Tutorial Problems: Bipolar Junction Transistor (Basic BJT Amplifiers)

Part A. Common-Emitter Amplifier

1. For the circuit shown in Figure 1, the transistor parameters are $\beta = 100$ and $V_A = \infty$. Design the circuit such that $I_{CQ} = 0.25$ mA and $V_{CEQ} = 3$ V. Find the small-signal voltage gain $A_v = v_o / v_s$. Find the input resistance seen by the signal source v_s.

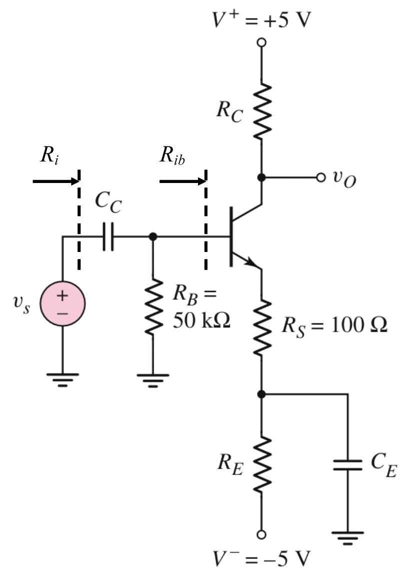


Figure 1

Solution:

For dc analysis, the capacitors C_C and C_E both act as *open circuit*.

Given the desired operating point $I_{CQ} = 0.25$ mA and $V_{CEQ} = 3$ V, we have:

$$
\begin{aligned}
0 - V^- &= I_{BQ} R_B + V_{BE(\text{on})} + I_{EQ}\left(R_S + R_E\right) \\
&= \frac{I_{CQ}}{\beta} R_B + V_{BE(\text{on})} + \left(\frac{1+\beta}{\beta}\right) I_{CQ}\left(R_S + R_E\right) \\
0 - (-5) &= \left(\frac{0.25}{100}\right)(50) + 0.7 + \left(\frac{101}{100}\right)(0.25)(0.1 + R_E) \\
&\Rightarrow R_E = 16.43\ (\text{k}\Omega)
\end{aligned}
$$

$$
\begin{aligned}
V^+ - V^- &= I_{CQ} R_C + V_{CEQ} + I_{EQ}\left(R_S + R_E\right) \\
5 - (-5) &= (0.25) R_C + 3 + \left(\frac{101}{100}\right)(0.25)(0.1 + 16.43) \\
&\Rightarrow R_C = 11.30\ (\text{k}\Omega)
\end{aligned}
$$

The small-signal parameters are:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.25} = 10.4\ (\text{k}\Omega)$$
$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.25}{0.026} = 9.6154\ (\text{mA/V})$$
$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

For small-signal ac analysis, all dc voltages and capacitors act as *short circuit*. The following expressions are obtained:

$$v_o = -\beta i_b R_C$$
$$v_s = i_b r_\pi + (1+\beta) i_b R_S$$
$$A_v = \frac{v_o}{v_s} = -\frac{\beta R_C}{r_\pi + (1+\beta) R_S}$$
$$= -\frac{(100)(11.30)}{10.4 + (101)(0.1)}$$
$$= -55.12$$

The input resistance R_i seen by the signal source v_s is:

$$R_i = R_B \parallel R_{ib}$$
$$= R_B \parallel \left(r_\pi + (1+\beta) R_S\right)$$
$$= 50 \parallel 20.5$$
$$= 14.54\ (\text{k}\Omega)$$

2. Consider the circuit shown in Figure 2. The transistor parameters are β = 100 and V_A = 100 V. Determine R_i, $A_v = v_o / v_s$ and $A_i = i_o / i_s$.

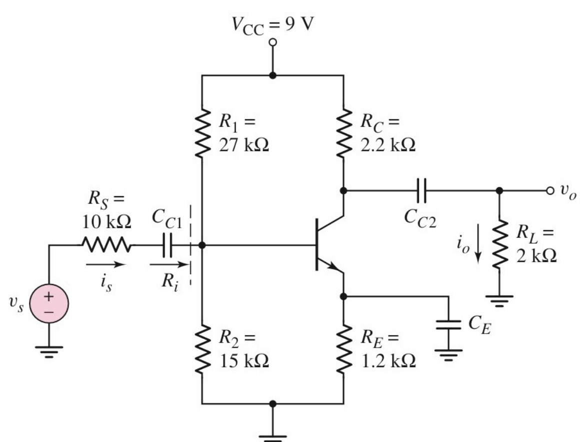


Figure 2

Solution:

A dc analysis is performed to determine the dc operating point by treating all capacitors as *open circuit*.

$$V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC} = \left(\frac{15}{15+27}\right)(9) = 3.2143 \text{ (V)}$$

$$R_{TH} = R_1 \,||\, R_2 = \frac{(15)(27)}{15+27} = 9.6429 \text{ (k}\Omega\text{)}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})}}{R_{TH} + (1+\beta)R_E} = \frac{3.2143 - 0.7}{9.6429 + (101)(1.2)} = 19.22 \text{ } (\mu\text{A})$$

$$I_{CQ} = \beta I_{BQ} = 1.922 \text{ (mA)}$$

$$I_{EQ} = (1+\beta) I_{BQ} = 1.941 \text{ (mA)}$$

$$V_{CEQ} = V_{CC} - I_{CQ}R_C - I_{EQ}R_E = 9 - (1.922)(2.2) - (1.941)(1.2) = 2.44 \text{ (V)}$$

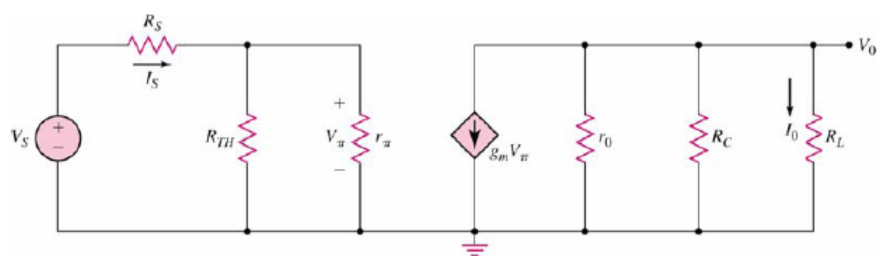
The small-signal parameters are:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{1.922} = 1.353 \text{ (k}\Omega\text{)}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.922}{0.026} = 73.923 \text{ (mA/V)}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{100}{1.922} = 52.029 \text{ (k}\Omega\text{)}$$

For small-signal ac analysis, all dc voltages and capacitors act as *short circuit*. The following small-signal ac equivalent circuit is obtained:



