Response Document for Qr-Hint: Actionable Hints Towards Correcting Wrong SQL Queries

MAIN CHANGES

We thank the reviewers for giving us the opportunity to revise the paper, which helped us significantly improve it. We first describe a summary of changes in the revision below and then list detailed responses to comments from each reviewer.

Updates made in response to specific comments are color-coded in the paper as follows:

- Changes in response to multiple reviewers are in Purple.
- Other generic improvements are also in Purple.
- Reviewer #1 in Blue.
- Reviewer #2 in Magenta.
- Reviewer #3 in Red.

META-REVIEW

All changes listed in Q14 of the individual reviews and all changes you promised in the author feedback. All other opportunities for improvement noted in Q8 of the individual reviews.

We have significantly revised our paper as follows:

- We have changed the title to "Qr-Hint: Actionable Hints Towards Correcting Wrong SQL Queries".
- We have adjusted **Section 2** so that the most relevant previous work is discussed more thoroughly. We also included all the related work pointed out by the reviewers.
- We have clearly stated the limitations of Qr-Hint at the end of **Section 3**.
- At the beginning of **Section 5.2**, we sketched the strategies for pushing down target bounds in DeriveFixes with a concrete example to convey the intuition better.
- (Not requested by reviewers, but as a general improvement) We have come up with a simpler and cleaner way of handling `HAVING` in **Section 7** (with no impact on experimental results).
- In the experiment section (**Section 9**), we have made the following changes:
  - (Not requested by reviewers, but as a general improvement) Changed our Boolean minimization primitive from Quine-McCluskey to ESPRESSO (see **Implementation/Test Environment**). This significantly improved the runtime of Qr-Hint (by a factor of approximately 4-5). We have rerun the performance experiments to reflect this change (see Figures 2, 3, 4).
  - As suggested by the reviewers, we have systematically gone through errors/issues in [12] and expanded our dataset of real student queries to include additional queries with errors/issues not already covered (**Test Data Preparation**); we call this new, augmented dataset Students+.
  - Besides using TPC-H to test performance, we now also use Students+ to test both coverage and performance of Qr-Hint (**Results and Discussion: Student+**).
- In user study (**Section 10**), following the suggestions of reviewers, we have made the following changes:
  - Clarifications on the design and context (please see individual comments for specific changes).
  - We combined the old 4 pie charts for hint categorization (i.e., obvious, unhelpful, helpful) into two stacked column charts (Figure 6) for a better side-by-side comparison of the vote cast result between TAs' hints and hints from Qr-Hint.
- We have fixed the typos/styling issues mentioned in the minor remarks from reviewers.

REVIEWER #1

Comment R1-O1. There are limitations in the language surface supported. While this is something that currently limits the applicability of the system it opens up opportunities for more research and improvement so I won't hold it against the paper.

Response. Thank you. We mentioned this as a future work in **Section 11**, and the limitations/unsupported SQL constructs are discussed at the end of **Section 3**.

Comment R1-O2. Using SMT solvers does make the technique less compelling, as the authors themselves mention about it returning "unknown" in several scenarios. However, this is a technique that most of the related work in this space use, so I again won't hold it against. It would be good to explore if there are alternate approaches to achieve these results without using SMT solvers, wherever possible.

Response. Thank you. We mentioned this as a future work in **Section 11**.

Comment R1-O3. It would be good to show experiments that showcase where Qr-Hint was unable to produce hints or produced unsatisfactory hints.

Response. To clarify, for all SQL constructs we support, Qr-Hint is guaranteed to provide hints leading to correct fixes. They may be unsatisfactory in not being globally optimal – including when we suggest a fix for a query that's correct (hard to avoid due to undecidability of query equivalence in general).

We have explicitly documented our coverage test in Section 9 (**Test Data Preparation**, the details of the dataset are in [28]). We used the coverage dataset to test 1) if Qr-Hint is able to fix wrong queries correctly and 2) how well Qr-Hint covers the errors listed in [12]. We also clearly stated the limitations/unsupported SQL constructs at the end of **Section 3**, including an explanation of when Qr-Hint will produce false positives (i.e., potential unsatisfactory hints).

REVIEWER #2

Comment R2-O1. The work is not about SQL debugging at all (find out how a query is evaluated), but about nudging learners towards the solution. Please note the line of research on SQL debugging, e.g. by Torsten Grust, and reconsider the title.

Response. We have changed our title accordingly and discussed the work by Torsten Grust in **Section 2**.

Comment R2-O2. The experiments are partly obscure, it's not clear to me that they show that this approach really helps students. After reading the paper, I am simply not convinced. Worse yet, I am left with no intuition of the sweet spots and limitations of your approach. The approach has limitations, but this information is scattered throughout the paper. Please address them early and openly (e.g., you may produce false positives, or not be able to provide any hints at all).

Response. To clarify, Qr-Hint is guaranteed to produce hints leading to correct fixes for single-block SFWGH queries without outer-join. We have listed our limitations openly at the end of **Section 3**.

In the experiments on the TPC-H queries, you "arbitrarily tweaked two predicates from each query". I am afraid this does not come across as a well-principled, systematic and scientific evaluation. You provide the queries, but as mentioned in the discussion of the artifacts, they are not well-consumerable. In the discussion in Section 9.1, you mention that "most real queries do not contain many predicates" - I agree to the point where most real queries *written by students under exam conditions* do not contain many predicates. I suggest you also evaluate runtime and costs for realistic student queries. In fact, I suggest to use the student queries from your user study.

Response. We have the set of student queries for the coverage experiment and we have added performance tests on them (**Student+**). They stressed our algorithms much less than synthetic TPC-H errors: it took on average 0.2 seconds to fix each query. As mentioned earlier in author feedback, the TPC-H queries with randomly injected errors were designed to exhaustively test a) Qr-Hint's efficiency and b) its ability to fix single- and multi-site errors optimally. In addition, we have updated the **Feasibility** paragraph in **Section 9** to remove the claim "most real queries do not contain many predicates".

I inspected the queries from the user study in the long version of the document. I suggest to work with more comprehensive collections of student queries, such as provided by Stefan Brass: http://dbs.informatik.uni-halle.de/sqllint/ He provides the solution queries, lists the student mistakes, he has statistics on which mistakes are common.

Response. Brass's work [12] doesn't make wrong student queries available; it only describes the types of errors. Nonetheless, we have gone through their list and designed coverage tests based on their list of errors in **Section 9** (**Test Data Preparation**, **Student+**). Also, many of their errors are not applicable to us because they are stylistic or efficiency issues (e.g., redundant predicate or GROUP BY column) and may not cause logical inequivalence. We have summarized how Qr-Hint reacts to those errors in **Student+**. The detailed breakdown is at the end of full version [28] due to the page limit of the paper.

It is not clear to me that the queries from your use study (over just 15 students) cover the same breadth of common mistakes as his collections. I am aware that user studies are tricky. However, just 15 students as a baseline is very small. I would like to point out Wolfgang Gatterbauer's recent work with user studies conducted on MechanicalTurk, maybe this is inspiring: https://osf.io/mycr2/wiki/home/

Response. QueryVis user study is indeed beautifully done. We also considered MTurk but deemed it less suitable to us. First, it is hard to control for MTurkers SQL expertise level to match our target population (students). Second, our task is much harder. QueryVis asks subjects to pick what a query does in English with/without visualization aid. In contrast, our subjects must spot sometimes subtle errors in a query that looks right; some took 1+ hour! This makes it hard to incentivize/vet MTurkers: a low reward turns them away, while a high reward encourages undesirable behavior. In our user study, we recruited 38 students, with both a $10 gift card and practicing SQL as incentive. In the end, only 15 produced complete and usable responses. A larger-scale user study will certainly be valuable, but availability of students during the winter break made it impossible for revision.

Instead, as promised in the author feedback, we strengthened the coverage experiment discussed above (based on real student queries) and add queries crafted according to Brass's work [12]. We also highlighted the scale of our user study as a limitation and added the context above to the **Participants** paragraph in **Section 10**.

