20192 CIVIL ENGINEERING FACULTY - DEPARTMENT OF CIVIL ENGINEERING

INS2942 NUMERICAL ANALYSIS FINAL EXAM **16.06.2020**

Name Surname	
Number	
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Group	**1**	**2**	**3**	**4**	**Total**

Good luck ☺

Grading: 25-25-25-25 **Duration : 90 minutes (Solution + System Upload)**

- **Preferably each question should be answered on its own page.**
- **Solutions must be readable and understandable.**
- **You must write your name and number at the top of each page, and you must sign the paper.**

QUESTIONS

1) Evaluate the integral given by $I=\int_1^7 x^2\sin(x)dx$ with 6 uniform subintervals using 3/8 Simpson Rule . Use 3 decimal digits, and fill the table shown below. (25p)

x	$x^2\sin(x)$	Coefficient

$$\int_i^{i+2} f(x)dx \cong \frac{\Delta x}{3}\left(f_i+4f_{i+1}+f_{i+2}\right), \qquad \int_i^{i+3} f(x)dx \cong \frac{3\Delta x}{8}\left(f_i+3f_{i+1}+3f_{i+2}+f_{i+3}\right)$$

2-a) Knowing that $y = y(x)$, find a solution to the differential equation given by $y'' + 4x - 8 = 0$ with the initial conditions $y(0) = 2$, and $y'(0) = 1$ in the domain $0 \le x \le 4$ by means of FDM. Use $N = 4$. (19p)

N: number of subintervals (or "number of segments")
FDM: finite difference method

2-b) Write "true" or "false", for the following statements. (6p)

	Statement	Answer
2-b-1)	FDM is a numerical integration method.	
2-b-2)	Differential equations higher than second order cannot be solved by means of FDM.	
2-b-3)	Boundary conditions should never be used in the solution of differential equations by means of FDM.	

Derivative	Coefficient	$i-2$	$i-1$	i	$i+1$	$i+2$
y_i'	$\frac{1}{2h}$		-1		+1	
y_i''	$\frac{1}{h^2}$		+1	-2	+1	

3) Knowing that $y = y(x)$, find a solution to the differential equation given by $y'' - y' - 2y = 4 + 3x + x^2$ with the initial conditions $y(0) = 0$, and $y'(0) = -1$ using the fourth order Taylor series method (i.e., use 5 terms in the series).

3-a) Write the function y. (20p)

3-b) Use 5 decimal digits, and compute the value of $y(0.5)$. (5p)

$$y(x) = y(x_0) + y'(x_0)x + \frac{1}{2!}y''(x_0)x^2 + \cdots\cdots + \frac{1}{n!}y^{(n)}(x_0)x^n$$

4) A set of data in terms of x and Y where $Y = Y(x)$ is given below.

4-a) Find the Lagrange interpolation polynomial. (5p)

4-b) Using the method of least squares find the straight line $y = a_0 + a_1 x$ which fits the set of data best. (16p)

x	0	1	3	4
$Y(x)$	0	2	6	5

4-c) Write "true" or "false", for the following statements. (4p)

	Statement	Answer
4-c-1)	The Lagrange interpolation polynomial passes through all the points in the set of data.	
4-c-2)	If $x_0, x_n, f(x_0)$ and $f(x_n)$ are known such that $x_0 < x < x_n$, the estimation of the value of $f(x)$ is called "extrapolation".	

$$P(x) = L_0(x) f_0 + L_1(x) f_1 + L_2(x) f_2 + \ldots$$

$$L_i = \frac{(x - x_0)(x - x_1)\ldots\ldots\ldots(x - x_{i-1})(x - x_{i+1})\ldots\ldots\ldots\ldots(x - x_n)}{(x_i - x_0)(x_i - x_1)\ldots\ldots\ldots(x_i - x_{i-1})(x_i - x_{i+1})\ldots\ldots\ldots\ldots(x_i - x_n)}$$

$$\begin{bmatrix} N & \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \ldots & \ldots \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \ldots & \ldots \\ \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i^3 & \sum_{i=1}^{N} x_i^4 & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \begin{Bmatrix} a_0 \\ a_1 \\ a_2 \\ \ldots \\ \ldots \end{Bmatrix} = \begin{Bmatrix} \sum_{i=1}^{N} Y_i \\ \sum_{i=1}^{N} Y_i x_i \\ \sum_{i=1}^{N} Y_i x_i^2 \\ \ldots \\ \ldots \end{Bmatrix}$$