

Part 2 Course Project

Yigit Ozan Berk

7/2/2019

Overview

Part 1 : In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with `rexp(n, lambda)` where `lambda` is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. Set `lambda = 0.2` for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Part 2: Now in the second portion of the project, we're going to analyze the `ToothGrowth` data in the `R datasets` package.

Load the `ToothGrowth` data and perform some basic exploratory data analyses Provide a basic summary of the data. Use confidence intervals and/or hypothesis tests to compare tooth growth by `supp` and `dose`. (Only use the techniques from class, even if there's other approaches worth considering) State your conclusions and the assumptions needed for your conclusions.

Part 2 - ToothGrowth Data Analysis

At first look, the data has 60 observations with 3 variables, tooth growth length, supplement type, used dosage:

```
library(datasets)
library(dplyr)

##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##   filter, lag

## The following objects are masked from 'package:base':
##
##   intersect, setdiff, setequal, union

library(ggplot2)
str(ToothGrowth)

## 'data.frame':   60 obs. of  3 variables:
##  $ len : num  4.2 11.5 7.3 5.8 6.4 10 11.2 11.2 5.2 7 ...
##  $ supp: Factor w/ 2 levels "OJ","VC": 2 2 2 2 2 2 2 2 2 2 ...
##  $ dose: num  0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 ...

head(ToothGrowth)

##   len supp dose
## 1  4.2   VC  0.5
## 2 11.5   VC  0.5
## 3  7.3   VC  0.5
## 4  5.8   VC  0.5
```

```
## 5 6.4 VC 0.5
## 6 10.0 VC 0.5
```

When we look at the composition of the observed data, we can see that there are length variables for 10 observations of half dosage, 10 obs of 1 dose, and 10 obs of 2 doses for each OJ supplement and VC supplement.

```
table(ToothGrowth[ToothGrowth$supp == "OJ", 3])
```

```
##
## 0.5 1 2
## 10 10 10
```

```
table(ToothGrowth[ToothGrowth$supp == "VC", 3])
```

```
##
## 0.5 1 2
## 10 10 10
```

We can compare the difference in tooth growth between different supplements and different doses of supplements.

Firstly, let's see the range of the tooth growth in each supplement type

```
myVC <- ToothGrowth %>% filter(supp == "VC")
myOJ <- ToothGrowth %>% filter(supp == "OJ")
```

```
range(myVC$len)
```

```
## [1] 4.2 33.9
```

```
range(myOJ$len)
```

```
## [1] 8.2 30.9
```

VC results seem to have a wider range than OJ.

The overall difference of means is as follows:

```
mean(myOJ$len) - mean(myVC$len)
```

