Yifei Chen, Nov 2024

Part 1

I wrote a main function to use the function to get the result. I also write a function called plot-result. Here is a screen shot for my main function.

```python
if __name__ == '__main__':
    H1 = .089 # 1519
    H2 = .095 #08535
    W1 = .109 # 11235
    W2 = .082
    L1 = .425 # 24365
    L2 = .392 # 21325

    B1 = np.array([0,1,0,W1+W2,0,L1+L2])
    B2 = np.array([0,0,1,H2,-L1-L2, 0])
    B3 = np.array([0,0,1,H2,-L2,0])
    B4 = np.array([0,0,1,H2,0,0])
    B5 = np.array([0,-1,0,-W2,0,0])
    B6 = np.array([0,0,1,0,0,0])
    Blist = np.array([B1,B2,B3,B4,B5,B6])
    Blist = Blist.T


    thetalist0_short = np.array([-2, -1.5, -1.2, 2.4, 1.4, 1.2]) # np.array([-2.5536, -2.0013, -1.6068, 2.2761, 1.4830, 0.9889])
    thetalist0_long = np.array([-2.5601, -1.0485, -1.8119, -0.4478, 2.9107, -2.0351]) #np.array([1.707,-1.578,0,-1.514,-0.032,1.514])


    # M
    M = np.array([[-1, 0, 0, L1+L2],
                  [0, 0, 1, W1+W2],
                  [0, 1, 0, H1-H2],
                  [0,0,0,1]])

    T = np.array([[1, 0, 0, 0.3],
                  [0, 1, 0, 0.3],
                  [0, 0, 1, 0.4],
                  [0, 0, 0, 1]])

    eomg = 0.001
    ev = 0.0001

    iter_thetas_s, err_s, positions_s, linear_errors_s, angular_errors_s = IKinBodyIterates(Blist,M,T,thetalist0_short,eomg,ev)
    # iter_thetas_l, err_short_l, positions_l, linear_errors_l, angular_errors_l = IKinBodyIterates(Blist,M,T,thetalist0_long,eomg,ev)

    # plot result
    # plot_results(positions_s, linear_errors_s, angular_errors_s, positions_l, linear_errors_l, angular_errors_l)
```

Screen log

This log show how your code is called and the text output. The way it call is using the **Ass2code.py**. Inside, there is the \textbf{main} function It runs the short and long iterates of **IKinBodyIterates** with my initial guess. (The output is long, so it takes maney images)

For the short interation, it takes 4 interates.

```
(446) (base) ericchen@EricdeMacBook-Pro ME449 % python -u "/Users/ericchen/ME449/yifei_chen_ass2/Ass2code.py"
 Iteration 0:

        Joint Vector:
        -2.0000, -1.5000, -1.2000, 2.4000, 1.4000, 1.2000

        SE(3) End-Effector Configuration:
        0.4638 -0.8536 -0.2372 0.2029
        0.1554 0.3420 -0.9268 0.1480
        0.8722 0.3930 0.2912 0.6136
        0.0000 0.0000 0.0000 1.0000

        Error Twist Vb:
        -1.0057, 0.8454, -0.7688, 0.0287, -0.0090, -0.3011

        Angular Error Magnitude || omega_b ||: 1.52228
        Linear Error Magnitude || v_b ||: 0.30261

 Iteration 1:

                Joint Vector:
                -2.0618, -2.1763, -1.1229, 1.8266, 1.9720, 0.6484

                SE(3) End-Effector Configuration:
                0.9160 -0.1054 -0.3870 0.3164
                0.1572 0.9820 0.1045 0.4285
                0.3690 -0.1566 0.9161 0.4427
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                0.1347, 0.3901, -0.1355, -0.0337, -0.1240, -0.0470

                Angular Error Magnitude || omega_b ||: 0.43443
                Linear Error Magnitude || v_b ||: 0.13685

 Iteration 2:

                Joint Vector:
                -2.1234, -1.8976, -1.7316, 2.0433, 1.5736, 0.5252

                SE(3) End-Effector Configuration:
                0.9996 0.0274 0.0055 0.2968
                -0.0275 0.9995 0.0143 0.2740
                -0.0051 -0.0144 0.9999 0.3913
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                0.0144, -0.0053, 0.0274, 0.0028, 0.0260, 0.0089

                Angular Error Magnitude || omega_b ||: 0.03141
                Linear Error Magnitude || v_b ||: 0.02762
```

```
Iteration 3:

                Joint Vector:
                -2.0956, -1.9334, -1.6530, 2.0156, 1.5706, 0.5248

                SE(3) End-Effector Configuration:
                1.0000 -0.0000 0.0002 0.2996
                0.0000 1.0000 -0.0001 0.2999
                -0.0002 0.0001 1.0000 0.3997
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.0001, -0.0002, -0.0000, 0.0004, 0.0001, 0.0003

                Angular Error Magnitude || omega_b ||: 0.00022
                Linear Error Magnitude || v_b ||: 0.00054

Iteration 4:

                Joint Vector:
                -2.0964, -1.9340, -1.6513, 2.0145, 1.5708, 0.5256

                SE(3) End-Effector Configuration:
                1.0000 0.0000 0.0000 0.3000
                -0.0000 1.0000 0.0000 0.3000
                -0.0000 -0.0000 1.0000 0.4000
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                0.0000, -0.0000, 0.0000, -0.0000, 0.0000, 0.0000

                Angular Error Magnitude || omega_b ||: 0.00000
                Linear Error Magnitude || v_b ||: 0.00000
```

