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1. Prove the convolution theorem

$$\begin{cases} F(k) = \mathcal{F}(f(x))(k) = \int_{R^n} f(x) e^{-2\pi i x\cdot k} dx \\ G(k) = \mathcal{F}(g(x))(k) = \int_{R^n} g(x) e^{-2\pi i x\cdot k} dx \end{cases}$$

$$h(x) = f(x) * g(x) = \int_{R^n} f(z) g(x-z) dz$$

$$\Rightarrow h(z) = \int_{R^n} f(x) g(z-x) dx$$

$$\iint |f(x) g(z-x)| dz\, dx = \int \left(|f(x)| \int |g(z-x)| dz \right) dx = \int |f(x)|\, \|g\|_1 dx$$

$$= \|f\|_1 \|g\|_1$$

By Fubini's theorem, $h \in L^1(\mathbb{R}^n)$, so

$$H(k) = \mathcal{F}(h) = \int_{R^n} h(z) e^{-2\pi i z\cdot k} dz$$

$$= \int_{R^n} \int_{R^n} f(x) g(z-x) dx\, e^{-2\pi i z\cdot k} dz$$

$$= \int_{R^n} f(x) \left(\int_{R^n} g(z-x) e^{-2\pi i z\cdot k} dz \right) dx$$

Substituting $y = z - x$ yields $dy = dz$

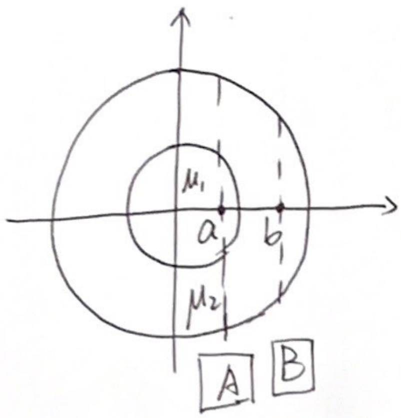
$$\Rightarrow H(k) = \int_{R^n} f(x) \left(\int_{R^n} g(y) e^{-2\pi i (y+x)\cdot k} dy \right) dx$$

$$= \int_{R^n} f(x) e^{-2\pi i x\cdot k} dx \int_{R^n} g(y) e^{-2\pi i y\cdot k} dy$$

$$= F(k)\, G(k)$$

QED

2.



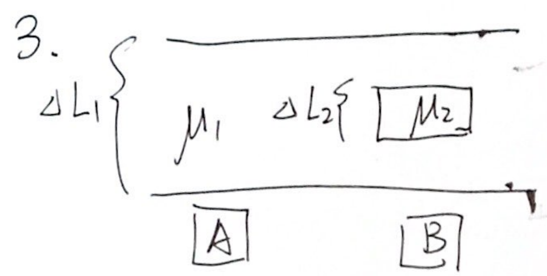
$$N_A = N_0 e^{-\mu_1 \cdot 2\sqrt{r^2-a^2}}\, e^{-\mu_2 \cdot 2(\sqrt{R^2-a^2}-\sqrt{r^2-a^2})}$$

$$= 2\mu_1\sqrt{r^2-a^2} + 2\mu_2\left(\sqrt{R^2-a^2}-\sqrt{r^2-a^2}\right)$$

$$N_B = N_0 e^{-\mu_2 \cdot 2\sqrt{R^2-b^2}}$$

$$= 2\mu_2\sqrt{R^2-b^2}$$

3.



$$N_A = N_0 e^{-\mu_1 \Delta L_1}$$

$$N_B = N_0 e^{-\mu_1(\Delta L_1 - \Delta L_2)} e^{-\mu_2 \Delta L_2}$$

$$1 - N_A/N_B = 1 - \frac{e^{-\mu_1 \Delta L_1}}{e^{-\mu_1(\Delta L_1 - \Delta L_2)} \cdot e^{-\mu_2 \Delta L_2}}$$

$$= 1 - \frac{1}{e^{\mu_1 \Delta L_2} \cdot e^{-\mu_2 \Delta L_2}}$$

$$= \frac{e^{(\mu_1 - \mu_2)\Delta L_2} - 1}{e^{(\mu_1 - \mu_2)\Delta L_2}}$$

$$\approx \frac{1 - (\mu_1 - \mu_2)\Delta L_2 - 1}{1 - (\mu_1 - \mu_2)\Delta L_2}$$

$$= \frac{-(\mu_1 - \mu_2)\Delta L_2}{1 - (\mu_1 - \mu_2)\Delta L_2}$$

$1 - \frac{N_A}{N_B}$ is proportional to $(\mu_1 - \mu_2)$ and ΔL_2.