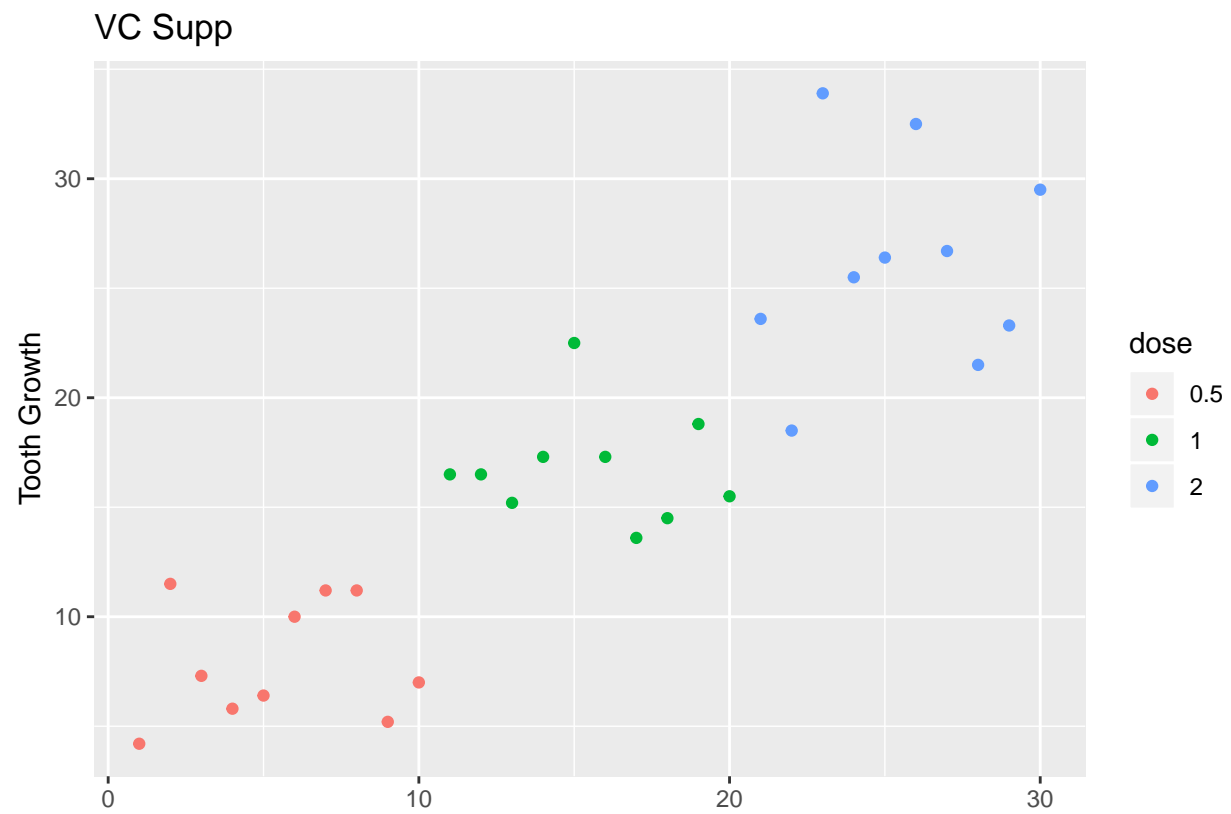
```
## [1] 3.7
```

On average, it looks like the tooth with OJ supplement grew 3.7 units more than tooth with VC supplement.

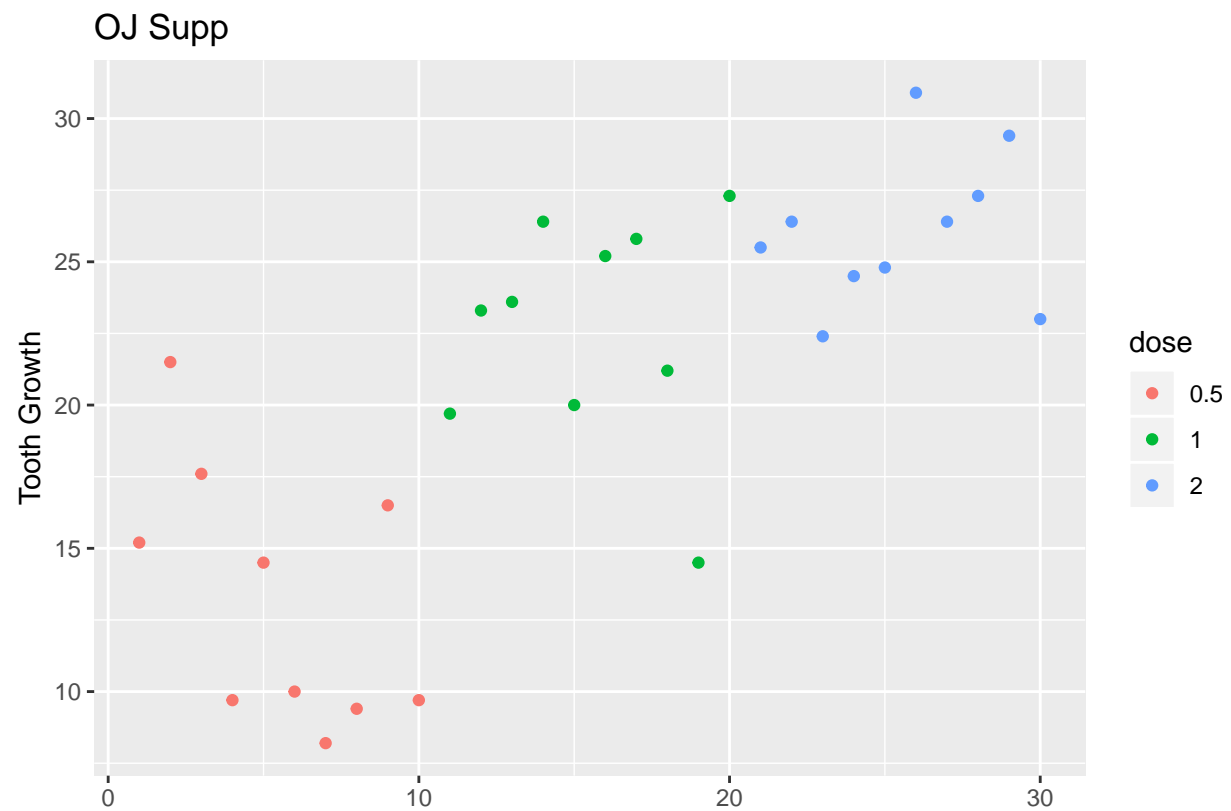
However, this generalisation is definitely not unbiased, because we know that the data contains length results of different doses. The average difference of tooth growth may be variable depending on the dosage.