For the long interation, it takes 14 interates.

```
(446) (base) ericchen@EricdeMacBook-Pro ME449 % python -u "/Users/ericchen/ME449/yifei_chen_ass2/Ass2code.py"
Iteration 0:

        Joint Vector:
        -2.5601, -1.0485, -1.8119, -0.4478, 2.9107, -2.0351

        SE(3) End-Effector Configuration:
        -0.2916 0.8917 -0.3461 0.1822
        -0.0690 0.3413 0.9374 0.0848
        0.9540 0.2973 -0.0380 0.6567
        0.0000 0.0000 0.0000 1.0000

        Error Twist Vb:
        0.7686, 1.5610, 1.1535, -0.2439, 0.3156, -0.1516

        Angular Error Magnitude || omega_b ||: 2.08765
        Linear Error Magnitude || v_b ||: 0.42670

Iteration 1:

                Joint Vector:
                2.2637, -2.9858, -1.7424, -1.2976, -0.5470, 2.5341

                SE(3) End-Effector Configuration:
                -0.8546 -0.3961 -0.3358 0.1683
                0.3608 0.0123 -0.9326 -0.4830
                0.3735 -0.9181 0.1324 -0.3180
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.0361, 1.7752, -1.8942, 1.4266, 0.1767, 0.1251

                Angular Error Magnitude || omega_b ||: 2.59625
                Linear Error Magnitude || v_b ||: 1.44291

Iteration 2:

                Joint Vector:
                0.6525, -2.2915, 1.4789, 2.2877, -0.6270, -0.1868

                SE(3) End-Effector Configuration:
                0.1427 0.8319 -0.5362 -0.1940
                -0.6164 0.4986 0.6094 0.0725
                0.7744 0.2436 0.5840 0.7318
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                0.2684, 0.9615, 1.0625, 0.1229, 0.4925, -0.4779

                Angular Error Magnitude || omega_b ||: 1.45788
                Linear Error Magnitude || v_b ||: 0.69716

Iteration 3:

                Joint Vector:
                -1.1908, -1.6686, -2.4593, -1.6168, -1.0976, 0.2363

                SE(3) End-Effector Configuration:
                -0.9002 0.4124 0.1398 -0.0010
                -0.0792 -0.4709 0.8786 0.3971
                0.4282 0.7799 0.4565 0.1409
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                0.4863, 1.4198, 2.4207, 0.3949, 0.3687, -0.0331

                Angular Error Magnitude || omega_b ||: 2.84818
                Linear Error Magnitude || v_b ||: 0.54129

Iteration 4:

                Joint Vector:
                -0.6332, 0.1713, -2.6949, 3.0509, -2.1131, 2.4075

                SE(3) End-Effector Configuration:
                0.3810 -0.2025 -0.9021 0.0320
                0.5092 0.8604 0.0219 0.0592
                0.7717 -0.4677 0.4310 0.1969
                0.0000 0.0000 0.0000 1.0000
```

