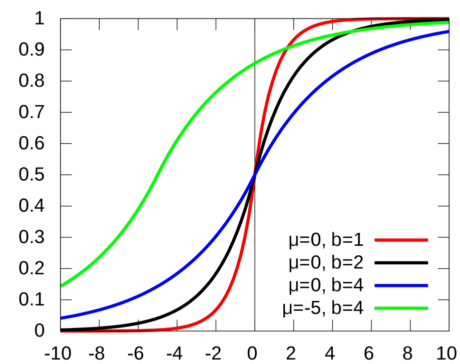
Paper Summary Volume Rendering of Neural Implicit Surfaces

Abstract

Improve geometry representation and reconstruction in neural volume rendering. We achieve that by **modeling the volume density as a function of geometry**. This is in contrast to previous work modeling the geometry as a function of the volume density.

Define the volume density function as **Laplace's CDF applied to a signed distance function** representation.



$$\text{Laplace CDF} = \begin{cases} \frac{1}{2}exp(\frac{x-\mu}{b}) & \text{if } x \leq \mu \\ 1 - \frac{1}{2}exp(\frac{x-\mu}{b}) & \text{if } x \geq \mu \end{cases}$$

where we should treat the value of signed distance function as x in this CDF. Three benefits for this representation:

1. Providing a useful inductive bias to the geometry learned in the neural volume rendering process.
2. Facilitating a bound on the opacity approximation error, leading to an accurate sampling of the viewing ray.
3. Allowing efficient unsupervised disentanglement of shape and appearance in volume rendering.

Understanding for disentanglement:

1. Introduction

Recall the volume rendering integral:

$$C(r) = \int_{t_n}^{t_f} T(t)\sigma(r(t))c(r(t),d)dt$$

Motivation: the density part $\sigma(\mathbf{x})$ is not as successful in faithfully predicting the scene's actual geometry, often producing noisy, low fidelity geometry approximation.

We propose VolSDF. The **key idea** is representing the density $\sigma(x)$ as a function of the signed distance to the scene's surface.

Benefits:

1. Guarantees the existence of a well-defined surface that generates the density.
2. Bound the approximation error of the opacity along rays when sampling the viewing ray.

2. Related Work

a. Neural Scene Representation & Rendering: It is non-trivial to find a proper threshold to **extract surfaces** from the predicted density.

3. Method

a. Density as transformed SDF

$$\mathbf{1}_\Omega(\mathbf{x}) = \begin{cases} 1 & \text{if } x \in \Omega \\ 0 & \text{if } x \text{ } not \in \Omega \end{cases}$$

and $d_\Omega(\mathbf{x}) = (-1)^{1_\Omega(x)} \min_{y \in M} ||\mathbf{x} - \mathbf{y}||$

$$\sigma(x) = \alpha \Psi_\beta(-d_\Omega(x))$$

where

$$\Psi_\beta(s) = \begin{cases} \frac{1}{2} exp(\frac{s}{\beta}) & \text{if } s \leq 0 \\ 1 - \frac{1}{2} exp(\frac{-s}{\beta}) & \text{if } x > 0 \end{cases}$$

$\alpha, \beta > 0$ are learnable parameters, and Ψ_β is the CDF of Laplace distribution with zero mean and β scale. A constant density α smoothly decreases near the boundary, where the smoothing amount is controlled by β. The benefit of defining the density in this way is:

1. Providing a principled way to reconstruct the surface, i.e., as the zero level set of d_Ω.
2. Bounding error.

3.2 Volume Rendering of σ

Volume rendering is all about approximating the integrated (**summed**) light radiance along this ray reaching the camera. Two important quantity in computation: **volume's *opacity*** O, or equivalently, its *transperancy* T, and **radiance field** L, (or $c(r(t), d)$ in NeRF).

$$T(t) = exp(-\int_0^t \sigma(\mathbf{x}(s))ds)$$

and the *opacity* O is the complement probability

$$O(t) = 1 - T(t)$$

Note that O is a monotonic increasing function where $O(0) = 0$ and assuming that every ray is eventually occulded $O(\infty) = 1$. Then we can think of O as a CDF, and

$$\tau(t) = \frac{dO}{dt}(t) = \sigma(\mathbf{x}(t))T(t)$$

is its PDF. The volume rendering equation is the expected light along the ray, **Used in this Paper**

$$I(\mathbf{c},\mathbf{v}) = \int_0^\infty L(\mathbf{x}(t),\mathbf{n}(t),\mathbf{v})\tau(t)dt$$

where L is the radiance field. Comparing what I know from NeRF:

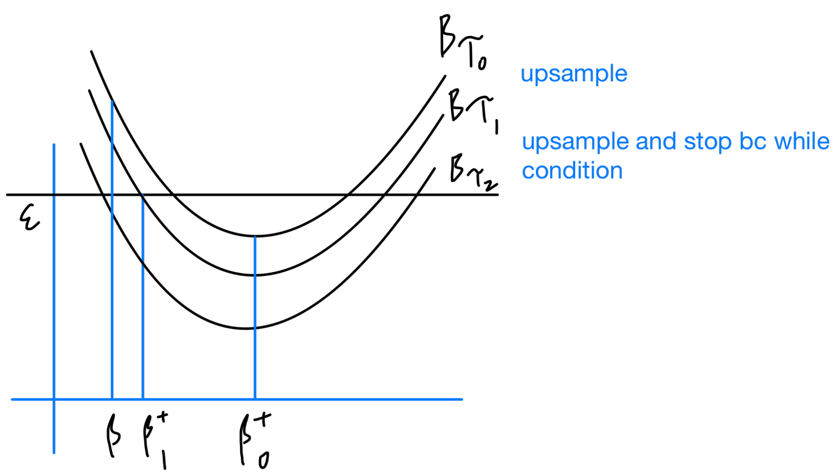
$$C(r) = \int_{t_n}^{t_f} T(t)\sigma(r(t))\mathbf{c}(\mathbf{r}(t),\mathbf{d})dt$$

where $\mathbf{c}(\mathbf{r}(t),\mathbf{d})$ is the radiance field

$I(\mathbf{c},\mathbf{v})$ and $\mathbf{C}(\mathbf{r})$ are actually the same expressions, just using different name.

3.4 Sampling Algorithm

In a nutshell, we start with a uniform sample $T = T_0$, and use Lemma 2 to initially set a $\beta_+ > \beta$ that satisfies $B_{T,\beta_+} \leq \varepsilon$. Then, we repeatedly upsample T to reduce β_+ while maintaining $B_{T,\beta_+} \leq \varepsilon$. The explicit method is shown in below figure.



3.5 Training

Also positional encoding for the position $\mathbf{x}$ and view direction $\mathbf{v}$ in the geometry and radiance field.