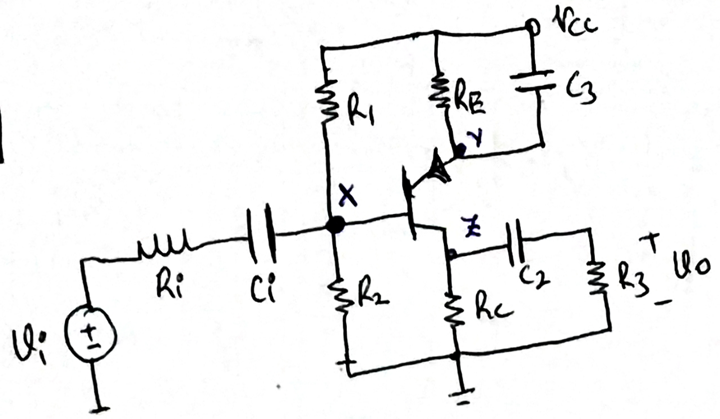
1]



$V_A < \infty$

C_1, C_2 : coupling caps.

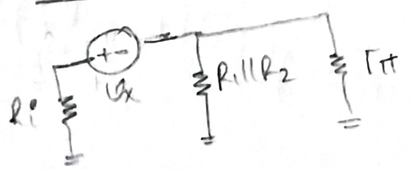
C_3 : bypass cap.

a) Each node introduces one pole

$v_{sig} = v_i = 0$;

We're treating C_i and thinking others as short circuit because in the frequency of interest, impedances of coupling and bypass caps are small (caps values are large)

At node X



Since C_3 is shorted, R_E is also shorted.

Let's call $R_1 \| R_2 = R_B$

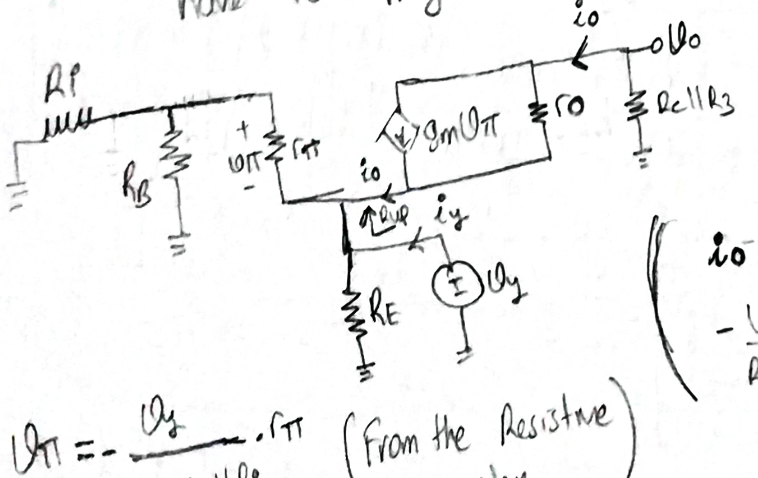
$R_x = R_i + R_B \| r_\pi \Rightarrow \tau_x = R_x C_i \Rightarrow w_{p_1} = \frac{1}{\tau_x}$

$$w_{p_1} = \frac{1}{\tau_x} = \frac{1}{R_x C_i} \Rightarrow f_1 = \frac{1}{2\pi C_i (R_i + R_B \| r_\pi)}$$

At node Y

* Since $V_A < \infty$, we cannot simply determine R_y by dividing base resistances, we have to apply small-signal model

Let us exclude R_E for now (to find R_{up})



$$i_y = -\left(\frac{v_\pi}{r_\pi} + g_m v_\pi + \frac{v_o - v_y}{r_o}\right)$$

$$i_y = -\left(\frac{1}{r_\pi} + g_m\right) v_\pi + \frac{v_y}{r_o} - \frac{v_o}{r_o}$$

$$i_o - g_m v_\pi = \frac{v_o - v_y}{r_o}$$

$$-\frac{v_o}{R_c \| R_3} - g_m v_\pi = \frac{1}{r_o} v_o - \frac{1}{r_o} v_y$$

$$v_\pi = -\frac{v_y}{r_\pi + R_B \| R_i} \cdot r_\pi \quad \text{(From the Resistive Divider)}$$

Now we have 3 equations

1) $i_y = -\left(\frac{1}{r_\pi} + g_m\right) v_\pi + \frac{1}{r_o} v_y - \frac{1}{r_o} v_o$