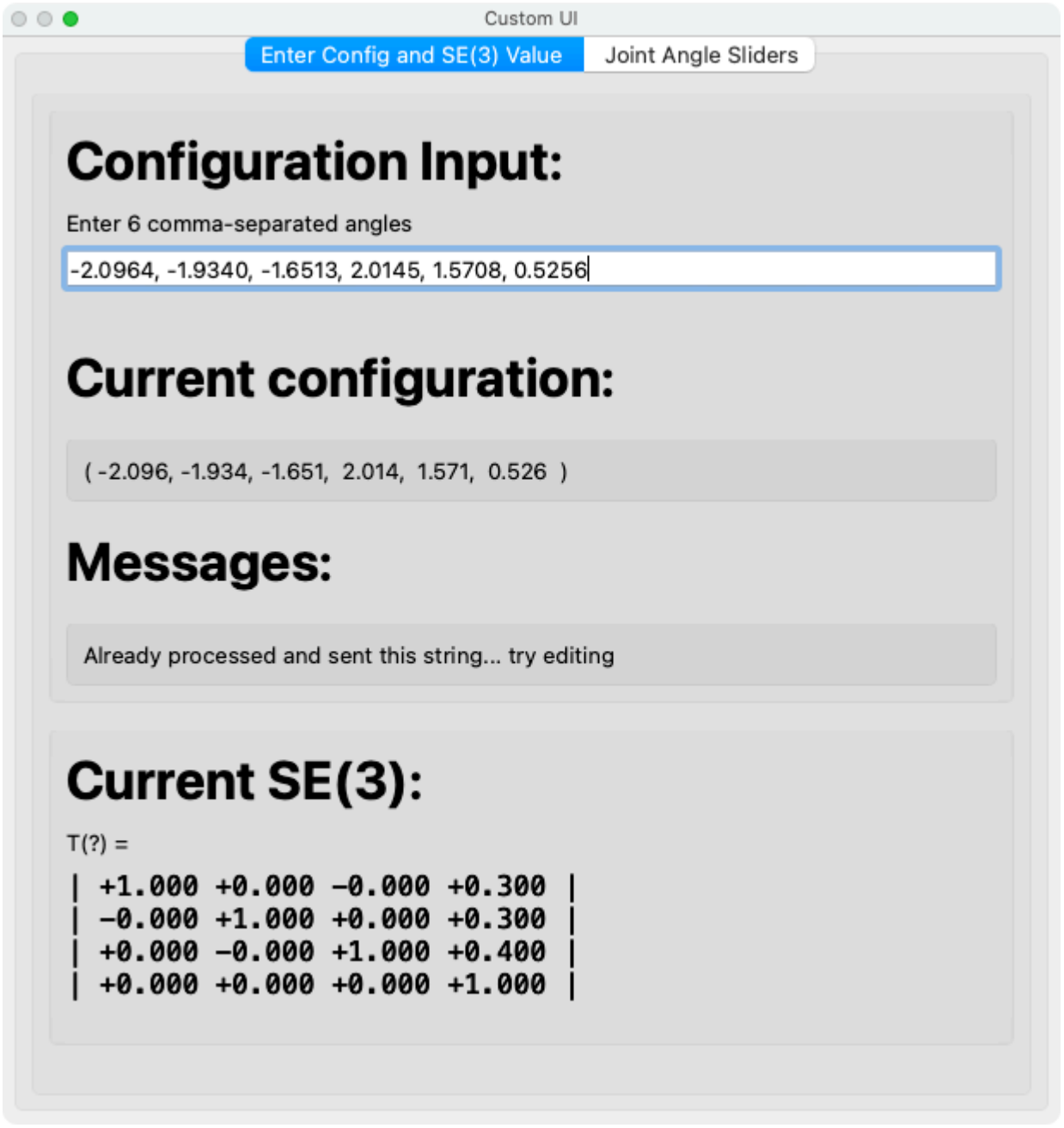
```
Iteration 5:

                Joint Vector:
                -1.7455, 1.1596, -1.7173, 0.6473, 2.9519, -0.5107

                SE(3) End-Effector Configuration:
                0.0213 -0.0059 -0.9998 -0.0605
                -0.8259 -0.5637 -0.0142 -0.5066
                -0.5635 0.8260 -0.0169 -0.1891
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -1.6536, 0.8587, 1.6137, -0.4029, 1.2115, -0.4086

                Angular Error Magnitude || omega_b ||: 2.46490
                Linear Error Magnitude || v_b ||: 1.34058

Iteration 6:

                Joint Vector:
                -0.8900, 0.1351, 2.4906, -1.2799, 2.2467, 2.4638

                SE(3) End-Effector Configuration:
                -0.1559 -0.9132 -0.3766 0.0460
                -0.7730 0.3501 -0.5290 0.0349
                0.6149 0.2086 -0.7605 -0.2452
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -1.4652, 1.9695, -0.2784, 0.7857, 0.5460, -0.1656

                Angular Error Magnitude || omega_b ||: 2.47048
                Linear Error Magnitude || v_b ||: 0.97107

Iteration 7:

                Joint Vector:
                0.2649, 0.7074, -0.5055, -0.0206, 0.8020, 0.3929

                SE(3) End-Effector Configuration:
                -0.7170 0.4855 0.5003 0.6784
                0.4935 -0.1534 0.8561 0.3560
                0.4924 0.8607 -0.1296 -0.3699
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -1.1795, -2.0400, -2.0685, -0.7218, 1.0582, -0.1332

                Angular Error Magnitude || omega_b ||: 3.13554
                Linear Error Magnitude || v_b ||: 1.28781

Iteration 8:

                Joint Vector:
                -0.7864, -1.0746, 1.2757, -0.2659, 2.1041, -1.6408

                SE(3) End-Effector Configuration:
                -0.0221 0.9687 0.2472 0.5160
                -0.0631 0.2454 -0.9674 -0.4218
                -0.9978 -0.0369 0.0557 0.2942
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.9673, -1.2943, 1.0728, -0.5465, 0.4857, -0.4770

                Angular Error Magnitude || omega_b ||: 1.93955
                Linear Error Magnitude || v_b ||: 0.87303

Iteration 9:

                Joint Vector:
                0.2567, -1.8439, 1.4379, -1.1195, 2.0896, -1.3687

                SE(3) End-Effector Configuration:
                0.9067 -0.3887 0.1640 0.3150
                0.4182 0.7776 -0.4696 0.1534
                0.0550 0.4943 0.8675 0.7199
                0.0000 0.0000 0.0000 1.0000
```

