① $f(x)$, $x=2$ de türevlenebilen bir fonk. ve $f'(2)=1$ olsun.

$$\lim_{h\to 1} \frac{f(h+1)-f(3-h)}{h-1}$$ limitinin sonucu nedir?

a) 2 b) 4 c) 6 d) 8 e) 10

$\frac{0}{0}$ $\Rightarrow$ $$\overset{L}{=} \lim_{h\to 1} \frac{f'(h+1)+f'(3-h)}{1} = f'(2)+f'(2) = 1+1 = 2$$

② $f(x) = (1+x).(1+x^2).(1+x^3)\dots(1+x^{99})$ ise $f'(0) = ?$

a) 0 b) 1 c) 2 d) -1 e) Hiçbiri

I. Yol Çarpım Türevi:

$$f'(x) = 1.(1+x^2)(1+x^3)\dots(1+x^{99}) + 2x.(1+x)(1+x^3)\dots(1+x^{99})$$
$$+ 3x^2.(1+x)(1+x^2)\dots(1+x^{99}) + \dots + 99.x^{98}(1+x)\dots(1+x^{98})$$

$$f'(0) = 1.1.1\dots1 + 0 + 0 + \dots + 0 = \underline{\underline{1}}$$

II. Yol : Logaritmik Türev:

$$\ln f(x) = \ln(1+x) + \ln(1+x^2) + \dots + \ln(1+x^{99})$$

$\downarrow$ Türev

$$\frac{f'(x)}{f(x)} = \frac{1}{1+x} + \frac{2x}{1+x^2} + \dots + \frac{99x^{98}}{1+x^{99}}$$

$\downarrow x=0$

$$\frac{f'(0)}{f(0) \to 1} = 1 + 0 + \dots + 0 \Rightarrow \boxed{f'(0) = 1}$$

③ $f(x) = \dfrac{1+\text{Tanh}x}{1-\text{Tanh}x} = ?$

a) Sinhx b) Coshx c) e^{2x} d) e^{-2x} e) Hiçbiri

$\text{Cosh}x = \dfrac{e^x + e^{-x}}{2}$ $\text{Sinh}x = \dfrac{e^x - e^{-x}}{2}$ $\text{Tanh}x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

$$f(x) = \frac{1+\text{Tanh}x}{1-\text{Tanh}x} = \frac{1+\dfrac{e^x - e^{-x}}{e^x + e^{-x}}}{1-\dfrac{e^x - e^{-x}}{e^x + e^{-x}}} = \frac{\dfrac{\cancel{2}e^x}{\cancel{e^x + e^{-x}}}}{\dfrac{\cancel{2}e^{-x}}{\cancel{e^x + e^{-x}}}} = \frac{e^x}{\dfrac{1}{e^x}} = e^{2x}$$

④ $\lim\limits_{x \to 0} (\sin x^2)^{1/\ln x} = ?$

a) -2 b) 2 c) e^{-2} d) e^2 e) Hiçbiri

$\lim\limits_{x \to 0^+} (\sin x^2)^{1/\ln x} = ?$

$y = (\sin x^2)^{1/\ln x}$

$\downarrow$ ln al

$\ln y = \frac{1}{\ln x} \cdot \ln(\sin x^2) = \frac{\ln(\sin x^2)}{\ln x}$

$\downarrow$ lim al

$\lim\limits_{x \to 0^+} \ln y = \lim\limits_{x \to 0^+} \frac{\ln(\sin x^2)}{\ln x} \to \frac{\infty}{\infty} \to$ L'H.

$= \lim\limits_{x \to 0^+} \frac{\frac{2x \cdot \cos x^2}{\sin x^2}}{\frac{1}{x}} = \lim\limits_{x \to 0^+} 2 \cdot \overbrace{\frac{x^2}{\sin x^2}}^{1} \cdot \overbrace{\cos x^2}^{1} = 2$

$\lim\limits_{x \to 0^+} \ln y = 2 \Rightarrow \ln\left(\lim\limits_{x \to 0^+} y\right) = 2 \Rightarrow \lim\limits_{x \to 0^+} y = e^2$

⑤ Aşağıdakilerden hangisi yanlıştır?

