HW3 - 2020

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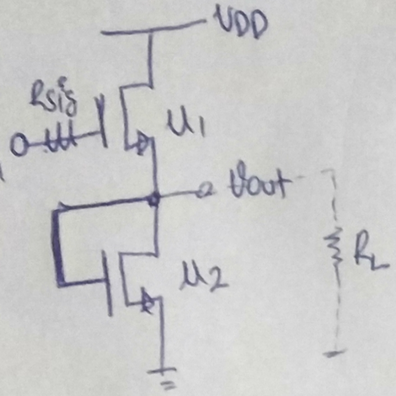
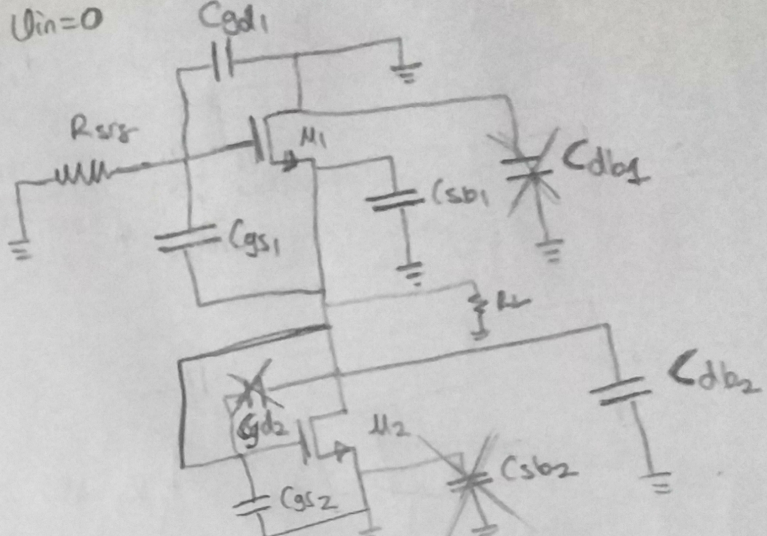
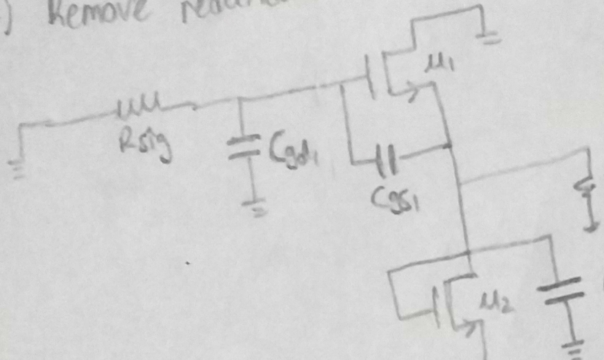
1]



1) $V_{in}=0$



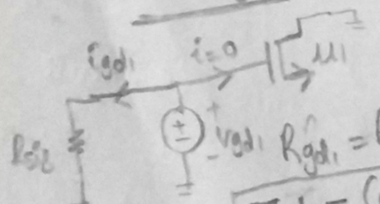
2) Remove redundancies



$C_{gs_2} + C_{db_2} + C_{sb_1} = C_{out}$

3) Consider one capacitor at time and replace others as open circuit.

For C_{gd_1}

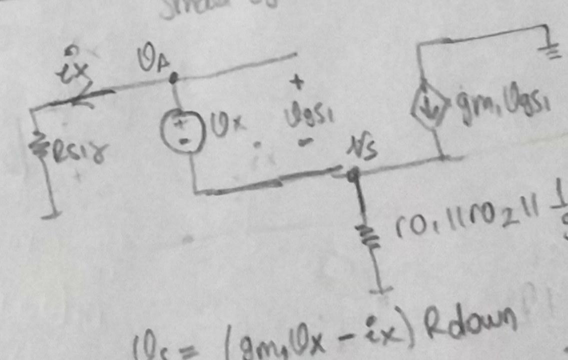


$R_{gd_1} = R_{sig}$

$$\boxed{\tau_{gd_1} = C_{gd_1} R_{gd_1}}$$

For C_{gs_1} (Since it is a floating capacitor we need to do small signal analysis)

