

- 1.) You must work on your own.
- 2.) You may consult 4 pages of notes, and any tables we hand out.
- 3.) I will be available during the test for clarifications of questions. **No hints.**
- 4.) Each sub-problem has equal weight.
- 5.) Please show all work on this exam book.

1D FT's

$$\text{FT} \{ \text{sinc}(x) \} = \text{rect}(k)$$

$$\text{FT} \{ \exp(-\pi x^2) \} = \exp(-\pi k^2)$$

$$\text{FT} \{ \text{comb}(x) \} = \text{comb}(k)$$

$$\text{FT} \{ \delta(x) \} = 1$$

$$\text{FT} \{ \exp(+i2\pi x k_0) \} = \delta(k-k_0)$$

$$\text{FT} \{ \exp(-a|x|) \} = 2a/(4\pi^2 k^2 + a^2)$$

$$\text{FT} \{ f(x/a) \} = |a| F(a k)$$

$$\text{FT} \{ f(x) * h(x) \} = F(k) H(k)$$

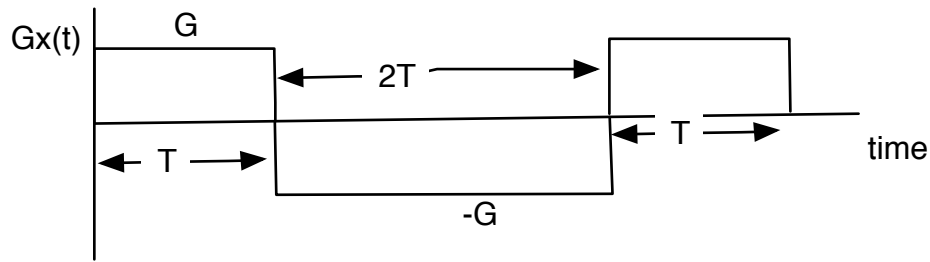
2D FT's

$$\text{FT} \{ \text{circ}(r) \} = \text{jinc}(\rho)$$

$$\text{FT} \{ f(x/W, y/Z) \} = |W Z| F(W K_x, Z K_y)$$

Signature: _____

Name (Printed neatly): _____



Problem 1 Suppose we wish to image an unknown complex-valued 1D object
 $\rho(x) = 1 + 2 \exp(i 2 \pi k_0 x)$

(a) Find the FT of $\rho(x)$.

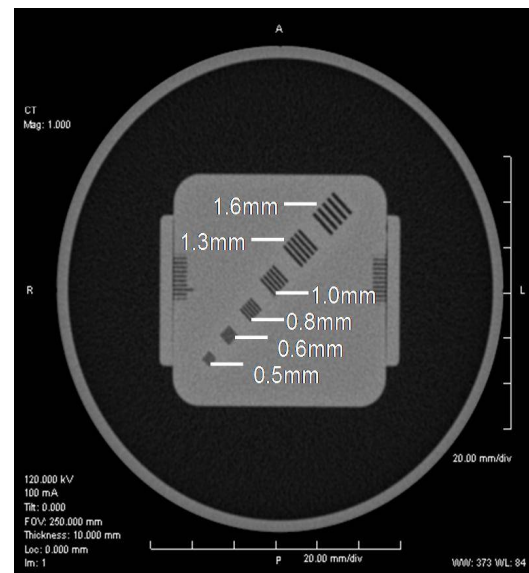
(b) **Plot** $K_x(t)$ versus time over the period $(0, 4T)$. Assume $\gamma/(2 \pi) = 10 \text{ MHz/T}$. $G = 10 \text{ mT/m}$ and $T = 10 \text{ ms}$. Make sure to find the min and max of $K_x(t)$.

(c) **Sketch** the demodulated signal over the period $(0, 4T)$. Assume $k_0 = 1/(2 \text{ mm})$ and neglect T_2 decay.

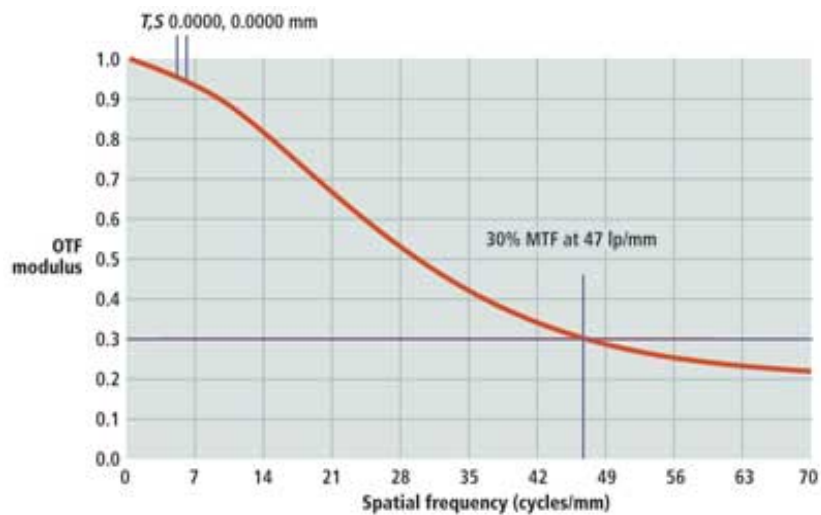
(d) Write an expression for the *reconstructed* 1D MRI image, neglecting T_2 decay.

Problem 2. Resolution phantom data

(a) The image to the right is a resolution phantom for CT. Estimate the spatial resolution and explain your answer.



(b) The OTF is similar to the MTF but for optical imaging systems. Estimate the FWHM spatial resolution of the imaging system below.



3. Imaging short answers

(a) Give **two** reasons why nuclear medicine is **not** used for angiography?

(b) How do you expect the SNR in CT to vary with the number of projections, N ?

(c.) Give **two** reasons why CT is **not** preferred in the brain?

4. New Imaging Systems

(a) Recall the HW problem on Fluorescence CT, where the "bing-ping" molecule is irradiated by a line source at energy E_1 and the molecule fluoresces isotropically at energy E_2 . Assume $E_2 < E_1$. Sketch the attenuation spectrum of electromagnetic radiation through the human body, and label where you would choose E_1 and E_2 to optimize imaging fundamentals (SNR, contrast, resolution). Explain your choice.

(b) Dr. Bore proposes a new way to deliver nuclear medicine agents to diagnose cancer. He wants to use a nanoparticle carrier instead of the conventional small-molecule like FDG, where each molecule has its own metabolic targeting. He prefers to coat the **outside** of the nanoparticle carrier with cancer-targeting chemistry (e.g., a sugar), and fill the inside of the nanoparticle with positron-emitting nuclei. Name **two tradeoffs** with this new method.

5. Circularly symmetric 2D PSF: State the necessary/sufficient conditions under which the 2D point spread function $h(x,y)$ has **circular symmetry** for each imaging modality:

(a) MRI

(b) CT

(c) X-ray

(d) PET

6 Sampling & Nyquist Rate: For each modality, explain in which domain sampling occurs (**time, x-space, or k-space**) and give the 1D Nyquist formula to prevent aliasing.

(a) MRI:

Sampling Domain: _____ (time, x-space, or k-space)

Nyquist Formula:

(b) CT

Sampling Domain: _____ (time, x-space, or k-space)

Nyquist Formula:

(c) X-ray

Sampling Domain: _____ (time, x-space, or k-space)

Nyquist Formula: