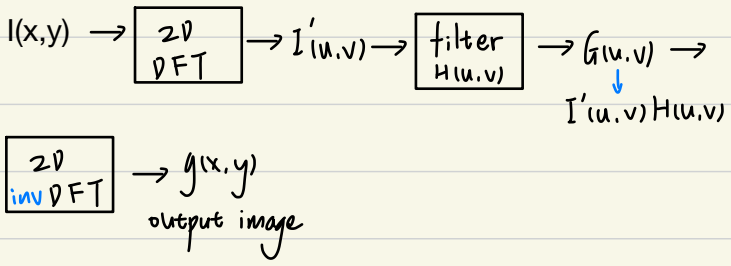
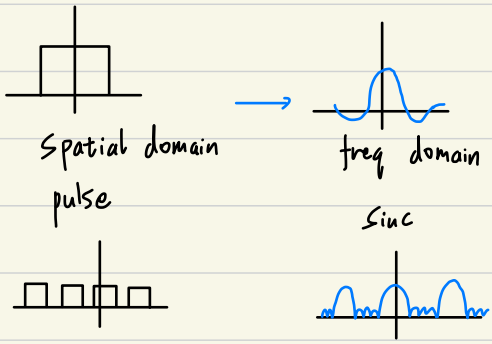
frequency domain filtering; sampling and aliasing



spatial domain convolution $\longleftrightarrow$ frequency domain multiplication

filter in frequency domain



discrete pulse train

kind of like a periodic sinc

smaller filter cutoff in freq domain => more blurring
bigger/more obvious ringing
so to avoid ringing, we often prefer simple spatial domain filter
a good exception: Gaussian low pass filter

$H(u,v) = e^{-\frac{u^2+v^2}{2\sigma^2}}$ Gaussian in spatial domain

$\Updownarrow$

Gaussian in frequency domain

high pass filter

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ - low pass eg: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\text{ones}(3)$

could be gaussian

laplacian:

$\nabla^2 I = \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}$

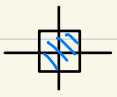
$\mathcal{F}(\nabla^2 I) = -4\pi^2(u^2+v^2)\mathcal{F}(I)$

$(2\pi^2)(ju)^2\mathcal{F}(I)$ $(2\pi^2)(jv)^2\mathcal{F}(I)$

$H(u,v)$ of Laplacian filter

just like in 1D, we have to deal with sampling, aliasing, band-limited
eg: a 2D function(image) is band-limited, if its FT is 0 outsides some rectangular region, i.e.

$F(u,v) = 0$ for $|u| \geq u_{max}$, $|v| \geq v_{max}$



anti-aliasing: it's not removing aliasing from existed image, it is blurring digital image prior to resampling to avoid visual artifacts

original image → blur (LPF) → downsample

in video games, anti-aliasing has many flavors

moire pattern