

# Homework - 6 Solution

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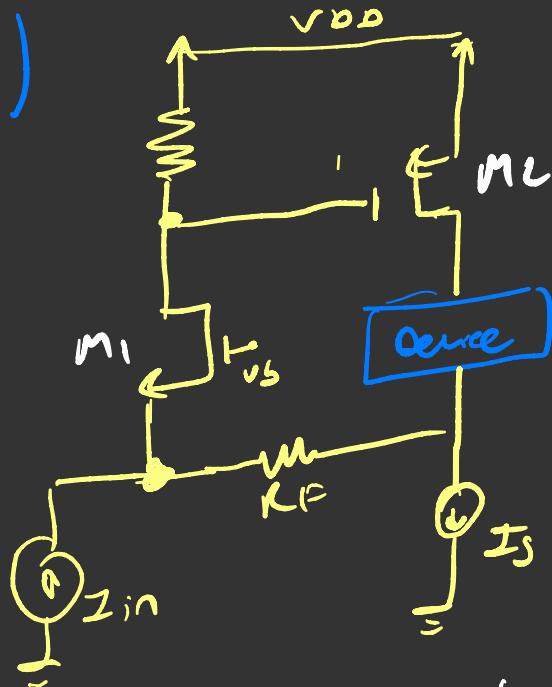
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1)

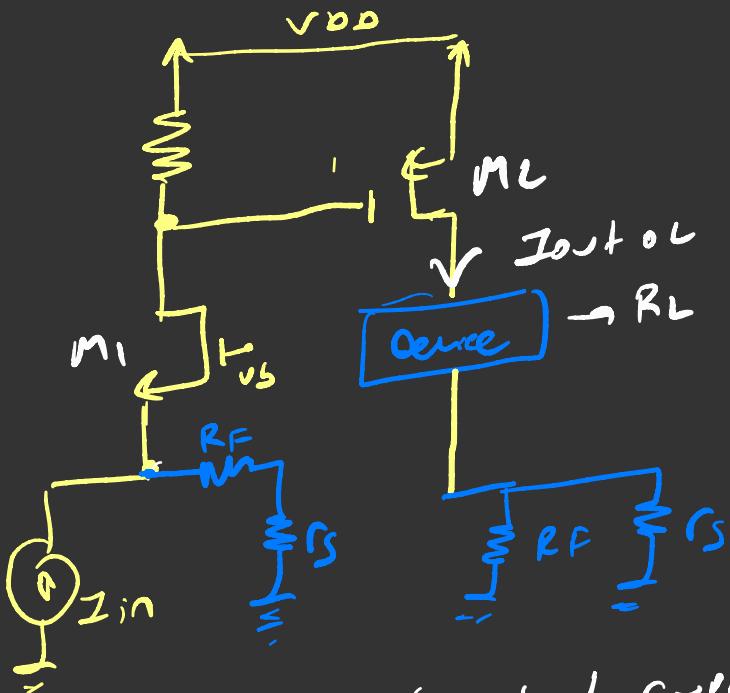


The feedback returns to the same terminal as input current source. Thus, we have shunt feedback at the input.

The output device processes the current of  $M_2$ . Thus the output is series.  
we have shunt-series feedback.

→ Assuming  $I_S$  has the output resistance  $r_S$ , let's draw open-loop circuit with loading.

Open-loop circuit with loading.



Assuming  $r_S = \infty$  (ideal current source)

$$I_{in} \cdot R_D \cdot g_{m2} \frac{r_{DS}}{r_{DS} + R_L + R_F} = I_{out\,OL}$$