```
Iteration 10:

                Joint Vector:
                0.8246, -2.0170, 2.5087, -2.5281, 1.6162, -2.5513

                SE(3) End-Effector Configuration:
                0.9585 0.0879 -0.2711 0.0654
                -0.1857 0.9142 -0.3601 0.2259
                0.2162 0.3955 0.8926 0.4032
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.3933, 0.2537, 0.1424, 0.2266, 0.0872, -0.0483

                Angular Error Magnitude || omega_b ||: 0.48923
                Linear Error Magnitude || v_b ||: 0.24757

Iteration 11:

                Joint Vector:
                0.2657, -1.8378, 2.2371, -1.9445, 1.8589, -1.8876

                SE(3) End-Effector Configuration:
                0.9928 0.0680 0.0983 0.3114
                -0.0394 0.9627 -0.2678 0.1735
                -0.1128 0.2620 0.9585 0.4227
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.2687, -0.1071, 0.0545, -0.0132, 0.1223, -0.0402

                Angular Error Magnitude || omega_b ||: 0.29435
                Linear Error Magnitude || v_b ||: 0.12941

Iteration 12:

                Joint Vector:
                0.5683, -1.6483, 2.1675, -2.1729, 1.5780, -2.1540

                SE(3) End-Effector Configuration:
                0.9977 0.0129 -0.0659 0.2748
                -0.0163 0.9986 -0.0506 0.3041
                0.0652 0.0516 0.9965 0.4078
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.0511, 0.0656, 0.0146, 0.0249, -0.0042, -0.0085

                Angular Error Magnitude || omega_b ||: 0.08447
                Linear Error Magnitude || v_b ||: 0.02667

Iteration 13:

                Joint Vector:
                0.5243, -1.6315, 2.1516, -2.0906, 1.5751, -2.0950

                SE(3) End-Effector Configuration:
                1.0000 -0.0001 0.0025 0.3000
                0.0001 1.0000 -0.0035 0.2990
                -0.0025 0.0035 1.0000 0.4004
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
                -0.0035, -0.0025, -0.0001, -0.0000, 0.0010, -0.0004

                Angular Error Magnitude || omega_b ||: 0.00428
                Linear Error Magnitude || v_b ||: 0.00104

Iteration 14:

                Joint Vector:
                0.5256, -1.6294, 2.1508, -2.0923, 1.5708, -2.0964

                SE(3) End-Effector Configuration:
                1.0000 0.0000 -0.0000 0.3000
                -0.0000 1.0000 -0.0000 0.3000
                0.0000 0.0000 1.0000 0.4000
                0.0000 0.0000 0.0000 1.0000

                Error Twist Vb:
```