2) $-\frac{v_o}{R_C \| R_3} - g_m v_\pi = \frac{1}{r_o} v_o - \frac{1}{r_o} v_y$

3) $v_\pi = -\frac{r_\pi}{r_\pi + R_i \| R_B} v_y$

consider equation (2), and put v_π from (3) into (2)

$$\frac{1}{r_o} v_y - \left(\frac{1}{r_o} + \frac{1}{R_C \| R_3}\right) v_o - g_m\left(-\frac{r_\pi}{r_\pi + R_i \| R_B}\right) v_y = 0$$

$$\left(\frac{1}{r_o} + \frac{g_m r_\pi}{r_\pi + R_i \| R_B}\right) v_y = -\left(\frac{1}{r_o} + \frac{1}{R_C \| R_3}\right) v_o$$

$$\Rightarrow v_o = \frac{\left(\frac{1}{r_o} + \frac{\beta}{r_\pi + R_i \| R_B}\right)}{\frac{1}{r_o} + \frac{1}{R_C \| R_3}} v_y = \left(\frac{1}{r_o} + \frac{\beta}{r_\pi + R_i \| R_B}\right)(r_o \| R_C \| R_3)\, v_y$$

Now consider equation (1)

$i_y = -\left(\frac{1}{r_\pi} + g_m\right) v_\pi + \frac{1}{r_o} v_y - \frac{1}{r_o} v_o$ (put v_o and v_π values obtained above)

$$i_y = \left(\frac{1}{r_\pi} + g_m\right)\left(\frac{r_\pi}{r_\pi + R_i \| R_B}\right) v_y + \frac{1}{r_o} v_y - \frac{1}{r_o}\left[\left(\frac{1}{r_o} + \frac{\beta}{r_\pi + R_i \| R_B}\right)(r_o \| R_C \| R_3)\right] v_y$$

$$\frac{v_y}{i_y} = R_{up} = \frac{1}{\frac{\beta+1}{r_\pi + R_i \| R_B} + \frac{1}{r_o}\left[1 - \left(\frac{1}{r_o} + \frac{\beta}{r_\pi + R_i \| R_B}\right)(r_o \| R_C \| R_3)\right]}$$

(we can see that $R_{up} = \frac{1}{g_m} + \frac{R_i \| R_B}{\beta + 1}$ as $r_o \to \infty$, which is satisfying)

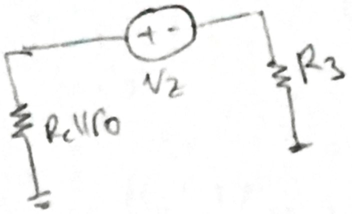
$$R_y = R_E \| R_{up} \Rightarrow \tau_y = R_y C_3 \Rightarrow w_{p1} = \frac{1}{\tau_y}$$

$$w_{p1} = \frac{1}{R_y C_3} \Rightarrow \boxed{f_3 = \frac{1}{2\pi R_y C_3}}$$

At node Z

Since emitter is grounded due to C_3.
r_o becomes parallel to R_C

$$R_Z = R_3 + R_C \| r_o$$
$$\tau_Z = C_2 R_Z$$
$$f_2 = \frac{1}{2\pi \tau_Z R_Z} = \frac{1}{2\pi R_Z C_2}$$



Since we're applying the short-circuit time constant method, 3-dB lower frequency is approximately

$$f_{L,3dB} \approx f_1 + f_2 + f_3$$

$$f_{L,3dB} \approx \frac{1}{2\pi R_X C_1} + \frac{1}{2\pi R_Z C_2} + \frac{1}{2\pi R_Y C_3}$$

b) At midband frequencies, all capacitors are shorted due to low impedance then the amplifier type becomes typical CE amplifier whose gain is

$$\frac{v_{out}}{v_x} = -g_m (R_C \| R_3 \| r_o)$$

and since the signal is attenuated due to source resistance R_i

$$v_{th} = \frac{R_1 \| R_2}{R_1 \| R_2 + R_i} \cdot v_i$$

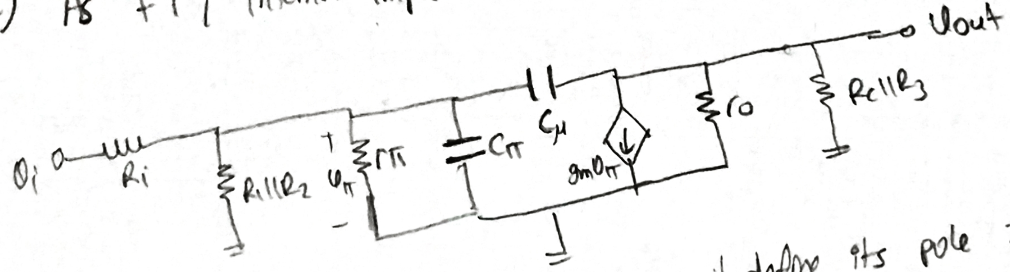
$$R_{th} = R_i \| R_1 \| R_2$$

$$\frac{v_{out}}{v_{th}} = \frac{r_\pi}{r_\pi + R_{th}} \cdot \left[-g_m (R_C \| R_3 \| r_o)\right]$$