a) $\lim_{x \to \infty} x . \operatorname{Sin} \frac{1}{x} = 1$

b) $\lim_{x \to \infty} \left(1 - \frac{3}{x}\right)^{x} = e^{-3}$

c) $\lim_{x \to 1} \operatorname{Sin} \frac{1}{1-x}$ limiti mevcut değildir

d) $\lim_{x \to 0} \left(\frac{\operatorname{Sin} x}{3x}\right)^{1/x} = 0$

e) $\lim_{x \to 0^-} e^{1/x} = 0$

$$\lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = 1 \quad \left(\frac{\sin 0}{0} \text{ durumu} \right) \checkmark$$

$$\lim_{x\to\infty} \left(1+\frac{-3}{x}\right)^{x} = e^{-3} \quad \left(\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{x} = e^{a} \right) \checkmark$$

$$\lim_{x\to 1} \sin\frac{1}{1-x} \to \text{Mevcut değil} \quad \checkmark$$

$$\lim_{x\to 0^-} e^{1/x} = 0 \quad \left(\lim_{x\to 0^-} \frac{1}{x} \to -\infty \quad (e^{-\infty}) \to 0 \right) \checkmark$$

$$\lim_{x\to 0} \left(\frac{\overbrace{\sin x}^{1/3}}{3x} \right)^{1/x}$$

$$\nearrow \lim_{x\to 0^+} \left(\frac{1}{3}\right)^{1/x \to \infty} \Rightarrow \left(\frac{1}{3}\right)^{\infty} \to 0 \searrow$$

$$\searrow \lim_{x\to 0^-} \left(\frac{1}{3}\right)^{1/x \to -\infty} \Rightarrow \left(\frac{1}{3}\right)^{-\infty} \to \infty \nearrow$$

$\neq$ Limit yok

⑥ $f(x)=\begin{cases} \dfrac{g(x)}{x} & , x\neq 0 \\ 0 & , x=0 \end{cases}$ ve $g(0)=g'(0)=0$, $g''(0)=16$ ise $f'(0)=?$

a) 4 b) 8 c) 0 d) 16 e) Hiçbiri

$$f'(0)=\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}=\lim_{h\to 0}\frac{\frac{g(h)}{h}-0}{h}=\lim_{h\to 0}\frac{g(h)}{h^2}\to\frac{0}{0}\to L'H$$

$$=\lim_{h\to 0}\frac{g'(h)}{2h}=\lim_{h\to 0}\frac{g''(h)}{2}=\frac{g''(0)}{2}=\frac{16}{2}=8$$

$\frac{g'(h)}{2h}$ ↓ 0/0 L'H

(Türev tanımı ile değil de direkt yapsaydık: $f'(x)=\dfrac{g(x)-xg'(x)}{x^2} \Rightarrow f'(0)=\dfrac{0}{0}$ olurdu.

Bu sonuç bize türev tanımı kullanmamız gerektiğini söyler)

⑦ $y = \left(\dfrac{(x+1)(x^2+1)}{(x^3+1).(x^4+1)} \right)^{1/3} \Rightarrow y'\Big|_{x=1} = ?$

a) $-\dfrac{1}{3}$ b) $-\dfrac{2}{3}$ c) $\dfrac{1}{4}$ d) $\dfrac{1}{6}$ e) $-\dfrac{1}{2}$

$$\ln y = \frac{1}{3}\left[\ln(x+1) + \ln(x^2+1) - \ln(x^3+1) - \ln(x^4+1)\right]$$

$$\frac{y'}{y} = \frac{1}{3}\left[\frac{1}{x+1} + \frac{2x}{x^2+1} - \frac{3x^2}{x^3+1} - \frac{4x^3}{x^4+1}\right]$$

$$\frac{y'(1)}{\underbrace{y(1)}_{1}} = \frac{1}{3}\left[\frac{1}{2} + 1 - \frac{3}{2} - 2\right] \Rightarrow y'(1) = -\frac{2}{3}$$