As the sample size is not very large, we can check all the data we have on a scatterplot:

```
myVC$dose <- as.factor(myVC$dose)
qplot(y = len, data = myVC, col = dose, ylab = "Tooth Growth", main = "VC Supp")
```



```
myOJ$dose <- as.factor(myOJ$dose)
qplot(y = len, data = myOJ, col = dose, ylab = "Tooth Growth", main = "OJ Supp")
```



At first look, VC supp results had a wider range than OJ supp results, but upon closer look, we see that there is greater variation in OJ supplements.

We can test this with a hypothesis.

H_0 : $sd_{oj} > sd_{vc}$, for every dose subset

But first, let's see the more basic hypothesis until this point:

$\mu_{VC0.5} < \mu_{VC1}$ $\mu_{VC1} < \mu_{VC2}$

$\mu_{OJ0.5} < \mu_{OJ1}$ $\mu_{OJ1} < \mu_{OJ2}$

and see the confidence intervals in which these are true.

Analyzing VC Supp Effects on Tooth Growth

assumptions

To calculate the confidence interval of this hypothesis, equal variance among groups is assumed.

The sample distribution is assumed to be normal. As the sample size is small, T distribution methods are applied.

H_0

The null hypothesis is that the mean of dose 0.5 is equal to the mean of dose 1.

The second null hypothesis is that the mean of dose 1 is equal to the mean of dose 2.

calculation

First we need to organize the groups into subsets

```
VC0.5 <- myVC %>% filter(dose == "0.5")
VC1 <- myVC %>% filter(dose == "1")
VC2 <- myVC %>% filter(dose == "2")
#example outlook
VC1
```

```
##      len supp dose
## 1  16.5   VC    1
## 2  16.5   VC    1
## 3  15.2   VC    1
## 4  17.3   VC    1
## 5  22.5   VC    1
## 6  17.3   VC    1
## 7  13.6   VC    1
## 8  14.5   VC    1
## 9  18.8   VC    1
## 10 15.5   VC    1
```

Then, calculating the confidence intervals:

Testing dose 0.5 and dose 1:

H_0 : The mean of dose 0.5 is equal to the mean of dose 1.

```
t.test(VC1$len - VC0.5$len, paired = F, var.equal = T)
```

```
##
## One Sample t-test
##
## data: VC1$len - VC0.5$len
## t = 6.1364, df = 9, p-value = 0.0001715
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 5.549601 12.030399
## sample estimates:
## mean of x
## 8.79
```

According to the T test, p value is smaller than .05 so we reject the null hypothesis that the mean differences is equal to zero. The mean of VC0.5 is 6.13 estimated standard errors away from the mean of VC1. Of which the probability of happening if the means are equal is 0.0001715.