$$A_{OL} = g_{m2} R_D \cdot \frac{r_{DS}}{r_{DS} + R_L + R_F}$$

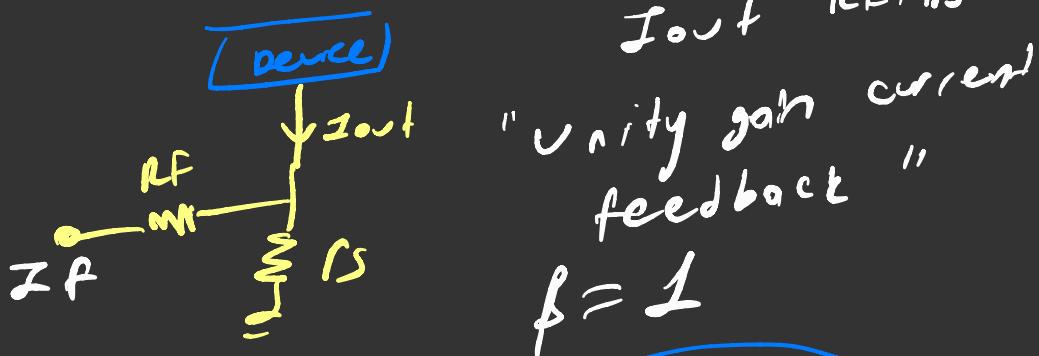
→ open loop current gain

$$R_{in\,OL} = \frac{1}{g_m 1} \rightarrow \text{open loop } R_{in}$$

$$R_{out\,OL} = R_F + R_L + r_{o2} \rightarrow \begin{matrix} \text{open loop} \\ \text{post} \end{matrix}$$

Let's zoom in to the feed back network.

$$\frac{I_F}{I_{out}} = \frac{r_s}{R_F + R_s} = 1$$



$$A_{CL} = g_m 2 \frac{R_o}{R_{in} + R_L + R_F} \frac{r_{o2}}{r_{o1} + R_L + R_F}$$

$$1 + g_m 2 \frac{R_o}{R_{in} + R_L + R_F} \frac{r_{o2}}{r_{o1} + R_L + R_F}$$

$$R_{inCL} = \frac{R_{inCL}}{1 + A_{oL}}$$

$$R_{outCL} = (1 + A_{oL}) R_{outCL}$$

→ you can indent  $A_{oL}$  to the equations above.

2) First, we can use the formula,

$$\phi = - \left[ \arctan \left( \frac{f}{10^5} \right) + \arctan \left( \frac{f}{3 \times 10^5} \right) + \arctan \left( \frac{f}{10^6} \right) \right]$$

for a  $5^\circ$  phase margin  $\phi$  should be  $-135^\circ$ .

we need to find "f" frequency  
which provide  $-135^\circ$  for  $\phi$ .  
 $\phi$  means "phase shift".  
Using calculator, the f  
value is approximately  $3.16 \times 10^5$  Hz  
which is not a coincidence.

### Comment

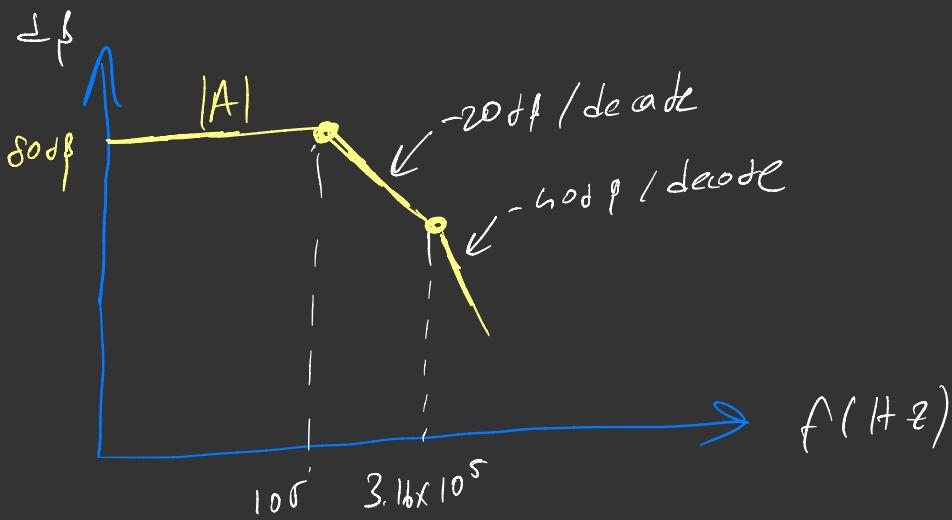
- The harmonic average of  $10^6$  and  $10^5$  is around  $3.16 \times 10^5$ .
- The second pole  $3.16 \times 10^5$  exactly brings a  $45^\circ$  phase shift at f.
- The first pole and third pole brings  $90^\circ$  phase shift at f.

