DSAI510 Assignment 04

October 24, 2025

0.1 Assignment 4 - Deadline: Oct 26, 2026, Sun 11pm

DSAI 510 Fall 2025 Complete the assignment below and upload both the .ipynb file and its pdf to https://moodle.boun.edu.tr by the deadline given above. The submission page on Moodle will close automatically after this date and time.

To make a pdf, this may work: Hit CMD+P or CTRL+P, and save it as PDF. You may also use other options from the File menu.

[16]:
```
# Run this cell first

import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt

# Set the display option to show all rows scrolling with a slider
# pd.set_option('display.max_rows', None)
# To disable this, run the line below:
# pd.reset_option('display.max_rows')
```

0.2 Note:

In the problems below, if it asks, "show the number of records that are nonzero", the answer is a number; so you don't need to show the records themselves. But if it asks, "show the records with NaN", it wants you to print those records (rows) containing NAN and other entries, not asking how many such records there are. So be careful about what you're asked.

0.3 Total: 60 pts

0.4 Problem 1 (10 pts)

(a) Load the Ames house dataset from the file **train.csv**.

(b) By using one-hot encoding, create bins for the SalePrice variable wrt the intervals `bins_SalePrice = [0, 250000, 500000, 1000000]`. The result should look like this:

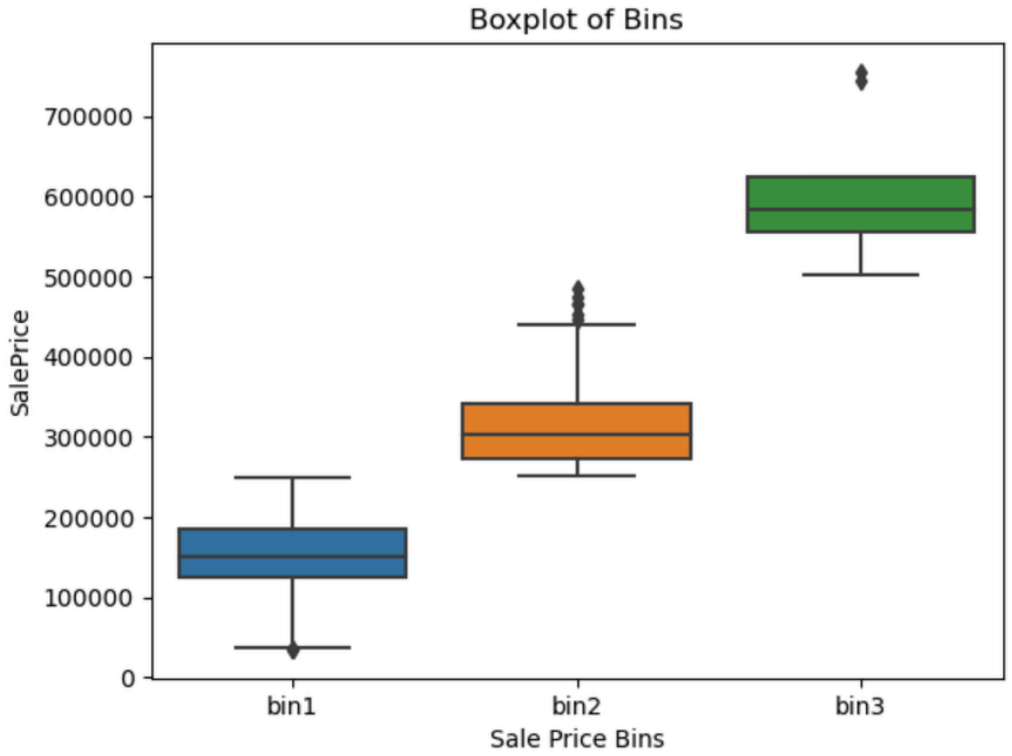
Y \ X	"1"	"2"	"3"
"1"	$\frac{9}{100}$	$\frac{3}{50}$	$\frac{7}{50}$
"2"	$\frac{3}{20}$	$\frac{3}{50}$	$\frac{29}{200}$
"3"	$\frac{27}{200}$	$\frac{13}{100}$	$\frac{9}{100}$