Comment R2-O3. Discussion of related work is very superficial. When citing Chandra et al [13], it seems that this is rather related. Please discuss this work in comparison to yours, don't just list it. Please discuss existing work on SQL debugging, e.g. by Torsten Grust. Please discuss the large body of work on SQL code smells, e.g. as conducted in the ICSE/ISSTA community, or SQL Linting by Stefan Brass http://dbs.informatik.uni-halle.de/sqllint/.

Response. We have added related work from Brass and Grust. We have also revamped **Section 2** to discuss more related work thoroughly.

Comment R2-O4. Problems with the presentation. The font in the SQL statements is way too small, also in the algorithms presented. The tiny charts are near-unreadable on my paper printout. Starting with section 4, the paper becomes very technical and therefore hard to read.

Response. We have adjusted the fonts in algorithms and tables. We have also adjusted the plots to be more visible.

REVIEWER #3

Comment R3-O1, R3-O2. The paper relies a bit too much on references to the technical report (citation [21] in the paper). Some of these references are reasonable, such as cases where an intuitive explanation is provided for a lemma, but the full proof is delegated to [21]. However, other cases go a bit too far, such as the "optimized" algorithm for DeriveFixes. In fact, I would suggest removing all references to DeriveFixes-optimized from the paper (including from Example 9 and the evaluation). According to Figures 3 and 4, DeriveFixes-optimized does not seem to result in a significant cost improvement over DeriveFixes, and the runtime is much worse. Given that the algorithm is not described in any detail, I think it would be better to leave it out. Another example of some complexity in the paper that is mostly left to [21] and seems potentially unnecessary to mention is DistributeFixes. The authors should also consider whether there are any other parts of the paper that can be simplified by removing references to [21].

The logic used in Algorithm 3 and described in Section 5.2 is quite complex and difficult to reason about. I think this section would benefit from additional examples and discussion to help readers understand the formulas.

Response. We didn't have enough space to cover the optimized DeriveFixes (now called DeriveFixesOPT in this revision), but we mentioned it because it does much better than DeriveFixes on the running example (it is now reflected in **Example 8**). We have significantly reduced the number of references to the technical report (from 10 to 4). We believe the following references are necessary:

- **Reference 1** in Section 4, mentioning all proofs are in the technical report.
- **Reference 2** at the end of **Section 5**, mentioning DeriveFixes-optimized is in the technical report.
- **Reference 3** in the result and discussion of the experiment, mentioning the details of the students' queries and the detailed support of Qr-Hint for [12] are in the technical report.
- **Reference 4** in **Section 10**, mentioning the survey questions and DBLP schemas are in the technical report.

We have rewritten part of **Section 5.2**. Instead of trying to convey intuition with symbols from the algorithm box, we used the derivation of target bounds on the root node from Example 5 to show the reasoning behind our strategy in DeriveFixes. We also have compared the fixes returned by DeriveFixes and DeriveFixesOPT on Example 5 to show DeriveFixesOPT outperforms DeriveFixes on cost. Finally, we added an explanation of what DistributeFixes does in **Section 5.2**.

Comment R3-O3. The complexity analysis seems to under-represent the size of S (set of <= n disjoint subtrees in the predicate x). Given this definition, I believe |S| is at least theta($|x|^n$) if not exponential. But the "Complexity and Optimality" discussion in Section 5.2 implies that $|\mathcal{S}|$ is linear in the size of the predicate ("let kappa denote the combined size of formulae P and P* ... $|\mathcal{S}|$, which is $O(\kappa)$ but is usually a small constant in practice"). However, later in that paragraph, it says "the number of calls to DeriveFixes by RepairWhere can be worst-case exponential in kappa, but in practice it will be O(κ^3)". It seems there is a discrepancy here.

Response. $\mathcal{S}$ is one set of repair sites to be considered (not the set of all possible sets of repair sites); thus, $|\mathcal{S}|$ is bounded by the $O(\kappa)$. We cannot completely dodge the exponential factor, which comes later in "Finally...": we need to invoke DeriveFixes for each possible S. If all possible sets are considered, that would be $O(2^\kappa)$ invocations. Thanks to the stopping condition of Algorithm 1, in practice we don't examine them all; we start with candidates with 1 repair site, then 2 repair sites, etc. If we find a good enough fix with 3 repair sites, we end up with $O(\kappa^3)$ calls. (Please also refer to **Complexity and Optimality** at the end of **Section 5**).

Comment R3-O4. Some of the evaluation section needs more explanation. For example, it's not clear in Figure 3(b) why the cost decreases as the number of predicates increases (assuming this is the cost from Definition 3). This should be discussed. It's also not clear how the Ground Truth was calculated in Figures 3(b) and 4(a). Finally, it's not clear what is happening in each iteration of Figure 5 or why the cost varies over time. This needs a lot more explanation.

Response. We have added the following clarifications in **Section 9** under **TPCH, conjunctive WHERE with varying number of atomic predicates.** and **TPCH, WHERE with nested AND/OR and varying number of injected errors**: 1) cost is normalized by tree size Eq. (1); 2) ground truth came from how we injected the errors in the first place; 3) variation comes from the order in which repair sites are considered: different sites may lead to varying-size fixes.

Comment R3-O5. The Query Error Analysis in Section 10 mentions that Qr-Hint can correct 89% of the 341 wrong student queries analyzed. It would be interesting to know why it was unable to correct the remaining 11%.

Response. See R1O3.

Comment R3-O6. The user study was performed with four manually constructed wrong queries rather than using the real errors committed by students. Why did the authors not use real incorrect queries? This could use some explanation.

Response. a) Recruited students are from past classes, so real wrong queries from the past may bias results, as subjects have done same/similar homework. b) We could not find any suitable existing benchmarks that contain many real wrong queries (those from SQLRepair [48] are mostly SFW and too simple). c) In constructing the survey, we indeed designed wrong queries based on the common mistakes students made.

We have added this discussion in the **Preparation** paragraph of **Section 10**. In the same paragraph we also clarify the errors in the crafted queries are based on our observations of common student mistakes, consistent with the coverage dataset as well as [2, 12].

Qr-Hint: Actionable Hints Towards Correcting Wrong SQL Queries

ABSTRACT

We describe a system called Qr-Hint that, given a (correct) target query $Q^\star$ and a (wrong) working query Q, both expressed in SQL, provides actionable hints for the user to fix the working query so that it becomes semantically equivalent to the target. It is particularly useful in an educational setting, where novices can receive help from Qr-Hint without requiring extensive personal tutoring. Since there are many different ways to write a correct query, we do not want to base our hints completely on how $Q^\star$ is written; instead, starting with the user's own working query, Qr-Hint purposefully guides the user through a sequence of steps that provably lead to a correct query, which will be equivalent to $Q^\star$ but may still "look" quite different from it. Ideally, we would like Qr-Hint's hints to lead to the "smallest" possible corrections to Q. However, optimality is not always achievable in this case due to some foundational hurdles such as the undecidability of SQL query equivalence and the complexity of logic minimization. Nonetheless, by carefully decomposing and formulating the problems and developing principled solutions, we are able to provide provably correct and locally optimal hints through Qr-Hint. We show the effectiveness of Qr-Hint through quality and performance experiments as well as a user study in an educational setting.

1 INTRODUCTION

In an era of widespread database usage, SQL remains a fundamental skill for those working with data. Yet, SQL's rich features and declarative nature can make it challenging to learn and understand. When students encounter difficulties in debugging their SQL queries, they often turn to instructors and teaching assistants for guidance. However, this one-on-one approach is limited in scalability. Syntax errors are easy to fix, but many queries contain subtle semantic errors that may require careful and time-consuming debugging. To save time, the teaching staff is often tempted to give hints based on how the reference solution query is written, ignoring what students have written themselves, but doing so misses opportunities for learning. A SQL query can be written in many ways that are different in syntax but nonetheless equivalent semantically. Seasoned teaching staff knows how to guide students through a sequence of steps that, starting with their own queries, lead them to a corrected version that is equivalent to the solution query but without revealing the solution query. Our goal is to build a system to help provide this service to students in a more scalable manner.

