**– EHB 335E – ANALOG ELECTRONICS – 11219 –**

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***040170025***

# HW3 - 2020

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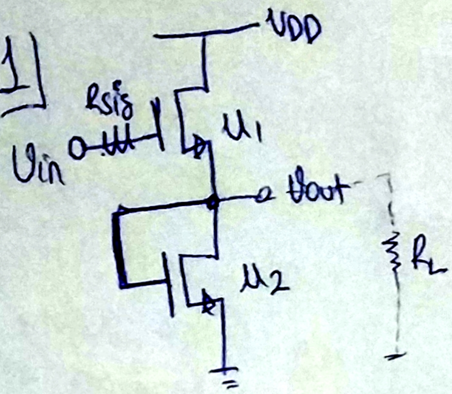
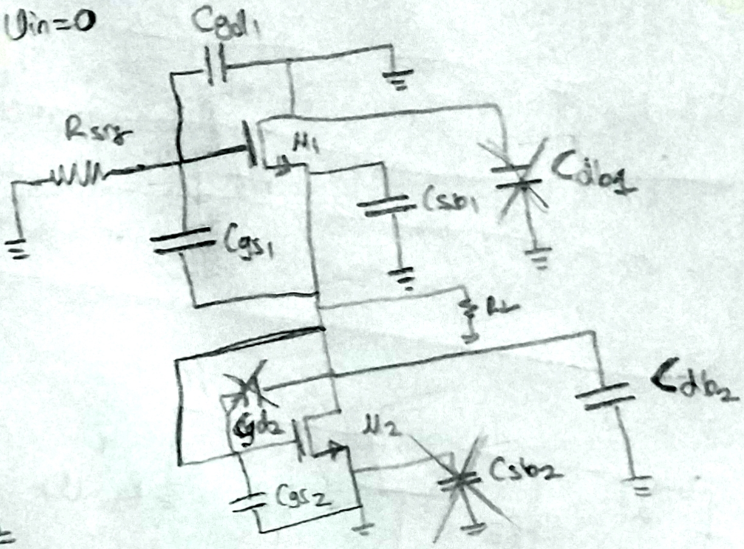
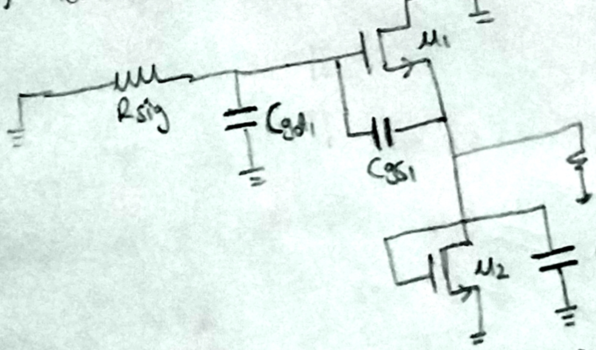
1)




1) $V_{in}=0$



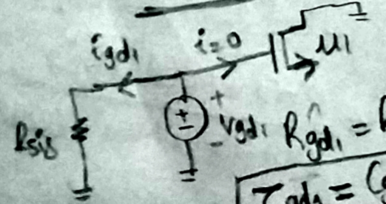

2) Remove redundancies




$C_{gs_2}+C_{db_2}+C_{sb_1}=C_{out}$

3) Consider one capacitor at time and replace others as open circuit.

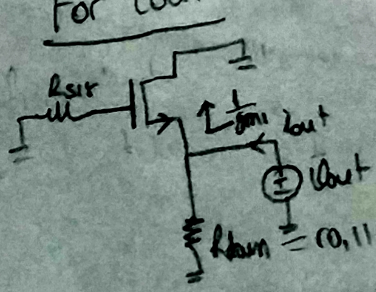
For $C_{gd_1}$




$R_{gd_1}=R_{sig}$

$\boxed{\tau_{gd_1}=C_{gd_1}R_{gd_1}}$

For Cout



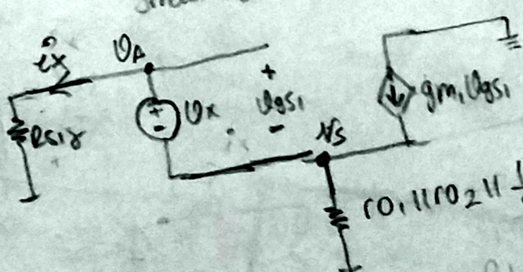

$R_{down}=r_{o_1}\|r_{o_2}\|\frac{1}{g_{m_2}}\|R_L$

$R_{out}=r_{o_1}\|r_{o_2}\|\frac{1}{g_{m_1}}\|\frac{1}{g_{m_2}}\|R_L$

$\tau_{out}=C_{out}R_{out}$

For $C_{gs1}$ (since it is a floating capacitor we need to do small signal analysis)