$$A_v = \frac{v_{out}}{v_{th}} \cdot \frac{v_{th}}{v_i}$$

$$A_v = \frac{v_{out}}{v_{in}} = -\frac{R_1 \| R_2}{R_1 \| R_2 + R_i} \cdot \frac{g_m r_\pi (R_C \| R_3 \| r_o)}{r_\pi + R_1 \| R_2 \| R_i}$$

c) As $f \uparrow$, internal impedances becomes effective in gain expression.



Since C_μ is a intact capacitor, we can't define its pole just looking the resistance it sees. By miller theorem we can seperate C_μ into two ground touching capacitors.

$$A_v = -g_m (R_C \| R_3 \| r_o)$$

$$C_{\mu B} = C_\mu \big(1 - \underbrace{(-g_m (R_C \| R_3 \| r_o))}_{A_v}\big) = C_\mu \big(1 + g_m (R_C \| R_3 \| r_o)\big)$$

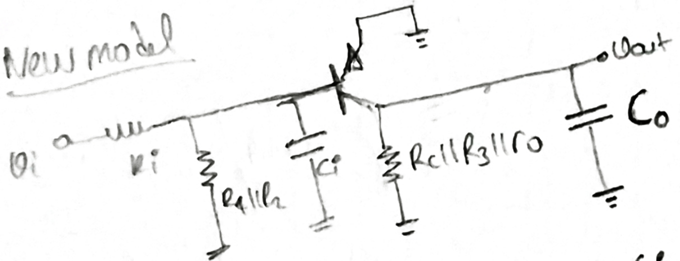
$$C_{\mu C} = C_\mu \left(1 - \frac{1}{A_v}\right) = C_\mu \left(1 - \frac{1}{-g_m (R_C \| R_3 \| r_o)}\right) = C_\mu \left(1 + \frac{1}{g_m (R_C \| r_o \| R_3)}\right)$$

& C_π and $C_{\mu B}$ are in shunt, we can see them as a one capacitor

$C_i = C_\pi + C_{\mu B}$; $C_o = C_{\mu C} + C_{CS}$ → collector-substrate depletion capacitance.

→ These are also in shunt

New model



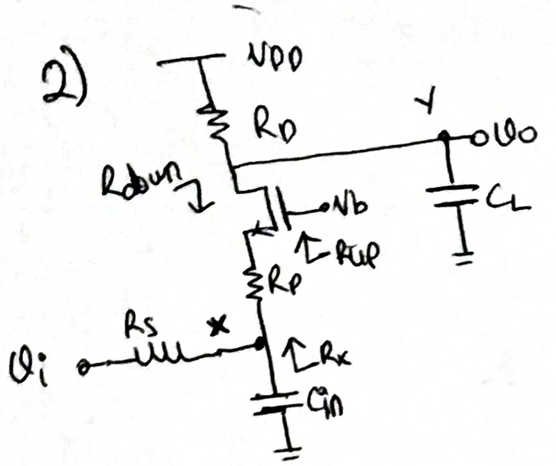
$R_i = R_i \| R_1 \| R_2 \| r_\pi$; $Z_i = R_i C_i \Rightarrow f_1 = \dfrac{1}{2\pi R_i C_i}$

$$\Rightarrow f_1 = \frac{1}{2\pi (R_i \| R_1 \| R_2 \| r_\pi)\big(C_\pi + C_\mu (1 + g_m (R_C \| R_3 \| r_o))\big)}$$

$R_o = R_C \| R_3 \| r_o$; $Z_o = R_o C_o \Rightarrow f_2 = \dfrac{1}{2\pi R_o C_o}$

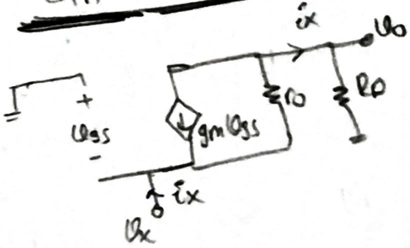
$$\Rightarrow f_2 = \frac{1}{2\pi (R_C \| R_3 \| r_o)\left(C_{CS} + C_\mu \left(1 + \frac{1}{g_m (R_C \| R_3 \| r_o)}\right)\right)}$$

2)



In order to determine pole frequencies, we need the resistances seen by capacitors

Cin (input pole)



