR and comparing with EWMA VaR, we can clearly see how recent data may affect the VaR calculation. According to the historical model, the model is subadditive since the sum of each individual VaR value is higher than that of the total. Besides, the VaR values for historical method is significantly lower than that of the EWMA, indicating that recent returns were more volatile.