8) $f(x)=\sqrt{x+\sqrt{x+\sqrt{x}}}$ ve $f'(1)=\dfrac{1+\dfrac{a}{b\sqrt{c}}}{c\sqrt{1+\sqrt{c}}}$ ise $a+b+c=?$

a) 0 b) 4 c) 7 d) 9 e) 5

$$\ln f(x)=\frac{1}{2}\ln\left[x+\sqrt{x+\sqrt{x}}\right]$$

$$\frac{f'(x)}{f(x)}=\frac{1}{2}\,\frac{1+\dfrac{1+\dfrac{1}{2\sqrt{x}}}{2\sqrt{x+\sqrt{x}}}}{x+\sqrt{x+\sqrt{x}}}\Rightarrow\frac{f'(1)}{\underbrace{f(1)}_{\sqrt{1+\sqrt{2}}}}=\frac{1}{2}\,\frac{1+\dfrac{1+\frac{1}{2}}{2\sqrt{2}}}{1+\sqrt{2}}$$

$$f'(1)=\frac{1+\dfrac{3}{4\sqrt{2}}}{2\sqrt{1+\sqrt{2}}}\qquad\left.\begin{array}{l}a=3\\b=4\\c=2\end{array}\right\}\quad a+b+c=3+4+2=9$$

9) f ve g, $f'=g$ ve $g'=f$ koşullarını sağlayan iki fonk. olsun. $f-g$ aşağıdakilerden hangisi olabilir?

a) x b) e^{-x} c) e^{x} d) $\ln(1+x)$ e) $\ln x$

$(\cosh x)' = \sinh x$ $(\sinh x)' = \cosh x$

$g = \sinh x = \dfrac{e^{x}-e^{-x}}{2}$ $f = \cosh x = \dfrac{e^{x}+e^{-x}}{2}$ olsun.

$$f-g = \frac{e^{x}+e^{-x}}{2} - \frac{e^{x}-e^{-x}}{2} = \underline{\underline{e^{-x}}}$$

(10) $f(x) = \text{Arctan}(\text{Cosh}(2x-2)) \Rightarrow f'(1) = ?$

a) 1 b) -1 c) 0 d) e e) Hiçbiri

$$f'(x) = \frac{2\sinh(2x-2)}{1+\cosh^2(2x-2)} \Rightarrow f'(1) = \frac{2 \cdot \sinh 0}{1+\cosh^2 0} = \frac{0}{2} = 0$$

11) $f(x) = \dfrac{\ln(\ln(\ln x))}{x-3} + \sin 2x + \sqrt[3]{x-2} + \cos(x-1)$ fonksiyonunun tanım kümesi?

a) $(3,\infty)$ b) $(e,3)$ c) $(0,3)$ d) $(e,3) \cup (3,\infty)$

e) $(e,2) \cup (2,\infty)$

$\ln(\ln(\ln x))$ için

$\ln(\ln x) > 0 \Rightarrow \ln x > 1 \Rightarrow \boxed{x > e}$

$\ln x > 0 \Rightarrow \boxed{x > 1}$

$\boxed{x > 0}$

$\boxed{x>e} \longrightarrow \boxed{x>e}$

$\sin 2x$: $\forall x \in \mathbb{R}$ tanımlı

$\sqrt[3]{x-2}$: $\forall x \in \mathbb{R}$ tanımlı

$\cos(x-1)$: $\forall x \in \mathbb{R}$ tanımlı

Payda : $x-3 \Rightarrow x \neq 3$ olmalı

Sonuç: $(e,3) \cup (3,\infty)$

(12) $e^x + y^e = e^y + x^e$ fonksiyonunun $(0,0)$ daki teğeti?

a) $y = -x$ b) $y = 4x$ c) $y = \frac{x}{4}$ d) $y = x$ e) $y = 2x$

(12) $e^x + y^e = e^y + x^e$

I. Yol . Kapalı Türetme:

$$e^x + e.y^{e-1}.y' = e^y.y' + e.x^{e-1} \xrightarrow[y=0]{x=0} 1 = y' \xrightarrow{\text{Teğet}} \boxed{y = x}$$

II. Yol formül: $f: e^x + y^e - e^y - x^e = 0$

$$y' = -\frac{f_x}{f_y} = -\frac{e^x - e.x^{e-1}}{e.y^{e-1} - e^y} \xrightarrow[y=0]{x=0} y' = -\frac{1}{-1} = 1 \xrightarrow{\text{Teğet}} \boxed{y = x}$$

13) $\lim\limits_{x\to 0^+} \dfrac{(\ln x)^2}{\ln(\sin x)} = ?$

a) 0 b) Limit mevcut değil c) ∞ d) $-\infty$ e) 1

$$\lim_{x\to 0^+} \frac{(\ln x)^2}{\ln(\sin x)} = \lim_{x\to 0^+} \frac{2.\ln x.\frac{1}{x}}{\frac{\cos x}{\sin x}} = \lim_{x\to 0^+} \underbrace{2.\ln x}_{-\infty}.\underbrace{\frac{\sin x}{x}}_{1}.\underbrace{\frac{1}{\cos x}}_{1} = \underline{-\infty}$$

$\infty/\infty \rightarrow$ L'H.

