Homework - 1

05.11.20[illegible]
I.Ş. [illegible]
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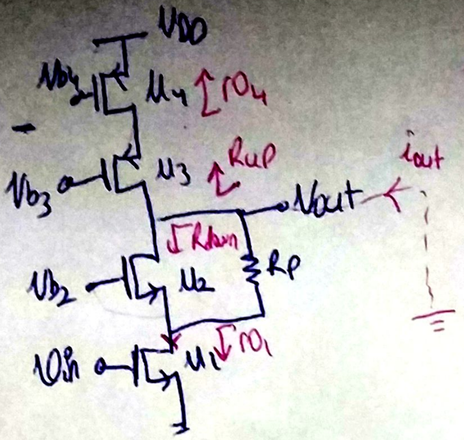
1 -



Gm

$$i_{out} = g_{m1} v_{in}$$

$$G_m = \frac{i_{out}}{v_{in}}\Big|_{v_{out}=0} = g_{m1}$$

Rout

$$R_{up} = r_{o4} + r_{o3} + g_{m3} r_{o3} r_{o4}$$

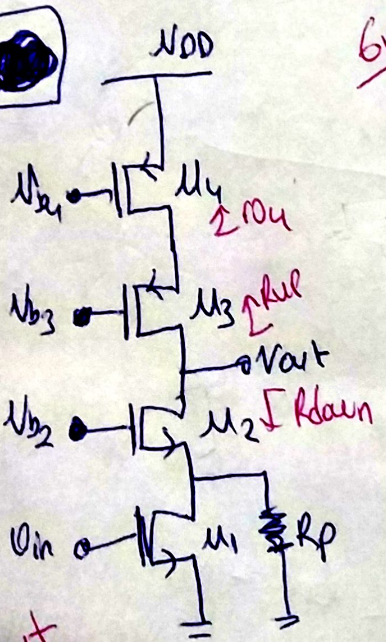
$$R_{down} = r_{o1} + (r_{o2} \| R_P) + g_{m2} (r_{o2} \| R_P) r_{o1}$$

$$R_{out} = R_{up} \| R_{down}$$

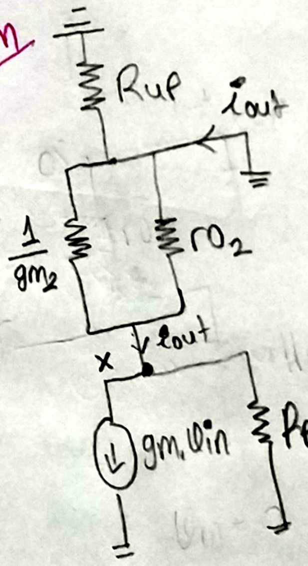
$$A_v = -G_m R_{out} = -g_{m1} \Big[(r_{o4} + r_{o3} + g_{m3} r_{o3} r_{o4}) \| \big(r_{o1} + (r_{o2} \| R_P) + g_{m2} (r_{o2} \| R_P) r_{o1} \big) \Big]$$

$$A_v \simeq -g_{m1} \big[g_{m3} r_{o3} r_{o4} \| g_{m2} (r_{o2} \| R_P) r_{o1} \big]$$

[redacted]



Gm



@ X:

$$0 - V_X = i_{out} \left(\frac{1}{g_{m2}} \| r_{o2} \right)$$

$$V_X = (R_P \| r_{o1}) (i_{out} - g_{m1} v_{in})$$

$$-i_{out} \left(\frac{1}{g_{m2}} \| r_{o2} \right) = (R_P \| r_{o1}) (i_{out} - g_{m1} v_{in})$$

$$G_m = \frac{i_{out}}{v_{in}}\Big|_{v_{out}=0} = \frac{(R_P \| r_{o1}) g_{m1}}{(R_P \| r_{o1}) + \left(\frac{1}{g_{m2}} \| r_{o2} \right)}$$