- what should be the  $\beta$  value  
 at this "f" frequency?  
 • we always investigate for  
 $|A\beta| = 1$  or  $20 \log |A\beta| = 0dB$   
 behaviour for phase margin and stability  
 analysis.

$$20 \log |A\beta| = 20 \log |A(f)| - 20 \log \frac{1}{\beta}$$

So  
 we need to investigate the  
 frequency which provides

$$\boxed{20 \log |A(f)| = 20 \log \frac{1}{\beta}}$$



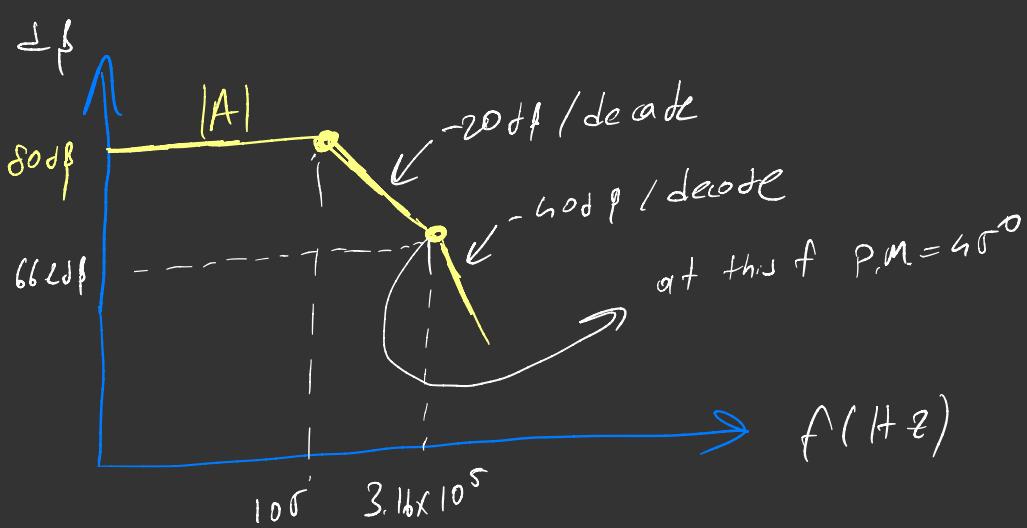
What is the  $|A|$  at +high frequency?

$$|A| = \left| \frac{10^4 \rightarrow \text{low frequency gain}}{\left( 1 + j \frac{3.16 \times 10^5}{10^5} \right) \left( 1 + j \frac{3.16 \times 10^5}{3.16 \times 10^5} \right) \left( 1 + j \frac{3.16 \times 10^5}{10^6} \right)} \right|$$

$\downarrow \quad \quad \quad \downarrow \quad \quad \quad \downarrow$   
 $P_1 \quad P_2 \quad P_3$

$$|A| = \frac{10^4}{3.31 \times 1.61 \times 1.05}$$

$\uparrow \quad \quad \quad$   
 $3.16 \times 10^5 \text{ Hz} \quad \approx 66.2 \text{ dB}$



$$20 \log \frac{1}{\beta} = 66.2 \text{ dB}$$

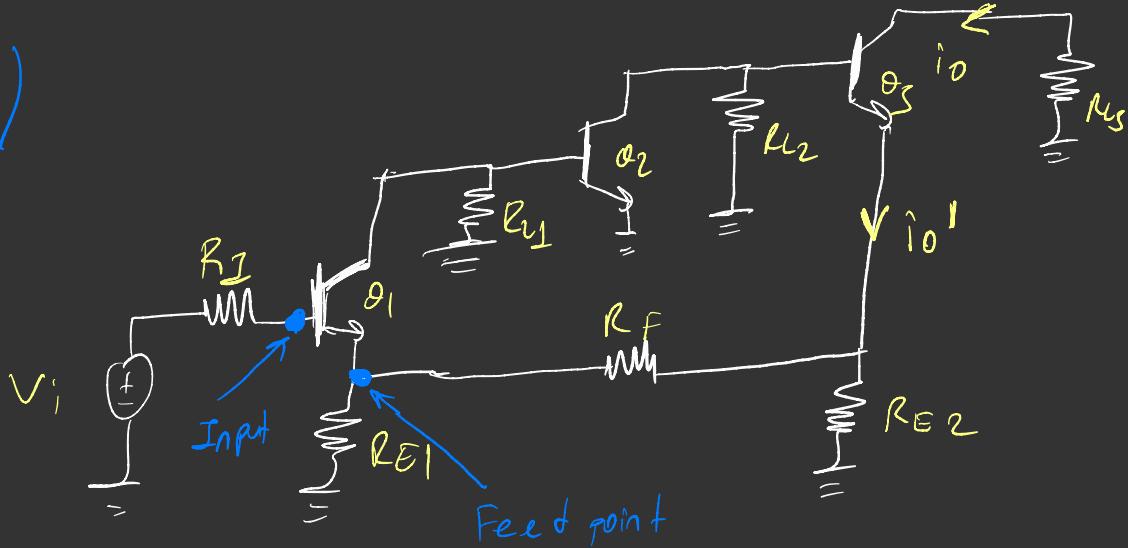
$$\frac{1}{\beta} = 2000, \quad \beta = 5 \times 10^{-5}$$