(14) $\lim_{\theta \to 0} \dfrac{3^{\sin\theta} - 1}{\theta} = ?$ a) 1 b) Mevcut değil c) 0 d) ln3 e) 3ln3

$$\lim_{\theta \to 0} \frac{3^{\sin\theta} - 1}{\theta} = \lim_{\theta \to 0} \frac{\overset{1}{\cos\theta} \cdot 3^{\overset{1}{\sin\theta}} \cdot \ln 3}{1} = \ln 3$$

$\frac{0}{0} \to L'H$

⑮ $\lim_{x \to 0^+} \sin x . \ln x = ?$ a) ∞ b) Yok c) 0 d) 1 e) 2

$$\lim_{x\to 0^+} \underbrace{\sin x . \ln x}_{0.(-\infty)} = \lim_{x\to 0^+} \underbrace{\frac{\ln x}{\frac{1}{\sin x}}}_{\frac{-\infty}{\infty} \to L'H.} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}}$$

$$= \lim_{x\to 0^+} \underbrace{\frac{\sin x}{x}}_{1} . \underbrace{\sin x}_{0} . \underbrace{\frac{-1}{\cos x}}_{-1} = 0$$

(17) $x^y = y^x \Rightarrow \dfrac{dy}{dx} = ?$

a) $\dfrac{1-\ln x}{1-\ln y}$ b) $x^{y-x}.\ln x$ c) $(1-\ln x)\dfrac{y}{x}$ d) $(1-\ln y)\dfrac{y}{x}$

e) $\left(\dfrac{y}{x}\right)^2 \cdot \dfrac{1-\ln x}{1-\ln y}$

$x^y = y^x$

$\downarrow$ ln al

$\ln x^y = \ln y^x$

$\downarrow$

$y.\ln x = x.\ln y$

$\downarrow$ Türev (Kapalı Türetme)

$y'.\ln x + y.\dfrac{1}{x} = \ln y + x.\dfrac{y'}{y}$

$y'\left[\ln x - \dfrac{x}{y}\right] = \ln y - \dfrac{y}{x}$

$y' = \dfrac{x\ln y - y}{x} \cdot \dfrac{y}{y\ln x - x} = \left(\dfrac{y}{x}\right) \dfrac{\dfrac{y\ln x}{x\ln y - y}}{\dfrac{y\ln x - x}{x\ln y}} = \left(\dfrac{y}{x}\right) \cdot \dfrac{y\ln x - y}{x\ln y - x}$

$= \left(\dfrac{y}{x}\right) \cdot \dfrac{y(\ln x - 1)}{x(\ln y - 1)}$

$= \boxed{\left(\dfrac{y}{x}\right)^2 \cdot \dfrac{1-\ln x}{1-\ln y}}$

16. $y = x^{\cos x} \Rightarrow y'\left(\frac{\pi}{2}\right) = ?$ a) $\frac{\pi}{2}$ b) $\frac{\pi}{2}+1$ c) Mevcut değil

d) $\ln\left(\frac{2}{\pi}\right)$ e) $\ln\left(\frac{\pi}{2}\right)$

$y = x^{\cos x} \Rightarrow \ln y = \cos x . \ln x$

↓ Türev

$\frac{y'}{y} = (-\sin x) . \ln x + \cos x . \frac{1}{x}$

$y\left(\frac{\pi}{2}\right) = \left(\frac{\pi}{2}\right)^0 = 1$

↓ $x = \pi/2$

$\frac{y'}{1} = -1 . \ln\left(\frac{\pi}{2}\right) + 0 \Rightarrow y'\left(\frac{\pi}{2}\right) = -\ln\frac{\pi}{2}$

$= \ln\frac{2}{\pi}$

⑱ $f\left(\frac{\pi}{2}\right)=6$, $f'\left(\frac{\pi}{2}\right)=3$ ve $g(x)=(f(x))^{\sin x}$ $\Rightarrow$ $g'\left(\frac{\pi}{2}\right)=?$

a) 3 b) $\frac{1}{2}$ c) 2 d) $\frac{1}{3}$ e) $\ln 3$

$\ln g(x) = \sin x . \ln f(x)$

$g\left(\frac{\pi}{2}\right) = \left(f\left(\frac{\pi}{2}\right)\right)^{\sin \pi/2} = 6$