Small-signal model of transistor circuit ($g_mV_\pi = \beta i_b$)*

$$v_o = -\beta i_b \left(R_L \,||\, R_C \,||\, r_o\right)$$

$$V_\pi = \frac{R_{TH} \,||\, r_\pi}{R_S + R_{TH} \,||\, r_\pi} v_s = i_b r_\pi$$

$$A_v = \frac{v_o}{v_s} = -\frac{\beta\left(R_L \,||\, R_C \,||\, r_o\right)}{r_\pi}\left[\frac{R_{TH} \,||\, r_\pi}{R_S + R_{TH} \,||\, r_\pi}\right]$$

$$= -\frac{(100)(1.027)}{1.353}\left(\frac{1.187}{10+1.187}\right) = -8.05$$

$$i_s = \frac{v_s}{R_S + R_{TH} \parallel r_\pi}$$

$$i_o = \frac{v_o}{R_L}$$

$$\begin{aligned}
A_i = \frac{i_o}{i_s} &= \frac{R_S + R_{TH} \parallel r_\pi}{R_L} \cdot \frac{v_o}{v_s} \\
&= \left[\frac{R_S + R_{TH} \parallel r_\pi}{R_L}\right]\left[-\frac{\beta\left(R_L \parallel R_C \parallel r_o\right)}{r_\pi}\left(\frac{R_{TH} \parallel r_\pi}{R_S + R_{TH} \parallel r_\pi}\right)\right] \\
&= -\frac{\beta\left(R_L \parallel R_C \parallel r_o\right)\left(R_{TH} \parallel r_\pi\right)}{R_L r_\pi} \\
&= -\beta\left(\frac{R_C \parallel r_o}{R_L + R_C \parallel r_o}\right)\left(\frac{R_{TH}}{r_\pi + R_{TH}}\right) \\
&= -(100)\left(\frac{2.111}{2 + 2.111}\right)\left(\frac{9.6429}{1.353 + 9.6429}\right) = -45.05
\end{aligned}$$

The input resistance R_i is:

$$\begin{aligned}
R_i &= R_{TH} \parallel r_\pi \\
&= (9.6429) \parallel (1.353) = 1.187 \text{ (k}\Omega\text{)}
\end{aligned}$$

3. The parameters of the transistor in Figure 3 are $\beta = 100$ and $V_A = 100$ V.
   (a) Find the dc voltages at the base and emitter terminals.
   (b) Find R_C such that $V_{CEQ} = 3.5$ V.
   (c) Assuming C_C and C_E act as short circuits, determine the small-signal voltage gain $A_v = v_o / v_s$.
   (d) Repeat part (c) if a 500 Ω source resistor is in series with the v_s signal source.

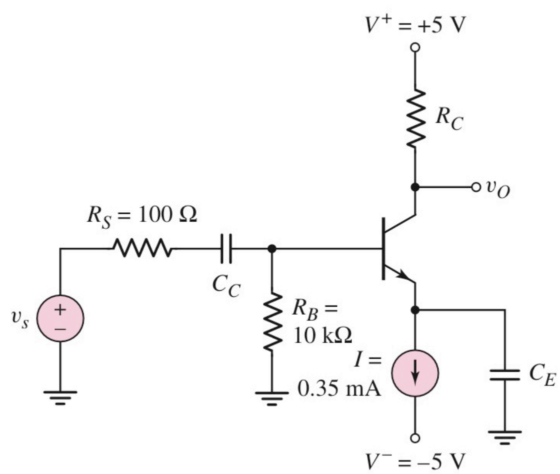


Figure 3

Solution:

(a)

A dc analysis is performed to determine the dc operating point by treating all capacitors as *open circuit*.

$$
\begin{aligned}
I_{CQ} &= \frac{\beta}{1+\beta} I_{EQ} = \left(\frac{100}{1+100}\right)(0.35) = 0.347 \text{ (mA)} \\
I_{BQ} &= \frac{I_{EQ}}{1+\beta} = \frac{0.35}{1+100} = 3.47\ (\mu\text{A}) \\
V_B &= 0 - I_{BQ} R_B = -\left(3.47\times 10^{-3}\right)(10) = -0.0347 \text{ (V)} \\
V_E &= V_B - V_{BE(\text{on})} = -0.737 \text{ (V)}
\end{aligned}
$$

(b)

Given V_{CEQ} is desired to be 3.5 V, hence:

$$
\begin{aligned}
V^+ &= V_E + V_{CEQ} + I_{CQ} R_C \\
R_C &= \frac{V^+ - V_E - V_{CEQ}}{I_{CQ}} \\
&= \frac{5 - (-0.737) - 3.5}{0.347} = 6.45 \text{ (k}\Omega\text{)}
\end{aligned}
$$

(c)

The small-signal parameters are:

$$
\begin{aligned}
r_\pi &= \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.347} = 7.493 \text{ (k}\Omega\text{)} \\
g_m &= \frac{I_{CQ}}{V_T} = \frac{0.347}{0.026} = 13.346 \text{ (mA/V)} \\
r_o &= \frac{V_A}{I_{CQ}} = \frac{100}{0.347} = 288.184 \text{ (k}\Omega\text{)}
\end{aligned}
$$

Using the small-signal ac equivalent circuit, the following expressions are obtained:

$$
\begin{aligned}
v_o &= -\beta i_b \left(R_C \,\|\, r_o\right) \\
V_\pi &= \frac{R_B \,\|\, r_\pi}{R_S + R_B \,\|\, r_\pi} v_s = i_b r_\pi \\
A_v &= \frac{v_o}{v_s} = -\frac{\beta\left(R_C \,\|\, r_o\right)}{r_\pi}\left(\frac{R_B \,\|\, r_\pi}{R_S + R_B \,\|\, r_\pi}\right) \\
&= -\frac{(100)(6.309)}{7.493}\left(\frac{4.283}{0.1 + 4.283}\right) = -82.28
\end{aligned}
$$

(d)

If the source resistor is changed to 500 Ω, the new value of A_v is:

$$A_v = \frac{v_o}{v_s} = -\frac{\beta\left(R_C \parallel r_o\right)}{r_\pi}\left(\frac{R_B \parallel r_\pi}{R_S + R_B \parallel r_\pi}\right)$$
$$= -\frac{(100)(6.309)}{7.493}\left(\frac{4.283}{0.5+4.283}\right) = -75.40$$

Therefore the voltage gain A_v decreases as the source resistance R_S increases due to a larger voltage drop across the source resistor.

4. The transistor in the circuit in Figure 4 has a dc current gain of $\beta = 100$.
   (a) Determine the small-signal voltage gain $A_v = v_o / v_s$.
   (b) Find the input and output resistances R_i and R_o.

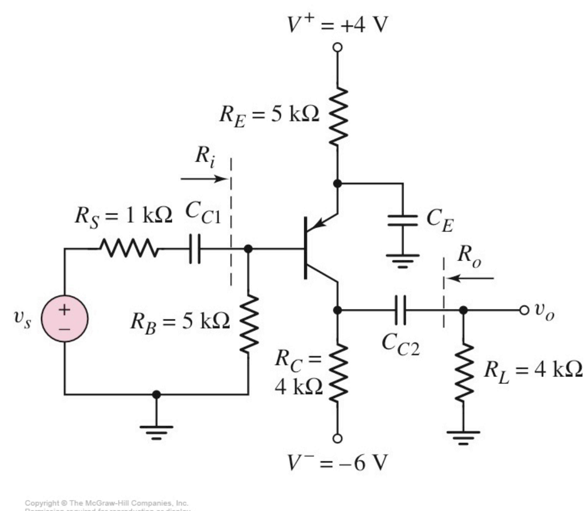


Figure 4

Solution:

(a)

A dc analysis is performed to determine the dc operating point by treating all capacitors as *open circuit*.

$$V^+ = I_{EQ}R_E + V_{EB(\text{on})} + I_{BQ}R_B$$
$$I_{BQ} = \frac{V^+ - V_{EB(\text{on})}}{R_B + (1+\beta)R_E} = \frac{4-0.7}{5+(101)(5)} = 6.47\ (\mu\text{A})$$
$$I_{CQ} = \beta I_{BQ} = (100)\left(6.47\times10^{-3}\right) = 0.647\ (\text{mA})$$
$$I_{EQ} = (1+\beta)I_{BQ} = 0.654\ (\text{mA})$$
$$V_{ECQ} = V^+ - V^- - I_{EQ}R_E - I_{CQ}R_C - = 4-(-6)-(0.654)(5)-(0.647)(4) = 4.14\ (\text{V})$$