Rout

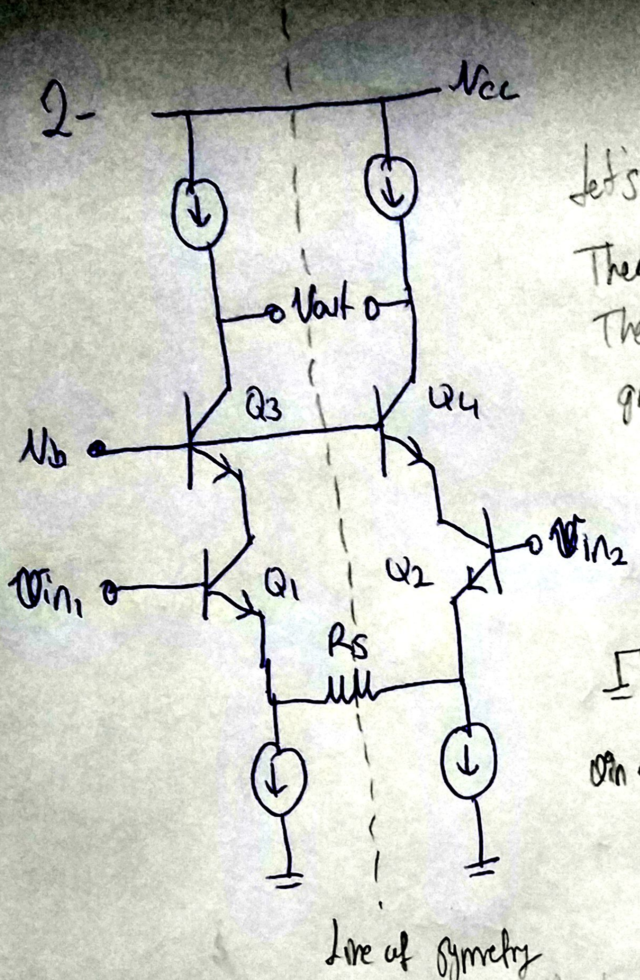
$$R_{up} = r_{o4} + r_{o3} + g_{m3} r_{o3} r_{o4} \; ; \; R_{down} = r_{o2} + (r_{o1} \| R_P) + g_{m2} r_{o2} (r_{o1} \| R_P)$$

$$R_{out} = R_{up} \| R_{down} = \big[r_{o4} + r_{o3} + g_{m3} r_{o3} r_{o4} \big] \| \big[r_{o2} + (r_{o1} \| R_P) + g_{m2} r_{o2} (r_{o1} \| R_P) \big]$$

$$A_v = -G_m R_{out} = - \frac{(R_P \| r_{o1}) g_{m1}}{(R_P \| r_{o1}) + \left(\frac{1}{g_{m2}} \| r_{o2} \right)} \Big\{ \big[r_{o3} + (1 + g_{m3} r_{o3}) r_{o4} \big] \| \big[r_{o2} + (1 + g_{m2} r_{o2}) (r_{o1} \| R_P) \big] \Big\}$$

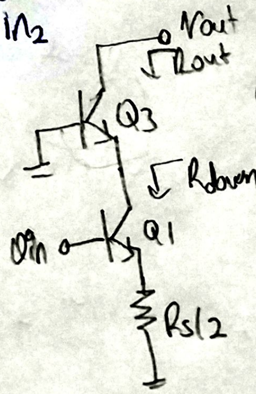
$$A_v \simeq - \frac{g_{m1} (R_P \| r_{o1})}{(R_P \| r_{o1}) + \frac{1}{g_{m2}}} \Big[(g_{m3} r_{o3} r_{o4}) \| \big(g_{m2} r_{o2} (r_{o1} \| R_P) \big) \Big]$$

2-



Let's assume $Q_3 = Q_4$ and $Q_1 = Q_2$
Then if the input voltages are differential
Then the line of symmetry is the virtual ground. (ac ground)

<u>Half-Circuit</u>



$$G_m = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0} = \frac{g_{m1}}{1 + g_{m1}\frac{R_S}{2}}$$

$$R_{down} = r_{o1} + (1 + g_{m1} r_{o1})\left(\frac{R_S}{2} \,||\, r_{\pi 1}\right)$$

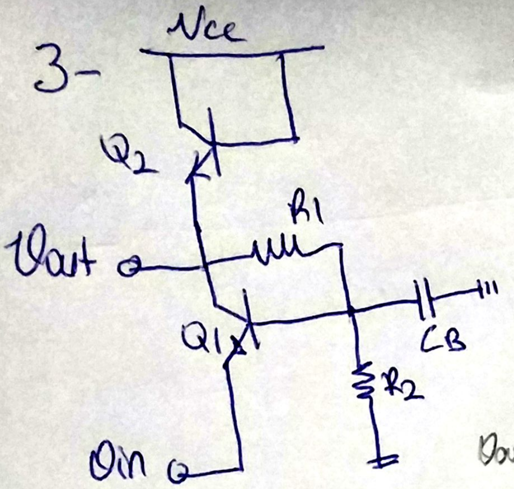
$$R_{out} = r_{o3} + (1 + g_{m3} r_{o3})(R_{down} \,||\, r_{\pi 3})$$