$V_{gs_1}=V_x$




$r_{o_1}\|r_{o_2}\|\frac{1}{g_{m_2}}\|R_L=R_{down}$

$V_s=(g_{m_1}V_x-i_x)R_{down}$

$i_xR_{sig}-V_x=g_{m_1}V_xR_{down}-i_xR_{down}$

$V_x(1+g_{m_1}R_{down})=i_x(R_{sig}+R_{down})$

$$\frac{V_x}{i_x}=R_{gs_1}=\frac{R_{sig}+R_{down}}{1+g_{m_1}R_{down}}$$

$$\Rightarrow R_{gs_1}=\frac{R_{sig}+\left(r_{o_1}\|r_{o_2}\|\frac{1}{g_{m_2}}\|R_L\right)}{1+g_{m_1}\left(r_{o_1}\|r_{o_2}\|\frac{1}{g_{m_2}}\|R_L\right)}$$

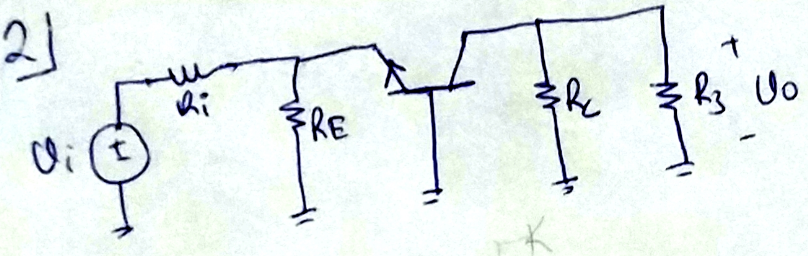
$\boxed{\tau_{gs_1}=C_{gs_1}R_{gs_1}}$

Then according to open-circuit time constant methodology

$$f_{3\text{-}dB} = \frac{1}{\sum_{i=1}^{3} \tau_i} = \frac{1}{(\tau_{out} + \tau_{gs1} + \tau_{gd1})2\pi}$$

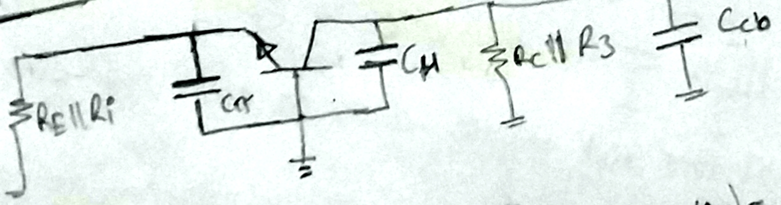
$$\boxed{f_{3\text{-}dB} = \frac{1}{2\pi\left(C_{out}R_{out} + C_{gs1}R_{gs1} + C_{gd1}R_{gd1}\right)}}$$

2)

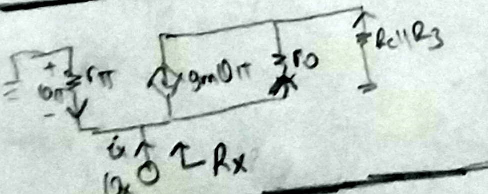



Since $C_1$ and $C_2$ are coupling capacitors, at high frequencies, they becomes short circuit.

1-) $V_i = 0$




For $C_\pi$




$$v_x - \left(i_x + \frac{v_\pi}{r_\pi} + g_m v_\pi\right) r_o - \left(i_x + \frac{v_\pi}{r_\pi}\right)(R_C \| R_3) = 0$$

$$v_x - \left(i_x - \frac{v_x}{r_\pi} - g_m v_x\right) r_o - \left(i_x - \frac{v_x}{r_\pi}\right)(R_C \| R_3) = 0$$

since $v_x = -v_\pi$ $\Rightarrow$ $v_x - \left(i_x - \frac{v_x}{r_\pi} - g_m v_x\right) \ldots$