The difference of means is of 8.79 units.

Our related power is

```
power.t.test(n = 20, delta = (mean(VC1$len) - mean(VC0.5$len)), sd = sd(myVC$len), type = "one.sample", a
```

```
## [1] 0.9983221
```

Testing dose 2 and dose 1:

H₀ : The mean of dose 1 is equal to the mean of dose 2.

```
t.test(VC2$len - VC1$len, paired = F, var.equal = T)
```

```
##
## One Sample t-test
##
## data: VC2$len - VC1$len
## t = 5.346, df = 9, p-value = 0.0004648
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## 5.405082 13.334918
## sample estimates:
## mean of x
## 9.37
```

According to the T test, p value is smaller than 0.05 so we reject the null hypothesis that the mean differences is equal to zero. The mean of VC2 is 5.346 estimated standard errors away from the mean of VC1. Of which the probability of happening if the means are equal is 0.0004648

The difference of means is of 9.37 units.

Our related power is

```
power.t.test(n = 20, delta = (mean(VC2$len) - mean(VC1$len)), sd = sd(myVC$len), type = "one.sample", a
```

```
## [1] 0.9993905
```

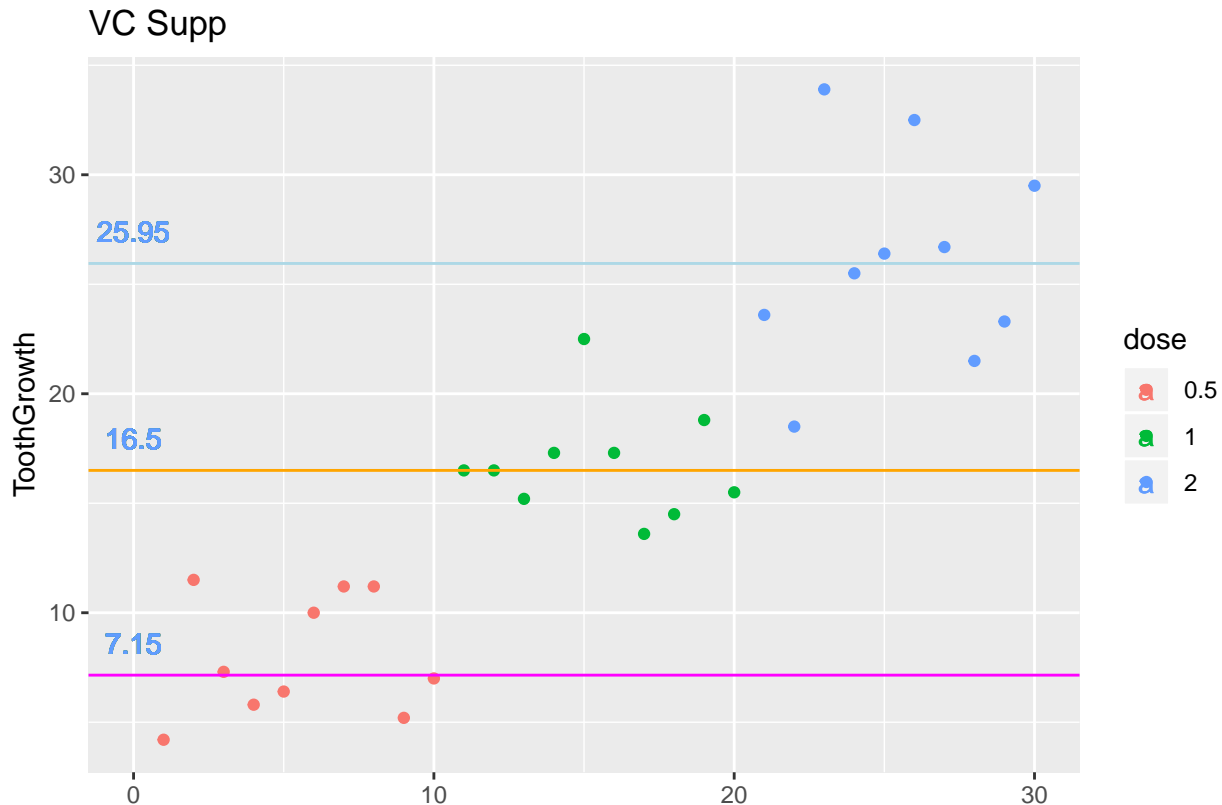
To visualize the medians in a graph together:

```
g <- qplot(y = myVC$len, data = myVC, col = dose, ylab = "ToothGrowth", main = "VC Supp")
g <- g + geom_hline(yintercept = median(VC0.5$len), col = "magenta")
g <- g + geom_hline(yintercept = median(VC1$len), col = "orange")
```

```

g <- g + geom_hline(yintercept = median(VC2$len), col = "lightblue")
g <- g + geom_text(aes(0,median(VC0.5$len),label = median(VC0.5$len), vjust = -1))
g <- g + geom_text(aes(0,median(VC1$len),label = median(VC1$len), vjust = -1))
g <- g + geom_text(aes(0,median(VC2$len),label = median(VC2$len), vjust = -1))
g

