Paper Summary BACON

Abstract

BACON: Band-limited Coordinate Networks for Multiscale Scene Representation. Coordinate-based network are trained to map continuous input coordinate to the value of a signal at each point. **Training at a single scale** will result in artifacts when naive downsampling and upsampling. BACON can be designed based on the **spectral characteristic of the represented signal** at unsupervised signal. Demonstrate BACON for

1. Multiscale neural representation of images.
2. radiance fields.
3. 3D scenes using **signed distance functions**

1. Introduction

Neural representations approximate signals using a continuous function that is embedded in the learned weights of a fully-connected NN. Since it is designed to represent signals at a single scale, the behavior of the NN at unsupervised coordinate is difficult to predict.

The **key properties** of BACON architecture are:

1. the maximum frequency at each layer can be manipulated analytically.
2. The behavior of a trained network is entirely characterized by its **Fourier spectrum**.

Contribution:

1. Introducing BACON for representing and optimizing
2. Developing methods for spectral analysis of the architecture and proposing initialization scheme

BACON is suited to **multiscale signal representation** because band-limited output layers can be designed with an inductive bias towards a particular resolution or scale.

2. Related Work

Architectures for Scene Representation

NN architectures for scene representation networks can be classified as **feature-based**, **coordinate-based**, and **hybrid**.

Feature-based: quickly evaluated, but have a large memory footprint.

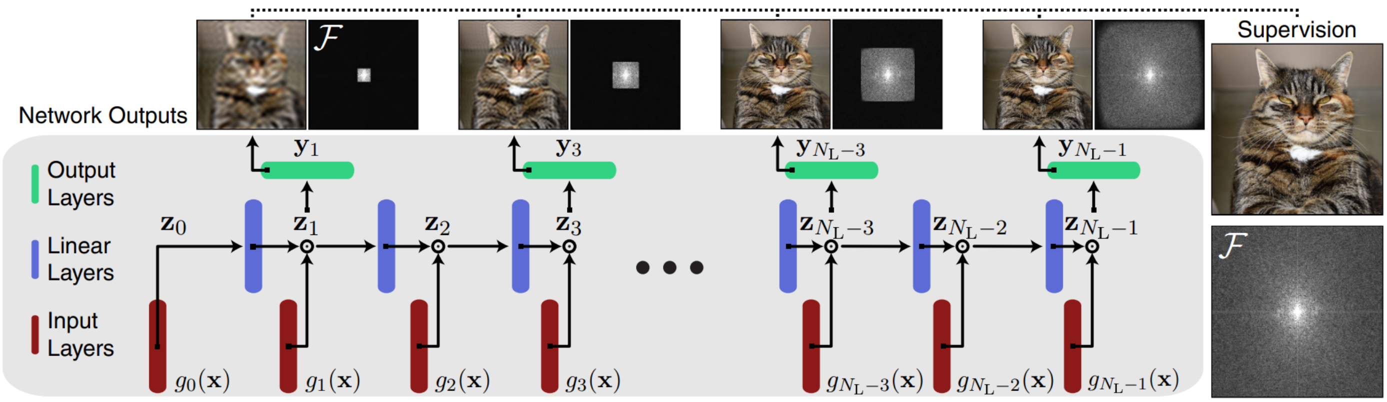
Coordinate-based: Map from an input coordinate to a signal value, and the proposed one in this paper is coordinate-based. Didn't use MLP but **multiplicative filter networks**(MFNs, recently proposed).

3. Method

3.1 Band-limited Coordinate Networks

MLP employ a Hadamard product b/t linear layers and sine activation functions. Extending the theoretical understanding and practicality of MFN by:

1. Architecture change to achieve multi-scale, band-limited outputs
2. Deriving formulas to quantify the expected frequencies in the representation
3. Deriving initialization scheme preventing vanishing activations in deep networks