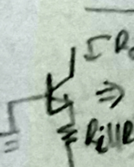
$$v_x\left(1 + \frac{r_o}{r_\pi} + g_m r_o + \frac{R_C \| R_3}{r_\pi}\right) - i_x\left(r_o + R_C \| R_3\right) = 0$$

$$R_x = \frac{r_o + (R_C \| R_3)}{1 + \frac{r_o}{r_\pi} + g_m r_o + \frac{R_C \| R_3}{r_\pi}} = \frac{r_\pi r_o + r_\pi (R_C \| R_3)}{r_\pi + r_o + g_m r_\pi r_o + (R_C \| R_3)}$$

$$R_x = \frac{r_\pi\left(r_o + R_C \| R_3\right)}{r_\pi + (\beta+1) r_o + (R_C \| R_3)} \Rightarrow R_\pi = R_x \| R_E \| R_i$$

$$\tau_\pi = C_\pi R_\pi = C_\pi\left(R_x \| R_E \| R_i\right)$$

For $C_\mu + C_{cb}$



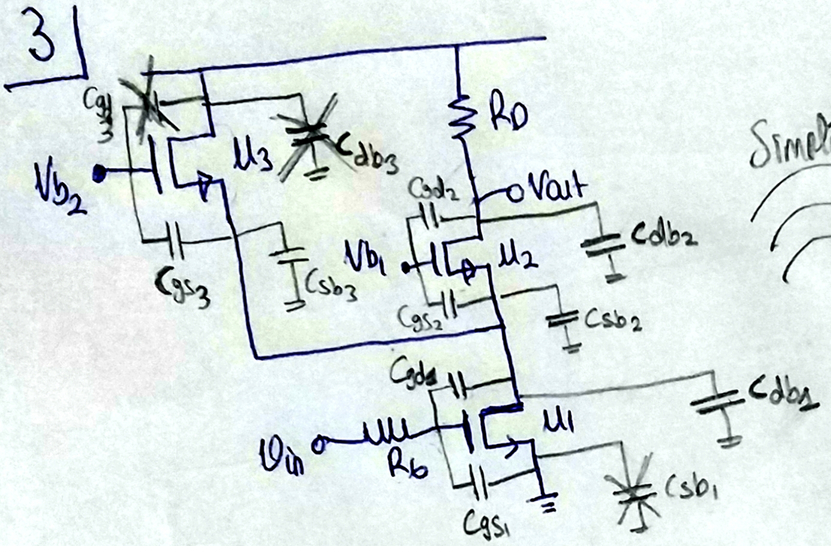

$\Rightarrow R_d = r_o + (R_i \| R_E \| r_\pi) + g_m r_o (R_i \| R_E \| r_\pi)$

$$R_\mu = R_d \| R_C \| R_3 \approx R_C \| R_3 \| \left(g_m r_o (R_i \| R_E \| r_\pi)\right)$$

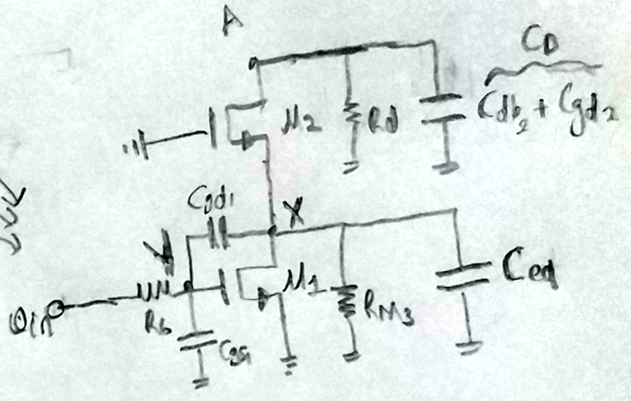
$$\tau_\mu = (C_\mu + C_{cb})\left[R_C \| R_3 \| \left(g_m r_o (R_i \| R_E \| r_\pi)\right)\right]$$

$$\boxed{f_{3\text{-}dB} = \frac{1}{2\pi(\tau_\mu + \tau_\pi)}}$$

3)



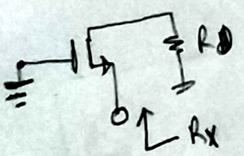

Simplify

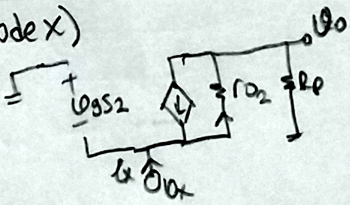



where
$C_{eq} = C_{db1} + C_{gs2} + C_{sb2} + C_{gs3} + C_{sb3}$

There is only one floating capacitor, $C_{gd1}$, and since we're asked to apply Miller's approximation, we need to calculate the DC gain between the terminals of $C_{gd1}$. (Between the terminals Y-X)

In order to find DC gain we need to find Rout for which Vout is taken from the drain of $M_1$. (Node X)



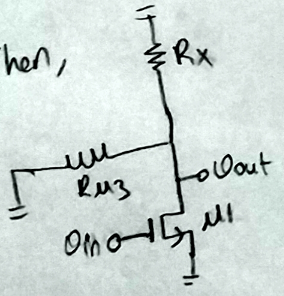

$v_x - (i_x + g_{m2} v_{gs2}) r_{o2} - i_x R_D = 0$ ($-v_x$)