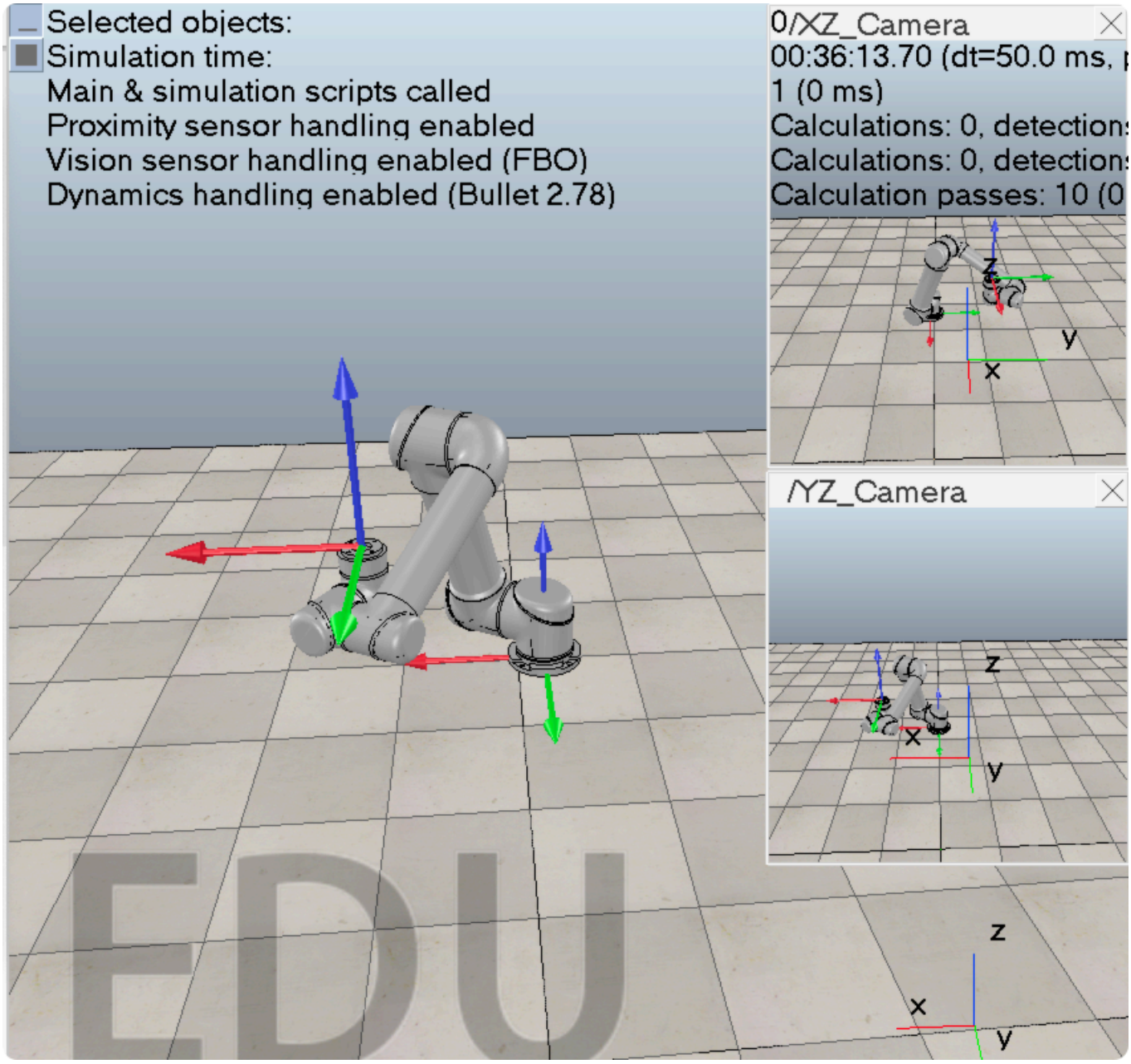
CoppliaSim Screenshot

A CoppeliaSim screenshot showing the UR5 at the solution configuration calculated after 2–4 iterations for my “good” initial guess. This screenshot clearly show the UR5’s end-effector configuration as well as the SE(3) configuration reported by the scene’s interface, confirming that my code calculated a good solution.