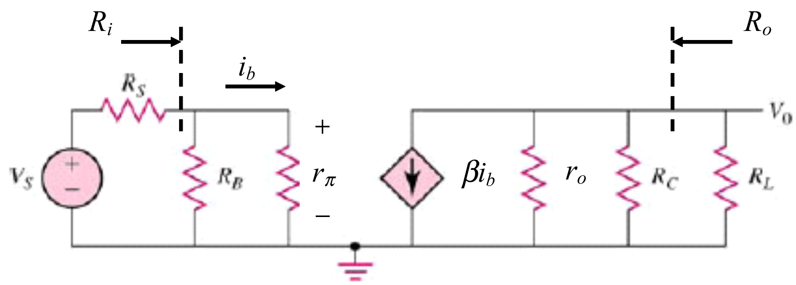
The small-signal parameters are:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.647} = 4.019\ (\text{k}\Omega)$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.647}{0.026} = 24.885\ (\text{mA/V})$$

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

Using the small-signal ac equivalent circuit, the following expressions are obtained:



$$v_o = -\beta i_b \left(R_L \parallel R_C \parallel r_o\right)$$

$$V_\pi = \frac{R_B \parallel r_\pi}{R_S + R_B \parallel r_\pi} v_s = i_b r_\pi$$

$$A_v = \frac{v_o}{v_s} = -\frac{\beta\left(R_L \parallel R_C \parallel r_o\right)}{r_\pi}\left(\frac{R_B \parallel r_\pi}{R_S + R_B \parallel r_\pi}\right)$$

$$= -\frac{(100)(2)}{4.019}\left(\frac{2.228}{1+2.228}\right) = -34.35$$

(b)

The input resistance R_i is:

$$R_i = R_B \parallel r_\pi$$
$$= (5) \parallel (4.019) = 2.228\ (\text{k}\Omega)$$

To calculate the output resistance R_o, the signal source v_s is short-circuited and this gives $i_b = 0$. The following equation can be written by KCL at node v_o:

$$-i_o + \frac{v_o}{R_C} = 0$$

$$\frac{v_o}{i_o} = R_o = R_C$$
$$= 4\ (\text{k}\Omega)$$

Part B. Common-Collector Amplifier (Emitter Follower)

5. The transistor parameters for the circuit in Figure 5 are $\beta = 180$ and $V_A = \infty$.
   (a) Find I_{CQ} and V_{CEQ}.
   (b) Plot the dc and ac load lines.
   (c) Calculate the small-signal voltage gain.
   (d) Determine the input and output resistances R_{ib} and R_o.

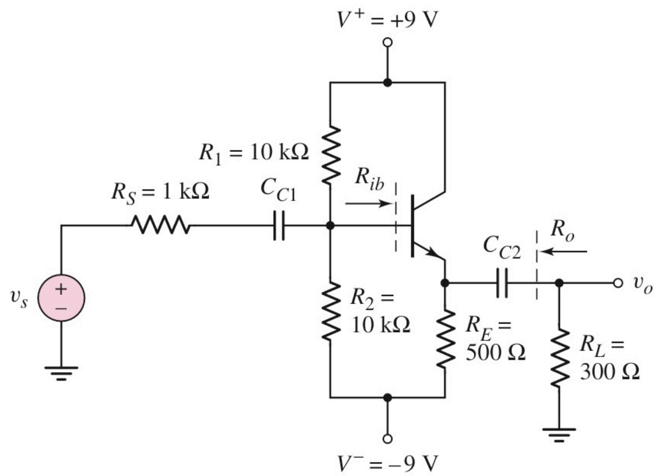


Figure 5

Solution:

(a)

For dc analysis, the capacitors C_{C1} and C_{C2} act as *open circuit*.

$$V_{TH} = \frac{R_2}{R_1 + R_2}\left(V^+ - V^-\right) + V^- = \left(\frac{10}{10+10}\right)(18) + (-9) = 0 \text{ (V)}$$

$$R_{TH} = R_1 \,||\, R_2 = \frac{(10)(10)}{10+10} = 5.0 \text{ (k}\Omega\text{)}$$

$$I_{BQ} = \frac{V_{TH} - V_{BE(\text{on})} - V^-}{R_{TH} + (1+\beta)R_E} = \frac{0 - 0.7 - (-9)}{5 + (181)(0.5)} = 86.91 \text{ } (\mu\text{A})$$

$$I_{CQ} = \beta I_{BQ} = 15.644 \text{ (mA)}$$

$$I_{EQ} = (1+\beta) I_{BQ} = 15.731 \text{ (mA)}$$

$$V_{CEQ} = V^+ - V^- - I_{EQ} R_E = 9 - (-9) - (15.731)(0.5) = 10.13 \text{ (V)}$$

(b)

The dc load line is given by:

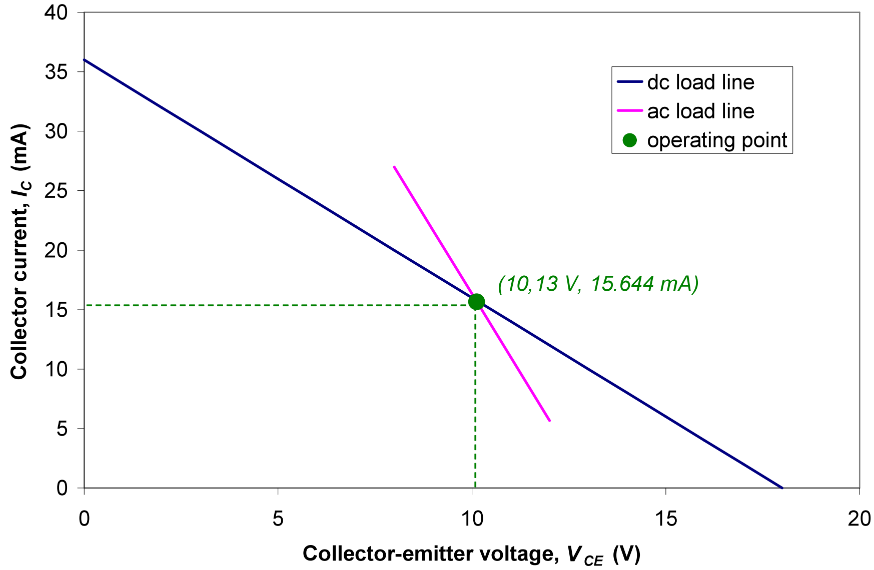
$$V^+ - V^- = V_{CE} + I_E R_E \approx V_{CE} + I_C R_E \text{ (for large } \beta\text{)}$$

$$I_C = -\frac{V_{CE}}{R_E} + \frac{\left(V^+ - V^-\right)}{R_E} = -2V_{CE} + 36$$

The ac load line is given by:

$$v_{ce} + i_e\left(R_E \parallel R_L\right) = 0$$

$$i_e \approx i_c = -\frac{v_{ce}}{R_E \parallel R_L} = -5.333 v_{ce}$$



(c)