(c) Use simple multiplication between the bin columns and SalePrice to generate this table:

	bin1	bin2	bin3	SalePrice
0	208500	0	0	208500
1	181500	0	0	181500
2	223500	0	0	223500
3	140000	0	0	140000
4	250000	0	0	250000
...	...	...	...	...
1455	175000	0	0	175000
1456	210000	0	0	210000
1457	0	266500	0	266500
1458	142125	0	0	142125
1459	147500	0	0	147500

1460 rows × 4 columns

(d) Remove the SalePrice column, melt the final dataframe and by using `sns.boxplot()` create the boxplot below. Make sure you're getting exactly the same plot as below with the same axes names and title.



[17]:
```
# Break your computations into multiple cells
# part a)
df = pd.read_csv("house-prices/train.csv")
df
```

[17]:

	Id	MSSubClass	MSZoning	LotFrontage	LotArea	Street	Alley	LotShape	\
0	1	60	RL	65.0	8450	Pave	NaN	Reg	
1	2	20	RL	80.0	9600	Pave	NaN	Reg	
2	3	60	RL	68.0	11250	Pave	NaN	IR1	
3	4	70	RL	60.0	9550	Pave	NaN	IR1	
4	5	60	RL	84.0	14260	Pave	NaN	IR1	
…	…	…	…	…	…	…	…	…	
1455	1456	60	RL	62.0	7917	Pave	NaN	Reg	
1456	1457	20	RL	85.0	13175	Pave	NaN	Reg	
1457	1458	70	RL	66.0	9042	Pave	NaN	Reg	
1458	1459	20	RL	68.0	9717	Pave	NaN	Reg	
1459	1460	20	RL	75.0	9937	Pave	NaN	Reg	

	LandContour	Utilities	…	PoolArea	PoolQC	Fence	MiscFeature	MiscVal	\
0	Lvl	AllPub	…	0	NaN	NaN	NaN	0	
1	Lvl	AllPub	…	0	NaN	NaN	NaN	0	
2	Lvl	AllPub	…	0	NaN	NaN	NaN	0	
3	Lvl	AllPub	…	0	NaN	NaN	NaN	0	

```
4            Lvl    AllPub  …         0    NaN    NaN         NaN       0
…            …      …       …         …    …      …           …         …
1455         Lvl    AllPub  …         0    NaN    NaN         NaN       0
1456         Lvl    AllPub  …         0    NaN  MnPrv         NaN       0
1457         Lvl    AllPub  …         0    NaN  GdPrv        Shed    2500
1458         Lvl    AllPub  …         0    NaN    NaN         NaN       0
1459         Lvl    AllPub  …         0    NaN    NaN         NaN       0

     MoSold YrSold  SaleType  SaleCondition  SalePrice
0         2   2008        WD         Normal     208500
1         5   2007        WD         Normal     181500
2         9   2008        WD         Normal     223500
3         2   2006        WD        Abnorml     140000
4        12   2008        WD         Normal     250000
…     …    …       …              …          …
1455      8   2007        WD         Normal     175000
1456      2   2010        WD         Normal     210000
1457      5   2010        WD         Normal     266500
1458      4   2010        WD         Normal     142125
1459      6   2008        WD         Normal     147500

[1460 rows x 81 columns]
```

[18]:
```
# part b)
bins_SalePrice = [0, 250000, 500000, 1000000]
labels = ['bin1', 'bin2', 'bin3']

bins = pd.cut(df['SalePrice'], bins=bins_SalePrice, labels=labels)
df_bins = pd.get_dummies(bins).astype(int)
final_df = pd.concat([df_bins, df[['SalePrice']]], axis=1)
final_df
```

[18]:
```
      bin1  bin2  bin3  SalePrice
0        1     0     0     208500
1        1     0     0     181500
2        1     0     0     223500
3        1     0     0     140000
4        1     0     0     250000
…    …   …     …     …          …
1455     1     0     0     175000
1456     1     0     0     210000
1457     0     1     0     266500
1458     1     0     0     142125
1459     1     0     0     147500

[1460 rows x 4 columns]
```

[19]:
```
# part c)

for i in range(1, 4):
    final_df[f'bin{i}'] = final_df[f'bin{i}'] * final_df['SalePrice']

final_df
```

