# Threshold

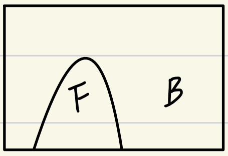
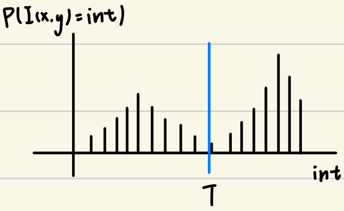



image segmentation
easiest method:

$$g(x,y) = \begin{cases} 1 & \text{if } I(x,y) > T \text{, object} \\ 0 & \text{if } I(x,y) \le T \text{, background} \end{cases}$$




histogram

global threshold for all pixels
vs
local/adaptive/pixel dependent threshold

global thresholding
assume we use the same T, how to find the best threshold?
Classical method: OTSU's algorithm
idea: maximize between-class variance
a good threshold should separate pixel into tight cluster.

Image PMF: $p_i$ = probability that $I(x,y) = i$, $i = 0, \cdots L-1$

$m_g = \sum_{i=0}^{L-1} i p_i$ global mean

$\sigma_g^2 = \sum_{i=0}^{L-1} (i - m_G)^2 p_i$ global variance

suppose we select a threshold T

$C_1 = \{(x,y) \mid I(x,y) \le k\}$ $C_2 = \{(x,y) \mid I(x,y) > k\}$

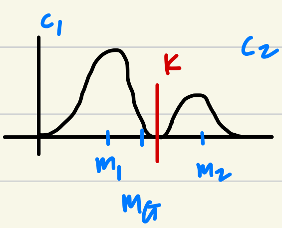
$P_1 = \sum_{i=0}^{k} p_i$ $P_2 = \sum_{i=k-1}^{L-1} p_i = 1 - P_1$

class-conditional mean/variance

$$m_1 = \frac{\sum_{i=1}^{k} i p_i}{P_1} \qquad m_2 = \frac{\sum_{i=k+1}^{L-1} i p_i}{P_2}$$

Otsu's criterion: maximize the between-class variance

$$\sigma_B^2 = P_1 (m_1 - m_G)^2 + P_2 (m_2 - m_G)^2$$



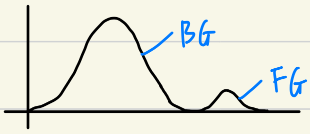

the ratio $\sigma_B^2 / \sigma_G^2$
is a good measure of separability.
higher is better
(more separable)

in practice, we just consider all possible threshold and choose final threshold T as the k that maximize $\sigma_B^2$

In general, we can extend this to finding k-1 thresholds to separate k classes

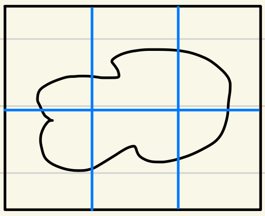
$$\sigma_B^2 = \sum_{k=1}^{K} P_k (m_k - m_G)^2$$

Otsu can fail when:
- no strong peaks in histogram
- object is small respect to background



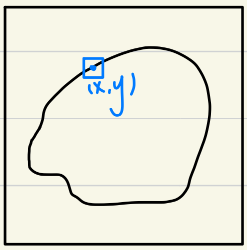

remedies: - low-pass filter, then apply Otsu
- only consider pixels near edges when computing the threshold

variable/adaptive thresholding



blockproc - applies a specified function to each M*N block of an image

works ok in cases, but choosing block side is tricky. blocking artifact.
better: adapt threshold on a per-pixel basis.




at every (x, y), build neighbourhood Sxy, computer the mean and variance.
$\mu_{xy}$, $\sigma_{xy}$

we can make a rule like

$$g(x,y) = \begin{cases} 1, & I(x,y) > \mu_{xy} + 2\sigma_{xy} \\ 0, & o/w \end{cases}$$

can also apply threshold to RGB colour images
- threshold independency on R,G,B, I channels
- combine the channels, e.g

$$\| \underset{\begin{bmatrix} R \\ G \\ B \end{bmatrix}}{I(x,y)} - \underset{\begin{bmatrix} R_c \\ G_c \\ B_c \end{bmatrix}}{C} \| \leq \tau$$