$v_x(1 + g_{m2} r_{o2}) = i_x(r_{o2} + R_D)$

$R_x = \frac{v_x}{i_x} = \frac{r_{o2} + R_D}{1 + g_{m2} r_{o2}} = \frac{(1/g_{m2})(r_{o2} + R_D)}{\frac{1}{g_{m2}} + r_{o2}}$

$R_x \simeq \frac{r_{o2} + R_D}{g_{m2} r_{o2}}$ (since $r_{o2} \gg \frac{1}{g_{m2}}$)

Then,




$G_m = g_{m1}$

$R_{out} = R_x \parallel R_{M3} \parallel r_{o1}$ $\Rightarrow$

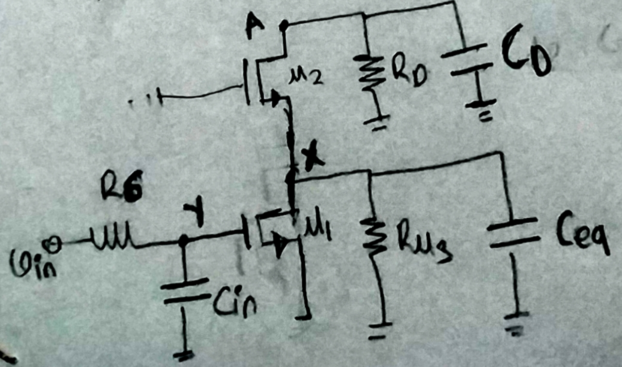
$R_{out} = \left(\frac{r_{o2} + R_D}{g_{m2} r_{o2}}\right) \parallel \left(\frac{1}{g_{m3}} \parallel r_{o3}\right) \parallel r_{o1}$

$\Rightarrow$ DC gain between the terminals of the floating capacitor is $A_V = G_m R_{out}$

$A_V = g_{m1}\left[\left(\frac{r_{o2} + R_D}{g_{m2} r_{o2}}\right) \parallel \left(\frac{1}{g_{m3}} \parallel r_{o3}\right) \parallel r_{o1}\right]$

Now we seperate $C_{gd1}$ into two standart capacitors by using Miller's theorem, and draw the new model

$C_Y = C_{gd1}(1 + A_V)$ ; $C_X = C_{gd1}\left(1 + \frac{1}{A_V}\right)$



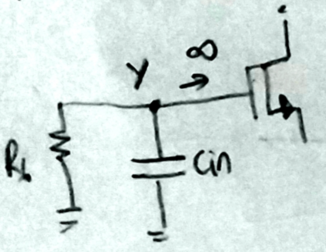

where $C_D = C_{db2} + C_{gd2}$

$C_{in} = C_Y + C_{gs1}$

$C_{eq} = C_X + C_{db1} + C_{gs2} + C_{sb2} + C_{gs3} + C_{sb3}$

Now, we obtained the model that we can apply the pole identification on

Let us first examine input pole which stems from the capacitor $C_{in}$

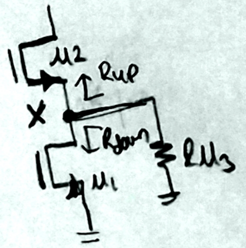



$R_Y = R_6$

$\tau_Y = C_{in} R_6$

$f_{in} = \dfrac{1}{2\pi C_{in} R_6}$

Then, examine $C_{eq}$




we have found that $R_{up} \simeq \dfrac{r_{o2} + R_O}{g_{m2} r_{o2}}$

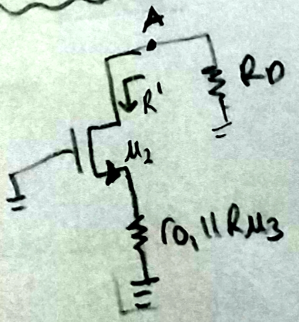
and we know $R_{down} = r_{o1}$

Then $R_X = R_{M3} \parallel R_{up} \parallel R_{down}$

$\Rightarrow R_X = \left(\dfrac{1}{g_{m3}} \parallel r_{o3}\right) \parallel \left(\dfrac{r_{o2} + R_O}{g_{m2} r_{o2}}\right) \parallel r_{o1}$

$\Rightarrow \tau_X = C_{eq} R_X \Rightarrow f_{eq} = \dfrac{1}{2\pi C_{eq} R_X}$

Lastly, examine $C_{up}$




$R' = r_{o2} + (r_{o1} \parallel R_{M3}) + g_{m2} r_{o2} (r_{o1} \parallel R_{M3})$

$R_A = R_O \parallel R' = R_O \parallel \left[r_{o2} + (1 + g_{m2} r_{o2})(r_{o1} \parallel R_{M3})\right]$

$\tau_A = R_A C_O$

$f_A = \dfrac{1}{2\pi (R_A C_O)}$

Node's names

X: Drain of $M_1$ and Source of $M_2$

Y: Gate of $M_1$

A: Drain of $M_3$

$R_{M3}$: The resistance seen looking into the source of $M_3$

4th question is simulated using LTSPICE

First of all, we need to models that are given to us.

