$$\frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial z^2}$$

We can write the T as a function of a unitless variable y:

$$T = f(y)$$
$$y = \frac{z}{\sqrt{K_T t}}$$

Now, we can further write the partial differential equation, where:

$$\frac{\partial T}{\partial t} = \frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial t} = -\frac{z}{2t\sqrt{t}}\left(\frac{\partial T}{\partial y}\right) = -\frac{y}{2t}\left(\frac{\partial T}{\partial y}\right)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial z}\right) = \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial y} \cdot \frac{1}{\sqrt{K_T t}}\right) = \frac{1}{\sqrt{K_T t}} \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial y}\right) = \frac{1}{\sqrt{K_T t}} \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y} \cdot \frac{\partial y}{\partial z}\right)$$
$$= \frac{1}{\sqrt{K_T t}} \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y} \cdot \frac{1}{\sqrt{K_T t}}\right) = \frac{1}{K_T t}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

According to the heat flow equation:

$$\frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial z^2}$$

We can have:

$$-\frac{y}{2t}\left(\frac{\partial T}{\partial y}\right) = \frac{1}{t}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

$$-\frac{y}{2}\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right) = \left(\frac{\mathrm{d}^2 T}{\mathrm{d}y^2}\right)$$

Assume that $\frac{\mathrm{d}T}{\mathrm{d}y} = p$, this will output an ODE:

$$-\frac{y}{2}p = \frac{\mathrm{d}p}{\mathrm{d}y}$$

$$\frac{\mathrm{d}p}{p} = -\frac{y}{2}\mathrm{d}y$$

We can integral both sides of the upper equation:

$$\ln p = -\frac{y^2}{4} + \ln A$$

$$p = A\exp\left(\frac{-y^2}{4}\right)$$

Then, we can derive the function $T(y)$:

$$T(y) = \int \quad A\exp\left(\frac{-y^2}{4}\right)\mathrm{d}y + C$$

The upper integration gives an output of error function:

$$T(y) = A\,\mathrm{erf}\left(-\frac{y}{2}\right) + C = A\int_0^{-\frac{y}{2}} \exp(-u^2)\,\mathrm{d}u + C \quad (y < 0)$$

$$T(z,t) = A\,\mathrm{erf}\left(-\frac{z}{2\sqrt{K_T t}}\right) + C = A\int_0^{-\frac{z}{2\sqrt{K_T t}}} \exp(-u^2)\,\mathrm{d}u + C$$

Given the condition that at $z = 0, T = T_0$, and $z = -\infty, T = 0$, we can have:

$$\begin{cases} C = T_0 \\ A = -T_0 \end{cases}$$

Thus, we can write the function as:

$$T(z,t) = -T_0\,\mathrm{erf}\left(-\frac{z}{2\sqrt{K_T t}}\right) + T_0 = T_0\,\mathrm{erfc}\left(-\frac{z}{2\sqrt{K_T t}}\right)$$