BME357/BME381J/EE385J **Learning Opportunity #3** **Due 18 November 2019**

1. a. Take $\boldsymbol{B} = B_0\mathbf{z}$ and show that the vector differential equation $\frac{d\boldsymbol{\mu}}{dt} = \gamma\boldsymbol{\mu} \times \boldsymbol{B}_0$ (from slide 31 in the MRI powerpoints) decomposes into:

$$\frac{d\mu_x}{dt} = \omega_0 \mu_y$$
$$\frac{d\mu_y}{dt} = -\omega_0 \mu_x$$
$$\frac{d\mu_z}{dt} = 0$$

b. By taking additional derivatives, show that the x and y components decouple to yield two second order differential equations.

c. Solve these two equations by your favorite method.

2. The signal intensity for a gradient echo sequence (a particular type of MRI acquisition) is given by the following equation:

$$SI = \frac{(1-\exp(-TR/T_1))\sin\theta}{1-\cos(\theta)\exp(-TR/T_1)}\exp(-TE/T_2^{*}).$$

Assume that $TE << T_2^*$ and show that the maximum signal is given by:

$$\theta = \cos^{-1}(\exp(-TR/T_1)).$$

This is called the Ernst Equation and is very important in designing certain pulse sequences. Why?

3. According to the Boltzmann law of statistical mechanics, the populations P_m of the energy levels are proportional to $\exp(-E_m/kT)$. Use this relation to derive an expression for the ratio of the number of protons in the anti-parallel state to the number in the parallel state.

4. (*grad students only*) The emf induced in a coil by a change in its magnetic flux environment can be calculated by Faraday's law of induction. Write down a differential equation for emf. Consider a fixed square coil of length L in a time-dependent sinusoidal magnetic field. The sinusoidal magnetic field with angular frequency ω makes an angle θ to the normal of the plane of a square coil show in the figure. The magnetic field is given by $\boldsymbol{B}(t) = B(\sin\theta\mathbf{y} + \cos\theta\mathbf{z})\sin\omega t$. The coil is chosen to lie in the x-y plane so that $d\boldsymbol{S} = dxdy\mathbf{z}$. Show that the emf generated in the coil by the time-varying magnetic field is given by: emf = $-L^2B\omega(\cos\theta)(\cos\omega t)$. Comment on this relationship; in particular, the significance of the ω term.