

Question 4

An urn contains four balls numbered from 1 to 4. If we add a new ball with the number 4, does the entropy associated with the extraction of a ball from the urn increase or decrease? And would it have happened if the added ball had the number 5?

Solution

Easy to know the initial situation is that

$$H_1(I_1(X_1)) = -4 \cdot \frac{1}{4} \left(\log_2 \left(\frac{1}{4} \right) \right) = 2\text{bits}$$

By adding a ball with the number 4, we then have

$$H_2(I_2(X_2)) = - \left(3 \cdot \frac{1}{5} \left(\log_2 \left(\frac{1}{5} \right) \right) + 2 \cdot \frac{1}{5} \left(\log_2 \left(\frac{2}{5} \right) \right) \right) \approx 1.92\text{bits}$$

And we have the situation when we add a ball with number 5

$$H_3(I_3(X_3)) = -5 \cdot \frac{1}{5} \left(\log_2 \left(\frac{1}{5} \right) \right) \approx 2.32\text{bits}$$

Finally we have the conclusion

$$H_2 < H_1 < H_3$$

Answer

$$\boxed{H_2 < H_1 < H_3}$$