Question 1

Calculate the entropy in bits for each of the following random variables:

Part 1: Pixel values in an image whose possible grey values are all the integers from 0 to 255 with uniform probability.

Solution

$$256 = 255 - 0 + 1$$

$$H(X) = \mathbb{E}(I(X)) = -256 \cdot \frac{1}{256}\left(log_2\left(\frac{1}{256}\right)\right) = 8\text{bits}$$

Answer

$$\boxed{H(X) = 8\text{bits}}$$

Part 2: Gender in a tri-sexed insect population whose three genders occur with probabilities $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{2}$.

Solution

$$H(X) = \mathbb{E}(I(X)) = -\left(2 \cdot \frac{1}{4}\left(log_2\left(\frac{1}{4}\right)\right) + \frac{1}{2}\left(log_2\left(\frac{1}{2}\right)\right)\right) = 1.5\text{bits}$$

Answer

$$\boxed{H(X) = 1.5\text{bits}}$$

Part 3: A population of persons classified by whether they are older, or not older, than the population's median age.

Solution

$$H(X) = \mathbb{E}(I(X)) = -\left(2 \cdot \frac{1}{2}\left(log_2\left(\frac{1}{2}\right)\right)\right) = 1\text{bits}$$

Answer

$$\boxed{H(X) = 1\text{bits}}$$