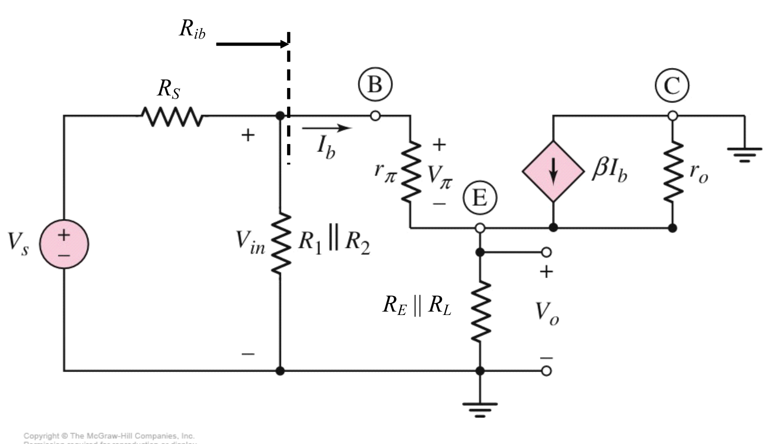
The small-signal parameters are:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(180)(0.026)}{15.644} = 0.299 \text{ (k}\Omega\text{)}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{15.644}{0.026} = 601.692 \text{ (mA/V)}$$

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

The small-signal ac equivalent circuit becomes:



$$v_o = (1+\beta) i_b (R_E \parallel R_L)$$

$$\Rightarrow \frac{v_o}{i_b} = (1+\beta)(R_E \parallel R_L) = (181)(0.1875) = 33.9375$$

$$v_b = V_\pi + v_o = i_b r_\pi + (1+\beta) i_b (R_E \parallel R_L)$$

$$\Rightarrow \frac{v_b}{i_b} = R_{ib} = r_\pi + (1+\beta)(R_E \parallel R_L) = 0.299 + 33.9375 = 34.2365$$

$$v_b = \frac{R_{ib} \parallel R_1 \parallel R_2}{R_S + R_{ib} \parallel R_1 \parallel R_2} v_s$$

$$\Rightarrow \frac{v_b}{v_s} = \frac{R_{ib} \parallel R_1 \parallel R_2}{R_S + R_{ib} \parallel R_1 \parallel R_2} = \frac{4.3628}{1 + 4.3628} = 0.8135$$

$$\begin{aligned}
\frac{v_o}{v_s} &= \frac{v_o}{i_b} \times \frac{i_b}{v_b} \times \frac{v_b}{v_s} \\
&= (1+\beta)(R_E \parallel R_L) \times \frac{1}{R_{ib}} \times \frac{R_{ib} \parallel R_1 \parallel R_2}{R_S + R_{ib} \parallel R_1 \parallel R_2} \\
&= \frac{(1+\beta)(R_E \parallel R_L)}{R_{ib}} \left(\frac{R_{ib} \parallel R_1 \parallel R_2}{R_S + R_{ib} \parallel R_1 \parallel R_2} \right) \\
&= (33.9375) \left(\frac{1}{34.2365} \right) (0.8135) = 0.8064
\end{aligned}$$

(d)

The input resistance R_{ib} is:

$$\begin{aligned}
R_{ib} &= r_\pi + (1+\beta)(R_E \parallel R_L) \\
&= 0.299 + 33.9375 = 34.24 \text{ (k}\Omega\text{)}
\end{aligned}$$

To calculate the output resistance R_o, the signal source v_s is short-circuited and the following equations can be written by KCL at node v_o and node v_b:

$$v_b = v_o + r_\pi i_b$$

$$\frac{v_b}{R_S \parallel R_1 \parallel R_2} + i_b = 0 \text{ (KCL at node } v_b\text{)}$$

$$\frac{v_o + r_\pi i_b}{R_S \parallel R_1 \parallel R_2} + i_b = 0 \Rightarrow \frac{v_o}{i_b} = -(r_\pi + R_S \parallel R_1 \parallel R_2)$$

$$i_o + (1+\beta) i_b = \frac{v_o}{R_E} \text{ (KCL at node } v_o\text{)}$$

$$i_o - (1+\beta)\left(\frac{v_o}{r_\pi + R_S \parallel R_1 \parallel R_2} \right) = \frac{v_o}{R_E}$$

$$\Rightarrow \frac{v_o}{i_o} = R_o = R_E \parallel \left(\frac{r_\pi + R_S \parallel R_1 \parallel R_2}{1+\beta} \right) = 6.18 \ (\Omega)$$

6. For the circuit shown in Figure 6, let V_{CC} = 5 V, R_L = 4 kΩ, R_E = 3 kΩ, R_1 = 60 kΩ, and R_2 = 40 kΩ. The transistor parameters are β = 50 and V_A = 80 V.
   (a) Determine I_{CQ} and V_{ECQ}.
   (b) Plot the dc and ac load lines.
   (c) Determine $A_v = v_o / v_s$ and $A_i = i_o / i_s$.
   (d) Determine R_{ib} and R_o.

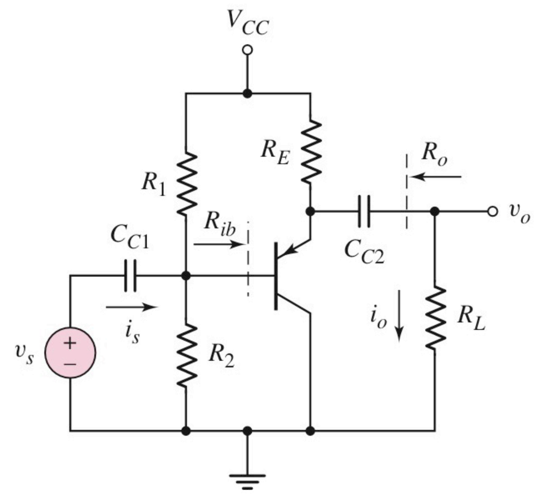


Figure 6

Solution:

(a)

For dc analysis, the capacitors C_{C1} and C_{C2} act as *open circuit*.

$$V_{TH} = \frac{R_2}{R_1 + R_2} V_{CC} = \left(\frac{40}{60+40}\right)(5) = 2.0 \text{ (V)}$$

$$R_{TH} = R_1 \parallel R_2 = \frac{(60)(40)}{60+40} = 24.0 \text{ (k}\Omega\text{)}$$

$$I_{BQ} = \frac{V_{CC} - V_{EB(\text{on})} - V_{TH}}{R_{TH} + (1+\beta)R_E} = \frac{5 - 0.7 - 2}{24 + (51)(3)} = 12.99 \text{ } (\mu\text{A})$$

$$I_{CQ} = \beta I_{BQ} = 0.650 \text{ (mA)}$$

$$I_{EQ} = (1+\beta) I_{BQ} = 0.663 \text{ (mA)}$$

$$V_{ECQ} = V_{CC} - I_{EQ} R_E = 5 - (0.663)(3) = 3.01 \text{ (V)}$$

(b)

The dc load line is given by:

$$V_{CC} = I_E R_E + V_{EC} \approx \left(\frac{1+\beta}{\beta}\right) I_C R_E + V_{EC}$$

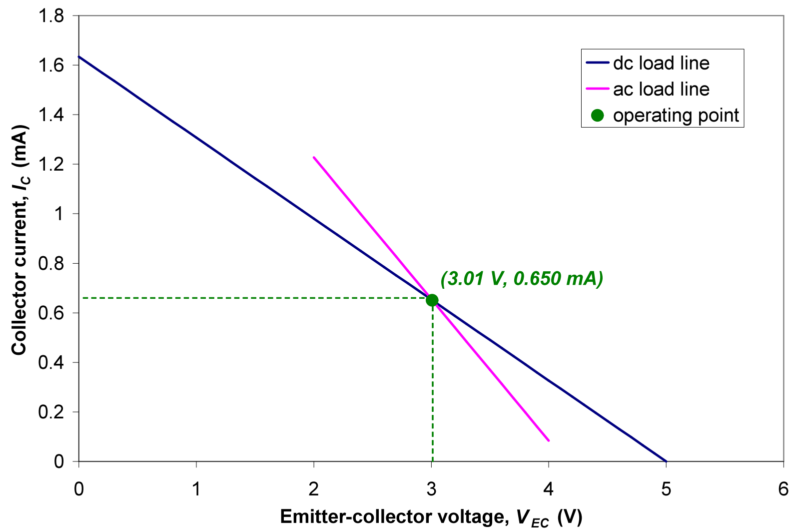
$$I_C = -\frac{V_{EC}}{\left(\frac{1+\beta}{\beta}\right) R_E} + \frac{V_{CC}}{\left(\frac{1+\beta}{\beta}\right) R_E} = -0.3268 V_{EC} + 1.6340$$