Example 1. *Consider the following database (keys are underlined) about beer drinkers and bars:* Likes(drinker, beer), Frequents(drinker, bar), Serves(bar, beer, price)*. Suppose we want to write a SQL query for the following problem:* For each beer b that Amy likes and each bar r frequented by Amy that serves b, show the rank of r among all bars serving b according to price (e.g., if r serves b at the highest price, r's rank should be 1). We assume that there are no ties.

The reference solution query $Q^\star$ is given as follows:

```
SELECT L.beer, S1.bar, COUNT(*)
FROM Likes L, Frequents F, Serves S1, Serves S2
WHERE L.drinker = F.drinker AND F.bar = S1.bar
  AND L.beer = S1.beer AND S1.beer = S2.beer
  AND S1.price <= S2.price
GROUP BY F.drinker, L.beer, S1.bar
HAVING F.drinker = 'Amy';
```

Now consider a wrong student query Q:

```
SELECT s2.beer, s2.bar, COUNT(*)
FROM Likes, Serves s1, Serves s2
WHERE drinker = 'Amy'
  AND Likes.beer = s1.beer AND Likes.beer = s2.beer
  AND s1.price > s2.price
GROUP BY s2.beer, s2.bar;
```

Suggesting good hints to help students fix Q is not easy. First, there are many ways to write a query that is equivalent to $Q^\star$, and queries that look very different syntactically might be semantically similar or equivalent, so relying solely on the syntactic difference between Q and $Q^\star$ to propose fixes is ineffective and potentially misleading. In Example 1, even though $Q^\star$ has a HAVING clause, it would be confusing to suggest add HAVING to Q, because the condition drinker='Amy' in Q's WHERE serves the same purpose. Also, even though Q has Likes.beer=s2.beer in WHERE while $Q^\star$ has S1.beer=S2.beer, the difference is non-consequential because of the transitivity of equality. Yet another example is s1.price>s2.price in Q versus S1.price≤S2.price in $Q^\star$. It would be wrong to suggest changing $>$ to $\leq$ in Q, because an examination of the entire Q would reveal that the student intends s2 (and s1) in Q to serve the role of S1 (and S2) in $Q^\star$. The correct fix is actually changing $>$ to $\geq$.[1]

Second, it is often impossible to declare a part of Q as "wrong" since one could instead fix the remainder of Q to compensate for it. For example, we could argue that s1.price>s2.price in Q is "wrong," but there exists a correct query containing precisely this condition, e.g., with (s1.price>s2.price OR s1.price=s2.price). Hence, it is difficult to formally define what "wrong" means. Instead of basing our approach heuristically on calling out "wrong" parts, we formulate the problem as finding the "smallest repairs" to Q that make it *correct*.

Third, hints are for human users, so for a query with multiple issues—which is often the case in practice—we must be aware of the cognitive burden on users and not overwhelm them by asking them to make multiple fixes simultaneously. This desideratum introduces the challenge of planning the sequence of hints and defining appropriate intermediate goals.

Finally, effective hinting faces several fundamental barriers. Realistically, we cannot hope to always provide "optimal" hints because doing so entails solving the query equivalence problem for SQL,

[1] Another wrong hint would be to suggest changing COUNT(*) to COUNT(*)+1 in Q's SELECT instead of changing the inequality because doing so misses the top-ranked bars. Qr-Hint will not make such a mistake.

which is undecidable [1, 27, 41, 53]; even for decidable query fragments, Boolean expression minimization is known to be on the second level of the polynomial hierarchy (precisely Σ_2^p [16]).

To address the challenges, we propose Qr-Hint, a system that, given a target query $Q^\star$ and a working query Q, follows the logical execution flow (i.e., FROM→WHERE→GROUPBY→HAVING→SELECT) and produces step-by-step hints for the user to edit the working query to eventually achieve $Q^\star$. The sequence of steps is guaranteed to lead the user on a correct path to eventual correctness. The following example shows Qr-Hint helps fix the query in Example 1.

Example 2. *Continuing with Example 1, Qr-Hint automatically generates the sequence of hints below. Currently built for the teaching staff, Qr-Hint only generates the "repairs" below; using these repairs, the teaching staff would then hint the user in natural language. With the recent advances in generative AI chatbots, it would not be difficult to automate the natural language hints as well; the advantage of using Qr-Hint in that setting would be to provide provable guarantees on the quality of hints, which otherwise would be difficult, if not impossible, for generative AI to achieve by itself.*

Stage	*Qr-Hint repair*	*Hint in natural language*
FROM	*Frequents needed*	It looks like you are missing one table—read the problem carefully and see what other piece of information you need.
WHERE	*s1.price>s2.price* ↦ *s1.price≥s2.price*	Your WHERE has a small problem with s1.price>s2.price. Think through some concrete examples and see how you may fix it.

Note the sequential nature of the hints above; the working query constantly evolves. Qr-Hint first focuses on FROM *and will only proceed to* WHERE *after* FROM *is "viable." After adding* Frequents *to* FROM, *the user will also need to add appropriate join conditions in* WHERE*; if these were not added correctly, the second step above would suggest additional repairs. It turns out that for this example, only the above two hints are needed to fix the query. In particular, Qr-Hint knows* not *to suggest spurious hints such as adding to* Frequents.drinker *to* GROUP BY *or changing* s2.beer *to* Likes.beer *in* SELECT.

We make the following contributions:

- We develop a novel framework that allows Qr-Hint to provide step-by-step hints to fix a working SQL query with the goal of making it equivalent to a target query. This framework formalizes the notion of "correctness" for a sequence of hints, allowing Qr-Hint to guarantee that every hint is actionable and is on the right path to achieve eventual correctness. Further, by formulating the hinting problem in terms of finding repair sites in Q with viable fixes, we are able to quantify the quality of the hints.
- Since the optimality of hints, in general, is impossible to achieve due to the foundational hurdles discussed earlier, we aim to provide guarantees on the "local" optimality of Qr-Hint in each step. We design practical algorithms with sensible trade-offs between optimality and efficiency.
- We evaluate the performance and efficacy of Qr-Hint experimentally. We further perform a user study involving students from current/past database courses offered at the authors' institution. Our findings indicate that Qr-Hint finds repairs that are optimal or close to optimal in practice under reasonable time, and they lead to hints that are helpful for students.

2 RELATED WORK

Debugging Query Semantics. There are two main lines of work toward debugging query semantics (as opposed to syntax or performance). The first line helps debug a query but without knowing the correct (reference) query; in this regard, it differs fundamentally from Qr-Hint. Qex [54] is a tool for generating input relations and parameter values for unit-testing parameterized SQL queries. SQLLint [10–13, 31] detects suspected semantic errors in a query, alerting users to what may be indicative of efficiency, logical, or runtime errors. The work highlights a list of common semantic errors made by students and SQL users [12], but it does not suggest edits, and fixing the suspected errors will not guarantee that the query is correct. Habitat [25, 32] is a query execution visualizer that allows users to highlight parts of a query and view their intermediate results. While it helps users spot possible errors, it gives no edit suggestions if errors exist. More recently, QueryVis [42] turns queries into intuitive diagrams, helping users better understand the semantics of the queries and spot potential errors.

The second line of work, more directly related to Qr-Hint, focuses on checking a query against a reference query and/or helping to explain their difference. However, previous work has not been able to suggest small fixes that will make the user query equivalent to the reference query. XData [19] checks the correctness of a query by running the query on self-generated testing datasets based on a set of pre-defined common errors, but it provides no guarantees beyond this pre-defined set. Cosette [21–23] uses constraint solvers and theorem provers to establish the equivalence of two queries or construct arbitrary instances that differentiate them. From a large database instance, RATest [45] utilizes data provenance to generate a small, illustrative instance to differentiate queries. C-instances [30] aims at constructing small abstract instances based on c-tables [37] that can differentiate two given queries in all possible ways. While Cosette, RATest, and c-instances can provide examples illustrating how two queries are semantically different, they can only indirectly help users pinpoint errors in the original query; none of them is able to suggest fixes. Chandra et al. [18] developed a grading system that canonicalizes queries by applying rewrite rules and then decides partial credits based on a tree-edit distance between logical plans. However, as query syntax differs signficiantly from canonicalized plans after rewrite, edits on a canonicalized plan do not translate naturally to small fixes on the original query, making it hard for users to use these edits as hints. Finally, SQLRepair [48] fixes simple errors in an SPJ query using constraint solvers to synthesize/remove WHERE conditions until the query produces correct outputs over all testing instances. Its scope of error is much narrower than what we consider, and its tests-driven nature offers no guarantee of query equivalence.

