HW3 - 2020

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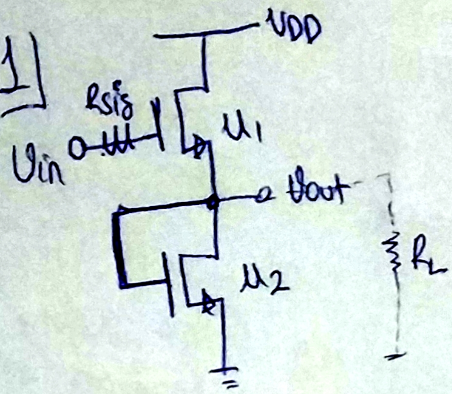
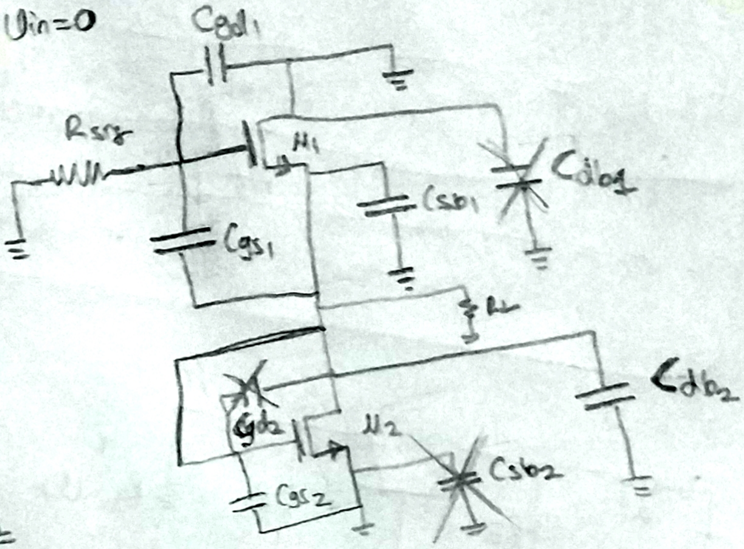
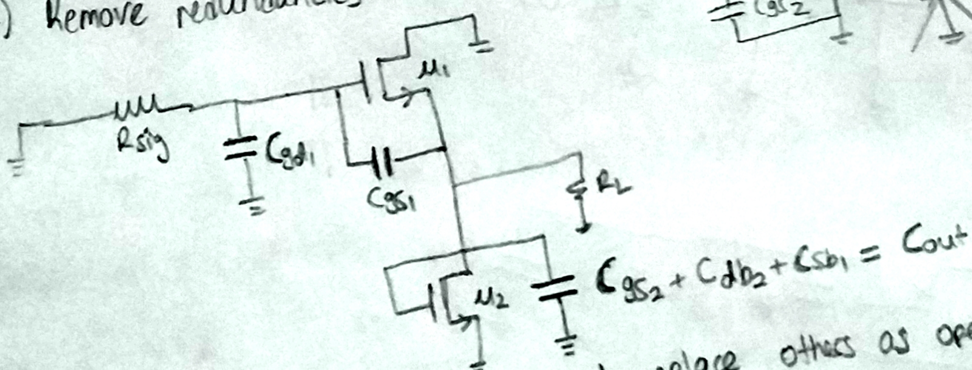
1)



1) Vin=0



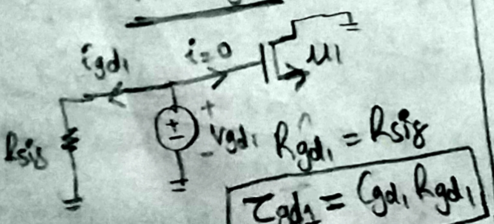
2) Remove redundancies



$C_{gs_2} + C_{db_2} + C_{sb_1} = C_{out}$

3) Consider one capacitor at time and replace others as open circuit.