$\downarrow$

$$\frac{g'(x)}{g(x)} = \cos x . \ln f(x) + \sin x . \frac{f'(x)}{f(x)}$$

$\downarrow$ $x = \frac{\pi}{2}$

$$\frac{g'\left(\frac{\pi}{2}\right)}{6} = 0 + 1 . \frac{3}{6}$$

$g'\left(\frac{\pi}{2}\right) = 3$

(19) $f(x) = \dfrac{1}{\sqrt{|x|-x}} + \ln\left(\dfrac{9-x^2}{x^2+x}\right)$ tanım kümesi?

a) $(-\infty, 0)$ b) $(0,3)$ c) $(-3,-1) \cup (0,3)$ d) $(-3,-1)$

$|x| - x > 0$

$\downarrow$

$|x| > x$

$\Downarrow$

$x < 0$ olmalı

$\boxed{(-\infty, 0)}$

$\dfrac{9-x^2}{x^2+x} > 0 \Rightarrow \dfrac{(3-x)(3+x)}{x(x+1)} > 0$ olmalı

x		-3		-1		0		3	
$9-x^2$	$-$	0	$+$		$+$		$+$	0	$-$
x^2+x	$+$		$+$	0	$-$	0	$+$		$+$
$\dfrac{9-x^2}{x^2+x}$	$-$		$+$		$-$		$+$		$-$

$\boxed{(-3,-1) \cup (0,3)}$

Kesişimi

$\boxed{(-3,-1)}$

(20) $f(x) = \ln(\ln(\ln x)) + \sqrt{9-x^2}$ => tanım kümesi?

a) (e, ∞) b) $[-3,3]$ c) $(1,\infty)$ d) $[e,3]$ d) $(e,3]$

$$\ln(\underbrace{\ln(\ln x)}_{>0})$$

$$\Downarrow$$

$$\ln(\ln x) > 0$$

$$\Downarrow$$

$$\ln x > 1$$

$$\Downarrow$$

$$\boxed{x > e}$$

$x > 0$

$\ln x > 0$

$x > 1$

$9 - x^2 \geq 0$

x		-3		3	
$9-x^2$	$-$	ϕ	$+$	ϕ	$-$

$[-3, 3]$

T.K. : $(e, 3]$

21) $f(x) = \begin{cases} \frac{\pi}{2} & , \; x=0 \\ \sin\frac{x}{3} & , \; 0<x<3 \\ 2^{\frac{1}{x-4}} & , \; 3<x<4 \text{ ve } 4<x\leq 5 \\ \frac{\pi}{2} & , \; x=4 \end{cases}$

[0,5] aralığında tanımlı f(x) fonk.u için aşağıdakilerden hangisi yanlıştır?

a) $x=0$ da kaldırılabilir süreksizdir

b) $x=3$ de sıçramalı süreksizdir

c) $x=4$ de sıçramalı süreksizdir

d) $x=1$ de süreklidir

e) $x=5$ de süreklidir

$\underline{x=0}$

$$\left.\begin{array}{l} \lim\limits_{x \to 0^+} \sin\left(\frac{x}{3}\right) = 0 \\ f(0) = \frac{\pi}{2} \end{array}\right] \neq \quad x=0 \text{ da kaldırılabilir süreksizdir.}$$

$\underline{x=3}$

$$\left.\begin{array}{l} \lim\limits_{x \to 3^+} 2^{\frac{1}{x-4}} = \frac{1}{2} = f(3) \\ \lim\limits_{x \to 3^-} \sin\left(\frac{x}{3}\right) = \sin 1 \end{array}\right] \neq \quad x=3 \text{ de sıçramalı süreksizdir.}$$

$\underline{x=4}$

$$\left.\begin{array}{l} \lim\limits_{x \to 4^+} 2^{\frac{1}{x-4}} = 2^{\infty} = \infty \\ \lim\limits_{x \to 4^-} 2^{\frac{1}{x-4}} = 2^{-\infty} = 0 \end{array}\right] \quad x=4 \text{ de sonsuz (esas) süreksizdir.}$$