Program Repair and Feedback for GPL. In the domain of program repair for general-purpose programming language (GPL), several types of approaches have been developed but none of them can be directly applied or easily transferred to cover SQL. First, a wrong program is usually aligned with reference program(s) ([3, 33, 55]) and fixes are generated based on the selected reference program using various techniques. Such an approach is similar to Qr-Hint, but SQL is essentially different from GPL as SQL is declarative and GPLs are usually procedural. While it is possible to write programs in GPL to simulate the execution of a specific SQL query, there is no well-defined mapping between the syntax of SQL and any GPL. As a result, it is impossible to apply such program repair techniques to SQL in general. Another approach is to leverage test cases to synthesize "patches" for the wrong program so that it returns the same output as the reference program for all test cases ([36, 46, 49, 52, 57]). However, such an approach heavily relies on the test cases to cover all possible errors and thus usually fails to guarantee semantic equivalence. Besides the traditional approaches, recent work explores ML algorithms to provide feedback and correction ([8, 9, 20, 34, 35, 43, 47]). In addition, large language models such as GPT-3 [15] have shown an ability to explain the semantics of SQL queries, but does not guarantee the correctness of fixes.

Testing query equivalence. While the query equivalence problem in general is undecidable [1, 5, 51, 53], tools and algorithms are developed to check the equivalence of various classes of queries with restrictions and assumptions [4, 17, 21–23, 38–40, 50, 56, 58]. Although they give a deterministic answer on equivalence, these tools/algorithms cannot provide any explanation on which parts of the users' queries cause semantic differences from the reference queries.

3 THE QR-HINT FRAMEWORK

Queries. We consider SQL queries that are select-project-join queries with an optional single level of grouping and aggregation. For simplicity of presentation, we assume these are single-block SQL queries with SELECT, FROM (without JOIN operators), and WHERE (with condition defaulting to TRUE if missing) clauses,[2] together with optional GROUP BY and HAVING clauses. We refer to such a query as an *SPJA* query if it contains grouping or aggregation or DISTINCT; otherwise, we will call it an *SPJ* query.

We assume the default bag (multiset) semantics of SQL. Given query Q, let $\mathsf{F}(Q)$ denote the cross product of Q's FROM tables (including multiple occurrences of the same table, if any); and let $\mathsf{FW}(Q)$ denote the query that further filters $\mathsf{F}(Q)$ by Q's WHERE condition (i.e., $\mathsf{FW}(Q)$ is a SELECT * query with the same FROM and WHERE clauses as Q). Furthermore, if Q is SPJA, let $\mathsf{FWG}(Q)$ denote the (non-relational) query[3] that further groups the result rows of $\mathsf{FW}(Q)$ according to Q's GROUP BY expressions (or $\emptyset$ if there are none but Q contains aggregation nonetheless, in which case all result rows belong to a single group). Finally, if Q is SPJA, let $\mathsf{FWGH}(Q)$ denote the (non-relational) query that filters the groups of $\mathsf{FWG}(Q)$ according to Q's HAVING conditions (which defaults to TRUE if missing). When discussing equivalence (denoted $\equiv$) among above queries, we require that they return the same bag of result rows (ignoring row and column ordering) for any underlying database instance, and additionally, for queries returning groups, they return the same partitioning of result rows (ignoring group ordering).

SMT Solvers. As with previous work [21, 45, 58], we leverage *satisfiability modulo theory* (*SMT*) solvers to implement various primitives used by our system. Such a solver can decide whether a formula, modulo the theories it references, is satisfiable, unsatisfiable, or unknown (beyond the solver's capabilities). Specifically, we use the popular SMT solver Z3 [24] to implement the following three primitives. Given two quantifier-free expressions, $\mathsf{IsEquiv}(e_1, e_2)$ tests whether $e_1 \Leftrightarrow e_2$ (for logic formulae such as those in WHERE) or $e_1 = e_2$ (for value experssions such as those in SELECT or GROUP BY). Given a logic formula p, $\mathsf{IsUnSatisfiable}(p)$ and $\mathsf{IsSatisfiable}(p)$ return, respectively, whether p is satisfiable or unsatisfiable, respectively. All above primitives may return "unknown" when Z3 is unsure about its answer. However, when they return true, Z3 guarantees that the answer is not a false positive. Our algorithms in subsequent sections act only on (true) positive answers from these primitives. For complex uses, it is often convenient to frame equivalence/satisfiability testing using a *context* C, or a set of logical assertions (e.g., types declaration, known constraints, and inference rules) under which testing is done. We use subscripts to specify the context: e.g., $\mathsf{IsUnSatisfiable}_C(p)$ is a shorthand for $\mathsf{IsUnSatisfiable}\left((\wedge_{c \in C} c) \wedge p\right)$.

Example 3. *Consider a query with a* WHERE *condition stipulating that $A > 100$ for an* INT*-typed column A, as well as a* HAVING *condition* $\mathsf{MAX}(A) \geq 101$. *We might wonder whether the* HAVING *condition is unnecessary. To this end, we call* $\mathsf{IsUnSatisfiable}_C(p)$ *with*

$$C : \left\{ \begin{array}{r} \mathbf{A} \textit{ has type } \mathsf{Array}(\mathbb{Z}) \\ \forall i \in \mathbb{N} : \mathbf{A}[i] > 100 \\ \mathsf{MAX} \textit{ has type } \mathsf{Array}(\mathbb{Z}) \to \mathbb{Z} \\ \forall i \in \mathbb{N}, \mathbf{X} \textit{ of type } \mathsf{Array}(\mathbf{Z}) : \mathsf{MAX}(\mathbf{X}) \geq \mathbf{X}[i] \end{array} \right\}, \quad p : \neg(\mathsf{MAX}(\mathbf{A}) \geq 101).$$

The first two assertions in C *are derived from the type of A and the* WHERE *conditions; here the array-typed* $\mathbf{A}$ *refers to a collection of A values. The last two specify (some) general inference rules on the SQL aggregate function* MAX. *Z3 correctly returns true, meaning that* $\mathsf{MAX}(A) \geq 101$ *must be true under C and is therefore unnecessary.*

Our use of Z3 for reasoning with SQL aggregation, such as the example above, goes beyond the practice in previous work, where aggregation functions are mostly treated as uninterpreted functions. For example, to test the equality of two aggregates, [58] conservatively checks whether input value sets or multisets for the aggregate function are equal. In contrast, we encode properties of SQL aggregation functions in a way that allows Z3 to reason with them. As formulae become more complicated, e.g., with quantifiers and arrays, Z3 no longer offers a complete decision procedure (as there exists no decision procedure for first-order logic) and may return "unknown" more often. Nonetheless, practical heuristics employed by Z3 allow it to handle many cases of practical uses to Qr-Hint.

3.1 Approach

Given a (*syntactically correct*) working query Q and a target query $Q^\star$, Qr-Hint provides hints in *stages* to help the user edit the working query incrementally until it becomes *semantically equivalent* to $Q^\star$. Each stage focuses on one specific syntactic fragment of the working query. Qr-Hint gives actionable hints for the user to edit this fragment with the aim of bringing Q a step "closer" to being equivalent to $Q^\star$. Qr-Hint strives to suggest the smallest edits possible and avoid suggesting unnecessary edits. Upon passing a *viability check*, the working query Q clears the current stage and moves on to the next. After clearing all stages, Qr-Hint guarantees that $Q \equiv Q^\star$ (even if syntactically they are still different).

We now briefly outline the concrete stages of Qr-Hint; the details will be presented in the subsequent sections.