For C_{gd_1}

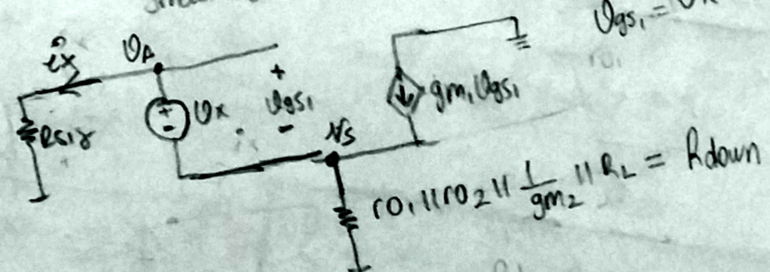


$R_{gd_1} = R_{sig}$

$\boxed{\tau_{gd_1} = C_{gd_1} R_{gd_1}}$

For C_{gs1} (Since it is a floating capacitor we need to do small signal analysis)

$v_{gs_1} = v_x$



$r_{o_1} \| r_{o_2} \| \frac{1}{g_{m_2}} \| R_L = R_{down}$

$v_s = (g_{m_1} v_x - i_x) R_{down}$

$i_x R_{sig} - v_x = g_{m_1} v_x R_{down} - i_x R_{down}$

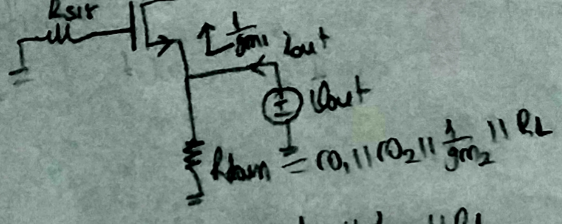
$v_x (1 + g_{m_1} R_{down}) = i_x (R_{sig} + R_{down})$

$\frac{v_x}{i_x} = R_{gs_1} = \frac{R_{sig} + R_{down}}{1 + g_{m_1} R_{down}}$

$\Rightarrow R_{gs_1} = \frac{R_{sig} + (r_{o_1} \| r_{o_2} \| \frac{1}{g_{m_2}} \| R_L)}{1 + g_{m_1}(r_{o_1} \| r_{o_2} \| \frac{1}{g_{m_2}} \| R_L)}$

$\boxed{\tau_{gs_1} = C_{gs_1} R_{gs_1}}$

For C_{out}



$R_{down} = r_{o_1} \| r_{o_2} \| \frac{1}{g_{m_2}} \| R_L$

$R_{out} = r_{o_1} \| r_{o_2} \| \frac{1}{g_{m_1}} \| \frac{1}{g_{m_2}} \| R_L$

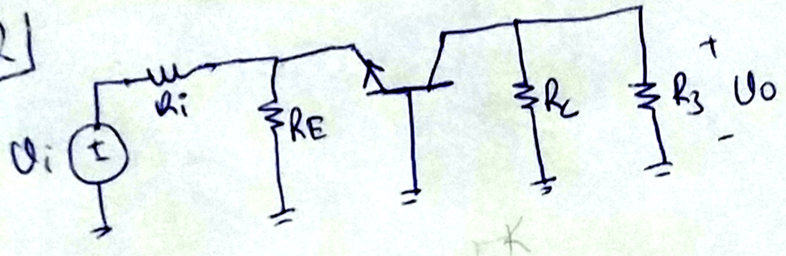
$\tau_{out} = C_{out} R_{out}$

Then according to open-circuit time constant methodology.

$$f_{3\text{-}dB} = \frac{1}{\sum_{i=1}^{3} \tau_i} = \frac{1}{(\tau_{out} + \tau_{gs1} + \tau_{gd1})2\pi}$$