```



The median values depending on the dosage is given in the graph above. With %95 confidence we can say the the tooth growth increases with the increase of dose of VC supp.

The average unit growths depending on the dosage with %95 confidence interval are given below:

```

rbind(
  t.test(VC0.5$len)$conf,
  t.test(VC1$len)$conf,
  t.test(VC2$len)$conf
)

```

```

##           [,1]      [,2]
## [1,]  6.015176  9.944824
## [2,] 14.970657 18.569343
## [3,] 22.707910 29.572090

```

In conclusion, according to our analysis, with %95 confidence, we can say that:

The tooth growth with VC supp dose 0.5 is between 6.01 and 9.94 units;

The tooth growth with VC supp dose 1 is between 14.97 and 18.56 units;

the tooth growth with VC supp dose 2 is between 22.70 and 29.57 units.

Analyzing OJ Supp Effects on Tooth Growth

assumptions

To calculate the confidence interval of this hypothesis, unequal variance among groups is assumed.

As the variances are unequal, the data do not follow a T distribution. But it can be estimated by a T distribution.

H₀

The null hypothesis is that the mean of dose 0.5 is equal to the mean of dose 1.

The second null hypothesis is that the mean of dose 1 is equal to the mean of dose 2.

calculation

First we need to organize the groups into subsets:

```
OJ0.5 <- myOJ %>% filter(dose == "0.5")
OJ1 <- myOJ %>% filter(dose == "1")
OJ2 <- myOJ %>% filter(dose == "2")
```

Then, calculating the confidence intervals:

H₀ : The mean of dose 0.5 is equal to the mean of dose 1.

```
t.test(OJ1$len - OJ0.5$len, paired = F, var.equal = F)
```

```
##
## One Sample t-test
##
## data: OJ1$len - OJ0.5$len
## t = 4.1635, df = 9, p-value = 0.002435
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  4.324616 14.615384
## sample estimates:
## mean of x
##      9.47
```

The p value is smaller than 0.05. So we reject the null hypothesis. The observed mean difference is 4.1635 estimated standard deviations away from the OJ1 mean. Of which the probability of happening if the null hypothesis is true is 0.002435.

The difference of means is 9.47 units.