[2] We can handle a query with common table expressions (WITH) and subqueries in FROM that are aggregation-free, as well as non-outer JOINs in FROM, by rewriting the query into single-block SQL.

[3] This query is non-relational because it returns, besides the underlying bag of rows from $\mathsf{FW}(Q)$, a partitioning of them into groups.

For an SPJ query, there are three stages. (1) We start with Q's FROM clause (Section 4) and make sure that its list of tables can eventually lead to a correct query; following this stage, $\texttt{F}(Q) \equiv \texttt{F}(Q^\star)$. (2) Next, we provide hints to repair Q's WHERE clause (Section 5) such that $\texttt{FW}(Q) \equiv \texttt{FW}(Q^\star)$, i.e., the repaired query returns the same sub-multiset of rows as $Q^\star$ that satisfy the WHERE clause, ignoring SELECT. (3) Finally, we handle Q's SELECT clause and ensure the working query returns correct output column values. Importantly, we make inferences of equivalence under the premise that all rows before SELECT already satisfy WHERE; this use of WHERE allows us to infer more equivalent cases and avoid spurious hints.

For an SPJA query, there are five stages. (1) The *first stage* handles FROM as in the SPJ case. (2) The *second stage* handles WHERE, but with a twist. As we have seen from Example 1, some condition can be either WHERE or HAVING, and it would be misleading to hint its absence from WHERE to be wrong; hence, Qr-Hint will look "ahead" at the two queries' HAVING and GROUP BY clauses to avoid misleading the user. At the end of this stage, instead of insisting that $\texttt{FW}(Q) \equiv \texttt{FW}(Q^\star)$ for the original $Q^\star$, we may rewrite $Q^\star$ (by legally moving some conditions between WHERE and HAVING) as needed first. (3) The *third stage* is GROUP BY, where we provide hints to edit Q's GROUP BY expressions to achieve equivalent grouping, i.e., $\texttt{FWG}(Q) \equiv \texttt{FWG}(Q^\star)$. Here, we infer equivalence under the premise that the rows to be grouped all satisfy WHERE. (4) The *fourth stage* is HAVING, where we provide hints to repair Q's HAVING condition in the same vein as WHERE; however, inferences in this stage would additionally consider both WHERE and GROUP BY, and they are more challenging because of aggregation functions. After this stage, we have $\texttt{FWGH}(Q) \equiv \texttt{FWGH}(Q^\star)$. (5) The *fifth and final stage* is SELECT, which is similar to the SPJ case, but with the challenge of handling aggregation functions while simultaneously considering WHERE, GROUP BY, and HAVING.

Progress and Correctness. Note that to clear a stage, the user only needs to come up with a fix to pass the viability checks up to this stage. Even though Qr-Hint may examine the queries in their entirety, the user does not have to think ahead about how to make the entire query correct.[4] Moreover, once a stage is cleared, Qr-Hint never requires the user to come back to fix the same fragment again. This stage-by-stage design with "localized" hints helps limit the cognitive burden on the user.

The following theorem formalizes the intuition that this stage-based approach leads to steady, forward progress toward the goal of fixing the working query. It follows from the observation that our solution for each stage ensures the properties asserted below, which we will show stage by stage in the subsequent sections.

Theorem 3.1. *Let $Q_0 = Q$ denote the initial working query and $Q^\star$ denote the target query. Let V_i denote the viability check for stage i, and Q_i denote the working query upon clearing stage i, where Q_i satisfies $V_1, V_2, \ldots, V_i$. We say that two queries are* stage-i consistent *if they are identical syntactically except in the fragments that stage $i+1$ and beyond focus on. For each stage i, the following hold:*

(Hint leads to fix) *If Q_{i-1} fails to satisfy V_i, there exists a query $\hat{Q}_i$ such that $\hat{Q}_i$ satisfies $V_1, V_2, \ldots, V_i$, $\hat{Q}_i$ is stage-$(i-1)$ consistent with Q_{i-1}, and $\hat{Q}_i$ follows the stage-i hint provided by Qr-Hint.*

(Fix leads to eventual correctness) *There exists a query $\hat{Q}$ such that $\hat{Q} \equiv Q^\star$ and $\hat{Q}$ is stage-i consistent with Q_i.*

We delegate all proofs in this paper to the full version [28].

Optimality. Ideally, we would like Qr-Hint to suggest the "best possible" hints, e.g., those leading to minimum edits to the working query. Unfortunately, it is impossible for any system to provide such a guarantee in general, because doing so entails being able to determine the equivalence of SQL queries: if $Q \equiv Q^\star$ to begin with, the system should not suggest any fix. It is well-known that the equivalence of first-order queries with only equality comparisons is undecidable [1]. Under bag semantics, even the decidability of equivalence of conjunctive queries has not been completely resolved [41]. Once we open up to the full power of SQL, which can express integer arithmetic, even equivalence of selection predicates becomes undecidable via a simple reduction to the satisfiability of Diophantine equations [27].

Given the foundational hurdles above, Qr-Hint seeks a pragmatic solution. Instead of offering any global guarantee on the optimality of its hints, which is impossible, Qr-Hint establishes, for each stage, guarantees on the necessity or minimality of its hints under certain assumptions. For example, for the FROM stage, Qr-Hint guarantees its suggested fixes are optimal for SPJ queries, but for some SPJA queries, it may suggest a fix that turns out to be unnecessary. As another example, for the WHERE stage, the optimality of Qr-Hint depends on, among other things, Z3-based primitives offering *complete* decision procedures. In each subsequence section, we will state any such assumption explicitly.

Finally, it is important to note that Qr-Hint's progress and correctness properties (Theorem 3.1) do *not* rely on these assumptions. In the worst case, the user may be hinted to make some fixes that are unnecessary or unnecessarily big, but Qr-Hint will still ensure that the user gets a correct working query in the end.

Limitations. Following Theorem 3.1, Qr-Hint is guaranteed to generate correct hints for select-project-join queries with an optional single level of grouping and aggregation. On the other hand, Qr-Hint currently has several limitations. 1) Qr-Hint may sometimes suggest suboptimal or even unnecessary fixes (even though they still lead to correct queries), as discussed above; the reason lies in fundamental hurdles due to the undecidability of SQL query equivalence and the use of heuristics to tame complexity. 2) Qr-Hint currently does not handle NULL values and assumes that all database columns are NOT NULL. With some additional effort and complexity, Qr-Hint can be extended to handle NULL using the technique in [58] of encoding each column with a pair of variables in Z3 (one for its value and the other a Boolean representing whether it is NULL). The same applies to OUTER JOIN. 3) Except the case of aggregation-free subqueries in FROM mentioned in Footnote 2, Qr-Hint does not support subqueries in general. Subqueries involving aggregation in general cannot be folded into the outer query block. Subquery constructs such as NOT EXISTS and NOT IN entail supporting queries involving the difference operator, which we have not yet studied. If we do not care about the number of duplicates in the result, positive subqueries with EXISTS and IN

[4] In some cases, just to maintain syntactic correctness, a fix may necessitate trivial edits to fragments handled in future stages: e.g., if we remove a table from FROM, we will need to remove references to this table in the rest of the query. However, the user never needs to worry about making those edits semantically correct—that responsibility falls on future stages.

could be rewritten as part of the join in the outer select-project-join query and supported as such. However, this approach is unsatisfactory, especially since our handling of FROM (Section 4) does assume that duplicates matter. In general, more work is needed to develop a comprehensive solution for subqueries. 4) Finally, Qr-Hint does not consider database constraints such as keys and foreign keys. While we can, in theory, encode some constraints as logical assertions and include them as part of the context when calling Z3, these assertions (with quantifiers) can significantly hamper Z3's performance. Future work is needed to develop more robust algorithms for incorporating constraints.