Closed loop gain

$$A_{CL} = \frac{A}{1 + A\beta} = \frac{10000}{1 + \frac{10000}{2000}} = \frac{10000}{6}$$

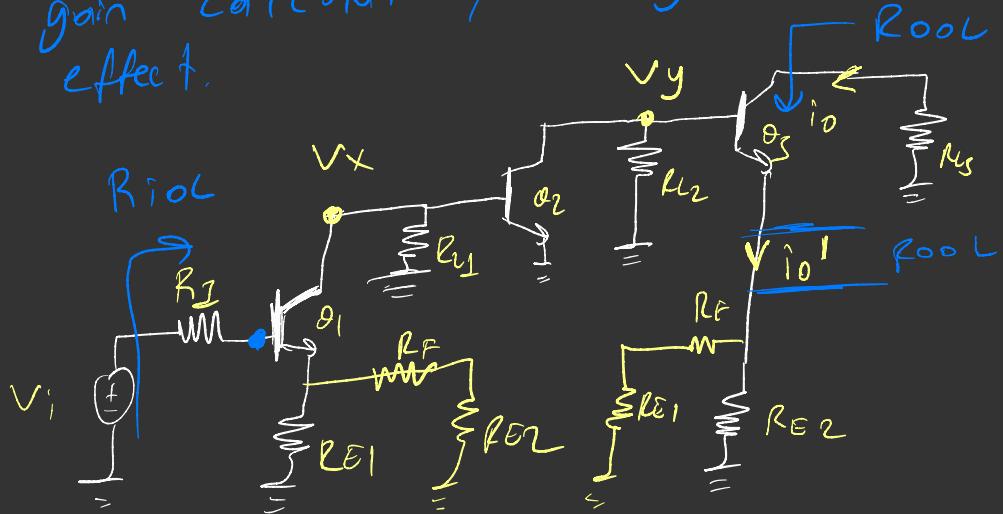
$$= 1666 \text{ or } \approx 64.4 \text{ dB}$$

3)



- Since input voltage signal and feeding network at point of the feedback we have series feedback at the input.
- At the output we are seeing of current (emitter current of  $\text{Q}_3$ ), we have series feedback at the output.
- Overall we have series-series or voltage-current feedback.

Equivalent circuit for open loop gain calculation, including the loading effect.



$$R_{E1} \parallel (R_F + R_{E2}) = R_A \quad \left. \begin{array}{l} \\ \end{array} \right\} \text{for simplicity}$$

$$R_{E2} \parallel (R_F + R_{E1}) = R_B \quad \left. \begin{array}{l} \\ \end{array} \right\}$$

$$\frac{V_x}{V_i} = \frac{\beta \left( \frac{1}{g_m 1} + R_A \right)}{\beta \left( \frac{1}{g_m 1} + R_A + R_I \right)} \cdot \frac{\left( R_{L1} \parallel \frac{\beta}{g_m 2} \right)}{\frac{1}{g_m 1} + R_A}$$

$$\frac{V_y}{V_x} = g_m 2 \cdot \left( R_{L2} \parallel \beta \left( \frac{1}{g_m 3} + R_B \right) \right)$$

