HUDM 5126 Linear Models and Regression Analysis Homework 1*

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1 Grade Point Average

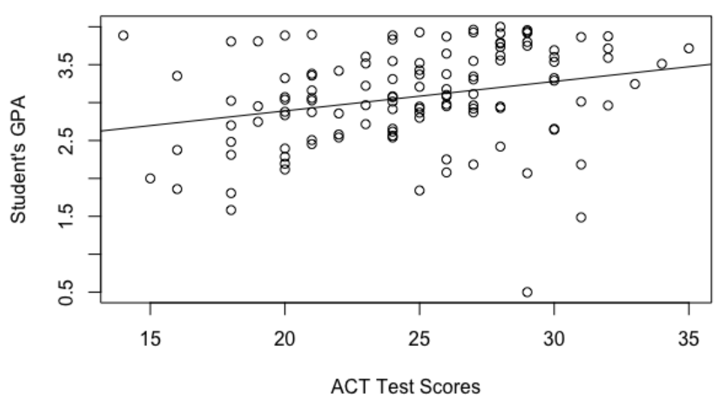
(a) Obtain the least squares estimates of β_0 and β_1, and state the estimated regression function.

$\beta_0 = 2.11405$, $\beta_1 = 0.03883$, $\overline{Y} = 2.11405 + 0.03883 * X$

(b) Plot the estimated regression function and the data. Does the estimated regression function appear to fit the data well?

You can see the plot in Figure 1. The estimated regression function does not appear to fit the data very well since the R-squared was very low, which is 0.07262.

Figure 1: This is a scatter plot with the regression line.



(c) Obtain a point estimate of the mean freshman GPA for students with ACT test score X=30.

$\overline{Y_h} = 2.11405 + 0.03883 * 30 = 3.27895$

(d) What is the point estimate of the change in the mean response when the entrance test score increases by one point?

For every one point increase in the entrance test scores, the grade point average (GPA) increases by 0.03883, which is β_1.

*This homework is written in LaTeX.

2 Least Squares Estimators

$$\sum Y_i = nb_0 + b_1 \sum X_i \tag{1.9a}$$

$$\sum X_i Y_i = b_0 \sum X_i + b_1 \sum X_i^2 \tag{1.9b}$$

$$b_1 = \frac{\sum(X_i - \overline{X})(Y_i - \overline{Y})}{\sum(X_i - \overline{X})^2} \tag{1.10a}$$

$$b_0 = \frac{1}{n}(\sum Y_i - b_1 \sum X_i) = \overline{Y} - b_1 \overline{X} \tag{1.10b}$$

(a) Derive the expression for b_1 in (1.10a) from the normal equation in (1.9).

First, solving 1.9a for b_0, then

$$b_0 = \frac{\sum Y_i - b_1 \sum X_i}{n} \tag{1.32.1}$$

Similarly, solving 1.9b for b_0, then

$$b_0 = \frac{\sum X_i Y_i - b_1 \sum X_i^2}{\sum X_i} \tag{1.32.2}$$

equating the results from 1.32.1 and 1.32.2:

$$\frac{\sum Y_i - b_1 \sum X_i}{n} = \frac{\sum X_i Y_i - b_1 \sum X_i^2}{\sum X_i} \tag{1.32.3}$$

and then solving for b_1 yields:

$$b_1 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2} = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\sum X_i^2 - \frac{(\sum X_i)^2}{n}} \tag{1.32.4}$$