BME357/BME381J/EE385J Learning Opportunity #2 Due 21 October 2019

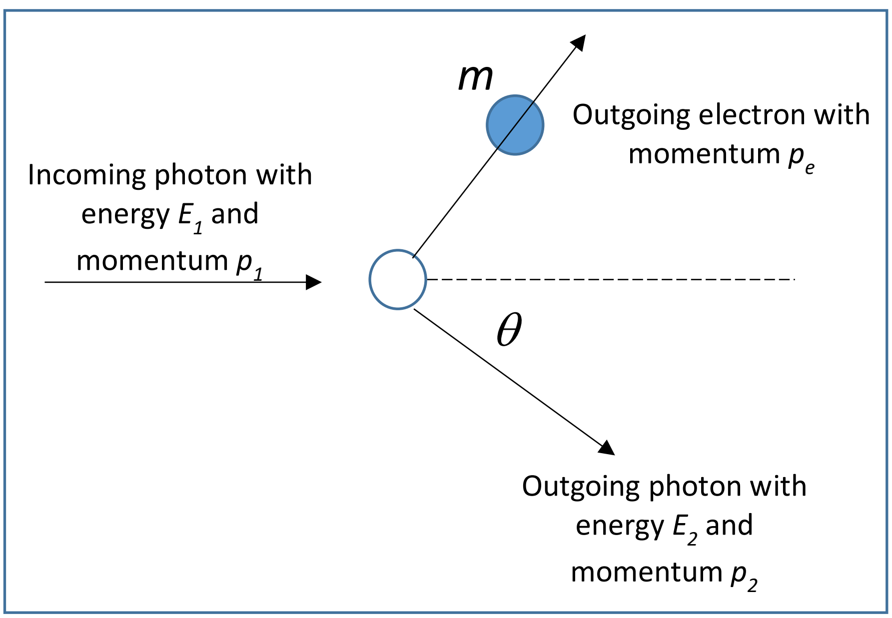
1) Let $f(t)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(\frac{-t^2}{\sigma^2}\right)$. Compute the Fourier transform of $f(t)$ where σ is such that the integral of $f(t)$ is 1. Interpret your answer in a sentence or two.

2) Let $f(t)=\left\{\begin{array}{c}\exp(+\alpha t), t<0 \\ \exp(-\alpha t), t>0\end{array}\right\}$. Compute the Fourier transform of $f(t)$. Interpret your answer in a sentence or two.

3) (Derivation of the Compton equation—grad students only) Let λ_1 and λ_2 and E_1 and E_2 be the wavelengths and energies of the incident and scattered x-rays, respectively. Use the laws of conservation of momentum and energy (and that $E = mc^2$) to show that

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1-\cos\theta) \quad ,$$

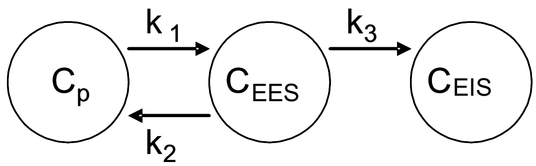
where h is Plank's constant, m is the mass of the electron, and θ is the scattering angle of the photon. Remember that momentum is a vector quantity and feel free (indeed, feel encouraged) to remind yourself what conservation of momentum and energy means from other sources. The figure below might be of use.



<u>Hint:</u> The energy of the electron before the collision is $E = mc^2$ (i.e., it's rest energy). After the collision, the energy of the electron is $\sqrt{E_2 + p_e^2c^2}$.

*(**Note:** More learning opportunities on the next page!)*

4. Refer to the figure below for this fun learning opportunity. FDG goes from the blood plasma space (with concentration C_p) to the extravascular extracellular space (with concentration C_{EES}), and then into the extravascular intracellular space where it is phosphorylated (with concentration C_{EIS}) and trapped. a) Write down the set of differential equations that describes this process. b) Now simplify this process by assuming that the extravascular space is just one compartment and not two distinct ones; write down and solve the differential equation that describes this process. Assume $C_{EES}(t = 0) = 0$. c) How can your equation be used in an FDG-PET study?



5. a) Starting from $\phi(x,t) = f(x-ct)$, derive the wave equation by differentiating appropriately. Describe, briefly, what $\phi(x,t) = f(x-ct)$ is.

b) Show that $\phi(x,t) = A\exp[i(\omega t \pm kx)]$ is a solution of the wave equation.

6. Consider an acoustic wave encountering a fat/liver interface at normal incidence. What fraction of acoustic intensity (not pressure) is reflected back from the fat/liver interface at normal incidence? (You can take the acoustic impedances of fat and liver to be 1.35×10^6 kg m^{-2} s^{-1} and 1.66×10^6 kg m^{-2} s^{-1}, respectively.) Does it matter from which direction the incident energy arrives? Why or why not?

7. a) Suppose a 5 MHz acoustic pulse travels from a transducer through 2 cm of fat, then encounters an interface with the liver at normal incidence. At what time after the transmitted pulse will the reflected pulse (i.e., the echo) arrive back at the transducer?

b) Taking both attenuation and reflection losses into account, what will be the amplitude loss (as a fraction of the original amplitude, A_0) of the returning waveform?