$$\frac{i_o}{V_y} = \frac{1}{\frac{1}{g_m} + R_B} \quad , i_o \approx i_0$$

So, open loop transconductance

$$A_{OL} = \frac{V_x}{V_i} \frac{V_y}{V_x} \frac{i_o}{V_y}$$

I won't write the whole expression

You got the point ✓

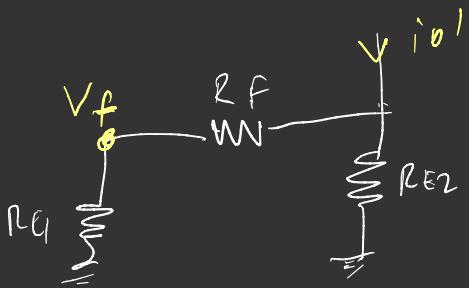
$$R_{iOL} = R_I + \beta \left( \frac{1}{g_m} + R_A \right)$$

Output resistance is a little bit tricky. Because we are applying the feedback from emitter of Q3 rather than

output collector of  $\text{Q}_3$ . we  
must find  $R_{\text{OOC}'}^1$  first.

$$R_{\text{OOC}'}^1 = R_B + \frac{1}{g_m} \quad \begin{aligned} & \text{(output resistance} \\ & \text{at emitter of } Q_3 \\ & \text{for open loop} \\ & \text{condition} \end{aligned}$$

## The feedback network



$$\boxed{\beta = \frac{i_o'}{V_f} = \frac{R_{E2}}{R_{E1} + R_{E2} + R_F} \cdot \frac{R_{E1}}{R_{E2}}}$$

## Closed Loop Gain

$$A_{CL} = \frac{A_{OL}}{1 + A_{OL}\beta} \rightarrow \text{put } A_{OL} \text{ and } \beta$$

## Closed loop input resistance

$$R_{in}^{CL} = (1 + A_{OL}\beta) \cdot R_{iOL}$$

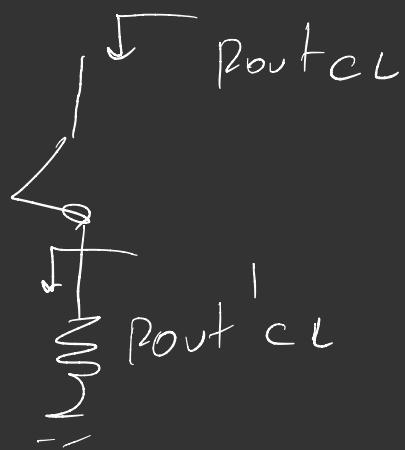
increases due to series feedback

$$R_{out}^{CL} = (1 + A_{OL}\beta) R_{oOL}$$

↳ output impedance when  
we look at from emitters of

Q3

A short discussion on  
 $r_{out\parallel CL}$  (looking from collector  
of  $\text{O}_3$ )