The ac load line is given by:

$$v_{ec} + i_e (R_E \| R_L) = 0$$

$$v_{ec} + i_c \left(\frac{1+\beta}{\beta}\right)(R_E \| R_L) = 0 \Rightarrow i_c = -\frac{v_{ce}}{\left(\frac{1+\beta}{\beta}\right)(R_E \| R_L)} = -0.5719 v_{ce}$$

(c)



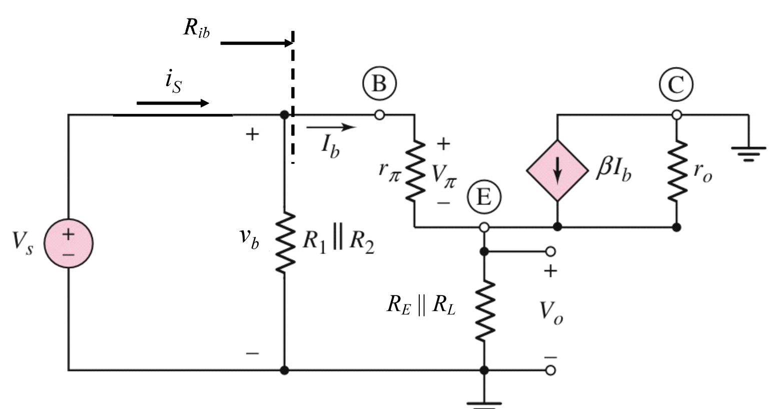
The small-signal parameters are:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.650} = 2.0 \text{ (k}\Omega)$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.650}{0.026} = 25.0 \text{ (mA/V)}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{80}{0.650} = 123.077 \text{ (k}\Omega)$$

The small-signal ac equivalent circuit becomes:



$$v_o = \left[i_b + \beta i_b + \frac{-v_o}{r_o} \right] (R_E \parallel R_L)$$

$$\Rightarrow \frac{v_o}{i_b} = \frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} = \frac{(51)(1.7143)}{1 + 1.7143/123.077} = 86.2283$$

$$v_b = V_\pi + v_o = i_b r_\pi + \left[\frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \right] i_b$$

$$\Rightarrow \frac{v_b}{i_b} = R_{ib} = r_\pi + \left[\frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \right] = 2.0 + 86.2283 = 88.2283$$

$$v_b = v_s$$

$$\Rightarrow \frac{v_b}{v_s} = 1$$

$$\frac{v_o}{v_s} = \frac{v_o}{i_b} \times \frac{i_b}{v_b} \times \frac{v_b}{v_s}$$

$$= \frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \times \frac{1}{R_{ib}} \times 1$$

$$= \frac{1}{R_{ib}} \left[\frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \right]$$

$$= (86.2283) \left(\frac{1}{88.2283} \right) = 0.9773$$

$$v_o = i_o R_L$$

$$\Rightarrow \frac{v_o}{i_o} = R_L = 4$$

$$v_o = \left[i_b + \beta i_b + \frac{-v_o}{r_o} \right] (R_E \parallel R_L)$$

$$\Rightarrow \frac{v_o}{i_b} = \frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} = \frac{(51)(1.7143)}{1 + 1.7143/123.077} = 86.2283$$

$$i_b = \frac{R_1 \parallel R_2}{R_{ib} + R_1 \parallel R_2} i_s$$

$$\Rightarrow \frac{i_b}{i_s} = \frac{R_1 \parallel R_2}{R_{ib} + R_1 \parallel R_2} = \frac{24}{88.2283 + 24} = 0.2138$$

$$\frac{i_o}{i_s} = \frac{i_o}{v_o} \times \frac{v_o}{i_b} \times \frac{i_b}{i_s}$$

$$= \frac{1}{R_L} \times \frac{(1+\beta)(R_E \parallel R_L)}{1 + (R_E \parallel R_L)/r_o} \times \frac{R_1 \parallel R_2}{R_{ib} + R_1 \parallel R_2}$$

$$= \left(\frac{1}{4} \right) (86.2283)(0.2138) = 4.61$$

(d)

The input resistance R_{ib} is:

$$R_{ib} = r_\pi + \left[\frac{(1+\beta)(R_E \parallel R_L)}{1+(R_E \parallel R_L)/r_o} \right] = 2.0 + 86.2283 = 88.23 \text{ (k}\Omega)$$

To calculate the output resistance R_o, the signal source v_s is short-circuited and the following equations can be written by KCL at node v_o:

$$i_b = -\frac{v_o}{r_\pi}$$

$$i_o + \beta i_b + i_b = \frac{v_o}{r_o \parallel R_E}$$

$$i_o + (1+\beta)\left(-\frac{v_o}{r_\pi}\right) = \frac{v_o}{r_o \parallel R_E}$$

$$\begin{aligned} \frac{v_o}{i_o} = R_o &= \frac{1}{\dfrac{1+\beta}{r_\pi} + \dfrac{1}{r_o \parallel R_E}} \\ &= r_o \parallel R_E \parallel \frac{r_\pi}{1+\beta} \\ &= 38.70 \ (\Omega) \end{aligned}$$

7. For the transistor in Figure 7, the parameters are $\beta = 100$ and $V_A = \infty$.
   (a) Design the circuit such that $I_{EQ} = 1$ mA and the Q-point is in the center of the dc load line.
   (b) If the peak-to-peak sinusoidal output voltage is 4 V, determine the peak-to-peak sinusoidal signals at the base of the transistor and the peak-to-peak value of v_s.
   (c) If the load resistor R_L = 1 kΩ is connected to the output through a coupling capacitor, determine the peak-to-peak value in the output voltage, assuming v_s is equal to the value determined in part (b).

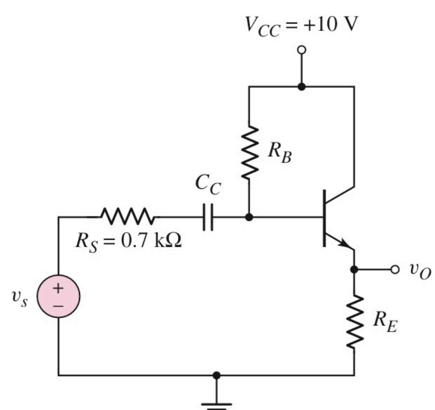


Figure 7

Solution:

(a)

For dc analysis, the capacitor C_C acts as *open circuit*.

$$
\begin{aligned}
V_{CC} &= I_{BQ}R_B + V_{BE(\text{on})} + I_{EQ}R_E \\
&= \left(\frac{R_B}{1+\beta} + R_E\right)I_{EQ} + V_{BE(\text{on})}
\end{aligned}
$$

$$\frac{R_B}{101} + R_E = \frac{V_{CC} - V_{BE(\text{on})}}{I_{EQ}} = \frac{10 - 0.7}{1} = 9.3\ (\text{k}\Omega) \qquad ...(1)$$

$$V_{CC} = V_{CEQ} + I_{EQ}R_E \ \left(V_{CEQ} = \frac{V_{CC}}{2} \text{ for } Q\text{-point is in the center of the dc load line}\right)$$

$$10 = 5 + (1)R_E$$

$$R_E = 5\ (\text{k}\Omega) \qquad ...(2)$$

$$\Rightarrow R_B = (101)(9.3 - R_E) = 434.3\ (\text{k}\Omega)$$

$$I_{CQ} = \left(\frac{\beta}{1+\beta}\right)I_{EQ} = \left(\frac{100}{101}\right)(1) = 0.990\ (\text{mA})$$

