Paper Summary Optical Models for Direct Volume Rendering

Abstract

Reviewing several different models for light intersection with volume densities of **absorbing**, **glowing**, **reflection**, and/or **scattering material**. In order of increasing realism,

2. absorption only

3. emission only

4. emission and absorption combined

5 6. single scattering of external illumination without shadows, single scattering with shadows

7. multiple scattering.

1. Introduction

Direct volume rendering refers to techniques which produce a projected image directly from the volume data, without intermediate constructs such as contour surface polygons. These techniques require some model of how the data volume **generates**, **reflects**, **scatter**s, or **occludes light**. This paper presents a seq of such optical models with **increasing degrees of physical realism**, which can **bring out different features of data**.

Assuming the interpolation is done somehow to give a scalar function $f(X)$ defined for all points X in the volume. Optical properties like **color** and **opacity** can then be assigned as functions of the interpolated value $f(X)$. The optical properties which affect passing through a "particular medium" are due to the absorption, scattering, or emission of light from small particles like water droplets, soot or other suspended solids, or individual molecules in the medium.

Author will write the equations taking the intensity and optical properties to be scalars, for black-and-white images. For multiple wavelength bands in a color image, the equations are repeated for each wavelength, so these quantities become vectors.

2. Absorption Only

Assume that the particles are identical spheres, of radius r and projected area $A = \pi r^2$, let ρ be the number of particles per unit volume. Consider a small cylindrical slab with a base B of area E, and thickness $\triangle s$, shown in figure 1.

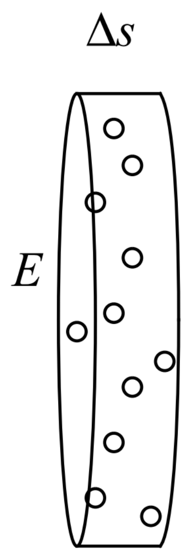

Fig. 1. A slab of base area E and thickness Δs.

with the light flowing along the direction $\triangle s$, perpendicular to the base. The slab has the volume $E\triangle s$, thus contains $N = \rho E\triangle s$ particles. If $\triangle s$ is small enough so that the particles projections on the base B have low probability of overlap, the total area that they occlude on B is approximated by $NA = \rho AE\triangle s$. Thus the fraction of the light flowing through B that is occluded is $\rho AE\triangle s/E = \rho A\triangle s$. This gives the DE:

$$\frac{dI}{ds} = \rho(s)AI(s) = -\tau(s)I(s)$$

where s is a length parameter along a ray in the direction of the light flow, and $I(s)$ **is the light intensity at distance** s. The quantity $\tau(s) = \rho(s)A$ is called the **extinction coef** and defines the rate that light is occluded. The solution to this DE is:

$$I(s) = I_0\ exp(-\int_0^s \tau(t)dt)$$

where I_0 is the intensity at $s = 0$, where the ray enters the volume. The quantity

$$T(s) = exp(-\int_0^s \tau(t)dt)$$

is the transparency of the medium between 0 and s. In the volume rendering literature the **extinction coefficient** τ is often simply called opacity. However, the **opacity** α **of a voxel of side** l, viewed parallel to one edge is actually

$$\alpha = 1 - T(l) = 1 - exp(-\int_0^l \tau(t)dt)$$

I_0 can represent a background intensity, varying from pixel to pixel, and resulting image represents the volume density as a cloud of black smoke obscuring the background.

3. Emission Only

In this section, we will model this case by assuming the small spherical particles discussed above transparent, and in next section we will include their absorption. If the particles in fig.1 are transparent, but glow diffusely with an **intensity C per unit projected area**, their projected area $\rho AE\triangle s$ derived above will contribute a glow flux $C\rho AE\triangle s$ to the base area E, for an added flux per unit area $C\rho A\triangle s$. Thus the DE for $I(s)$ is:

$$\frac{dI}{ds} = C(s)\rho(s)A = C(s)\tau(s) = g(s)$$

The term $g(s)$ is called the source term. Later, we will let it include reflection as well as emission. The solution to this DE is simply:

$$I(s) = I_0 + \int_0^s g(t)dt$$

Fig.3 shows the clouds of fig.2 drawn in this way, with g proportional to f.

4. Absorption Plus Emission

The particles in an actual cloud occlude incoming light, as well as add their own glow. Thus a realistic DE should include both source and attenuation terms:

$$\frac{dI}{ds} = g(s) - \tau(s)I(s)$$

Integrating from $s = 0$ at the edge of the volume to $s = D$ at the eye,we get:

$$I(D) = I_0\ exp(-\int_0^D \tau(t)dt) + \int_0^D g(s)exp(-\int_s^D \tau(t)dt)ds$$

The first term represents the light coming from the background, multiplied by the cloud's transparency. The second term is the integral for the contribution of the source term g(s) at each position s, multiplied by the transparency $T'(s) = exp(\int_s^D \tau(s)dx)$ between s and the eye. Thus,

$$I(D) = I_0T(D) + \int_0^D g(s)T'(s)ds$$

4.1 Calculation Methods: Using Riemann Sum

straightforward, omit.