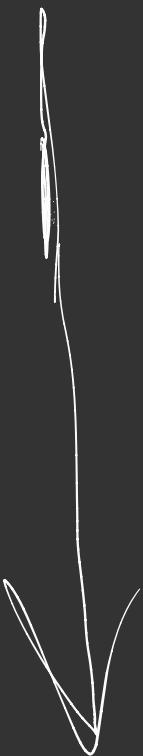


$$r_{out\parallel CL} = g_m r_{o3} r_{out\parallel CL}$$

$$\text{Since } r_{o3} = \infty \quad (V_A = \infty)$$

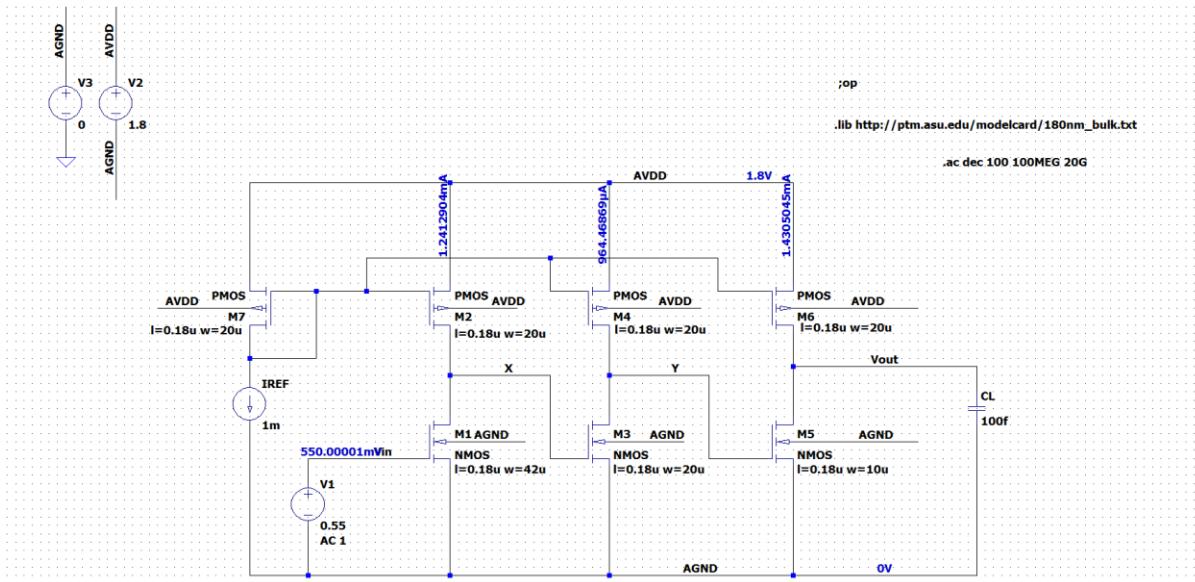
- $r_{out\parallel CL} = \infty$
- In reality it will be really high even if  $V_A \neq \infty$  !!