Our related probability of rejecting the null hypothesis when it is false is(power):

```
power.t.test(n = 20, delta = mean(OJ1$len) - mean(OJ0.5$len), sd = sqrt((9*sd(OJ1$len)^2 + 9*sd(OJ0.5$len)^2)/20)

## [1] 0.9999997
```

H₀ : The mean of dose 1 is equal to the mean of dose 2.

```
t.test(OJ2$len - OJ1$len, paired = F, var.equal = F)
```

```
##
## One Sample t-test
##
## data: OJ2$len - OJ1$len
## t = 1.9435, df = 9, p-value = 0.08384
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
## -0.5509376 7.2709376
## sample estimates:
## mean of x
## 3.36
```

The p value is larger than 0.05.

So we fail to reject the null hypothesis.

The observed mean difference is 1.9435 estimated standard deviations away from the OJ2 mean. Of which the probability of happening if the null hypothesis is true is 0.08384.

The difference of means is 3.36 units.

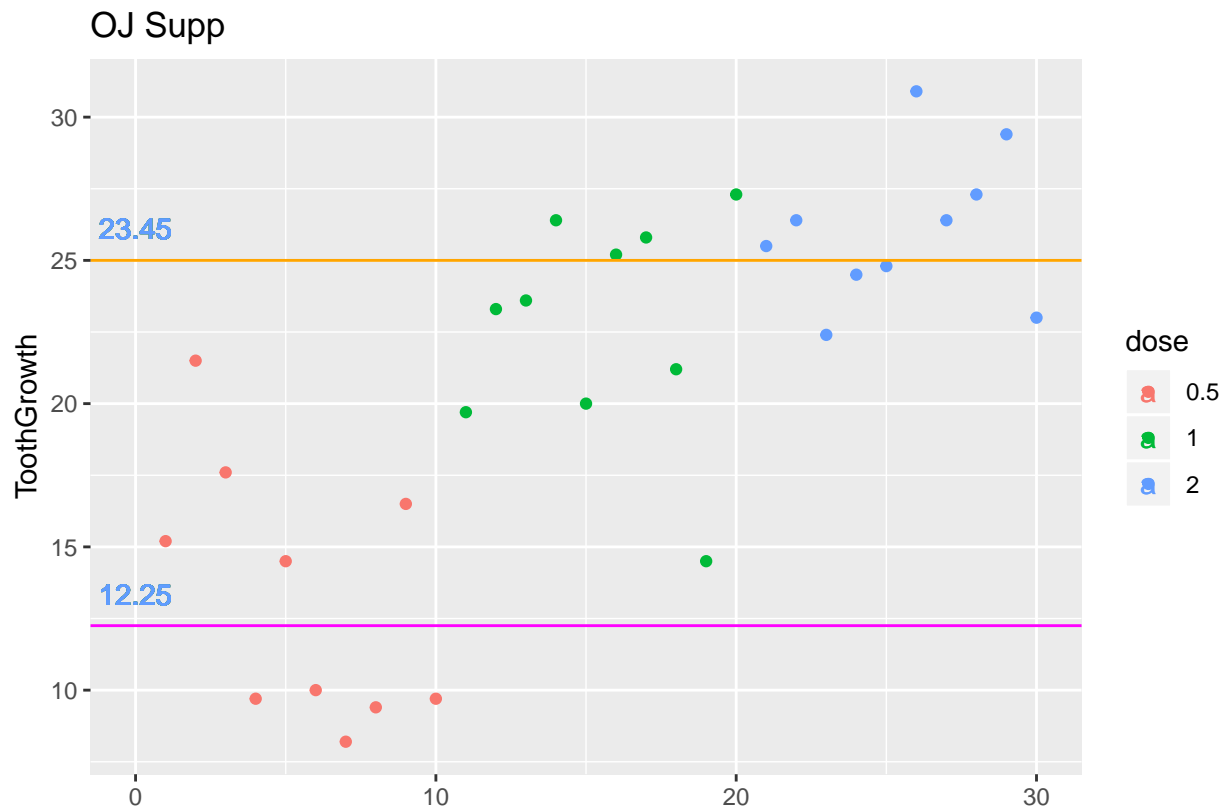
Our related power is:

```
power.t.test(n = 20, delta = mean(OJ2$len) - mean(OJ1$len), sd = sqrt((9*sd(OJ2$len)^2 + 9*sd(OJ1$len)^2)/
```

```
## [1] 0.8723473
```

