Rensselaer Polytechnic Institute
Department of Electrical, Computer, and Systems Engineering
ECSE 4540: Introduction to Image Processing, Spring 2018

Homework #2: due Monday, Feb. 5^{th}, at the beginning of class.
Show all work for full credit!

1. (15 points.) Sketch 4 images that all have the histogram given by:

D	$p(I(x,y)=D)$
0	0.25
128	0.25
255	0.5

   Now, suppose each of these images was blurred using an averaging filter such as `ones(9)/81`. Will the images have the same histogram after this operation? Why?

2. (15 points.) Suppose an image's histogram has two peaks at intensity values 90 and 180. Determine (i.e., give an explicit form for) a linear stretch function $f(D)$ so that the first peak is mapped to the new intensity value of 30, the second peak is mapped to the new intensity value of 210, and all the output intensity values are clipped to the range [0, 255]. Sketch the function $f(D)$. What is the largest intensity that gets mapped to 0, and the smallest intensity that gets mapped to 255?

3. (15 points.) Consider an image I with intensities between 0 and 255 such that the number of pixels having intensity n is equal to $2n+1$. After applying histogram equalization to the image, what new intensity value are pixels with intensity 100 mapped to? For this problem, it's OK to think of the intensity distribution as a continuous function and solve the problem with calculus. Sketch the function that maps input intensities to output intensities. Hint: you should first figure out how many total pixels there must be in the image. You may need to remember/derive the formula for the finite sum $\sum_{n=0}^{N-1} n$.

4. (15 points.) Consider the "mystery image" at `http://www.ecse.rpi.edu/~rjradke/4540/mystery2.png`. It appears to be a bunch of solid rectangles on a grey background. Apply histogram equalization and describe what you see. Then, apply local histogram equalization with an 80×80 neighborhood. This can be accomplished with the `blockproc` command; see `help blockproc` for how to set up and use this function. How do the global and local histogram equalization results differ, and why?

5. (10 points.) Determine the single 2×2 similarity transformation that sequentially (a) rotates the image clockwise by 30 degrees and (b) flips it horizontally (i.e., across the x axis). Here, we assume the transformation is applied in Cartesian coordinates. Will the answer be the same if (a) and (b) are switched?

6. (15 points.) It was determined that a given affine transformation $x' = Ax + b$ maps (1, 1) to (0, 1), (-1, 1) to (-1, 0), and (1, -1) to (2, 0). Compute A and b; show your work! You may need Matlab.

7. (15 points.) Suppose we transform the image described by $I(x,y) = \left\lfloor 255 \exp -\left(\frac{(x-250)^2+(y-250)^2}{10^2}\right)\right\rfloor$ using a given transformation T, so that $T^{-1}(128,\ 64) = (251.7,\ 252.2)$. If we create the new image I' using bilinear interpolation, what is its intensity at (128, 64)? (You'll need a calculator or Matlab for this problem. Note that $\lfloor\cdot\rfloor$ means to round down to the nearest integer. Assume we're using image coordinates, not Cartesian coordinates.)