4.2 The Particle Model $(g(s) = C(s)\tau(s))$

Makes the second integral in $I(D)$ much simpler:

$$\int_0^D g(s)exp(-\int_s^D \tau(t)dt)ds = C(1 - exp(-\int_0^D \tau(t)dt))$$

Substitute in expression $I(D)$ and using the total transparency $T(D)$ from equation (3), we get:

$$I(D) = I_0 T(D) + C(1 - T(D))$$

This is the simple compositing of the color C on top of the background I_0, using the transparency $T(D)$. Conceptually, the opacity $\alpha = (1. - T(D))$ represents the **probability that a ray from the eye will hit a particle, and "see" color C**.

5. Scattering and Shading

Include scattering of illumination external to the voxel. The external illumination is assumed to reach the voxel from a distant source, unimpeded by any intervening objects or volume absorption. We will consider this case first, and deal with shadows in the next section. A **general shading rule for the scattered light** $S(X, \omega)$ at position X in direction ω is:

$$S(X, \omega) = r(X, \omega, \omega')i(X, \omega')$$

where $i(X, \omega')$ is the incoming illumination reaching X flowing in direction ω', and $r(X, \omega, \omega')$ is the **bidirectional reflection distribution function**, which depends on the **direction** ω of the **reflected light**, the **direction** ω' of the **incoming light**, and on other properties like f or its gradient that vary with position X. For **light scattered** by a density of particles.

$$r(X, \omega, \omega') = a(X)\tau(X)p(\omega, \omega')$$

where $a(X)$ is the particle albedo, giving the fraction of the extinction τ which represents scattering rather than absorption, and p is the phase function, which specifies the directionality of the scattering. A common formula for p is:

$$p(\omega, \omega') = \frac{1}{4\pi} \frac{1 - c^2}{(1 + c^2 - 2cx)^{3/2}}$$

where c is an adjustable constant between -1 and 1, positive for **forward** scattering, negative for **backward** scattering, and **zero** for isotropic scattering (equal in all directions). An simpler formula by Blinn can be derived by geometric optics for a spherical particle much larger than the light wavelength, whose surface scatters diffusely by Lambert's law:

$$p(\omega, \omega') = (8/3\pi)(sin(\alpha) + (\pi - \alpha)cos(\alpha))$$

The most general source term $g(X, \omega)$ is the sum of a non-directional internal glow or emissivity $E(X)$ as in section 3, and reflection or scattering term $S(X, \omega)$ of this section: **eq12**

$$g(X, \omega) = E(X) + S(X, \omega)$$

6. Shadows

If L is the intensity from an infinite light source in the direction $-\omega'$, the **illumination** $i(X,\omega')$ which reaches X is:

$$i(X,\omega') = L\, exp(-\int_0^\infty \tau(X - t\omega')dt)$$

New meaning of the parameter s in old version equation: it starts at a viewpoint X and goes out in direction $-\omega$ opposite to the light flow, reaching $X - s\omega$ at distance s. Rewriting with this reversed ray parametrization, $s' = D - s$, $t' = D - t$, we have: **eq14**

$$I(X) = I_0\, exp(-\int_0^D \tau(X-t\omega)dt) + \int_0^D r(X-s\omega,\omega,\omega')\, L\, exp(-\int_0^\infty \tau(X-sw-t\omega')dt)\; exp(-\int_0^s \tau(X-t\omega)dt)ds$$

The factor $exp(-\int_0^\infty \tau(X - sw - t\omega')dt)$ corresponds to the "shadow feelers" used in recursive ray tracing.

7. Multiple Scattering

To correct unnaturally dark look in fig.6 and account for multiple scattering, one may apply the **"radiosity"** methods.

Emphasis again: 1. $I(X,\omega)$ is the intensity at each point X in each light flow direction ω

2. The point at distance s along the viewing ray from X, opposite to the light flow is $X - s\omega$

Integrating the scattering at $X - s\omega$ of light from all possible incoming directions ω' on the 4π unit sphere, the added scattered intensity gives the source term:

$$g(s,\omega) = \int_{4\pi} r(X - s\omega, \omega, \omega')I(X - s\omega, \omega')d\omega'$$

Substituting this into **eq14** gives: **eq15**

$$\begin{aligned} I(X,\omega) &= I_0(X - D\omega, \omega)\; exp(-\int_0^D \tau(X - t\omega)dt) \\ &+ \int_0^D \left(\int_{4\pi} r(X - s\omega, \omega, \omega')I(X - s\omega, \omega')d\omega'\right)\; exp(-\int_0^s \tau(X - t\omega)dt)ds \end{aligned}$$

where $X - D\omega$ is the point at the edge of the volume density, reached by the ray from X in direction $-\omega$, and $I_0(X - D\omega, \omega)$ is the external illumination there flowing in direction ω.

7.1 The Zonal Method

Zonal method assume if the scattering is isotropic, so that $g(X,\omega)$ depends only on X, then $g(X,\omega)$ will change to $g(X)$, and assume that $g(X)$ is **piecewise constant** on volume elements. The **total contribution** of all voxels X_j to the **isotropic scattering** $S(X_i)$ at X_i is:

$$S(X_i) = a(X_i) \sum_j F_{ij}\, g(X_j)$$

where the "form factor" F_{ij} represents the fraction of the flux originating at voxel X_j that is intercepted by voxel X_i, and the albedo $a(X_i)$ is the portion of this intercepted flux that is scattered. Using **eq12**, this then gives a system of simultaneous linear equations for the unknowns of $g(X_i)$: **eq16**

$$g(X_i) = E(X_i) + a(X_i) \sum_j F_{ij}\, g(X_j)$$

7.2 The Monte Carlo Method

7.3 The P-N Method

7.4 The Discrete Ordinates Method