image restoration and the Wiener filter
improving the visual quality of an image
- lpf/spatial averaging to remove Gaussian noise
- median filtering to remove impulsive or salt & pepper noise
- unsharp masking to enhance details

today: objective methods for undoing corruption

Basic model for degradation

$I(x,y)$ — [degradation PSF H (eg.blur)] — ⊕ — $\hat{I}(x,y)$
noise $\eta(x,y)$

$\hat{I}(x,y)$ — [restoration filter] — $\hat{\hat{I}}(x,y)$

spatial domain: $\hat{I}(x,y) = I(x,y)*h(x,y) + \eta(x,y)$
convolution
frequency dom: $I(u,v)H(u,v) + N(u,v)$

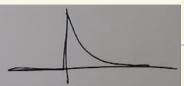
easier case: H = identity (no blur)
degraded image only contain additive noise
noise is typically described by a pmf
$P(\eta(x,y) = z)$ assume i.i.d
noise may be due to:
- non-ideal sensor elements
- environmental conditions (white level, temperature)
- corruption during transmission/compression

common noise PDFs:
- Gaussian $P(z) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$
- Rayleigh (range imaging)

$$P(z) = \begin{cases} \frac{2}{b}(z-a)e^{-(z-a)^2/b} & z \geq a \\ 0 & z < 0 \end{cases}$$

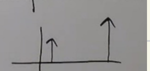
- exponential (laser imaging)

$$P(z) \begin{cases} ae^{-az} & z \geq 0 \\ 0 & z < 0 \end{cases}$$



- salt & pepper

$$P(z) = \begin{cases} P_w & z = w \\ P_b & z = b \\ 0 & \text{ELSE} \end{cases}$$



-uniform

periodic noise (non-iid) manifest as unusual peaks in the FFT; remove these w/ notch filter

How to determine what kind of noise is present?
- one way: find a region should be flat (constant intensity), look at image histogram

$\hat{I}(x,y) = I(x,y) + N(x,y)$ can not just subtract this
$\hat{I}(u,v) = I(u,v) + N(u,v)$

In the iid case, not much we can do beyond what we already know:
- mean filter $\hat{\hat{I}}(x,y) = \frac{1}{n^2} \sum_{(x',y') \in \text{nxn block around } (x,y)} \hat{I}(x',y')$
- median for impulsive noise

better method is adaptive filter:
changes depending on noise characteristics in local window around the pixel
suppose we know
- $\hat{I}(x,y)$
- σ_η^2 noise variance across entire image
- $\hat{\mu}_L$ local mean around window
- $\hat{\sigma}_L^2$ local variance around window

$$\hat{\hat{I}}(x,y) = \hat{I}(x,y) - \frac{\sigma_\eta^2}{\hat{\sigma}_L^2}(\hat{I}(x,y) - \hat{\mu}_L)$$

- if $\sigma_\eta^2 = 0 \Rightarrow \hat{\hat{I}}(x,y) = \hat{I}(x,y)$
- if $\hat{\sigma}_L^2 \gg \sigma_\eta^2 \Rightarrow \hat{\hat{I}}(x,y) \approx \hat{I}(x,y)$

why good? a high local variance means an edge —> preserve edges

- if $\hat{\sigma}_L^2 \approx \sigma_\eta^2 \Rightarrow \hat{\hat{I}}(x,y) \approx \hat{\mu}_L$

average intensities in “normal regions”

we need an estimate of σ_η^2 for this to work (eg. from previous method)

de-bluring process
what happens when we also have degradation

$$\hat{I}(x,y) = I(x,y) * h(x,y) + \eta(x,y)$$

$$\hat{I}(u,v) = I(u,v)\,H(u,v) + N(u,v)$$

how to estimate H(u,v)
- guessing, e.g take a piece of the degraded image and guess what the original image should have looked like

$\hat{I}_s(u,v)$ vs $G_s(u,v)$ $\qquad H(u,v) = \dfrac{\hat{I}_s(u,v)}{G_s(u,v)}$

s = guess area $\qquad$ manually guess

- experimentation (if you have access to the imaging device) - directly acquiring the impulsive response/point spread function

- estimate/model H(u,v) e.g Gaussian blur

we have the degraded image Î(x,y), we have the estimated blur, H(x,y)
Inverse filtering:

$$\hat{\hat{I}}(x,y) = \frac{\hat{I}(u,v)}{H(u,v)}$$ usually is bad why?

$$\hat{\hat{I}}(u,v) = \frac{I(u,v)H(u,v) + N(u,v)}{H(u,v)} = I(u,v) + \frac{N(u,v)}{H(u,v)}$$

if H(u,v) is very small for some (u,v), then $\frac{N(u,v)}{H(u,v)}$ is very large => poor reconstruction

One solution: only apply inverse filter at low freq

$$\hat{\hat{I}} = \begin{cases} \dfrac{\hat{I}(u,v)}{H(u,v)} & \text{if } \sqrt{u^2+v^2} \le r^2 \\ \hat{I}(u,v) & \text{if } \sqrt{u^2+v^2} > r^2 \end{cases}$$

The right thing to do: Wiener filter, minimum mean square error filter

$$e^2 = E\{ | I(x,y) - \hat{\hat{I}}(x,y) |^2 \}$$

the Wiener filter:

$S_F(u,v) = |I(u,v)|^2$

$S_n = |N(u,v)|^2$

$$\hat{\hat{I}}(u,v) = \left[\frac{H^*(u,v)\,S_f(u,v)}{S_f(u,v)\,|H(u,v)|^2 + S_n(u,v)} \right] \hat{I}(u,v)$$

$$= \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_n(u,v)}{S_f(u,v)}} \right] \hat{I}(u,v)$$

we may be able to estimate $S_n(u,v)$, but we don't know $S_F(u,v)$ because requires original image. Instead, we usually use

$$\hat{\hat{I}}(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + K} \right] \hat{I}(u,v)$$

tuning para