

# HW 1 Yifu Huang yh9692

1. Prove the convolution theorem

$$\begin{cases} F(k) = \mathcal{F}(f(x))(k) = \int_{\mathbb{R}^n} f(x) e^{-2\pi i x \cdot k} dx \\ G(k) = \mathcal{F}(g(x))(k) = \int_{\mathbb{R}^n} g(x) e^{-2\pi i x \cdot k} dx \end{cases}$$

$$h(x) = f(x) * g(x) = \int_{\mathbb{R}^n} f(z) g(x-z) dz$$

$$\Rightarrow h(z) = \int_{\mathbb{R}^n} f(x) g(z-x) dx$$

$$\int \int |f(x) g(z-x)| dz dx = \int (|f(x)| \int |g(z-x)| dz) dx = \int |f(x)| \|g\|_1 dx$$

$$= \|f\|_1 \|g\|_1$$

By Fubini's theorem,  $h \in L^1(\mathbb{R}^n)$ , so

$$H(k) = \mathcal{F}(h) = \int_{\mathbb{R}^n} h(z) e^{-2\pi i z \cdot k} dz$$

$$= \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} f(x) g(z-x) dx e^{-2\pi i z \cdot k} dz$$

$$= \int_{\mathbb{R}^n} f(x) \left( \int_{\mathbb{R}^n} g(z-x) e^{-2\pi i z \cdot k} dz \right) dx$$

Substituting  $y = z - x$  yields  $dy = dz$

$$\Rightarrow H(k) = \int_{\mathbb{R}^n} f(x) \left( \int_{\mathbb{R}^n} g(y) e^{-2\pi i (y+x) \cdot k} dy \right) dx$$

$$= \int_{\mathbb{R}^n} f(x) e^{-2\pi i x \cdot k} dx \int_{\mathbb{R}^n} g(y) e^{-2\pi i y \cdot k} dy$$

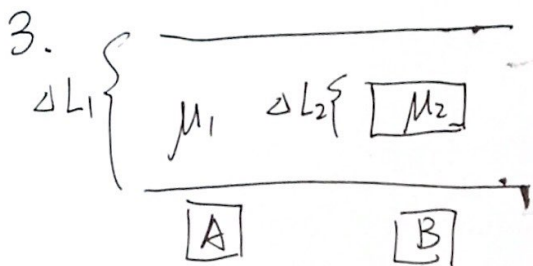
$$= F(k) G(k)$$

QED

A diagram showing two concentric circles centered at the origin of a Cartesian coordinate system. The inner circle has radius  $a$  and is labeled with  $\mu_1$ . The outer circle has radius  $b$  and is labeled with  $\mu_2$ . Below the circles are two boxes labeled 'A' and 'B'.

$$N_B = N_0 e^{-\mu_2 \cdot 2\sqrt{R^2 - b^2}}$$

$$= 2\mu_2 \sqrt{R^2 - b^2}$$



$$N_A = N_0 e^{-\mu_1 \Delta L_1}$$

$$N_B = N_0 e^{-\mu_1 (\Delta L_1 - \Delta L_2)} e^{-\mu_2 \Delta L_2}$$

$$1 - N_A/N_B = 1 - \frac{e^{-\mu_1 \Delta L_1}}{e^{\mu_1 (\Delta L_1 - \Delta L_2)} e^{-\mu_2 \Delta L_2}}$$

$$= 1 - \frac{1}{e^{\mu_1 \Delta L_2} \cdot e^{-\mu_2 \Delta L_2}}$$

$$= \frac{e^{(\mu_1 - \mu_2) \Delta L_2} - 1}{e^{(\mu_1 - \mu_2) \Delta L_2}}$$

$$\approx \frac{1 - (\mu_1 - \mu_2) \Delta L_2 - 1}{1 - (\mu_1 - \mu_2) \Delta L_2}$$

$$= \frac{-(\mu_1 - \mu_2) \Delta L_2}{1 - (\mu_1 - \mu_2) \Delta L_2}$$

$1 - \frac{N_A}{N_B}$  is proportional to  $(\mu_1 - \mu_2)$  and  $\Delta L_2$ .