Let's visualize the medians on a graph

```
myOJ12 <- rbind(OJ1, OJ2)
g2 <- qplot(y = myOJ$len, data = myOJ, col = dose, ylab = "ToothGrowth", main = "OJ Supp")
g2 <- g2 + geom_hline(yintercept = median(OJ0.5$len), col = "magenta")
g2 <- g2 + geom_hline(yintercept = median(myOJ12$len), col = "orange")
g2 <- g2 + geom_text(aes(0, median(OJ0.5$len), label = median(OJ0.5$len), vjust = -1))
g2 <- g2 + geom_text(aes(0, median(myOJ12$len), label = median(OJ1$len), vjust = -1))
g2
```

The median values depending on the dosage is given in the graph above. With %95 confidence we can say the the tooth growth increases if dosage of OJ is >0.5 .

The average unit growths depending on the dosage with %95 confidence interval are given below:

```
rbind(
  t.test(OJ0.5$len)$conf,
  t.test(myOJ12$len)$conf
)
```

```
##           [,1]      [,2]
## [1,] 10.03972 16.42028
## [2,] 22.65688 26.10312
```

In conclusion, according to our analysis, with %95 confidence, we can say that:

The tooth growth with OJ supp dose 0.5 is between 10.03 and 16.42 units;

The tooth growth with OJ supp dose 1 or dose 2 is between 22.65 and 26.10 units.

Comparing the Two Different Medications

Lastly, we can compare the two different medications.

```
rbind(
  c(mean(VC0.5$len), mean(OJ0.5$len)),
  c(mean(VC1$len), mean(OJ1$len)),
  c(mean(VC2$len), mean(OJ2$len))
)
```

```
##      [,1]  [,2]
## [1,]  7.98 13.23
## [2,] 16.77 22.70
## [3,] 26.14 26.06
```

At first look, we can see that for doses 0.5 and 1, OJ has a superior tooth growth output. However, for doses of 2, the difference is not that big.

Let's compare the supplements depending on dosage level

```
rbind(
  t.test(OJ0.5$len - VC0.5$len, paired = F, var.equal = F)$conf,
  t.test(OJ1$len - VC1$len, paired = F, var.equal = F)$conf,
  t.test(OJ2$len - VC2$len, paired = F, var.equal = F)$conf
)
```

```
##      [,1]      [,2]
## [1,]  1.263458 9.236542
## [2,]  1.951911 9.908089
## [3,] -4.328976 4.168976
```

As we predicted above, for doses 0.5 and 1, the confidence intervals are both positive, which means with 95% confidence we can say that OJ supplement gives out more tooth growth than VC supplement.

However, for doses of 2, the 95% confidence interval contains 0, and the distribution is more or less evenly distributed with a center close to 0.

Let's see more details about the test of hypothesis that the means of both groups are equal.

$H_0 : \mu_{OJ2} = \mu_{VC2}$

```
t.test(OJ2$len - VC2$len, paired = F, var.equal = F)
```

```
##
## One Sample t-test
##
## data:  OJ2$len - VC2$len
## t = -0.042592, df = 9, p-value = 0.967
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -4.328976  4.168976
## sample estimates:
## mean of x
##      -0.08
```

We can see that the p-value is nowhere near 0.05. Moreover, we can instead see that with a p-value of .967 we can say that the average tooth growth of OJ and VC supplements at doses of 2 are the same.