[19]:

	bin1	bin2	bin3	SalePrice
0	208500	0	0	208500
1	181500	0	0	181500
2	223500	0	0	223500
3	140000	0	0	140000
4	250000	0	0	250000
…	…	…	…	…
1455	175000	0	0	175000
1456	210000	0	0	210000
1457	0	266500	0	266500
1458	142125	0	0	142125
1459	147500	0	0	147500

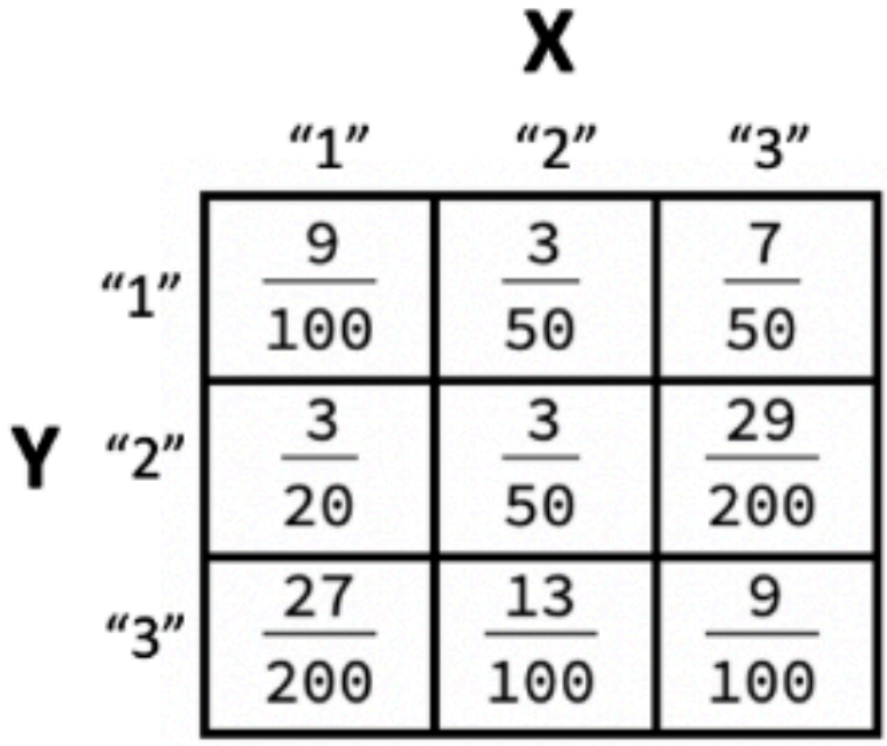
[1460 rows x 4 columns]

[ ]:

0.5 Problem 2 (5 pts)

Produce the plot of the sliced section of the correlation matrix as shown below.

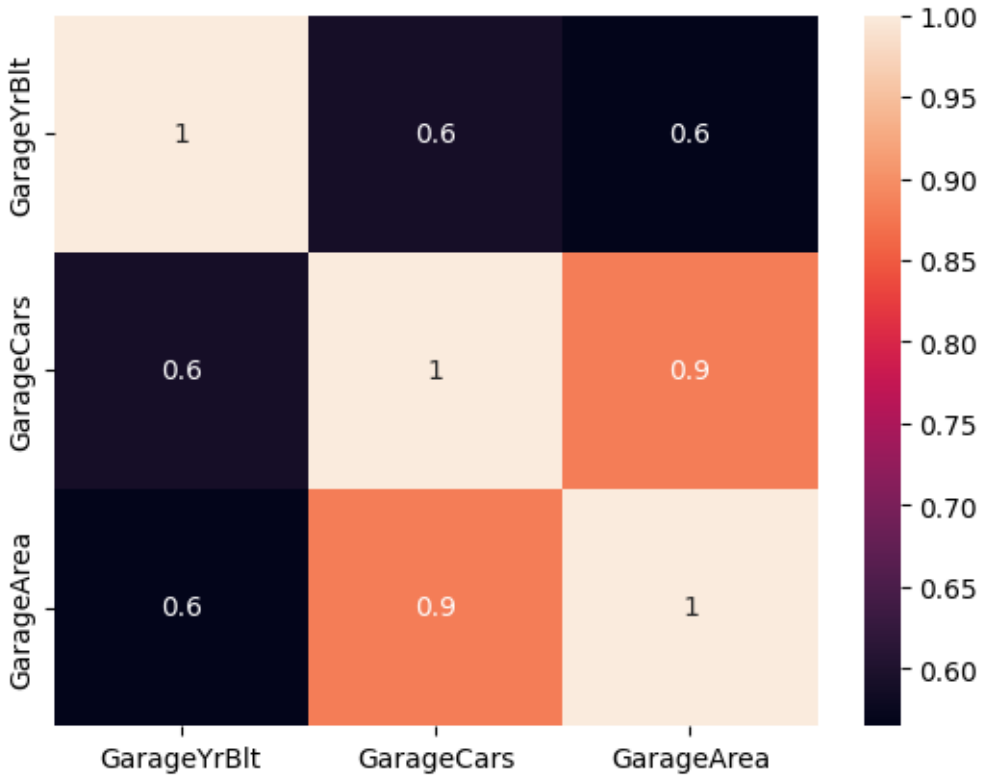
Hint: Just use the `correlation_matrix` we generated in the class and single out the relevant variables with `.loc()` method and then use `sns.heatmap()`to create the plot shown below.