$V_{gs_1} = V_x$



$r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}} \parallel R_L = R_{down}$

$$V_s = (g_{m_1} V_x - i_x) R_{down}$$

$$i_x R_{sig} - V_x = g_{m_1} V_x R_{down} - i_x R_{down}$$

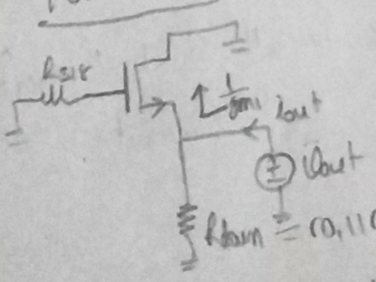
$$V_x (1 + g_{m_1} R_{down}) = i_x (R_{sig} + R_{down})$$

$$\frac{V_x}{i_x} = R_{gs_1} = \frac{R_{sig} + R_{down}}{1 + g_{m_1} R_{down}}$$

$$\Rightarrow R_{gs_1} = \frac{R_{sig} + (r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}} \parallel R_L)}{1 + g_{m_1}(r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}} \parallel R_L)}$$

$$\boxed{\tau_{gs_1} = C_{gs_1} R_{gs_1}}$$

For Cout



$R_{down} = r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_2}} \parallel R_L$

$$R_{out} = r_{o_1} \parallel r_{o_2} \parallel \frac{1}{g_{m_1}} \parallel \frac{1}{g_{m_2}} \parallel R_L$$

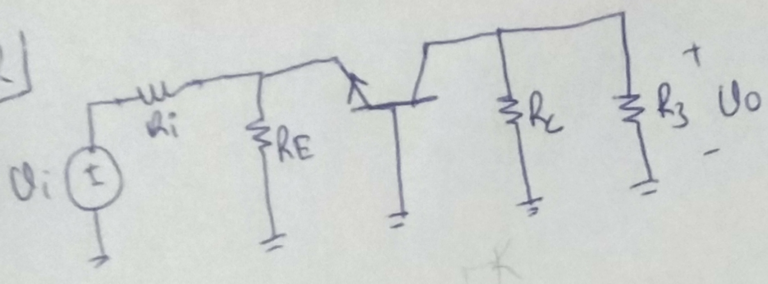
$$\tau_{out} = C_{out} R_{out}$$

Then according to open-circuit time constant methodology.

$$f_{3\text{-}dB} = \frac{1}{\sum_{i=1}^{3} \tau_i} = \frac{1}{(\tau_{out} + \tau_{gs1} + \tau_{gd1})2\pi}$$

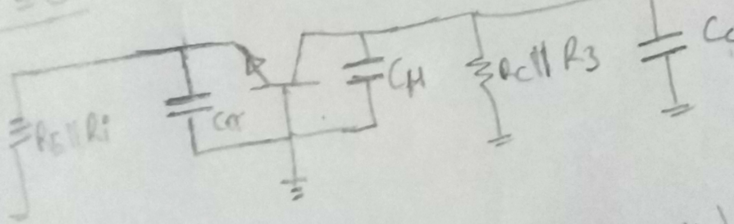
$$\boxed{f_{3\text{-}dB} = \frac{1}{2\pi\left(C_{out}R_{out} + C_{gs1}R_{gs1} + C_{gd1}R_{gd1}\right)}}$$

2)

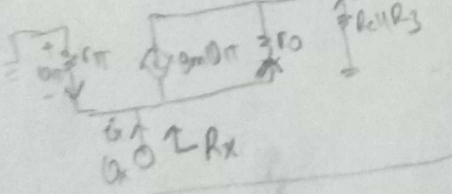


Since C_1 and C_2 are coupling capacitors, at high frequencies, they becomes short circuit.

1- $v_i = 0$



For C_π



$$v_x - \left(i_x + \frac{v_\pi}{r_\pi} + g_m v_\pi\right)r_o - \left(i_x + \frac{v_\pi}{r_\pi}\right)(R_c \| R_3) = 0$$

since $v_x = -v_\pi$

$$v_x - \left(i_x - \frac{v_x}{r_\pi} - g_m v_x\right)r_o - \left(i_x - \frac{v_x}{r_\pi}\right)(R_c \| R_3) = 0$$

$$\Rightarrow v_x - \left(i_x - \frac{v_x}{r_\pi} - g_m v_x\right)r_o - i_x\left(r_o + R_c \| R_3\right) = 0$$

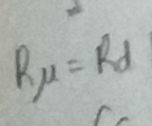
$$v_x\left(\frac{r_o}{r_\pi} + g_m r_o + \frac{R_c \| R_3}{r_\pi}\right)$$

$$R_x = \frac{r_o + (R_c \| R_3)}{\frac{r_o}{r_\pi} + g_m r_o + \frac{R_c \| R_3}{r_\pi}} = \frac{r_\pi r_o + (R_c \| R_3) r_\pi}{r_o + g_m r_\pi r_o + (R_c \| R_3)}$$

$$R_x = \frac{r_\pi\left(r_o + (R_c \| R_3)\right)}{(\beta + 1)r_o + (R_c \| R_3)} \Rightarrow \boxed{R_\pi = R_x \| R_E \| R_i}$$