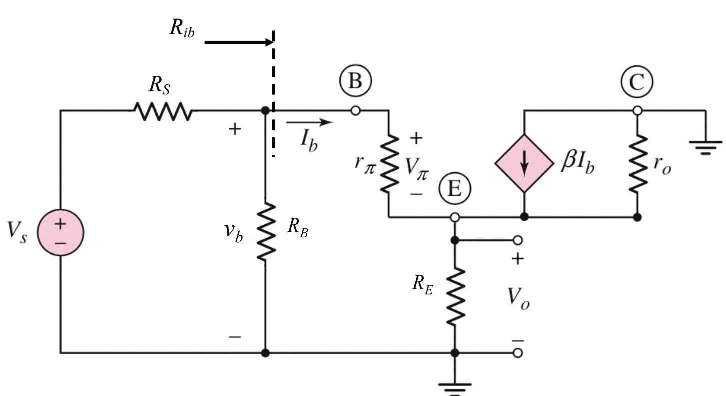
$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{(100)(0.026)}{0.990} = 2.6263\ (\text{k}\Omega)$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.990}{0.026} = 38.0769\ (\text{mA/V})$$

$$r_o = \frac{V_A}{I_{CQ}} = \infty$$

(b)

The small-signal ac equivalent circuit is given by:



$$r_o = \infty$$

$$v_o = (1+\beta) R_E i_b$$

$$\Rightarrow \frac{v_o}{i_b} = (1+\beta) R_E = (101)(5) = 505$$

$$v_b = V_\pi + v_o = i_b r_\pi + (1+\beta) R_E i_b$$

$$\Rightarrow \frac{v_b}{i_b} = R_{ib} = r_\pi + (1+\beta) R_E = 2.6263 + 505 = 507.6263$$

$$v_b = \frac{R_B \| R_{ib}}{R_S + R_B \| R_{ib}} v_s$$

$$\Rightarrow \frac{v_b}{v_s} = \frac{R_B \| R_{ib}}{R_S + R_B \| R_{ib}} = \frac{234.0545}{0.7+234.0545} = 0.9970$$

$$\frac{v_b}{v_o} = \frac{v_b}{i_b} \times \frac{i_b}{v_o}$$

$$= \frac{r_\pi + (1+\beta) R_E}{(1+\beta) R_E}$$

$$= \frac{507.6263}{505} = 1.0052 \qquad ...(3)$$

$$\frac{v_o}{v_s} = \frac{v_o}{i_b} \times \frac{i_b}{v_b} \times \frac{v_b}{v_s}$$

$$= (1+\beta) R_E \times \frac{1}{R_{ib}} \times \frac{R_B \| R_{ib}}{R_S + R_B \| R_{ib}}$$

$$= 505 \times \frac{1}{507.6263} \times 0.9970 = 0.9918 \qquad ...(4)$$

If the peak-to-peak output voltage $v_{o(peak\text{-}peak)}$ is 4 V,

$$\text{Eq.(3)} \Rightarrow v_{b(peak-peak)} = 1.0052 v_{o(peak-peak)} = 4.021 \text{ (V)}$$

$$\text{Eq.(4)} \Rightarrow v_{s(peak-peak)} = \frac{v_{o(peak-peak)}}{0.9918} = 4.033 \text{ (V)}$$

(c)

If the load resistor $R_L = 1$ kΩ is added in parallel to R_E, Eq. (4) must be modified accordingly:

$$\frac{v_o}{v_s} = \frac{(1+\beta)(R_E \| R_L)}{r_\pi + (1+\beta)(R_E \| R_L)} \left(\frac{R_B \| R_{ib}}{R_S + R_B \| R_{ib}} \right)$$

$$= \frac{(101)(0.8333)}{2.6263 + (101)(0.8333)} (0.9970) = 0.9668$$

$$\Rightarrow v_{o(peak-peak)} = 0.9668 v_{s(peak-peak)} = (0.9668)(4.033) = 3.90 \text{ (V)}$$

Therefore $v_{o(peak\text{-}peak)}$ becomes smaller due to the loading effect by R_L.

8. An emitter-follower amplifier, with the configuration shown in Figure 8, is to be designed such that an audio signal given by $v_s = 5\sin(3000t)$ V but with a source resistance of $R_S = 10\ \Omega$ can drive a small speaker. Assume the supply voltages are $V^+ = +12$ V and $V^- = -12$ V and $\beta = 50$. The load, representing the speaker, is $R_L = 12\ \Omega$. The amplifier should be capable of delivering approximately 1 W of average power to the load. What is the signal power gain of your amplifier?

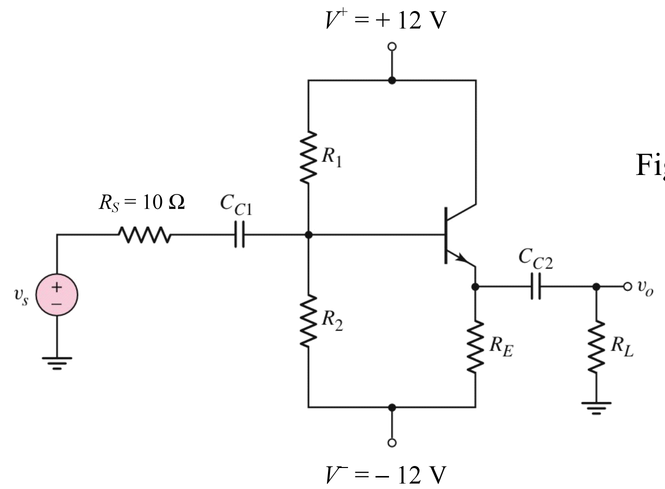


Figure 8

Solution:

To deliver 1 W of average power to the load, the peak-to-peak output voltage should be:

$$\frac{v_{o(rms)}^2}{R_L} = \frac{v_{o(peak)}^2}{2R_L} = 1$$

$$\Rightarrow v_{o(peak)} = 4.899\ \text{(V)}$$

$$\Rightarrow i_{o(peak)} = \frac{4.899}{12} = 0.408\ \text{(A)}$$

$$\Rightarrow v_{o(peak-peak)} = 9.798\ \text{(V)}$$

The required voltage gain A_v is:

$$A_v = \frac{v_{o(peak)}}{v_{s(peak)}} = \frac{4.899}{5.0} = 0.9798$$

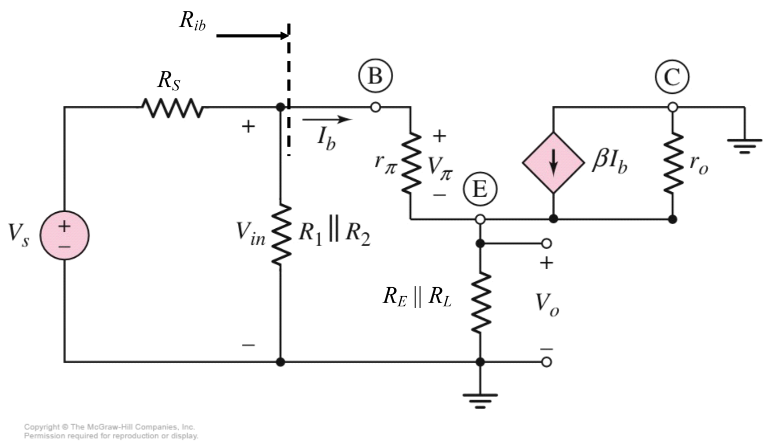
Choose $I_{EQ} = 0.8$ A and $V_{CEQ} = 12$ V,