[26]:
```
garage_df = df.loc[:, ['GarageYrBlt', 'GarageCars', 'GarageArea']]
g_corr = garage_df.corr()

sns.heatmap(g_corr, annot=True, fmt="0.1g")
```

[26]: <Axes: >



0.6 Problem 3 (5 pts)

(a) Load the `Breast_Cancer_Wisconsin.csv` into a dataframe and then **normalize** the numerical columns to the interval [0,1]. Check with `.describe()` and write a sentence or two to explain what exactly in the outcome of `.describe()` tells you the **normalization** you did was successful.

(b) This time, **standardize** the numerical columns. Check with `.describe()` and write a sentence or two to explain what exactly in the outcome of `.describe()` tells you the **standardization** you did was successful.

[ ]:
```
# part a)
cancer_df = pd.read_csv("Breast_Cancer_Wisconsin.csv")
norm_df = cancer_df.copy()
num_cols = norm_df.select_dtypes("number").columns
norm_df[num_cols] = (norm_df[num_cols] - norm_df[num_cols].min()) /␣
 ↪(norm_df[num_cols].max() - norm_df[num_cols].min())

norm_df.describe()
# for each numerical columns the maximum value is 1 and minimum value is 0,␣
 ↪clearly we fitted them into [0, 1] interval
```

[ ]:

	id	radius_mean	texture_mean	perimeter_mean	area_mean
count	569.000000	569.000000	569.000000	569.000000	569.000000
mean	0.033318	0.338222	0.323965	0.332935	0.216920
std	0.137187	0.166787	0.145453	0.167915	0.149274
min	0.000000	0.000000	0.000000	0.000000	0.000000
25%	0.000944	0.223342	0.218465	0.216847	0.117413
50%	0.000985	0.302381	0.308759	0.293345	0.172895
75%	0.009661	0.416442	0.408860	0.416765	0.271135
max	1.000000	1.000000	1.000000	1.000000	1.000000

	smoothness_mean	compactness_mean	concavity_mean	concave points_mean
count	569.000000	569.000000	569.000000	569.000000
mean	0.394785	0.260601	0.208058	0.243137
std	0.126967	0.161992	0.186785	0.192857
min	0.000000	0.000000	0.000000	0.000000
25%	0.304595	0.139685	0.069260	0.100944
50%	0.390358	0.224679	0.144189	0.166501
75%	0.475490	0.340531	0.306232	0.367793
max	1.000000	1.000000	1.000000	1.000000

	symmetry_mean	…	texture_worst	perimeter_worst	area_worst
count	569.000000	…	569.000000	569.000000	569.000000
mean	0.379605	…	0.363998	0.283138	0.170906
std	0.138456	…	0.163813	0.167352	0.139932
min	0.000000	…	0.000000	0.000000	0.000000
25%	0.282323	…	0.241471	0.167837	0.081130
50%	0.369697	…	0.356876	0.235320	0.123206
75%	0.453030	…	0.471748	0.373475	0.220901
max	1.000000	…	1.000000	1.000000	1.000000

