1- part A)

[1.9 -0. -0.03 0.335] → hammer handle positions

[0.3259 0. 0.945] hammer handle Quaternions

part D)

[2.6 -2. 0.52] → mug relative Position

Part D)

$$J(q) = \frac{\partial f(q)}{\partial q} = \begin{bmatrix} \partial y/\partial q_0 & \partial y/\partial q_1 \\ \partial z/\partial q_0 & \partial z/\partial q_1 \end{bmatrix} = \begin{bmatrix} \cos(q_0) + \cos(q_0+q_1) & \cos(q_0 \\ -\sin(q_0) - \sin(q_0+q_1) & -\sin(q_0 \end{bmatrix}$$

we need to find $\det(J(q))$

$\det(J) = (\cos(q_0)+\cos(q_0+q_1))(-\sin(q_0+q_1)) - (-\sin(q_0) - \sin(q_0+q_1))(\cos(q_0+q_1$

$\Rightarrow$ by long calculations and trigonometric relations

$\Rightarrow \det(J) = -\cos(q_0)\sin(q_0+q_1) - \cos(q_0+q_1)\sin(q_0+q_1) + \rightarrow$

$\rightarrow \cos(q_0+q_1)\sin(q_0) + \cos(q_0+q_1)\sin(q_0+q_1)$

$= \sin(q_0-(q_0+q_1)) = \sin(-q_1) = -\sin(q_1)$

$\Rightarrow \det(J) = 0$, when $\sin(q_1) = 0 \Rightarrow$ happens when $q_1 = 0$ and $q_1 = \pi$

Two classes of configuration are fully extended configuration w $q_1 = 0$ and links are aligned in a straight line.

Fully folded configuration where $q_1 = \pi$ and links are on top of each other.

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Part A)

$^{B}p^{C}(0) = [1,?,?]^T$, $^{A}p^{C}(0) = ?$

At, $t=0$ arm is aligned along y axis and frame B is aligned with frame A.

Thus, $^{A}R^{B}(0) = I$ } Identity Matrix

Joint rotates by angle $\theta(t)$, frame B rotates around z-axis of frame A

$$\Rightarrow \underline{^{A}R^{B}(t)} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ +\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$^{A}p^{B}(t) = [0, l_1, l_0]^T$ when $t=0$ since end of the arm is located at a distance l_1 along y axis.

when joint rotates by angle $\theta(t) \rightarrow$ arm swings with radius l_1

$$\Rightarrow \underline{^{A}p^{B}(t)} = \begin{bmatrix} -l_2\sin\theta(t) \\ (l_1 + l_2\cos\theta(t) \\ l_0 \end{bmatrix}$$

$^{A}p^{C}(t) = {^{A}R^{B}(t)}\, {^{B}p^{C}(0)} + {^{A}p^{B}(t)}$, $^{B}p^{C}(0) = [1,0,0]^T \Rightarrow$

$$\Rightarrow {^{A}R^{B}(0)}\,{^{B}p^{C}(0)} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ +\sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta(t) \\ \sin\theta(t) \\ 0 \end{bmatrix}$$

$$\Rightarrow \underline{^{A}p^{C}(t)} = \begin{bmatrix} \cos\theta(t) \\ \sin\theta(t) \\ 0 \end{bmatrix} + \begin{bmatrix} ^{A}p^{B} \\ l_1\cos\theta(t) \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta(t) + l_1\sin\theta(t) \\ \sin\theta(t) + l_1\cos\theta(t) \\ 0 \end{bmatrix}$$

Part B

$R R^T = I \Rightarrow R^{-1} = R^T$

(a) $\frac{d}{dt}(R R^T) = \dot{R} R^T + R \dot{R}^T = 0$

$\hat{\omega} = \dot{R} R^{-1}$ by $R^{-1} = R^T \Rightarrow \hat{\omega} = \dot{R} R^T$

we need to show $\hat{\omega} + \hat{\omega}^T = 0$

$\Rightarrow \hat{\omega}^T = (\dot{R} R^T)^T = R \dot{R}^T \Rightarrow \hat{\omega} + \hat{\omega}^T = \underbrace{\dot{R} R^T + R \dot{R}^T}_{= 0 \text{ by (a)}} = \underline{0}.$

$\Rightarrow \hat{\omega} + \hat{\omega}^T = 0.$

$R = {}^A R^B(t)$ where ${}^A R^B(t) = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

transpose this

$\Rightarrow \dot{R} = \begin{bmatrix} -\sin\theta(t)\dot{\theta}(t) & -\cos\theta(t)\dot{\theta}(t) & 0 \\ \cos\theta(t)\dot{\theta}(t) & -\sin\theta(t)\dot{\theta}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Then, $\hat{\omega} = \dot{R} R^T = \begin{bmatrix} -\sin\theta(t)\dot{\theta}(t) & -\cos\theta(t)\dot{\theta}(t) & 0 \\ \cos\theta(t)\dot{\theta}(t) & -\sin\theta(t)\dot{\theta}(t) & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \dot{R}^T$

$= \begin{bmatrix} 0 & -\dot{\theta}(t) & 0 \\ +\dot{\theta}(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$

Part C)

Angular velocity

$${}^{A}\omega^{B}(t) = \dot{\theta}(t)\hat{z}, \quad \hat{z} = [0, 0, 1]^T \Rightarrow {}^{A}\omega^{B}(t) = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}(t) \end{bmatrix}$$

Linear velocity

$${}^{A}p^{B}(t) = \begin{bmatrix} -l_1 \sin\theta(t) \\ l_1 + l_2\cos\theta(t) \\ 0 \end{bmatrix} \Rightarrow {}^{A}v^{B}(t) = \frac{d}{dt}\begin{bmatrix} -l_1 \sin\theta(t) \\ l_1 + l_2\cos\theta(t) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 \cos\theta(t)\dot{\theta}(t) \\ -l_2 \sin\theta(t)\dot{\theta}(t) \\ 0 \end{bmatrix}.$$

$$\Rightarrow {}^{A}V^{B}(t) = \begin{bmatrix} {}^{A}\omega^{B}(t) \\ {}^{A}v^{B}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}(t) \\ -l_1\cos\theta(t)\dot{\theta}(t) \\ -l_2\sin\theta(t)\dot{\theta}(t) \\ 0 \end{bmatrix}$$

Spatial velocity.