(22) $f(x) = \begin{cases} x^2 \cdot \sin \dfrac{1}{x^2}, & x<0 \\ a - \text{Arcsin}\left(\dfrac{x+1}{2}\right), & 0 \le x < 1 \\ \dfrac{a}{b} + \text{Arctan}\sqrt{3}x, & x \ge 1 \end{cases}$ fonk. her $x \in \mathbb{R}$ için sürekli ise a ve b değerleri?

a) $a=\frac{\pi}{6}$ $b=-\frac{1}{4}$ b) $a=1+\frac{\pi}{6}$ $b=\frac{6+\pi}{6}\cdot\frac{12-7\pi}{12}$ c) $a=\frac{\pi}{3}$ $b=-\frac{1}{2}$

$$\lim_{x\to 0^-} x^2 \cdot \sin\frac{1}{x^2} = 0$$

$$\lim_{x\to 0^+} a - \arcsin\left(\frac{x+1}{2}\right) = a - \frac{\pi}{6} = f(0)$$

$\Rightarrow a - \frac{\pi}{6} = 0 \Rightarrow \boxed{a = \frac{\pi}{6}}$ //

$$\lim_{x\to 1^-}\left(a - \arcsin\left(\frac{x+1}{2}\right)\right) = a - \frac{\pi}{2} = \frac{\pi}{6} - \frac{\pi}{2} = -\frac{\pi}{3}$$

$$\lim_{x\to 1^+}\left(\frac{a}{b} + \arctan\sqrt{3}x\right) = \frac{a}{b} + \frac{\pi}{3} = f(1)$$

$\frac{\pi/6}{b} + \frac{\pi}{3} = -\frac{\pi}{3}$

$\boxed{b = -\frac{1}{4}}$ //

(23) $g(1)=h'(1)=g'(1)=h(1)=2$, $f(x)=(g(x^2))^{h(x)}$ $\Rightarrow f'(1)=?$

a) $2\ln 2+4$ b) $8\ln 2+16$ c) $8\ln 2+2$ d) 8 e) $2+\ln 2$

$$f(x)=[g(x^2)]^{h(x)} \quad , \quad f(1)=(g(1))^{h(1)}=2^2=4$$

$$\ln f(x)=h(x)\cdot \ln g(x^2)$$

$$\frac{f'(x)}{f(x)}=h'(x)\cdot \ln g(x^2)+h(x)\cdot \frac{2x\, g'(x^2)}{g(x^2)}$$

$$\frac{f'(1)}{f(1)}=h'(1)\cdot \ln g(1)+h(1)\ \frac{2\, g'(1)}{g(1)}$$

$$f'(1)=4\cdot\left(2\cdot\ln 2+\cancel{2}\cdot\frac{2\cdot 2}{\cancel{2}}\right)$$

$$f'(1)=8\ln 2+16 \quad //$$

(24) $f: (-\infty, 1) \to \mathbb{R}$, $f(x) = \dfrac{1+x}{\sqrt{1+x^2}}$ $\Rightarrow (f^{-1})'(0) = ?$

a) $\sqrt{2}$ b) 1 c) $\dfrac{1}{\sqrt{2}}$ d) -1 e) 0

$$f(x) = \frac{1+x}{\sqrt{x^2+1}} \Rightarrow f'(x) = \frac{1 \cdot \sqrt{1+x^2} - (1+x) \cdot \dfrac{2x}{2\sqrt{1+x^2}}}{(1+x^2)}$$

$$(f^{-1})'(0) = \frac{1}{f'(\underbrace{f^{-1}(0)}_{-1})}$$

$$f'(-1) = \frac{1}{\sqrt{2}}$$

$f^{-1}(0) = a \Rightarrow f(a) = 0$

$$\frac{1+a}{\sqrt{1+x^2}} = 0 \Rightarrow a = -1 \quad \Rightarrow (f^{-1})'(0) = \frac{1}{f'(-1)} = \frac{1}{\frac{1}{\sqrt{2}}} = \underline{\underline{\sqrt{2}}}$$