Here is the SE(3) shown in the UI

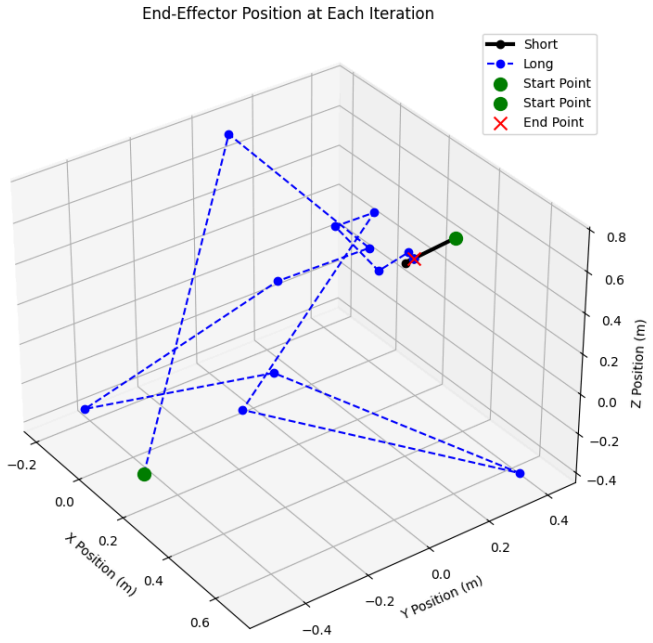


Here is the configuration of the desired position of UR5



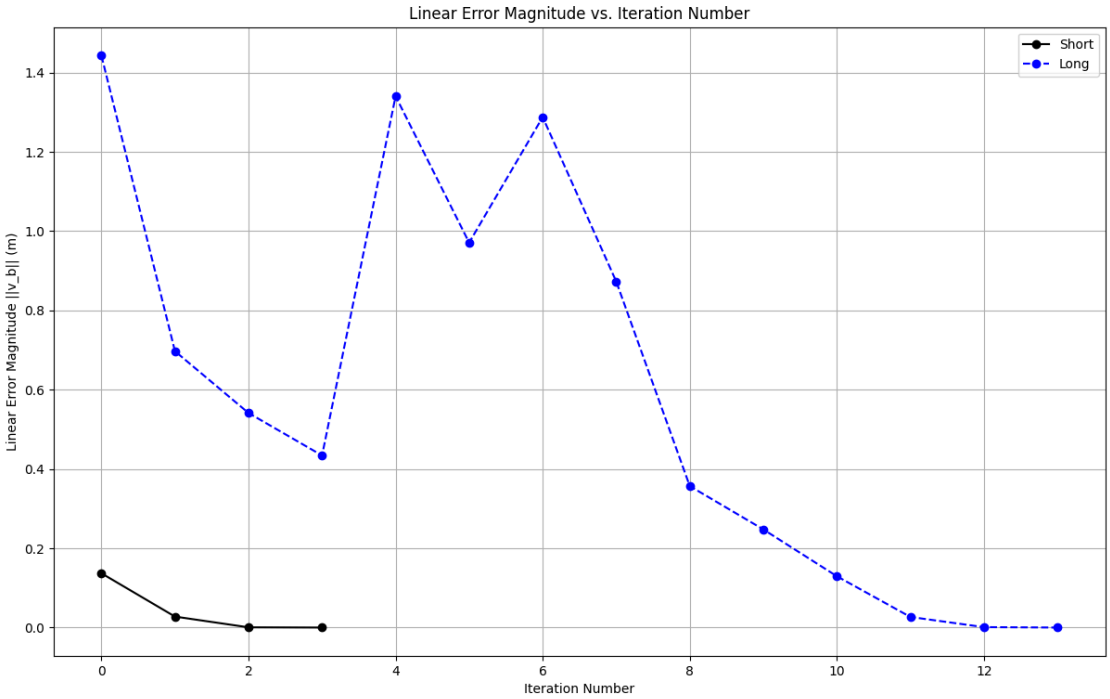
Progression of end-effector positions

A figure showing the progression of end-effector (x, y, z) positions during the solution process. (Plots of both initial guesses in the same figure.)



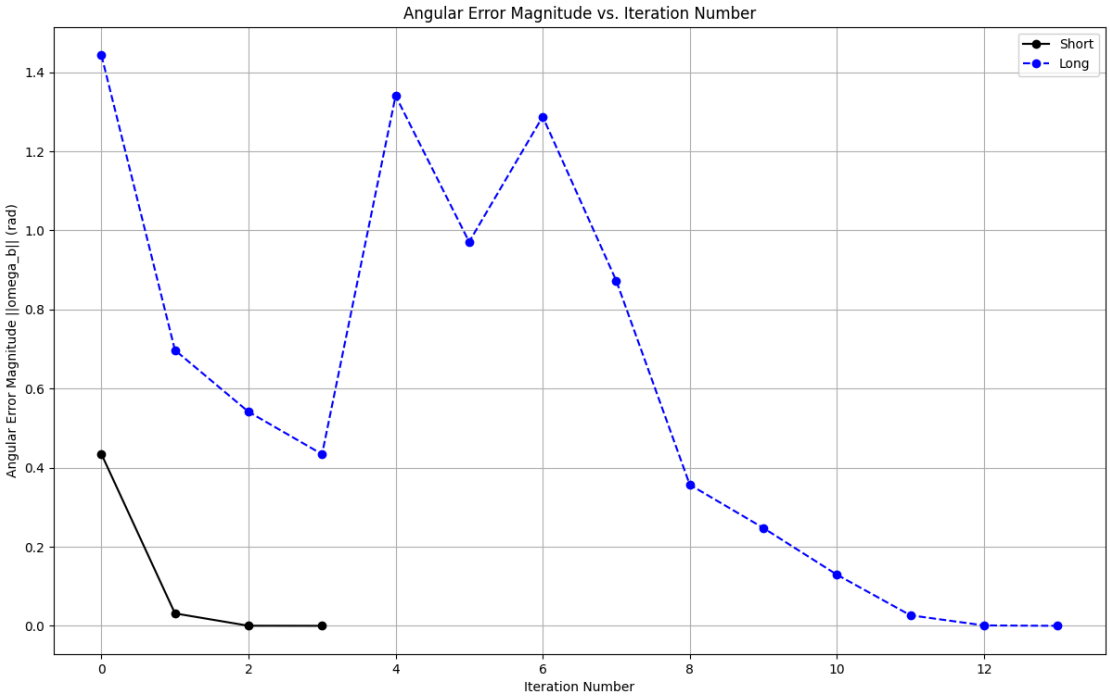
linear errors

A figure showing the magnitudes of the linear error as a function of iterations (both initial guesses).