	smoothness_worst	compactness_worst	concavity_worst
count	569.000000	569.000000	569.000000
mean	0.404138	0.220212	0.217403
std	0.150779	0.152649	0.166633
min	0.000000	0.000000	0.000000
25%	0.300007	0.116337	0.091454
50%	0.397081	0.179110	0.181070
75%	0.494156	0.302520	0.305831
max	1.000000	1.000000	1.000000

	concave points_worst	symmetry_worst	fractal_dimension_worst
count	569.000000	569.000000	569.000000
mean	0.393836	0.263307	0.189596
std	0.225884	0.121954	0.118466
min	0.000000	0.000000	0.000000
25%	0.223127	0.185098	0.107700
50%	0.343402	0.247782	0.163977

```
75%                0.554639        0.318155                 0.242949
max                1.000000        1.000000                 1.000000

       Unnamed: 32
count          0.0
mean           NaN
std            NaN
min            NaN
25%            NaN
50%            NaN
75%            NaN
max            NaN

[8 rows x 32 columns]
```

[ ]:
```
# part b)
std_df = cancer_df.copy()
num_cols = std_df.select_dtypes("number").columns
std_df[num_cols] = (std_df[num_cols] - std_df[num_cols].mean()) /␣
 ↪std_df[num_cols].std()

pd.set_option('display.float_format', '{:.6f}'.format)
std_df.describe()
# for each numerical columns we observe standart deviation is equal to 1 and
# mean is almost zero (there are some negligible -0.00 values, most likely␣
 ↪negative values close to 0)
```

[ ]:
```
                id  radius_mean  texture_mean  perimeter_mean  area_mean  \
count 569.000000   569.000000    569.000000      569.000000 569.000000
mean    0.000000     0.000000     -0.000000        0.000000  -0.000000
std     1.000000     1.000000      1.000000        1.000000   1.000000
min    -0.242865    -2.027864     -2.227289       -1.982759  -1.453164
25%    -0.235982    -0.688779     -0.725325       -0.691347  -0.666609
50%    -0.235688    -0.214893     -0.104544       -0.235773  -0.294927
75%    -0.172441     0.468980      0.583662        0.499238   0.363188
max     7.046429     3.967796      4.647799        3.972634   5.245913

       smoothness_mean  compactness_mean  concavity_mean  concave points_mean  \
count       569.000000        569.000000      569.000000           569.000000
mean          0.000000         -0.000000        0.000000             0.000000
std           1.000000          1.000000        1.000000             1.000000
min          -3.109349         -1.608721       -1.113893            -1.260710
25%          -0.710338         -0.746429       -0.743094            -0.737295
50%          -0.034860         -0.221745       -0.341939            -0.397372
75%           0.635640          0.493423        0.525599             0.646366
max           4.766717          4.564409        4.239858             3.924477
```

```
       symmetry_mean  …  texture_worst  perimeter_worst  area_worst  \
count     569.000000  …     569.000000       569.000000  569.000000
mean       -0.000000  …      -0.000000         0.000000   -0.000000
std         1.000000  …       1.000000         1.000000    1.000000
min        -2.741705  …      -2.222039        -1.691872   -1.221348
25%        -0.702621  …      -0.747971        -0.688972   -0.641571
50%        -0.071564  …      -0.043477        -0.285729   -0.340881
75%         0.530313  …       0.657762         0.539804    0.357275
max         4.480808  …       3.882489         4.283568    5.924959

       smoothness_worst  compactness_worst  concavity_worst  \
count        569.000000         569.000000       569.000000
mean          -0.000000           0.000000         0.000000
std            1.000000           1.000000         1.000000
min           -2.680337          -1.442609        -1.304683
25%           -0.690623          -0.680485        -0.755849
50%           -0.046802          -0.269264        -0.218040
75%            0.597020           0.539194         0.530674
max            3.951897           5.108382         4.696536

       concave points_worst  symmetry_worst  fractal_dimension_worst  \
count            569.000000      569.000000                569.000000
mean              -0.000000        0.000000                  0.000000
std                1.000000        1.000000                  1.000000
min               -1.743529       -2.159060                 -1.600431
25%               -0.755735       -0.641299                 -0.691303
50%               -0.223272       -0.127297                 -0.216254
75%                0.711884        0.449742                  0.450366
max                2.683516        6.040726                  6.840837

       Unnamed: 32
count     0.000000
mean           NaN
std            NaN
min            NaN
25%            NaN
50%            NaN
75%            NaN
max            NaN

[8 rows x 32 columns]
```