4 FROM STAGE

This stage aims to ensure $\mathsf{F}(Q) \equiv \mathsf{F}(Q^\star)$. Recall that a FROM clause may reference a table T multiple times, and each reference is associated with a distinct alias (which defaults to the name of T). Each column reference must resolve to exactly one of these aliases. Let $\mathit{Tables}(Q)$ denote the multiset of tables in the FROM clause of Q, and let $\mathit{Aliases}(Q)$ denote the set of aliases they are associated with in Q. With a slight abuse of notation, given table T, let $\mathit{Aliases}(Q, T)$ denote the subset of $\mathit{Aliases}(Q)$ associated with T (a non-singleton $\mathit{Aliases}(Q, T)$ implies a self-join involving T). Given an alias $t \in \mathit{Aliases}(Q)$, let $\mathit{Table}(Q, t)$ denote the table that t is associated with in Q.

The viability check (Theorem 3.1, stage 1) for FROM is simple:

$$V_1 : \text{Check if } \mathit{Tables}(Q) \stackrel{\sqcup}{=} \mathit{Tables}(Q^\star)$$

where $\stackrel{\sqcup}{=}$ denotes multiset equality. If the working query Q fails the viability check, Qr-Hint simply hints, for each table T whose counts in $\mathit{Tables}(Q)$ and $\mathit{Tables}(Q^\star)$ differ (including cases where T is used in one query but not the other), that the user should consider using T more or less to make the counts the same. It is straightforward to see that this hint leads to a fix that makes $\mathit{Tables}(Q) \stackrel{\sqcup}{=} \mathit{Tables}(Q^\star)$, which enables the user to further edit Q into some $\hat{Q} \equiv Q^\star$ without retouching FROM: at the very least, one can make $\hat{Q}$ isomorphic to $Q^\star$ up to the substitution of table references with those in $\mathit{Aliases}(Q)$. This observation establishes the progress and correctness properties (see Theorem 3.1) of FROM-stage hints, which we state below along with the remark that $\mathsf{F}(Q) \equiv \mathsf{F}(Q^\star)$ after this stage.

Lemma 4.1. *Qr-Hint's FROM-stage hint leads to a fixed working query Q_1 that (1) passes the viability check V_1 $\mathit{Tables}(Q_1) \stackrel{\sqcup}{=} \mathit{Tables}(Q^\star)$; (2) satisfies $\mathsf{F}(Q_1) \equiv \mathsf{F}(Q^\star)$; and (3) leads to eventual correctness.*

While the correctness of the FROM-stage hint is straightforward, its optimality is surprisingly strong. The following lemma states that the viability check is, in fact, necessary—regardless of what could be done in WHERE and SELECT—under reasonable assumptions.

Lemma 4.2. *Two SPJ queries $Q^\star$ and Q cannot be equivalent under bag semantics if $\mathit{Tables}(Q^\star) \stackrel{\sqcup}{\neq} \mathit{Tables}(Q)$ assuming no database constraints are present, and there exists some database instance for which either $Q^\star$ or Q returns a non-empty result.* [5]

Table Mappings. To facilitate analysis in subsequent stages, Qr-Hint needs a way to "unify" table and column references in Q and $Q^\star$ so that all of them use the same set of table aliases.

Definition 1. *Given queries $Q^\star$ and Q over the same schema where $\mathit{Tables}(Q^\star) \stackrel{\sqcup}{=} \mathit{Tables}(Q)$, a* table mapping *from $Q^\star$ to Q is a bijective function $\mathfrak{m} : \mathit{Aliases}(Q^\star) \longrightarrow \mathit{Aliases}(Q)$ with the property that two corresponding aliases are always associated with the same table, i.e., $\forall t \in \mathit{Aliases}(Q^\star) : \mathit{Table}(Q^\star, t) = \mathit{Table}(Q, \mathfrak{m}(t))$.*

If the queries have no self-joins, it is straightforward to establish this mapping by table names. With self-joins, however, it can be tricky because we must match multiple roles played by the same table across queries. The information contained in FROM alone would be insufficient for matching. One approach is to explore every possible table mapping and select the one that leads to the minimum fix. Doing so would blow up complexity by a factor exponential in the number of self-joined tables. Qr-Hint instead opts for a heuristic that picks the single most promising table mapping. Here we describe the heuristic briefly. For each alias, we build a "signature" that captures how its columns are used by various parts of the query in a canonical fashion. We define a distance (cost) metric for the signatures. Then, for each table involved in self-joins, to determine the mapping between its aliases in Q and $Q^\star$, we construct a bipartite graph consisting of these aliases and solve the minimum-cost bipartite matching problem. We illustrate the high-level idea using the example below.

Example 4. *Continuing with Example 1, the following are signatures (one per column) for S1 and S2 in $Q^\star$ and s1 and s2 in Q.*

		S1 in $Q^\star$	*S2 in $Q^\star$*	*s1 in Q*	*s2 in Q*
WHERE & HAVING	*bar:*	$=\{$*F.bar*$\}$	$=\{$*F.bar*$\}$	*None*	*None*
	beer:	$=\{$*L.beer, S2.beer*$\}$	$=\{$*L.beer, S2.beer*$\}$	$=\{$*Likes.beer, s2.beer*$\}$	$=\{$*Likes.beer, s2.beer*$\}$
	price:	$\leq\{$*S2.price*$\}$	$\geq\{$*S2.price*$\}$	$>\{$*s2.price*$\}$	$<\{$*s1.price*$\}$
GROUP BY		$\{$*bar, beer*$\}$	$\{$*beer*$\}$	$\{$*beer*$\}$	$\{$*beer*$\}$
SELECT	*bar:*	$\{2\}$	$\emptyset$	$\emptyset$	$\{2\}$
	beer:	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
	price:	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

For example, S1.beer's WHERE/HAVING *signature says that it is involved in an equality comparison with both L.beer and S2.beer; the latter is inferred—Qr-Hint automatically adds column references and constants that obviously belong to the same equivalence class. Likewise, S1's* GROUP BY *signature includes both bar and beer, with the latter added because of its equivalence to the* GROUP BY *column L.beer. When comparing signatures, all aliases are replaced by table names (which is a heuristic simplification); therefore, all four* WHERE/HAVING *signatures above for beer are considered the same. In this case, what makes the difference in bipartite matching turns out to be the* SELECT *signatures for bar, which clearly favors the mapping with S1 ⟼ s2 and S2 ⟼ s1.*

Once we have selected the table mapping $\mathfrak{m}$, we can then "unify" $Q^\star$ and Q. For convenience, we simply rename each alias a in $Q^\star$ to $\mathfrak{m}(a)$; in subsequent sections, we shall assume that $Q^\star$ and Q have consistent column references.

5 WHERE STAGE

WHERE is our most involved stage, aimed at making small edits to the WHERE condition of Q so that it becomes logically equivalent to that of $Q^\star$, thereby ensuring $\mathsf{FW}(Q^\star) \equiv \mathsf{FW}(Q)$ (recall from Section 3.1).

[5] The assumption of a not-always-empty result may seem out of the blue but is necessary. For example, queries SELECT 1 FROM R WHERE FALSE and SELECT 1 FROM R,R WHERE FALSE are equivalent—both always return empty results. However, if at least one of $Q^\star$ and Q can return non-empty results, $\mathit{Tables}(Q^\star) \stackrel{\sqcup}{=} \mathit{Tables}(Q)$ becomes necessary for equivalence. Our proof of Lemma 4.2, in fact, builds on such a non-empty result.

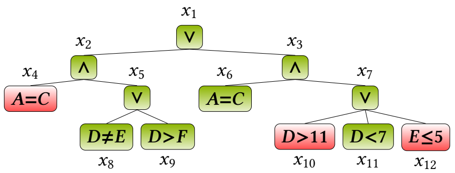


Figure 1: Syntax trees for P in Example 5.