$$A_{DM} = -G_m R_{out}$$

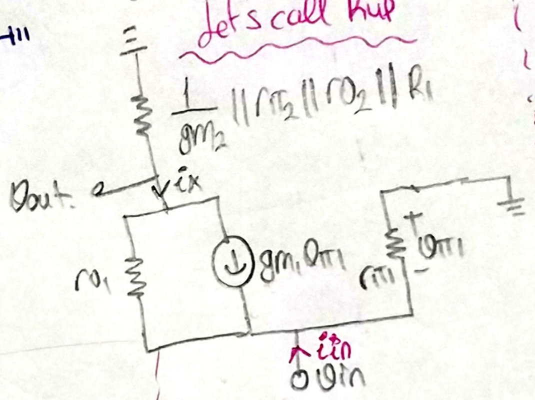
$$A_{DM} = -\frac{g_{m1}}{1 + g_{m1}\frac{R_S}{2}}\left[r_{o3} + (1 + g_{m3} r_{o3})\left(\underbrace{\left[r_{o1} + (1 + g_{m1} r_{o1})\left(\frac{R_S}{2} \,||\, r_{\pi 1}\right)\right]}_{R_{down}} \,||\, r_{\pi 3}\right)\right]$$

$$A_{DM} \approx -\frac{g_{m1}}{1 + g_{m1}\frac{R_S}{2}}\left[g_{m3} r_{o3}\left(g_{m1} r_{o1}\left(\frac{R_S}{2} \,||\, r_{\pi 1}\right) \,||\, r_{\pi 3}\right)\right]$$

3-



Assume we are operating in the midband frequency and C_B is very large. Then in the small-signal analysis C_B is short circuit.



Gain Calculation

1st way

$v_{out} = 0 \Rightarrow v_{Rup} = 0$

$i_{out} \simeq g_{m_1} v_{\pi_1}$; $v_{\pi_1} = -v_{in}$

$G_m = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0} = -g_{m_1}$

$R_{out} = R_{up} \| r_{o_1}$ $(v_{in} = 0)$

$A_v = -G_m R_{out}$

$A_v = g_{m_1}(R_{up} \| r_{o_1})$

$A_v = g_{m_1}\left(\frac{1}{g_{m_2}} \| r_{\pi_2} \| r_{o_2} \| R_1 \| r_{o_1}\right)$

2nd way

$v_{out} = -i_x R_{up} \Rightarrow i_x = -\frac{v_{out}}{R_{up}}$

$v_{out} - v_{in} = r_{o_1}(i_x - g_{m_1} v_{\pi_1})$; $v_{\pi_1} = -v_{in}$

$v_{out} - v_{in} = r_{o_1}\left(-\frac{v_{out}}{R_{up}} + g_{m_1} v_{in}\right)$

$v_{out}\left(1 + \frac{r_{o_1}}{R_{up}}\right) = (1 + g_{m_1} r_{o_1}) v_{in} \Rightarrow \frac{v_{out}}{v_{in}} = A_v = \frac{1 + g_{m_1} r_{o_1}}{1 + \frac{r_{o_1}}{R_{up}}} = \frac{1/r_{o_1} + g_{m_1}}{1/r_{o_1} + \frac{1}{R_{up}}}$

$\Rightarrow A_v \simeq \frac{g_{m_1}}{\frac{1}{r_{o_1}} + \frac{1}{R_{up}}} \Rightarrow A_v = g_{m_1}(R_{up} \| r_{o_1}) = g_{m_1}\left(\frac{1}{g_{m_2}} \| r_{\pi_2} \| r_{o_2} \| R_1 \| r_{o_1}\right)$

Rin Calculation

We found that : $v_{out} = g_{m_1}(R_{up} \| r_{o_1}) v_{in}$; $v_{\pi_1} = -v_{in}$

$\Rightarrow i_{in} = -\left(i_x + \frac{v_{\pi_1}}{r_{\pi_1}}\right) = \frac{v_{out}}{R_{up}} + \frac{v_{in}}{r_{\pi_1}} = \frac{1}{R_{up}}\left(g_{m_1}(R_{up} \| r_{o_1}) v_{in}\right) + \frac{v_{in}}{r_{\pi_1}}$

$\Rightarrow i_{in} = v_{in}\left(\frac{g_{m_1}(R_{up} \| r_{o_1})}{R_{up}} + \frac{1}{r_{\pi_1}}\right)$

$\Rightarrow R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{\frac{g_{m_1}(R_{up} \| r_{o_1})}{R_{up}} + \frac{1}{r_{\pi_1}}}$, where $R_{up} = \frac{1}{g_{m_2}} \| r_{\pi_2} \| r_{o_2} \| R_1$

R_{in} if $V_A \to \infty$, $R_{in} = \frac{1}{g_{m_1} + \frac{1}{r_{\pi_1}}} = \left(r_{\pi_1} \| \frac{1}{g_m}\right) \simeq \frac{1}{g_m}$