$$\tau_\pi = C_\pi R_\pi = C_\pi (R_x \| R_E \| R_i)$$

For $C_\mu + C_{cb}$



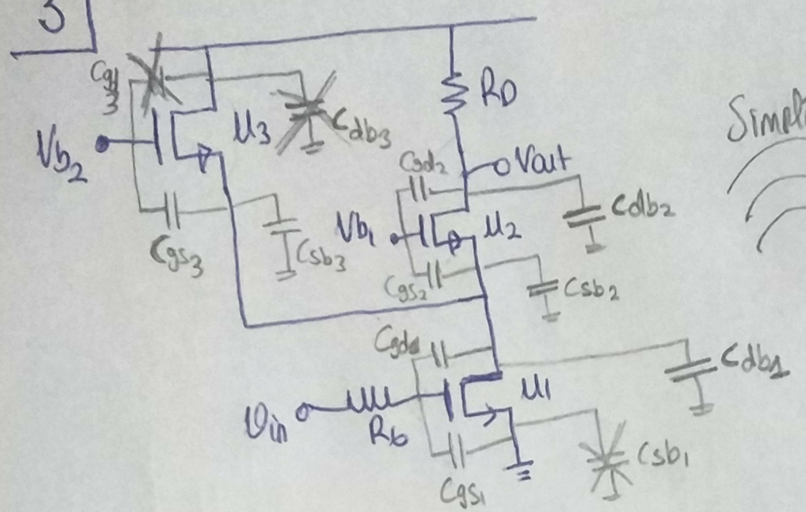
$$\Rightarrow R_d = r_o + (R_i \| R_E \| r_\pi) + g_m r_o (R_i \| R_E \| r_\pi)$$

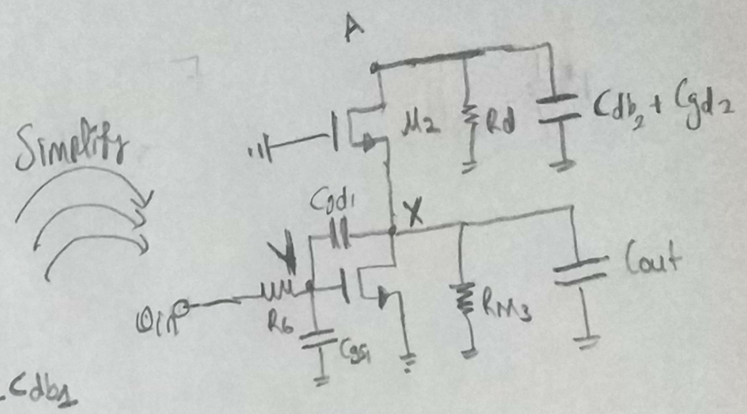
$$R_\mu = R_d \| R_c \| R_3 \approx R_c \| R_3 \| \left(g_m r_o (R_i \| R_E \| r_\pi)\right)$$

$$\tau_\mu = (C_\mu + C_{cb})\left[R_c \| R_3 \| \left(g_m r_o (R_i \| R_E \| r_\pi)\right)\right]$$

$$f_{3\text{-}dB} = \frac{1}{2\pi(\tau_\mu + \tau_\pi)}$$

3)



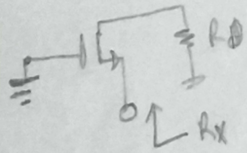


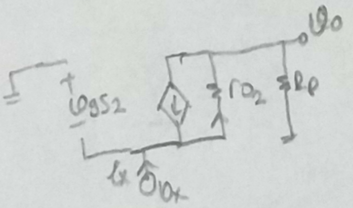
where

$C_{out} = C_{db_1} + C_{gs_2} + C_{sb_2} + C_{gs_3} + C_{sb_3}$

There is only one floating capacitor, C_{gd_1}, and since we're asked to apply Miller's approximation, we need to calculate the DC gain between the terminals of C_{gd_1}.

In order to find DC gain we need to find Rout for which Vout is taken from the source of M_1.





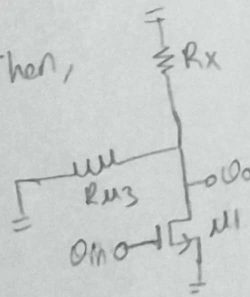
$v_x - (i_x + g_{m_2} v_{gs_2}) r_{o_2} - i_x R_D = 0$ ($v_{gs_2} = -v_x$)

$v_x (1 + g_{m_2} r_{o_2}) = i_x (r_{o_2} + R_D)$

$$R_X = \frac{v_x}{i_x} = \frac{r_{o_2} + R_D}{1 + g_{m_2} r_{o_2}} = \frac{(1/g_{m_2})(r_{o_2} + R_D)}{\frac{1}{g_{m_2}} + r_{o_2}}$$

$$R_X \approx \frac{r_{o_2} + R_D}{g_{m_2} r_{o_2}} \quad (\text{since } r_{o_2} \gg \tfrac{1}{g_{m_2}})$$