Let P and $P^\star$ denote the WHERE predicates in Q and $Q^\star$, respectively. We assume that they have already been unified by the selected table mapping to have the same set of column references, as discussed in Section 4. The viability check for the WHERE stage (Theorem 3.1, stage 2) is simply that P is logically equivalent to $P^\star$:

$$V_2 : \text{Check if } P \Leftrightarrow P^\star$$

As discussed in Section 1, if $P \not\Leftrightarrow P^\star$, there are many different ways to modify P so that becomes logically equivalent to $P^\star$, and it is impossible to declare any part of P as definitively "wrong." Therefore, we suggest the smallest possible edits on P to reduce the cognitive burden on the user. We formalize the notion of "small edits" below. We represent P and $P^\star$ using syntax trees, where:

- Internal (non-leaf) nodes represent logical operators $\wedge$, $\vee$, and $\neg$. Let $\mathsf{op}(x)$ denote the operator associated with node x, and $\mathsf{Children}(x)$ denote the x's child nodes. If $\mathsf{op}(x)$ is $\neg$, $|\mathsf{Children}(x)| = 1$. If $\mathsf{op}(x) \in \{\wedge, \vee\}$, $|\mathsf{Children}(x)| \geq 2$.
- Leaf nodes are atomic predicates involving column references and/or literals. We treat each unique column reference as a free variable over the domain of the referenced column. We support basic SQL types and as well as standard comparison, arithmetic, and string operators to the extent supported by Z3, e.g.: $A{>}5$, $B{\leq}2C{-}10$, D LIKE 'Eve%'.

Example 5. *Consider the following logical formulae $P^\star$ and P where A, B, C, D, E are integers:*

$P^\star$: $(A{=}C \wedge (E{<}5 \vee D{>}10 \vee D{<}7) \vee (A{=}B \wedge (D{\neq}E \vee D{>}F))$
P: $(A{=}C \wedge (D{\neq}E \vee D{>}F)) \vee (A{=}C \wedge (D{>}11 \vee D{<}7 \vee E{\leq}5))$

The syntax tree of P is shown in Figure 1. The syntax tree of $P^\star$ would have the positions of nodes x_2 and x_3 reversed and red nodes replace with $A{=}B$, $D{>}10$, and $E{<}5$ respectively.

Definition 2 (Repair for SQL Predicate). *Given a quantifier-free logical formulae P represented as a tree, a* repair *of P is a pair $(\mathcal{S}, \mathcal{F})$ where $\mathcal{S}$ is a set of disjoint subtrees of P called the* repair sites*, and $\mathcal{F}$ is function that maps each site $x \in \mathcal{S}$ to a new formulae $\mathcal{F}(x)$ called the* fix *for x. Given a target predicate $P^\star$, a repair $(\mathcal{S}, \mathcal{F})$ for P is* correct *if applying it to P—i.e., replacing each $x \in \mathcal{S}$ with $\mathcal{F}(x)$—results in a formulae P' such that $P' \Leftrightarrow P^\star$.*

Definition 3 (Cost of a repair). *Given target predicate $P^\star$, the* cost *of a repair $(\mathcal{S}, \mathcal{F})$ for P is:*

$$\mathit{Cost}(\mathcal{S}, \mathcal{F}) = w \cdot |\mathcal{S}| + \sum_{s \in \mathcal{S}} \frac{\mathit{dist}(s, \mathcal{F}(s))}{|P| + |P^\star|}, \text{ where} \quad (1)$$

$$\mathit{dist}(s, \mathcal{F}(s)) = |s| + |\mathcal{F}(s)|, \text{ and} \quad (2)$$

$w \in \mathbb{R}^+$ controls the relative weights of the cost components.

Here, we simply define $\mathsf{dist}(\cdot,\cdot)$ to be the number of nodes deleted and inserted by the repair; other notions of edit distance could be used too. The denominator under $\mathsf{dist}(\cdot,\cdot)$ serves to normalize the measure relative to the sizes of the queries. Also, note that the $w \cdot |\mathcal{S}|$ term adds a fixed penalty for each additional repair site. Intuitively, Qr-Hint will present all repair sites (without the associated fixes) to the user as a hint. Even a moderate number of repair sites will pose a significant cognitive challenge—if there were so many issues with P, we might as well ask the user to rethink the whole predicate (which would be a single repair site at the root). In our experiments (Section 9), we set $w = 1/6$, and the number of repair sites per WHERE rarely goes above two or three.

Algorithm 1: RepairWhere$(x, x^\star, n)$

```
Input  : a wrong predicate $x$, a correct predicate $x^\star$, and a cap $n$
         on the number of repair sites
Output : a repair $(\mathcal{S}, \mathcal{F})$ with minimum cost
let $\mathcal{S}_\circ = \emptyset, \mathcal{F}_\circ = \emptyset$;
let $c_\circ$ denote the minimum cost so far, and $\infty$ initially;
foreach set $\mathcal{S}$ of $\leq n$ disjoint subtrees in $x$, in ascending $|\mathcal{S}|$ order do
    if $Cost(\mathcal{S}, \cdot) \geq c_\circ$ then // cost due to # sites alone is already too big
        return $(\mathcal{S}_\circ, \mathcal{F}_\circ)$;                  // safe to stop now
    if $x^\star \in CreateBounds(x, \mathcal{S})$ then
        let $\_, \mathcal{F} = DeriveFixes(x, \mathcal{S}, x^\star, x^\star)$;
        if $c_\circ > Cost(\mathcal{S}, \mathcal{F})$ then
            let $\mathcal{S}_\circ = \mathcal{S}, \mathcal{F}_\circ = \mathcal{F}$;
return $(\mathcal{S}_\circ, \mathcal{F}_\circ)$;
```

Example 6. *Consider Figure 1. One correct repair for P consists of three sites (x_4, x_{10}, x_{12}) and the corresponding fixes $(A{=}B, D{>}10, E{<}5)$. The cost for this repair is $3w + \frac{3\times(1+1)}{12+12} = \frac{1}{2} + \frac{1}{4} = 0.75$.*

Another correct repair for P consists of two sites (x_5, x_3) and the corresponding fixes $E{<}5 \vee D{>}10 \vee D{<}7$ and $A{=}B \wedge (D{\neq}E \vee D{>}F)$. The cost for this repair is $2w + \frac{(4+3)+(5+6)}{12+12} = \frac{1}{3} + \frac{3}{4} \approx 1.08$.

A trivial single-site repair that replaces the entire P with $P^\star$ would have cost $1w + \frac{(12+12)}{12+12} \approx 1.16$.

Algorithm 1 is our overall procedure for computing a minimum-cost repair for a predicate. It considers all possible sets of repair sites, prioritizing smaller ones because the number of repair sites heavily influences the repair cost, and stopping once the lowest cost found so far is no greater than a conservative lower bound on the cost of the repairs to be considered. In the worst case, the number of repairs to be considered is exponential in the size of P, but in practice, the early stopping condition usually kicks in when the number of repair sites is 2 or 3, so the number of repairs considered is usually quadratic or cubic in $|P|$.

The two key building blocks of Algorithm 1 are CreateBounds and DeriveFixes, which we describe in more detail in the remainder of this section. Intuitively, CreateBounds (Section 5.1) provides a quick and "exact" test to determine whether a given set of repair sites could ever lead to a correct repair. If yes, DeriveFixes (Section 5.2) then finds the "optimal" fixes for these repair sites. Our algorithms use Z3, so their exactness and optimality depend on Z3's completeness for the types of predicates they are given. DeriveFixes's optimality further hinges on a Boolean minimization procedure (MinBoolExp) that it also uses. On the other hand, since Z3 inferences are sound, progress and correctness (Section 3.1) are guaranteed.