(25) $f(x) = 1 + x + \ln(1+x^2)$, $f: [0,\infty) \to \mathbb{R}$ $\Rightarrow (f^{-1})'(1) = ?$

a) 1   b) 2   c) $\frac{1}{2}$ d) $\sqrt{2}$ e) 0

$f(x) = 1 + x + \ln(1+x^2)$

$f'(x) = 1 + \frac{2x}{1+x^2}$

$f'(0) = 1$

$(f^{-1})'(1) = \frac{1}{f'(\underbrace{f^{-1}(1)}_{0})}$

$= \frac{1}{f'(0)}$

$= \frac{1}{1}$

$f^{-1}(1) = a$

$f(a) = 1$

$1 + a + \ln(1+a^2) = 1$

$\Downarrow$

$\boxed{a = 0}$

(26) $y=f(x)$ eğrisinin $x=1$ deki normal doğrusu $2x+y-1=0$ olsun. $f(x)$ in $x=-1$ de tersi mevcut olduğuna göre $(f^{-1})'(-1)=?$

a) -2 b) $\frac{1}{2}$ c) 2 d) $-\frac{1}{2}$ e) 1

$y=f(x)$, $x=1$, $2x+y-1=0$

$2x+y-1=0 \Rightarrow y=-2x+1 \Rightarrow m_N=-2$

$m_N \cdot m_T=-1 \Rightarrow \boxed{m_T=\frac{1}{2}=f'(1)}$

$x=1 \Rightarrow 2.1+y-1=0$

$y=-1$

$f(1)=-1$

$(f^{-1})'(-1)=\frac{1}{f'(1)}=\frac{1}{\frac{1}{2}}=2$ //

(27) $g(t)=t^3+7t+21$ ve $g(-2)=-1$ olsun. $g^{-1}(t)$ fonksiyonunun $t=-1$ deki teğet doğrusu?

a) $y=-2+\frac{1}{19}(x+1)$ b) $y=-2-\frac{1}{19}(x+1)$ c) $y=-2+\frac{1}{19}(x-1)$

d) $y=-2-\frac{1}{19}(x-1)$ e) $y=-2+19(x+1)$

$g(t)=t^3+7t+21$, $g(-2)=-1$

$g'(t)=3t^2+7$ $g^{-1}(-1)=-2$

$g^{-1}(t)\approx L(t)=g^{-1}(-1)+(g^{-1})'(-1)\,(x+1)$

$(g^{-1})'(-1)=\dfrac{1}{g'(g^{-1}(-1))}=\dfrac{1}{g'(-2)}=\dfrac{1}{12+7}=\dfrac{1}{19}$

$g^{-1}(t)\approx L(t)=-2+\frac{1}{19}(x+1)$

//

(28) $g: \mathbb{R} \to \mathbb{R}$, $g(1) = g'(1) = 4$ olsun. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \dfrac{g(x^2)}{1+x^2}$ ise $f(1.25)$ sayısının yaklaşık değeri?

a) 2,5 b) 0 c) 1,5 d) 2,25 e) 2,75

$g(1) = g'(1) = 4$ $\quad f(x) = \dfrac{g(x^2)}{1+x^2}$ $\quad , f(1,25)$

$a = 1$ $\quad , f(1) = \dfrac{g(1)}{2} = 2$ $\quad , f'(x) = \dfrac{2x\, g'(x^2)(1+x^2) - 2x\, g(x^2)}{(1+x^2)^2}$

$$f'(1) = \frac{4 g'(1) - 2 g(1)}{4}$$

$$f'(1) = \frac{16-8}{4} = 2$$

$$f(1,25) \approx L(1,25) = f(1) + f'(1)(1,25-1)$$

$$= 2 + 2(0,25) = 2,5$$

$$f(1,25) \approx 2,5 \;//$$

(29) $\lim\limits_{x \to \infty} \left(\dfrac{x+7}{x+3} \right)^{2x+3} = ?$

a) e^4 b) 1 c) ∞ d) e^8 e) 0

$$\lim_{x \to \infty} \left(\frac{x+7}{x+3} \right)^{2x+3} = \lim_{x \to \infty} \Bigg[\underbrace{\left(1 + \frac{4}{x+3} \right)^{x+3}}_{e^4} \Bigg]^{\frac{\frac{2x+3}{x+3}}{2}} = (e^4)^2 = e^8$$

(30) $\lim_{x \to \infty} \left(\frac{x}{x+2} \right)^{3x} = ?$ a) 0 b) 1 c) e^{6} d) e^{-6} e) ∞

$$\lim_{x \to \infty} \left(\frac{x}{x+2} \right)^{3x} = \lim_{x \to \infty} \frac{1}{\left(\underbrace{\left(1+\frac{2}{x}\right)^{x}}_{e^{2}} \right)^{3}} = \frac{1}{e^{6}}$$