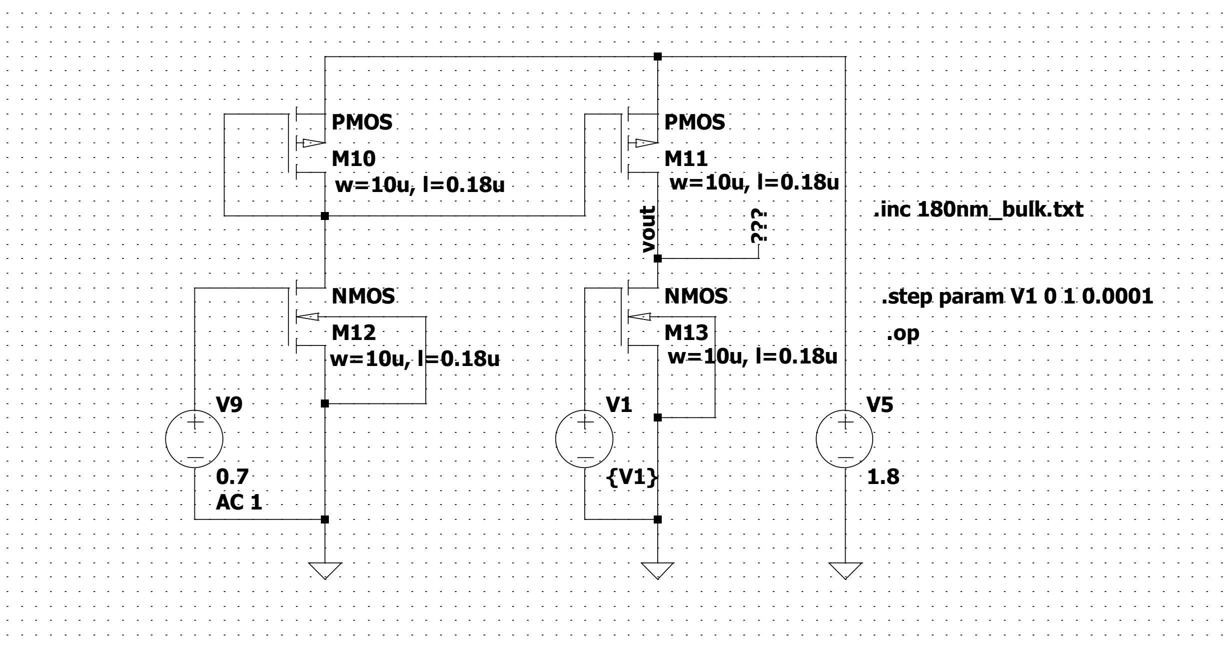
In order to add these models in LTspice,

1-) Models codes are copied into a txt file named 180nm_bulk.txt

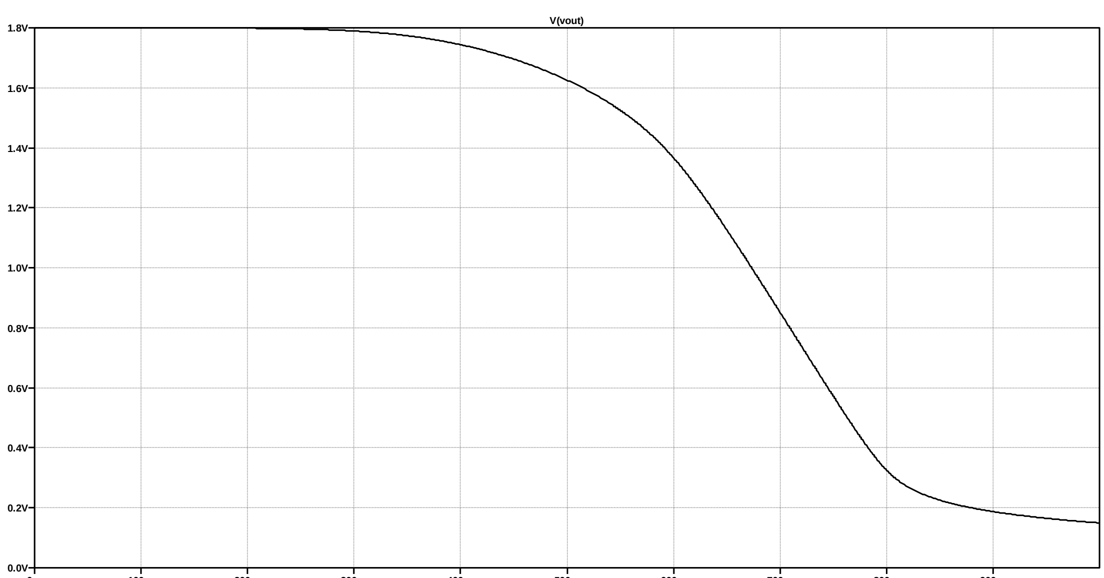
2-) We include this txt file using the command: .inc 180nm_bulk.txt