0.7 Problem 4 - Bayes' Theorem (10 pts)

Let's assume the probability of being infected by covid is 0.1. If you're infected, PCR tests show you're covid positive with 80% accuracy. If you're not infected, PCR tests show you're covid positive with probability 1/90 (false positive). If you test covid positive in a PCR test, what is the probability that you're really infected with covid?

Hint 1: You can use c+ and c- for covid infection, and pcr+ and pcr- for test results, like P(c+)=0.1.
Hint 2: The answer is 0.89. Show in the cell below your algebraic steps involved in your solution. You don't need to write a Python code.

P(c+) = 0.1

P(c-) = 0.9

P(pcr+|c+) = 0.8

P(pcr+|c-) = 1/90

Let c+ = A and pcr+ = B

Question becomes P(A|B)

From Bayes' Theorem:

P(A|B) = P(B|A) * P(A) / P(B)

but we do not know P(B) yet

from Law of Probability we know that P(Positive) = P(True Positive) + P(False Positive)

P(B) = P(B|A) * P(A) + P(B|~A) * P(~A)

P(B) = 0.8 * 0.1 + 1/90 * 0.9

P(B) = 0.09

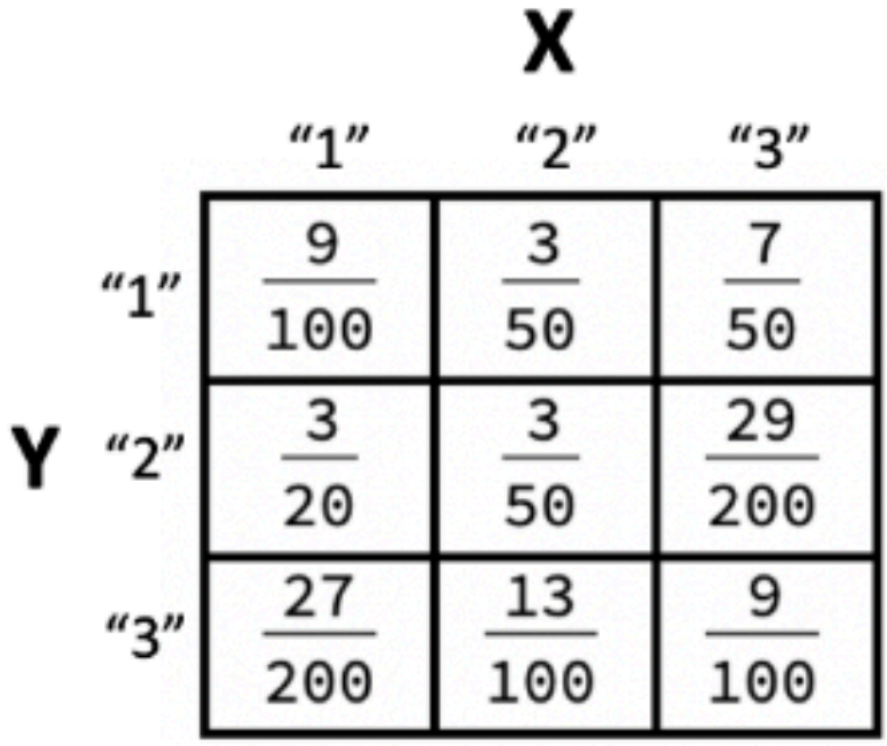
then

P(A|B) = 0.8 * 0.1 / 0.09

P(A|B) = 8/9 (0.88 but nearly 0.89)

0.8 Problem 5 (5 pts)

Calculate the mean of the random variable K which assumes the values k=1, 2, 3 and 4 by using the pmf given below.