Then,



$\Rightarrow$ $G_m = g_{m_1}$

$R_{out} = R_X \parallel R_{M_3} \parallel r_{o_1}$

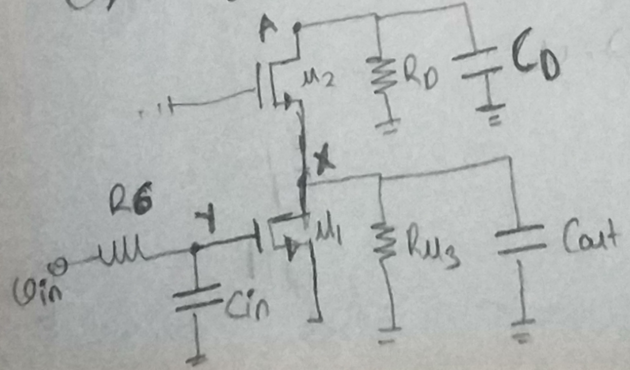
$$R_{out} = \left(\frac{r_{o_2} + R_D}{g_{m_2} r_{o_2}}\right) \parallel \left(\frac{1}{g_{m_3}} \parallel r_{o_3}\right) \parallel r_{o_1}$$

$\Rightarrow$ DC gain between the terminals of the floating capacitor is $A_V = G_m R_{out}$

$$A_V = g_{m_1}\left[\left(\frac{r_{o_2} + R_D}{g_{m_2} r_{o_2}}\right) \parallel \left(\frac{1}{g_{m_3}} \parallel r_{o_3}\right) \parallel r_{o_1}\right]$$

Now we seperate C_{gd_1} into two standart capacitors by using Miller's theorem, and draw the new model

$C_Y = C_{gd_1}(1 + A_V)$; $C_X = C_{gd_1}\left(1 + \frac{1}{A_V}\right)$, and draw the new model



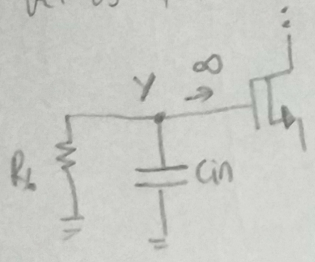
where $C_D = C_{db_2} + C_{gd_2}$

$C_{in} = C_Y + C_{gs_1}$

$C_{out} = C_X + C_{db_1} + C_{gs_2} + C_{sb_2} + C_{gs_3} + C_{sb_3}$

Now, we obtained the model that we can apply the pole identification on

Let us first examine input pole which stems from the capacitor C_{in}



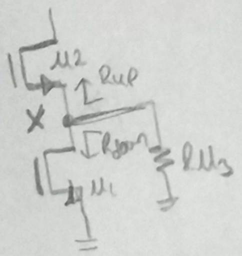
$$R_Y = R_6$$

$$\tau_Y = C_{in} R_6$$

$$f_{in} = \frac{1}{2\pi C_{in} R_6}$$

Then, examine Cout

we have found that $R_{up} \simeq \frac{r_{O2} + R_O}{g_{m2} r_{O2}}$



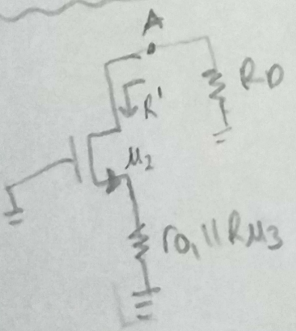
and we know $R_{down} = r_{O1}$

Then $R_X = R_{M3} \parallel R_{up} \parallel R_{down}$

$$\Rightarrow R_X = \left(\frac{1}{g_{m3}} \parallel r_{O3}\right) \parallel \left(\frac{r_{O2} + R_O}{g_{m2} r_{O2}}\right) \parallel r_{O1}$$

$$\Rightarrow \tau_X = C_{out} R_X \Rightarrow f_{out} = \frac{1}{2\pi C_{out} R_X}$$

Lastly, examine Cup



$$R' = r_{O2} + (r_{O1} \parallel R_{M3}) + g_{m2} r_{O2} (r_{O1} \parallel R_{M3})$$

$$R_A = R_O \parallel R' = R_O \parallel \left[r_{O2} + (1 + g_{m2} r_{O2})(r_{O1} \parallel R_{M3}) \right]$$

$$\tau_A = R_A C_O$$

$$f_A = \frac{1}{2\pi (R_A C_O)}$$

Node's names

X: Drain of M_1 and Source of M_2

Y: Gate of M_1

A: Drain of M_3

R_{M3}: The resistance seen looking into the source of M_3