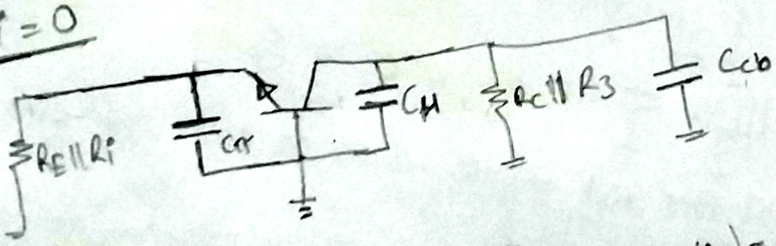
$$\boxed{f_{3\text{-}dB} = \frac{1}{2\pi\left(C_{out}R_{out} + C_{gs1}R_{gs1} + C_{gd1}R_{gd1}\right)}}$$

2]

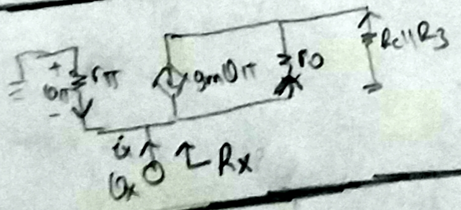


Since C_1 and C_2 are coupling capacitors, at high frequencies, they becomes short circuit.

1-) $v_i = 0$



For C_π



$$v_x - \left(i_x + \frac{v_\pi}{r_\pi} + g_m v_\pi\right) r_o - \left(i_x + \frac{v_\pi}{r_\pi}\right)(R_C \| R_3) = 0$$

$$v_x - \left(i_x - \frac{v_x}{r_\pi} - g_m v_x\right) r_o - \left(i_x - \frac{v_x}{r_\pi}\right)(R_C \| R_3) = 0$$

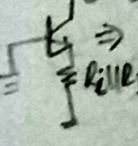
since $v_x = -v_\pi$ $\Rightarrow v_x\left(1 + \frac{r_o}{r_\pi} + g_m r_o + \frac{R_C \| R_3}{r_\pi}\right) - i_x\left(r_o + R_C \| R_3\right) = 0$

$$R_x = \frac{r_o + (R_C \| R_3)}{1 + \frac{r_o}{r_\pi} + g_m r_o + \frac{R_C \| R_3}{r_\pi}} = \frac{r_\pi r_o + r_\pi (R_C \| R_3)}{r_\pi + r_o + g_m r_\pi r_o + (R_C \| R_3)}$$

$$R_x = \frac{r_\pi (r_o + R_C \| R_3)}{r_\pi + (\beta + 1) r_o + (R_C \| R_3)} \Rightarrow R_\pi = R_x \| R_E \| R_i$$

$$\tau_\pi = C_\pi R_\pi = C_\pi (R_x \| R_E \| R_i)$$

For $C_\mu + C_{cb}$



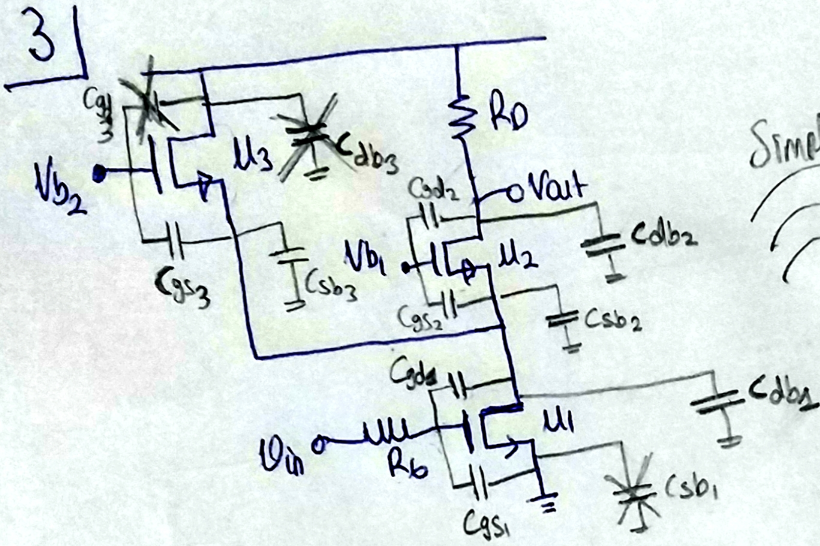
$\Rightarrow R_d = r_o + (R_i \| R_E \| r_\pi) + g_m r_o (R_i \| R_E \| r_\pi)$

$$R_\mu = R_d \| R_C \| R_3 \approx R_C \| R_3 \| (g_m r_o (R_i \| R_E \| r_\pi))$$

