Question 2

Consider a ternary source on $\{R, G, B\}$ with two distributions:

Source	R	G	B
Q_1	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$
Q_2	$\frac{1-p}{2}$	p	$\frac{1-p}{2}$

Part 1: Solution

We compute

$$H(Q_1) = -\sum_{x\in\{R,G,B\}} Q_1(x)\log_2 Q_1(x) = -\Big(\tfrac{1}{6}\log_2\tfrac{1}{6} + \tfrac{1}{2}\log_2\tfrac{1}{2} + \tfrac{1}{3}\log_2\tfrac{1}{3}\Big).$$

Numerically,

$$-\tfrac{1}{6}(-\log_2 6) - \tfrac{1}{2}(-1) - \tfrac{1}{3}(-\log_2 3) \approx 0.4308 + 0.5 + 0.5283 \approx 1.459 \text{ bits.}$$

Answer

$$\boxed{H(Q_1) \approx 1.459 \text{ bits.}}$$

Part 2: Solution

For $Q_2 = (\frac{1-p}{2}, p, \frac{1-p}{2})$,

$$H(Q_2) = -p\log_2 p - 2\cdot\tfrac{1-p}{2}\log_2\Big(\tfrac{1-p}{2}\Big) = -p\log_2 p - (1-p)\log_2(1-p) + (1-p).$$

Differentiate w.r.t. p:

$$\frac{dH}{dp} = -\log_2 p + \log_2(1-p) - 1 \stackrel{!}{=} 0 \implies \frac{1-p}{p} = 2 \implies p = \tfrac{1}{3}.$$

Answer

$$\boxed{p = \frac{1}{3}.}$$