$$R_E = \frac{V^+ - V^- - V_{CEQ}}{I_{EQ}} = \frac{12-(-12)-12}{0.8} = 15\ (\Omega)$$

$$R_{TH} = \frac{1}{10}(1+\beta)R_E = \left(\frac{51}{10}\right)(15) = 76.5\ (\Omega)\ \text{(for bias-stable circuit)}$$

$$
\begin{aligned}
V_{TH} &= V^- + I_{EQ}R_E + V_{BE(\text{on})} + I_{BQ}R_{TH} \\
&= -12 + (0.8)(15) + 0.7 + \left(\frac{0.8}{51}\right)(76.5) = 1.9 \text{ (V)} \\
&= \frac{R_1}{R_1 + R_2}\left(V^+ - V^-\right) + V^- = \frac{1}{R_1}\left(R_1 \parallel R_2\right)\left(V^+ - V^-\right) + V^- \\
&\Rightarrow R_1 = 132\ (\Omega) \qquad R_2 = 182\ (\Omega)
\end{aligned}
$$

The small-signal ac equivalent circuit is given by:



Choosing $I_{EQ} = 0.5$ A gives:

$$
\begin{aligned}
I_{CQ} &= \frac{\beta}{1+\beta} I_{EQ} = \left(\frac{50}{51}\right)(0.8) = 0.784 \text{ (A)} \\
r_\pi &= \frac{\beta V_T}{I_{CQ}} = \frac{(50)(0.026)}{0.784} = 1.658\ (\Omega)
\end{aligned}
$$

The small-signal voltage gain is taken from Q.7 with some modifications:

$$
\begin{aligned}
\frac{v_o}{v_s} &= \frac{(1+\beta)(R_E \parallel R_L)}{r_\pi + (1+\beta)(R_E \parallel R_L)}\left(\frac{R_1 \parallel R_2 \parallel R_{ib}}{R_S + R_1 \parallel R_2 \parallel R_{ib}}\right) \\
&= \frac{(51)(6.667)}{1.658 + (51)(6.667)}\left(\frac{62.5116}{10 + 62.5116}\right) \\
&= 0.8579
\end{aligned}
$$

Due to the presence of the source resistance R_S (loading effect) the required voltage gain of $A_v = 0.9798$ cannot be achieved. Note that $A_v = 0.9951$ if $R_S = 0$.

Therefore the maximum achievable peak output voltage is:

$$
\frac{v_{o(peak)}}{v_{s(peak)}} = 0.8579 \Rightarrow v_{o(peak)} = 4.290 \text{ (V)}
$$

Hence the output power delivered to the load R_L is:

$$P_L = \frac{v_{o(peak)}^2}{2R_L} = 0.767 \text{ (W)}$$

The input power delivered by the signal source v_s is:

$$P_S = v_{s(rms)} i_{s(rms)}$$

$$i_{s(rms)} = \frac{v_{s(rms)}}{R_i} = \frac{v_{s(rms)}}{R_S + R_1 \| R_2 \| R_{ib}} = \frac{5/\sqrt{2}}{10 + 62.5116} = 48.758 \text{ (mA)}$$

$$\Rightarrow P_S = v_{s(rms)} i_{s(rms)} = \left(\frac{5}{\sqrt{2}}\right)(48.758) = 172.386 \text{ (mW)}$$

Hence the signal power gain of the amplifier is:

$$G_{power} = \frac{P_L}{P_S} = \frac{0.767}{172.386 \times 10^{-3}} = 4.45$$

Part C. AC Load Line Analysis / Maximum Symmetrical Swing

9. For the circuit in Figure 9, the transistor parameters are $\beta = 100$ and $V_A = 100$ V. The values of R_C, R_E and R_L are as shown in the figure. Design a bias-stable circuit to achieve the maximum undistorted swing in the output voltage if the total instantaneous C-E voltage is to remain in the range $1 \le v_{CE} \le 8$ V and the minimum collector current is to be i_C (min) = 0.1 mA.

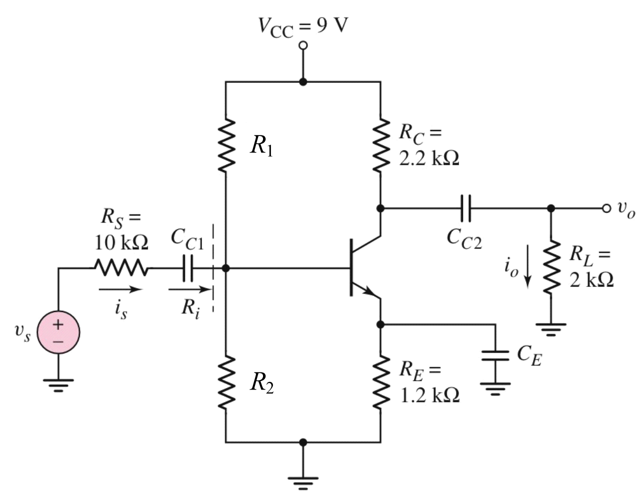


Figure 9

Solution:

To obtain a bias-stable circuit, let:

$$R_{TH} = R_1 \parallel R_2$$
$$= \frac{1}{10}(1+\beta)R_E = \left(\frac{101}{10}\right)(1.2) = 12.12 \text{ (k}\Omega\text{)}$$

The dc load line of the circuit is given by:

$$V_{CC} = I_C R_C + V_{CE} + I_E R_E$$
$$= I_C R_C + V_{CE} + \left(\frac{1+\beta}{\beta}\right) I_C R_E$$
$$I_C = -\frac{V_{CE}}{R_C + \left(\frac{1+\beta}{\beta}\right)R_E} + \frac{V_{CC}}{R_C + \left(\frac{1+\beta}{\beta}\right)R_E} = -0.2931V_{CE} + 2.6377 \qquad \text{...(1)}$$

The ac load line of the circuit is given by:

$$v_{ce} + i_c\left(R_C \parallel R_L\right) = 0$$
$$i_c = -\frac{v_{ce}}{R_C \parallel R_L} = -0.9545 v_{ce} \qquad \text{...(2)}$$

Given $v_{CE(\text{min})}$ = 1 V and $i_{C(\text{min})}$ = 0.1 mA, the maximum swing of v_{CE} and i_C from the Q-point (I_{CQ}, V_{CEQ}) would be:

$$\left|\Delta v_{CE(\text{max})}\right| = V_{CEQ} - v_{CE(\text{min})} = V_{CEQ} - 1$$
$$\left|\Delta i_{C(\text{max})}\right| = I_{CQ} - i_{C(\text{min})} = I_{CQ} - 0.1$$

Since $\left|\Delta v_{CE(\text{max})}\right|$ and $\left|\Delta i_{C(\text{max})}\right|$ are related by the ac load line,