$X \sim U(-0.5, 0.5)$

$g_0(x) = \sin(W_0 x + \phi_0)$

$z_0 = g_0(x)$

$g_1(x) = \sin(W_1 x + \phi_1)$

$z_1 = g_1(x) \circ (W_1 z_0 + b_1)$ linear layer

$y_1 = W_1^{out} z_1 + b_1^{out}$

$g_2(x) = \sin(W_2 x + \phi_2)$

$z_2 = g_2(x) \circ (W_2 z_1 + b_2)$

$y_2 = W_2^{out} z_2 + b_2^{out}$

All these intermediate steps can be transferred to two single expressions:

$$y_k = \sum_{j=0}^{N_{sine}^{(k)}-1} \bar{\alpha}_j \, sin(\bar{\omega}_j x + \bar{\phi}_j)$$

where $N_{sine}^{(k)} =$:

$$N_{sine}^{(k)} = \sum_{i=0}^{k-1} 2^i d_h^{i+1}$$

3.2 Frequency Spectrum

Designing the architecture so that the frequency of all represented sines never exceeds a desired threshold. Setting ω_i's to a bandwidth in $[-B_i, B_i]$ using random uniform initialization.

Introducing linear layers at intermediate stages throughout the network to extract band-limited outputs.

Because the outputs are band-limited, BACON can be trained in a semi-supervised fashion where the bandwidth of the **supervisory signal** need **not match** the **desired bandwidth** of the output of the network.

$*$ Products of sine result in summed frequencies. Why do we need this? Recall:

$$\begin{aligned} sin(a)sin(b) &= \frac{1}{2}(sin(a+b-\frac{\pi}{2}) + sin(a-b+\frac{\pi}{2})) \\ &= \frac{1}{2}(cos(a-b) - cos(a+b)) \end{aligned}$$

suppose $a, b \in [-\pi, \pi] \Rightarrow a-b, a+b \in [-2\pi, 2\pi] \Rightarrow$ summed freq

$$y_1 = W_1^{out} z_1 + b_1^{out}$$

$$z_1 = g_1(x) \circ (W_2 z_0 + b_1)$$

$$= \sin(\underbrace{w_1 x + \phi_1}_{d^h}) \circ (\underbrace{W_2 \sin(w_0 x + \phi_0) + b_1}_{d^h})$$

$$= \sin(\ )\sin(\ ) \in d^h$$

So suppose the bandwidth of w_0 is $[-B_0, B_0]$, and w_1 is also $[-B_0, B_0]$, then bandwidth of z_1 is $[-2B_0, 2B_0]$

3.3 Initialization Scheme

Assume the input to the network is uniformly distributed $x \sim U(-0.5, 0.5)$ with $\omega_i \sim U(-B_i, B_i)$ and $\phi_i \sim U(-\pi, \pi)$. Then, $\omega_i x + \phi_i$ is distributed uniformly as:

$$\begin{cases} 1/B_i \; log(B_i/min(|2x|, B_i)), & -B/2 \leq x \leq B/2 \\ 0 & \text{else} \end{cases}$$

$g_0(x)$ will be distributed uniformly, and $g_i(x)$ will be approximately arcsine distributed with variance of 0.5. Now let $W_i \sim U[-\sqrt{6/d_h}, \sqrt{6/d_h}]$. Then we have that $W_1 g_0(x) + b_1$ converges to std normal distribution with increasing d_h. Finally the Hadamard product $g_1(x) \circ (W_1 z_0 + b_1)$ is the product of **arcsine**

distributed and std normal random variable, which again has a variance of 0.5. Applying the next linear layers results in another standard normal distribution, which is also the case after all subsequent linear layers.

4. Experiments for **Neural Radiance Field** (4.2)

Adaption for BACON:

1. Setting the maximum bandwidth B to 64 cycles per unit interval allows fitting high frequency image detail.
2. Evaluate all methods without the viewing direction input originally used for NeRF => the input to all networks is a **3D coordinate corresponding to the position along the ray** $r(t)$.
3. BACON produces 4 outputs using B_i to constrain each output to $1/8$, $1/4$, $1/2$, and full resolution.
4. Hierarchical Sampling procedure of NeRF: the alpha compositing weights ω_j from an initial forward pass are used to resample the ray in regions of non-zero-opacity. To improve efficiency, using the lowest-resolution output of the network for this initial forward pass.

In BACON, y_i is actually the output which we can treat it as $C(r)$ in NeRF.