4) See comments for  
LT Space at the next  
page

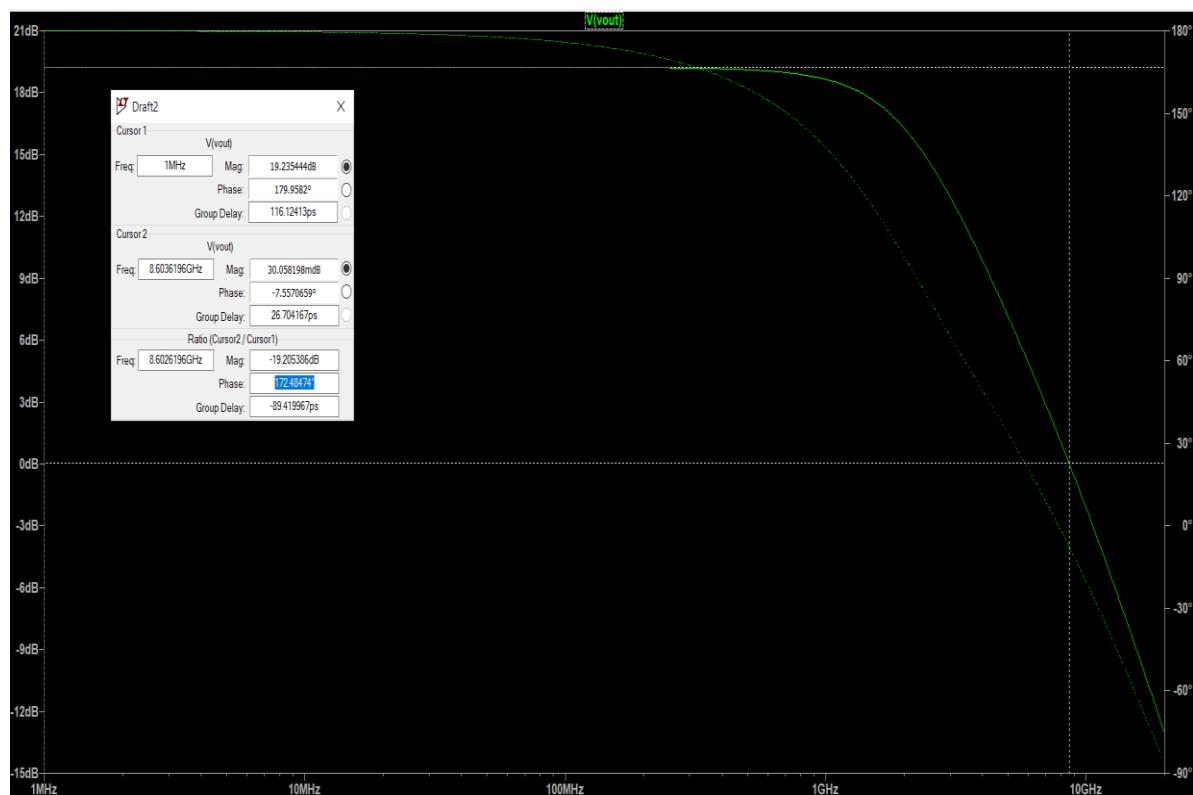


## QUESTION-4 (LTSPICE SOLUTIONS)

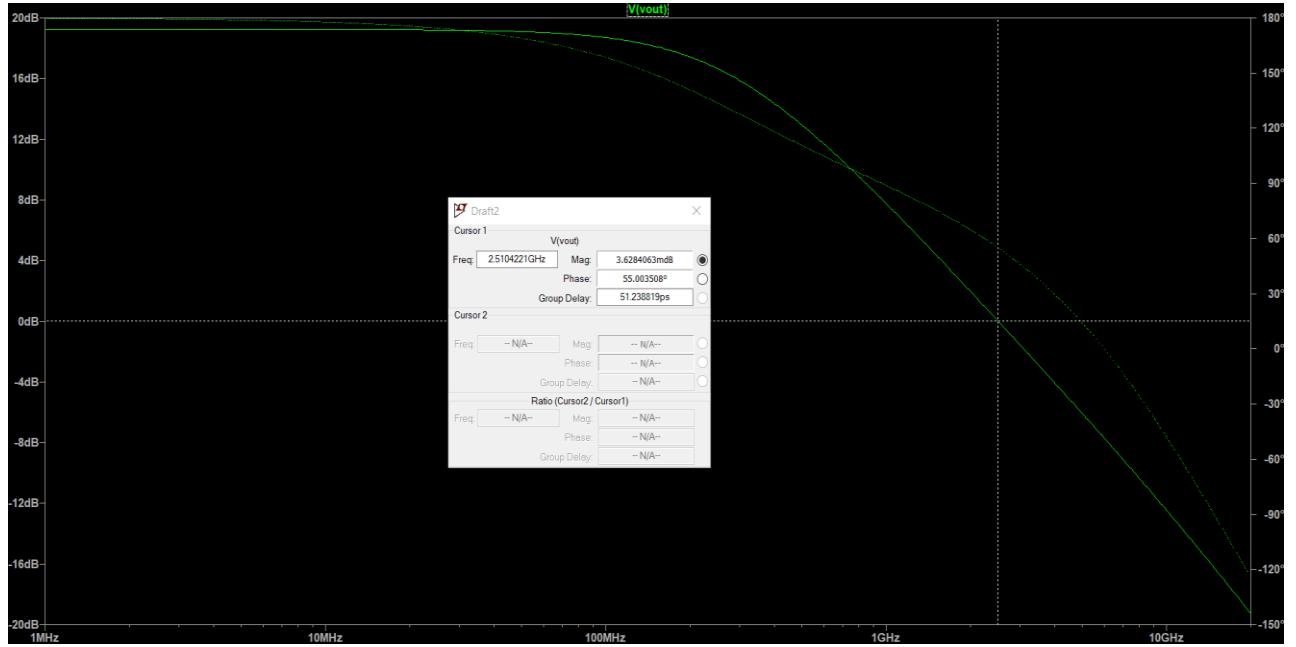
The DC conditions can be seen below in the schematic.



For the phase margin, let's discuss the frequency response. Zoom in to the cursor and you will observe approximately -8 degree phase margin, which means if you use this amplifier in the unity gain feedback mode you cannot provide the stability. It's expected as you have 3 poles that are very close to each other in frequency domain. So you need to use a compensation capacitor. (5 pts)



After putting a 83 fF capacitor between node X and Y. You can see the new phase margin and unity gain bandwidth below. Now the PM is 55 degree and GBW is 2.51 GHz. Actually we sacrificed from speed to provide stability to the amplifier. For the previous case GBW was around 8.6 GHz but we did not have stability. (10 pts)



After putting the 83 fF capacitor between X and ground, you can see the phase margin below. We expect less stability as we do not have the Miller effect. We have PM around 7 degree, which is not good since we could not even provide a 45 degree PM. In this practice you can see the benefit of Miller effect. Even though, in some cases we hate the Miller effect, for OPAMP or OTA design we exploit from that to provide stability while slowing down the operation. (10 pts)