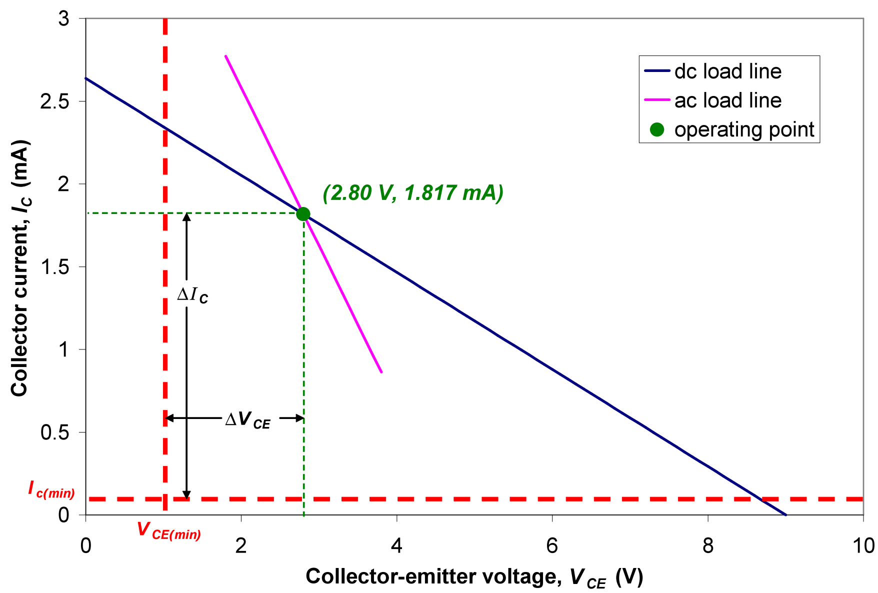
$$\left|\Delta i_C\right| = 0.9545\left|\Delta v_{CE}\right|$$
$$I_{CQ} - 0.1 = 0.9545\left(V_{CEQ} - 1\right)$$
$$I_{CQ} = 0.9545V_{CEQ} - 0.8545 \qquad \text{...(3)}$$

Solving (1) and (3) at the Q-point (I_{CQ}, V_{CEQ}):

$$0.9545V_{CEQ} - 0.8545 = -0.2931V_{CEQ} + 2.6377$$
$$1.2476V_{CEQ} = 3.4922$$
$$V_{CEQ} = 2.80 \text{ (V)}$$
$$I_{CQ} = 1.817 \text{ (mA)}$$

To decide the value for V_{TH}:

$$
\begin{aligned}
V_{TH} &= I_{BQ}R_{TH} + V_{BE(\text{on})} + I_{EQ}R_E \\
&= \left(\frac{1.817}{100}\right)(12.12) + 0.7 + \left(\frac{101}{100}\right)(1.817)(1.2) \\
&= 3.122\ (\text{V}) \\
&= \frac{R_2}{R_1+R_2}V_{CC} = \frac{1}{R_1}(R_1 \parallel R_2)V_{CC} \\
&\Rightarrow R_1 = 34.93\ (\text{k}\Omega) \qquad R_2 = 18.56\ (\text{k}\Omega)
\end{aligned}
$$



10. In the circuit in Figure 10 with transistor parameters $\beta = 180$ and $V_A = \infty$, design the bias resistors R_1 and R_2 to achieve maximum symmetrical swing in the output voltage and to maintain a bias-stable circuit. The total instantaneous C-E voltage is to remain in the range $0.5 \le v_{CE} \le 4.5$ V and the total instantaneous collector current is to be $i_C \ge 0.25$ mA.

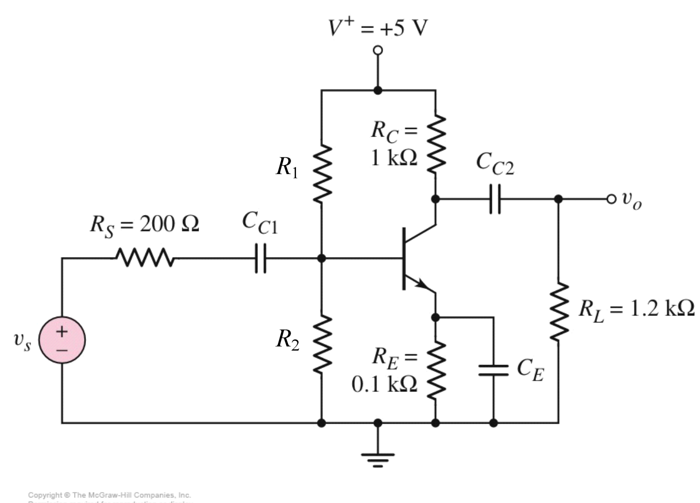


Figure 10

Solution:

To obtain a bias-stable circuit, let:

$$\begin{aligned} R_{TH} &= R_1 \parallel R_2 \\ &= \frac{1}{10}(1+\beta)R_E = \left(\frac{181}{10}\right)(0.1) = 1.81 \text{ (k}\Omega) \end{aligned}$$

The dc load line of the circuit is given by:

$$\begin{aligned} V_{CC} &= I_C R_C + V_{CE} + I_E R_E \\ &= I_C R_C + V_{CE} + \left(\frac{1+\beta}{\beta}\right) I_C R_E \end{aligned}$$

$$I_C = -\frac{V_{CE}}{R_C + \left(\frac{1+\beta}{\beta}\right)R_E} + \frac{V_{CC}}{R_C + \left(\frac{1+\beta}{\beta}\right)R_E} = -0.9086V_{CE} + 4.5432 \qquad ...(1)$$

The ac load line of the circuit is given by:

$$v_{ce} + i_c\left(R_C \parallel R_L\right) = 0$$

$$i_c = -\frac{v_{ce}}{R_C \parallel R_L} = -1.8333 v_{ce} \qquad ...(2)$$

Given $v_{CE(\min)}$ = 0.5 V and $i_{C(\min)}$ = 0.25 mA, the maximum swing of v_{CE} and i_C from the Q-point (I_{CQ}, V_{CEQ}) would be:

$$\begin{aligned} \left|\Delta v_{CE(\max)}\right| &= V_{CEQ} - v_{CE(\min)} = V_{CEQ} - 0.5 \\ \left|\Delta i_{C(\max)}\right| &= I_{CQ} - i_{C(\min)} = I_{CQ} - 0.25 \end{aligned}$$

Since $\left|\Delta v_{CE(\max)}\right|$ and $\left|\Delta i_{C(\max)}\right|$ are related by the ac load line,

$$\begin{aligned} \left|\Delta i_C\right| &= 1.8333\left|\Delta v_{CE}\right| \\ I_{CQ} - 0.25 &= 1.8333\left(V_{CEQ} - 0.5\right) \\ I_{CQ} &= 1.8333V_{CEQ} - 0.6667 \qquad ...(3) \end{aligned}$$

Solving (1) and (3) at the Q-point (I_{CQ}, V_{CEQ}):

$$\begin{aligned} 1.8333V_{CEQ} - 0.6667 &= -0.9086V_{CEQ} + 4.5432 \\ 2.7419V_{CEQ} &= 5.2099 \\ V_{CEQ} &= 1.90 \text{ (V)} \\ I_{CQ} &= 2.817 \text{ (mA)} \end{aligned}$$

To decide the value for V_{TH}:

$$
\begin{aligned}
V_{TH} &= I_{BQ}R_{TH} + V_{BE(\text{on})} + I_{EQ}R_E \\
&= \left(\frac{2.817}{180}\right)(1.81) + 0.7 + \left(\frac{181}{180}\right)(2.817)(0.1) \\
&= 1.012 \text{ (V)} \\
&= \frac{R_2}{R_1 + R_2}V_{CC} = \frac{1}{R_1}(R_1 \parallel R_2)V_{CC} \\
&\Rightarrow R_1 = 8.95 \text{ (k}\Omega\text{)} \qquad R_2 = 2.27 \text{ (k}\Omega\text{)}
\end{aligned}
$$