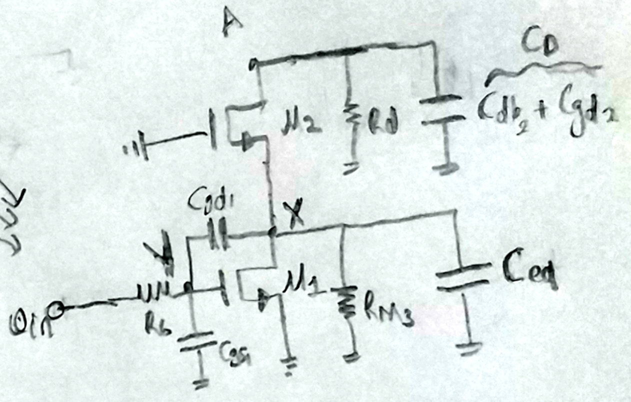
$$\tau_\mu = (C_\mu + C_{cb})\left[R_C \| R_3 \| (g_m r_o (R_i \| R_E \| r_\pi))\right]$$

$$\boxed{f_{3\text{-}dB} = \frac{1}{2\pi(\tau_\mu + \tau_\pi)}}$$

3|



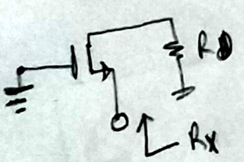
Simplify

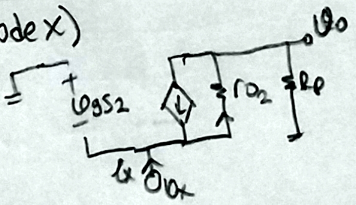


where
$C_{eq} = C_{db1} + C_{gs2} + C_{sb2} + C_{gs3} + C_{sb3}$

There is only one floating capacitor, C_{gd1}, and since we're asked to apply Miller's approximation, we need to calculate the DC gain between the terminals of C_{gd1}. (Between the terminals Y-X)

In order to find DC gain we need to find Rout for which Vout is taken from the drain of M_1. (Node X)





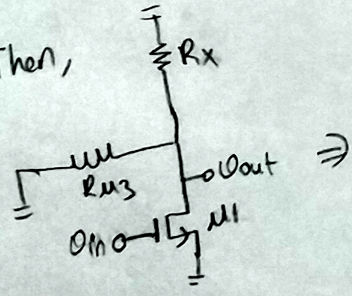
$v_x - (i_x + g_{m2} v_{gs2}) r_{o2} - i_x R_D = 0$ ($-v_x$)

$v_x (1 + g_{m2} r_{o2}) = i_x (r_{o2} + R_D)$

$R_x = \frac{v_x}{i_x} = \frac{r_{o2} + R_D}{1 + g_{m2} r_{o2}} = \frac{(1/g_{m2})(r_{o2} + R_D)}{\frac{1}{g_{m2}} + r_{o2}}$

$R_x \approx \frac{r_{o2} + R_D}{g_{m2} r_{o2}}$ (since $r_{o2} \gg \frac{1}{g_{m2}}$)