Angular errors

A figure showing the magnitudes of the angular error as a function of iterations (both initial guesses).



why convergence is difficult from the long iterates initial guess?

When the initial guess is far from the target solution, numerical inverse kinematics struggles to converge due to:

1. **Initial Guess Deviation**: Newton-Raphson relies on linear approximations, which become inaccurate when starting far from the solution, making it difficult to find the correct descent direction.
2. **Nonlinearity and Singularity**: High nonlinearity and potential singularity of the Jacobian can lead to large, unstable updates, causing divergence.
3. **Error Propagation and Large Steps**: Large initial errors can result in overly large step sizes, causing oscillations and instability.
4. **Local Minima**: A poor initial guess may cause the iteration to get stuck in local minima, preventing convergence to the global solution.

Part 2

Part 2 (a)

Suppose the robot moves from the initial configuration θ_0 to the target configuration θ^*, and each joint moves with a constant velocity. Let the total movement time be t_f. The velocity vector for each joint can be represented as:

1. **Joint Angle Difference**:

$$\Delta\theta = \theta^* - \theta_0$$

Constant Joint Velocity Vector:

$$\dot{\theta} = \frac{\Delta\theta}{t_f}$$

For each joint i, this can be extended to the following form:

$$\dot{\theta}_i = \frac{\theta_i^* - \theta_{0,i}}{t_f}, \quad i = 1, 2, \ldots, n$$

This expression represents that each joint linearly changes its value from the initial to the target position at a constant speed.

Part 2(b)

Constant Twist Movement of the End-Effector Suppose the end-effector moves from the initial configuration T_0 to the target configuration T_{sd} with a constant twist. 1. **Twist Representation**: - The twist V can be computed as:

$$V = \frac{\log(T_{sd} T_0^{-1})}{t_f}$$

where $\log$ represents the logarithm map that converts the transformation matrix into a twist vector.

Joint Velocity Calculation:

- At time $t = 0$, the joint velocity vector can be computed using the body Jacobian $J_b(\theta_0)$:

$$\dot{\theta}(0) = J_b(\theta_0)^{-1} V$$

- At time $t = t_f/2$, assuming the end-effector's position at $t_f/2$ is obtained by interpolation, the joint velocity can be similarly calculated as:

$$\dot{\theta}\left(\frac{t_f}{2}\right) = J_b(\theta(t_f/2))^{-1} V$$

Part 2(c)

1. **Constant Joint Velocity**

- **Advantages**:
  - Easy to implement, with linearly changing joint velocities that are straightforward to compute.
  - Low computational cost, suitable for real-time control and simple tasks.
- **Disadvantages**:
  - The end-effector trajectory may not be linear, which could lead to a deviation from the intended path.
  - Independent joint movements make the end-effector's speed and direction difficult to predict.

2. **Constant Twist Movement**:

- **Advantages**:
  - Better control of the end-effector trajectory, making the path smoother and closer to linear.
  - More suitable for tasks requiring precise control of the end-effector's path.
- **Disadvantages**:
  - Requires calculation of the Jacobian matrix and its inverse, which increases computational complexity.
  - Near singularities, the Jacobian matrix may become non-invertible, leading to control failure.

Constant Joint Velocity is suitable for tasks where the trajectory is not critical, such as simple pick-and-place operations.
Constant Twist Movement is more appropriate for tasks requiring precise control of the end-effector's trajectory, but it comes with higher computational costs and potential singularity issues.