After the inclusion of the models, draw the asked circuit and select the asked transistors sizing.

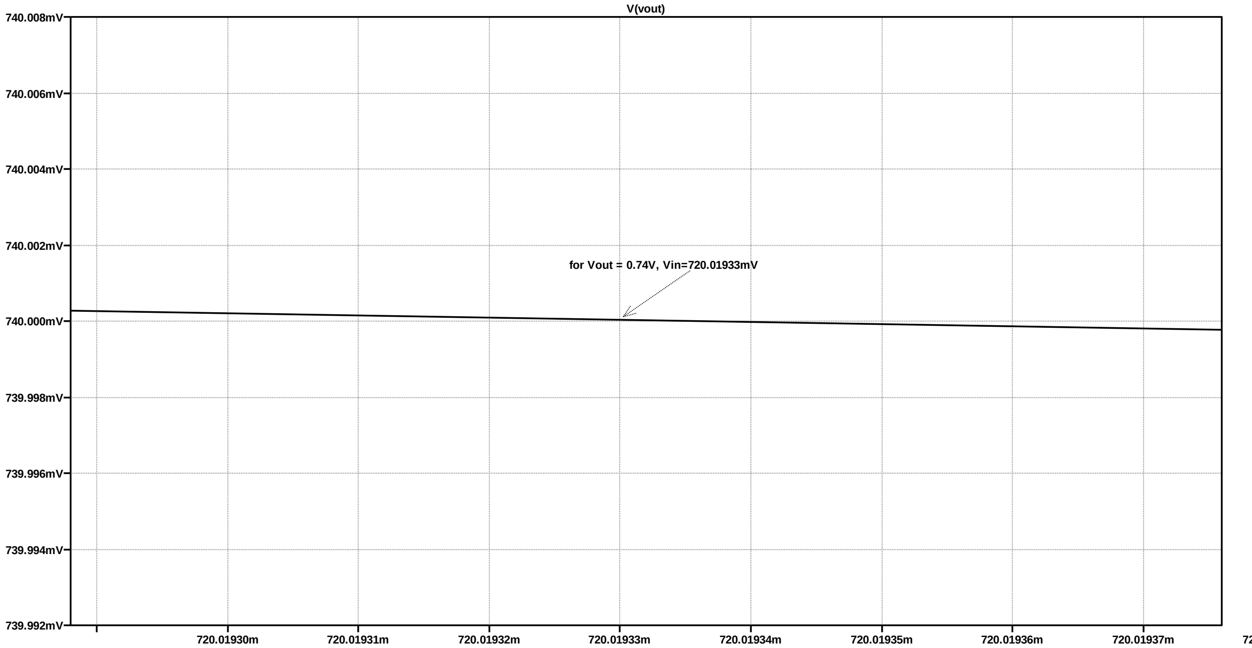
The circuit is drawn as shown:




Then, in order to determine the V1 voltage (input to M13) that corresponds to an output voltage of Vout = 0.74V, I increased V1 from 0V to 1V with small increments. The reason of increasing with small steps is the sensitivity of the circuit. It is very sensitive to small V1 voltages and the Vout – V1 relationship can be seen from the figure:

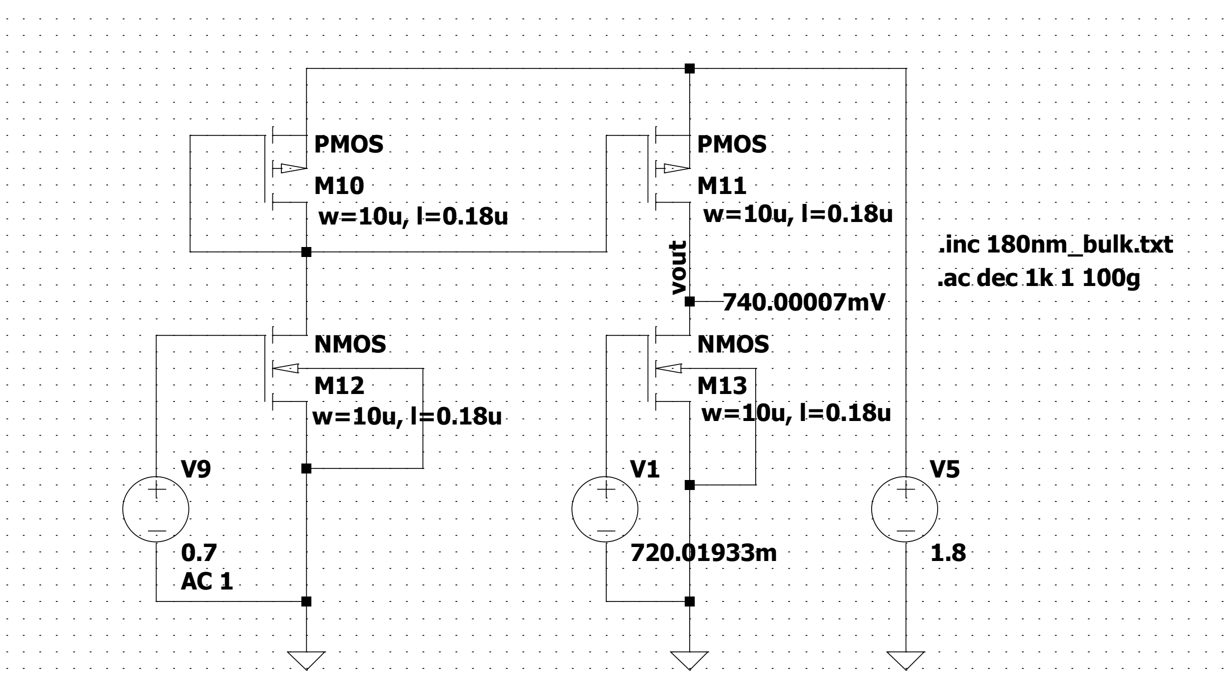


Then we can determine the V1 voltage that corresponds to 0.74V of output voltage from the figure. And to be precise let's zoom in the considered area. We can see the V1 voltage from the figure:

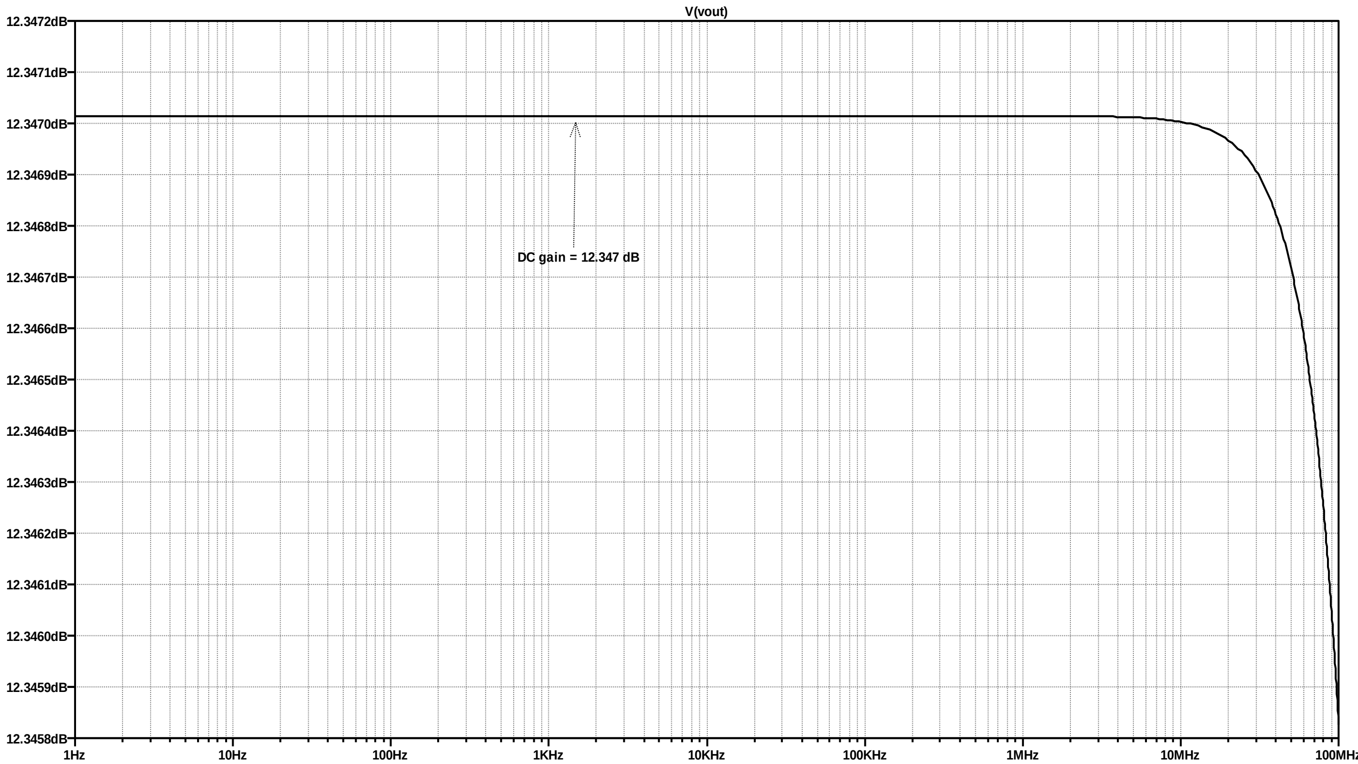



As it can be clearly seen V1 = 720.01933mV corresponds to Vout = 740mV

Then by selecting V1 = 720.01933, analyze the gain of this circuit structure. We are using ac analysis for 1k points from 1 Hz to 100Ghz. The code for this is shown in the figure.

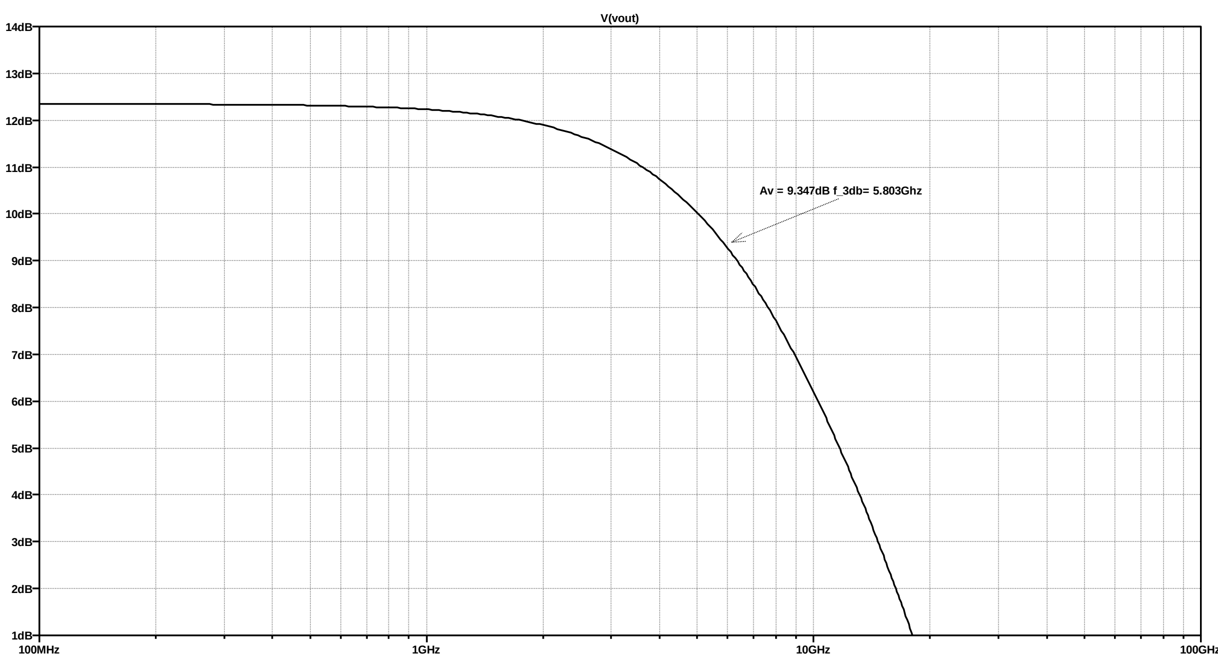


If we run the circuit above, we get a frequency response as shown:




From this figure we find the DC gain of this circuit as Av = 12.3470 dB

Then let’s zoom in to see at which frequency the circuit power halves. In other words, the gain drops by 3dB. The zoomed figure as shown:




Now from this figure we can say that $A_v = 12.3470\ dB\ and\ f_{3-db} = 5.0803\ Ghz$