Then,



$\Rightarrow$

$G_m = g_{m1}$

$R_{out} = R_x \parallel R_{M3} \parallel r_{o1}$

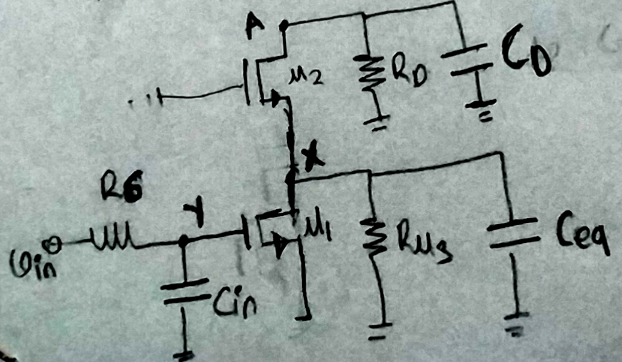
$R_{out} = \left(\frac{r_{o2} + R_D}{g_{m2} r_{o2}}\right) \parallel \left(\frac{1}{g_{m3}} \parallel r_{o3}\right) \parallel r_{o1}$

$\Rightarrow$ DC gain between the terminals of the floating capacitor is $A_V = G_m R_{out}$

$$A_V = g_{m1} \left[\left(\frac{r_{o2} + R_D}{g_{m2} r_{o2}}\right) \parallel \left(\frac{1}{g_{m3}} \parallel r_{o3}\right) \parallel r_{o1} \right]$$

Now we seperate C_{gd1} into two standart capacitors by using Miller's theorem, and draw the new model

$C_Y = C_{gd1}(1 + A_V)$; $C_X = C_{gd1}\left(1 + \frac{1}{A_V}\right)$, and draw the new model



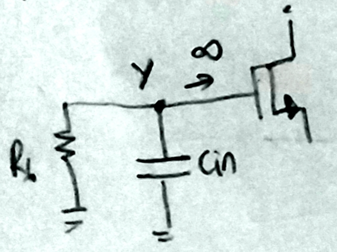
where $C_D = C_{db2} + C_{gd2}$

$C_{in} = C_Y + C_{gs1}$

$C_{eq} = C_X + C_{db1} + C_{gs2} + C_{sb2} + C_{gs3} + C_{sb3}$

Now, we obtained the model that we can apply the pole identification on

Let us first examine input pole which stems from the capacitor C_{in}

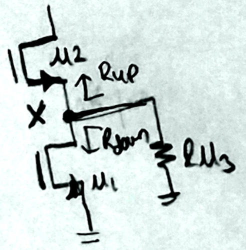


$R_Y = R_6$

$Z_Y = C_{in} R_6$

$f_{in} = \dfrac{1}{2\pi C_{in} R_6}$

Then, examine C_{eq}



we have found that $R_{up} \approx \dfrac{r_{o2} + R_O}{g_{m2} r_{o2}}$

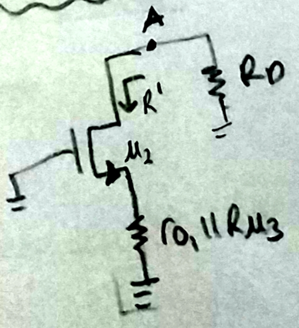
and we know $R_{down} = r_{o1}$

Then $R_X = R_{M3} \parallel R_{up} \parallel R_{down}$

$\Rightarrow R_X = \left(\dfrac{1}{g_{m3}} \parallel r_{o3}\right) \parallel \left(\dfrac{r_{o2} + R_O}{g_{m2} r_{o2}}\right) \parallel r_{o1}$

$\Rightarrow Z_X = C_{eq} R_X \Rightarrow f_{eq} = \dfrac{1}{2\pi C_{eq} R_X}$

Lastly, examine C_{up}



$R' = r_{o2} + (r_{o1} \parallel R_{M3}) + g_{m2} r_{o2} (r_{o1} \parallel R_{M3})$

$R_A = R_O \parallel R' = R_O \parallel \left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel R_{M3})\right]$

$Z_A = R_A C_O$

$f_A = \dfrac{1}{2\pi (R_A C_O)}$

Node's names

X: Drain of M_1 and Source of M_2

Y: Gate of M_1

A: Drain of M_3

R_{M3}: The resistance seen looking into the source of M_3