(You don't need to write a Python code; please show the algebraic steps involved in your solution.)

[ ]:
```
pmf = [(1, 0.2), (2, 0.1), (3, 0.6), (4, 0.1)]
sum = 0

for (k, prob) in pmf:
    sum += k*prob

print(sum)
```

```
2.5999999999999996
```

Expected Value E[X] is defined as SUM(x * P(X=x) for each x)

k = 1: 1 * 0.2 = 0.2

k = 2: 2 * 0.1 = 0.2

k = 3: 3 * 0.6 = 1.8

k = 4: 4 * 0.1 = 0.4

SUM = 2.6

0.9 Problem 6 (10 pts)

A joint pmf is provided on the table, where X and Y are not independent.

(a) You randomly select values for X and Y. What is the probability that X=2 and Y=3?

(b) Your friend randomly select values for X and Y, and tells you that X turned out to be 3, but he does not reveal what Y came out to be. What's the probability that Y=2 given that you know that X=3.

(c) What is the probability that Y=3 regardless of the value of X?

		X "1"	X "2"	X "3"
Y	"1"	$\frac{9}{100}$	$\frac{3}{50}$	$\frac{7}{50}$
Y	"2"	$\frac{3}{20}$	$\frac{3}{50}$	$\frac{29}{200}$
Y	"3"	$\frac{27}{200}$	$\frac{13}{100}$	$\frac{9}{100}$

(You don't need to write a Python code; please show the algebraic steps involved in your solution and explain your solution in words if necessary.)

[ ]:
```
# (a)
# Since this is a joint function already we can just choose the box that is X=2␣
 ↪and Y=3
13/100
```

[ ]:
```
0.13
```

[ ]:
```
# (b)
# Question asks conditional probability P(Y=2|X=3), this can be calculates as␣
 ↪their joint probabiliy divided by the condition margin.
# P(Y=2|X=3) = P(Y=2, X=3) / P(X=3)
joint = 29/200
margin = 7/50 + 29/200 + 9/100
```

```
joint/margin
```

[ ]:
```
0.38666666666666666
```

[43]:
```
# (c)
# This is marginal probability so we need to sum up the row where Y=3
27/200 + 13/100 + 9/100
```

[43]:
```
0.355
```

0.10 Problem 7 (5 pts)

We observe that ice-cream sales and sunburns on people's back co-occur. Are the statements below True or False?

(a) "These two phenomena are correlated" True/False?
(b) "There's a direct causal relationship between these two phenomena" True/False?

[ ]:
```
# Your answer (type True or False)
# a) True
# b) False
```

0.11 Problem 8 (10 pts)

A particle in a box can be somewhere between x=0 and x=1 m. The pdf about where it can be found is given by

$$f(x) = ax(1-x)$$

(a) What should be the value of a so that the pdf is normalized? (Hint 1: The area under the pdf should be equal to 1.)

(b) What's the probability that this particle can be found in the region 0 x 0.1?

Hint 2: For the second question, you'll take the integral. You can use https://www.wolframalpha.com for that or get help from fellow students. In any case, explain how you did it, or if you used wolframalpha, attach a screenshot by using a Markdown cell. The answer for part (b) is 0.028; but show your work on how you reached the answer.

a)

$$\int_0^1 ax(1-x)\,dx = a\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= a\left(\frac{1}{2} - \frac{1}{3} - 0\right) = a\left(\frac{3-2}{6}\right) = \frac{a}{6}$$

$$\frac{a}{6} = 1, a = 6$$

b) Since this is a density function, we can integrate the desired interval and find the probability. We already found a=6 from part a) so we can just substitute and continue

$$P(0 \leq x \leq 0.1) = \int_0^{0.1} f(x)\,dx$$

$$= \int_0^{0.1} 6x(1-x)\,dx$$

$$= 6\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{0.1}$$

$$= 6\left(\left(\frac{(0.1)^2}{2} - \frac{(0.1)^3}{3}\right) - (0)\right)$$

$$= 6\left(\frac{1}{200} - \frac{1}{3000}\right)$$

$$= \frac{84}{3000}$$

[57]:
```
84/3000
```

[57]: 0.028