BioEC165/EEC145B Fall | Midterm
Prof. Steve Conolly | Dec 1, 2011

1.) You must work on your own.
2.) You may consult 4 pages of notes, and any tables we hand out.
3.) I will be available during the test for clarifications of questions. **No hints.**
4.) Each sub-problem has equal weight.
5.) Please show all work on this exam book.

1D FT's

FT {sinc(x)} = rect(k)
FT{exp($-\pi x^2$) } = exp($-\pi k^2$)
FT{comb(x)} = comb(k)
FT{δ(x)} = 1
FT{exp(+i2π x k0)} = δ(k-k0)
FT{exp(-a|x|) }= $2a/(4\pi^2 k^2+a^2)$

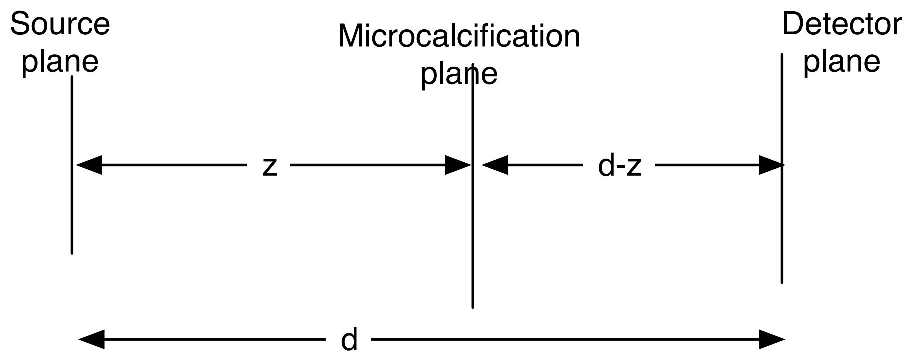
FT { f(x/a) } = |a| F(a k)
FT { f(x) * h(x) } = F(k) H(k)

2D FT's

FT { circ(r) } = jinc(ρ)
FT { f(x/W,y/Z) } = |W Z| F(W Kx, Z Ky)

Signature: ______________________________

Student ID: ______________________________



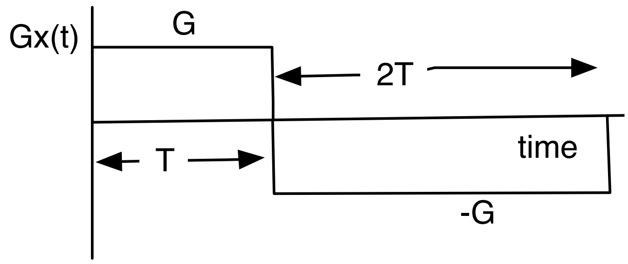
1. Consider the X-ray projection imaging shown above above.

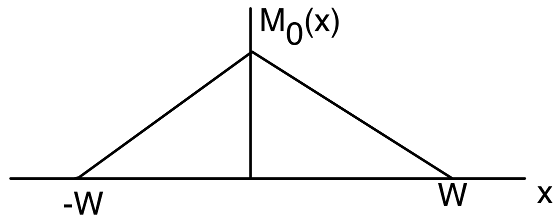
(a) What is the source magnification?

(b) What is the image magnification?

(c.) What is the FWHM spatial resolution of the imaging system if the source size is 1 mm?

(d) Dr. Bore says doubling voltage on the source will improve spatial resolution by a factor of two. Dr. Smore says this won't work, but that doubling the cooling of the anode will allow for doubling the resolution. Explain who is more correct.





Problem 2 Suppose we wish to image an unknown object $M_0(x) = \text{tri}(x/W)$ as shown above with 1D MRI.

(a) Find the FT of $M_0(x)$.

(b) **Plot** Kx(t) versus time over the period (0, 3T). Assume $\gamma/(2\pi) = 10$ MH/T. G= 10 mT/m and T = 10 ms. Make sure to find the min and max of Kx(t).

(b) **Sketch** the demodulated signal over the period (0, 3T). Assume W = 5 mm.

(c) Write an expression for the *reconstructed* 1D MRI image.

Problem 3. A new 1D imaging modality has the following imaging equation

$$s(t) = \int_{-\infty}^{\infty} L(k)\, M(k) \exp(+i\, 2\, \pi\, k\, X(t)\,)\, dk$$

where M(k) is the spatial FT of the **unknown image**, m(x). s(t) is the received signal, and L(k) is a physical property of the imaging system. X(t) is a user-controllable position waveform.

(a) Re-write this imaging equation in x-space as a convolution.

(b) Is this an LSI imaging system?

(c) What is the 1D point spread function and FWHM spatial resolution?

(d) How would you design X(t) to obtain full image data over a FOV with no aliasing?

4. Projection Slice Theorem.

(a) Prove that real-valued functions, $f(x,y)$, have conjugate symmetry for their 2D FT. That is, prove that $F(-Kx,-Ky) = F^*(Kx,Ky)$.

(b) Would there be conjugate symmetry in k-space data for CT? Why?

(c.) Dr. Bore insists that one can exploit this conjugate symmetry property for CT to use half as many projections -- and thereby reduce dose by a factor of 2! Do you agree? Explain your answer.

3. Imaging short answers

(a) What could collimators help with in planar X-ray or with CT? Explain why they are **not** typically used.

(b) Explain why 60 Hz noise in 2D FT MRI manifests as a discrete line along the y-direction.

(c) Could you replace the collimator in SPECT with a pinhole camera?

(d) Explain why it is hard to distinguish bone cancer metastases from broken bones with PET.