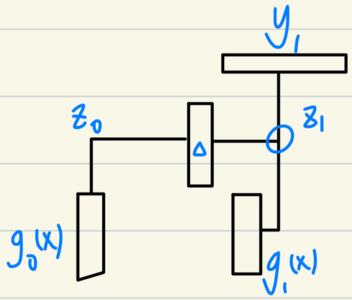
Pipeline :

initial input : $x \in d^{in}$, $d^{in} = 2$ or 3 $\Leftarrow$ 2d or 3d coordinate



$z_0 \to z_1$: $z_1 = g_1(x) \circ (W_1 z_0 + b_1)$

g_1

Δ : $W_1 z_0 + b_1$

$g_0(x) = \sin(w_0 x + \phi_0)$

$g_0 = z_0$

$\underset{d^h \times d^{in}}{\underline{w_0}} \underset{d^{in}}{\underline{x}} + \phi_0 \in d^h$

$w_0 \in [-B_0, B_0]$, if w_1 also $\in [-B_0, B_0]$

$2B_0 = B$ $\Rightarrow$ z_1's bandwidth will be $\leq 2B$

$y_1 = W_1^{out} z_1 + b_1^{out}$

$w_0 \; x$

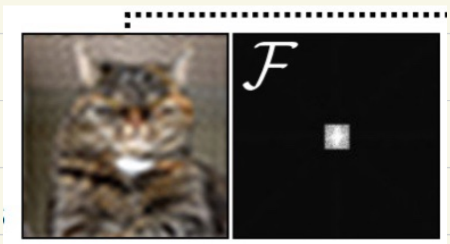
$\quad\;\; d_{in}$

$d_h \begin{bmatrix} \; \end{bmatrix} \underset{x}{[\;]} \; d_{in}$

w_0

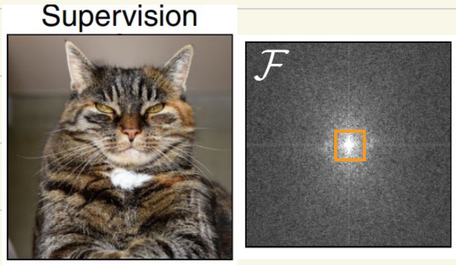
what are entries of ω_i?
random uniform initialization from [-B_0, B_0], or in coding
random.uniform(-B_i, B_i, d_h * d_in)

y_1 and its spectrum



Supervisory

Supervision



learn for each pixel coordinate

Guess_1: the output dimension should be 1, which represents grayscale value [0, 255].
because “the network is characterized entirely by its Fourier specturm”.
We should train in the Fourier frequency domain, and visualize the performance in image domain by inverse Fourier transform.

set manually

$$f_\theta(w_i \mathbf{X}) \xrightarrow{FFT} \tilde{F}(u,v) = \tilde{F} \; - \; F \longleftarrow F(u,v)$$

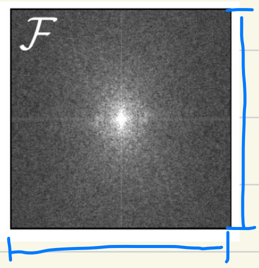
unknown
what we want to train

predicted output

real frequency value by
supervisory image

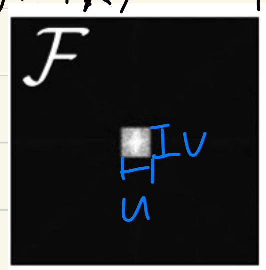
$\mathbf{X} = (x, y)$ $\quad w_i \mathbf{X}$ limit the possible coordinate in frequency domain (u, v) will not appear out of expected frequency domain

$f_\theta(\mathbf{X}) \xrightarrow{FFT} F(u,v)$



(u,v) domain

$f_\theta(w_i \mathbf{X}) \xrightarrow{FFT} F(u,v)$



(u,v) domain

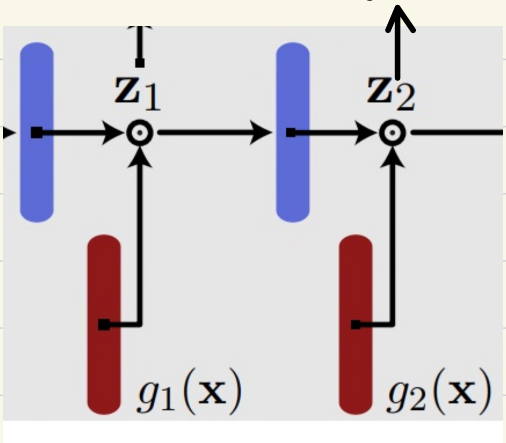
that’s how ω works

$$g_i(\mathbf{X}) = \sin(w_i \mathbf{X} + \phi_i)$$

$$z_i = g_i(\mathbf{X})(W_i z_{i-1} + b_i)$$ — : freeze

$$y_i = W_i^{out} z_i + b_i^{out}$$ — : trainable

$f_\theta(w_i \mathbf{X})$



$$z_2 = g_2(x) \circ \underbrace{(W_2 z_1 + b_2)}_{\leq 2B}$$

$$\downarrow$$

$$\underbrace{\sin(w_2 x + \phi_2)}_{B}$$

$\Rightarrow z_2$ and y_2 $\leq 3B$

frequency

$f_\theta(x) =$ the value of signal (maybe grayscale or RGB, need to be checked)

where x is input coordinate