$v_{gs} = -v_x$

$i_x = -\left(g_m v_{gs} + \frac{v_o - v_x}{r_o}\right)$

$i_x = \frac{v_o}{R_D}$

$$\frac{v_o}{R_D} = g_m v_x - \frac{1}{r_o} v_o + \frac{1}{r_o} v_x \quad \Rightarrow \quad v_o = \frac{g_m r_o + 1}{\cancel{r_o}} \cdot \frac{\cancel{r_o} \cdot R_D}{r_o + R_D} v_x$$

$$v_o\left(\frac{1}{R_D} + \frac{1}{r_o}\right) = \left(g_m + \frac{1}{r_o}\right) v_x$$

$$\Rightarrow i_x = \frac{1}{R_D} v_o = \frac{1}{\cancel{R_D}}\left(\frac{g_m r_o + 1}{r_o + R_D} \cdot \cancel{R_D}\right) v_x \quad \Rightarrow \frac{v_x}{i_x} = \frac{r_o + R_D}{g_m r_o + 1} \approx \frac{r_o + R_D}{g_m r_o}$$

$$\Rightarrow R_x = R_s \parallel \left(R_p + \frac{r_o + R_D}{g_m r_o}\right) \quad \Rightarrow f_1 = \frac{1}{2\pi C_{in}\left(R_s \parallel \left(R_p + \frac{r_o + R_D}{g_m r_o}\right)\right)}$$

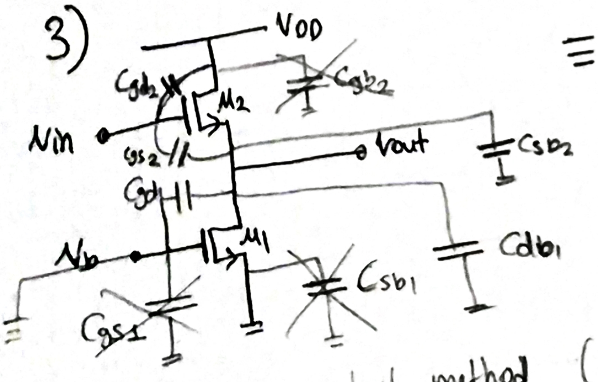
CL (output pole)

$$R_{down} = r_o + (R_p + R_s) + g_m r_o (R_p + R_s) \approx g_m r_o (R_p + R_s)$$

$$R_y = R_{down} \parallel R_D = [g_m r_o (R_p + R_s)] \parallel R_D$$

$$f_2 = \frac{1}{2\pi C_L [g_m r_o (R_p + R_s) \parallel R_D]}$$

3)



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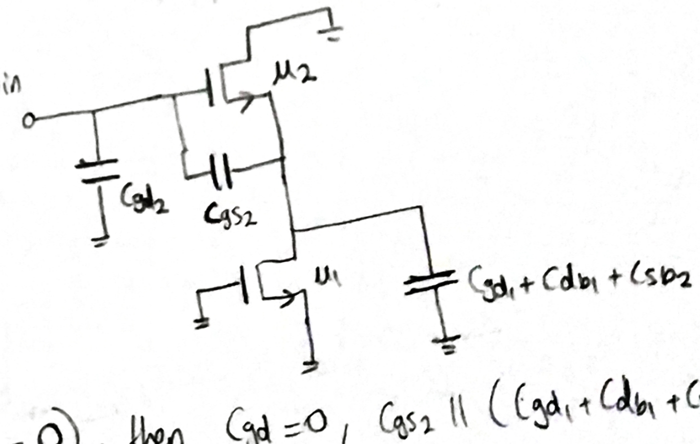
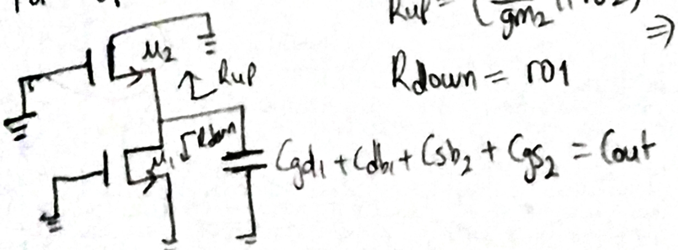


For open-time constant method ($v_{sig} = 0$), then $C_{gd} = 0$, $C_{gs2} \parallel (C_{gd1} + C_{db1} + C_{sb2})$



$R_{up} = \left(\frac{1}{g_{m2}} \parallel r_{o2}\right)$

$R_{down} = r_{o1}$

$\Rightarrow$ $R_{out} = r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}$

$$f_1 = \frac{1}{2\pi\left(r_{o1} \parallel r_{o2} \parallel \frac{1}{g_{m2}}\right)\left(C_{gd1} + C_{db1} + C_{sb2} + C_{gs2}\right)}$$

↳ only one pole, because all caps are tied to the same node.