Lemma 5.1. *WHERE-stage hint leads to a fixed working query Q_2 with WHERE condition that 1) passes the viability check $P \Leftrightarrow P^\star$; 2) satisfies $\mathit{FW}(Q_2) \equiv \mathit{FW}(Q^\star)$; and 3) leads to eventual correctness.*

Algorithm 2: CreateBounds$(x, \mathcal{S})$

```
Input  : a predicate $x$, and a set $\mathcal{S}$ of disjoint subtrees (repair
         sites) of $x$
Output : lower and upper bounds for $x$ achievable by fixing $\mathcal{S}$
if $x \in \mathcal{S}$ then return $[\text{false}, \text{true}]$ ;
else if $x$ is atomic then return $[x, x]$ ;
else if $op(x) \in \{\wedge, \vee\}$ then
    foreach $c \in Children(x)$ do
        let $[l_c, u_c]$ = CreateBounds$(c, \mathcal{S}[c])$;[6]
    return $[\Theta_{c \in \text{Children}(x)} l_c,\ \Theta_{c \in \text{Children}(x)} u_c]$ where $\Theta = \text{op}(x)$;
else // op(x) is ¬
    let $c$ = Children$(x)[0]$;                    // the only child of $x$
    let $[l_c, u_c]$ = CreateBounds$(c, \mathcal{S}[c])$;
    return $[\neg u_c,\ \neg l_c]$;
```

Lemma 5.2. *Given P and $P^\star$, assuming that Z3 inference is complete with respect to the logic exercised by P and $P^\star$, and that* MinBoolExp *finds a minimum-size Boolean formula equivalent to its given input, the repair returned by* RepairWhere$(P, P^\star, |P|)$ *is optimal (i.e., has the lowest possible cost) if there exists an optimal repair that either contains a single site or has all its sites sharing the same parent in P.*

Note that Lemma 5.2 provides optimality for two important cases that commonly arise in practice: 1) P makes a single (presumably small) mistake; 2) P is either conjunctive or disjunctive (because all atomic-predicate nodes share the same $\wedge$ or $\vee$ parent node).

5.1 Viability of Repair Sites

The key idea is that, given a set of repair sites in P, we can quickly compute a "bound" that precisely defines what can be accomplished by *any* fixes at these sites (and only at these sites). We first give the definition of bounds and introduce some notations. Give quantifier-free logical formulae $P_\perp$, P, and $P_\top$ such that $P_\perp \implies P \implies P_\top$, we say that $[P_\perp, P_\top]$ is a *bound* for P, denoted $P \in [P_\perp, P_\top]$. We call $P_\top$ an *upper bound* of P and $P_\perp$ a *lower bound* of P.

CreateBounds$(P, \mathcal{S})$ (Algorithm 2) computes a precise bound for any predicate that can be obtained by fixing P at the given set $\mathcal{S}$ of repair sites. It works by computing a bound for each node in P in a bottom-up fashion, starting from the repair sites or leaves of P. We call these bounds *repair bounds.* Intuitively, the repair bound at a repair site would be $[\text{false}, \text{true}]$, because a fix can change it to any logical formula. If a subtree contains no repair sites underneath, it would have a very tight repair bound of $[p, p]$, where p denotes the formulae corresponding to the subtree, which is unchangeable by the given repair. The internal logical nodes combine and transform these bounds in expected ways in Algorithm 2.

Example 7. *Given repair sites $\{x_4, x_{10}, x_{12}\}$ for P in Figure 1, CreateBounds computes the repairs bounds shown below.*

Node(s)	repair lower bound	repair upper bound
x_4	*false*	*true*
x_8, x_9, x_5	*original predicate in P*	
x_2	*false*	$D{\neq}E \vee D{>}F$
x_6	*original predicate in P*	
x_{10}	*false*	*true*
x_{11}	*original predicate in P*	
x_{12}	*false*	*true*
x_7	$D{<}7$	*true*
x_3	$A{=}C \wedge D{<}7$	$A{=}C$
x_1 (P)	$A{=}C \wedge D{<}7$	$D{\neq}E \vee D{>}F \vee A{=}C$

The following shows that repair bounds computed by CreateBounds are valid. The proof uses an induction on the structure of P.

Lemma 5.3 (Validity of Repair Bounds). *Given a predicate P and a set $\mathcal{S}$ of repair sites,* CreateBounds$(P, \mathcal{S})$ *outputs two predicates $P_\perp$ and $P_\top$, such that applying any repair $(\mathcal{S}, \mathcal{F})$ (with the given $\mathcal{S}$) will result in a predicate $P' \in [P_\perp, P_\top]$.*

Lemma 5.3 immediately yields a method for deciding whether a candidate set $\mathcal{S}$ of repair sites is viable: if the target formula $P^\star \notin [P_\perp, P_\top]$ given $\mathcal{S}$, then there does not exist a set of correct fixes $\mathcal{F}$ for $\mathcal{S}$. The next natural question to ask is: if the target formula $P^\star \in [P_\perp, P_\top]$ given $\mathcal{S}$, is it always possible to find some correct fixes? The answer to this question is yes—and Section 5.2 will provide constructive proof. Hence, repair bounds provide a *precise* test of whether a set $\mathcal{S}$ of repair sites is viable.

For example, continuing from Example 7, using Z3, it is easy to verify that $P^\star \in [A{=}C \wedge D{<}7,\ D{\neq}E \vee D{>}F \vee A{=}C]$; therefore, $\{x_4, x_{10}, x_{12}\}$ is a viable set of repair sites for P with respect to $P^\star$.

5.2 Derivation of Fixes

Suppose the target formula $P^\star$ falls within the repair bound $[P_\perp, P_\top]$ computed by CreateBounds$(P, \mathcal{S})$. We now introduce DeriveFixes (Algorithm 3) that computes correct fixes $\mathcal{F}$ for $\mathcal{S}$. The idea is to traverse P's syntax tree top-down and derive a *target bound* for each node x. As long as we repair subtrees rooted at x's children such that the resulting predicates fall within their respective target bounds, we will have a repair for x that makes its result predicate fall within x's target bound. We start from P's root with the desired target bound $[P^\star, P^\star]$ and "push it down"; whenever we reach a repair site, its fix would simply be the smallest formula (found by MinFix) that falls within the target bound we have derived for the repair site.

The intuition behind how to "push down" the target bound at node x to its children is as follows. First, the repair bound on a child c of x dictates what repairs are possible—the target bound we set for c must be bound by its repair bound. However, we want to tighten the repair bound as little as possible because a looser target bound gives MinFix more freedom in finding a small formula. As a simple example, consider the target bound $[a_1 \wedge a_2,\ (a_1 \wedge a_2) \vee a_3]$, where a_1, a_2, a_3 are independent atomic predicates. The smallest formula within this bound is $a_1 \wedge a_2$. However, if the target bound were looser, e.g., $[a_1 \wedge a_2 \wedge a_3,\ (a_1 \wedge a_2) \vee a_3]$, the smallest formula within this new bound would be just a_3, smaller than before.

Lines 15–22 of Algorithm 3 spells out our strategy. We will illustrate the key ideas with Example 7 and Figure 1. Consider pushing down the target bound of $[P^\star, P^\star]$ at x_1 to x_2 and x_3. Note that our choices of target bounds for x_2 and x_3 are constrained by

[6] For node x in P, $\mathcal{S}[x]$ denotes the subset of $\mathcal{S}$ that belong to the subtree rooted at x.

Algorithm 3: DeriveFixes$(x, \mathcal{S}, l^\star, u^\star)$

```
Input  : a predicate $x$, a set $\mathcal{S}$ of disjoint subtrees (repair sites) of
         $x$, and a target bound $[l^\star, u^\star]$ for $x$ to achieve by fixes
Output : a repair represented as a set of $(s, f)$ pairs, one for each
         $s \in \mathcal{S}$
1  if $x \in \mathcal{S}$ then return $\{(x, \mathsf{MinFix}(l^\star, u^\star))\}$ ;
2  else if $x$ is atomic then return $\emptyset$ ;
3  else if $op(x)$ is $\neg$ then
4      let $c = \mathsf{Children}(x)[0]$;                    // the only child of $x$
5      return DeriveFixes$(c, \mathcal{S}[c], \neg u^\star_0, \neg l^\star_0)$;
6  let $\Theta = \mathsf{op}(x)$ ;                          // either $\wedge$ or $\vee$ at this point
7  foreach $c \in Children(x)$ do
8      let $[l_c, u_c] = \mathsf{CreateBounds}(c, \mathcal{S}[c])$;
9  let $\mathcal{R} = \mathsf{Children}(x) \cap \mathcal{S}$ ;            // children of $x$ being repaired
10 if $\mathcal{R} = \emptyset$ then let $r = \emptyset$ and $C = \mathsf{Children}(x)$;
11 else // treat all children being repaired as one
12     let $r = \Theta_{c \in \mathcal{R}} c$ and $[l_r, u_r] = [\mathsf{false}, \mathsf{true}]$;
13     let $C = \mathsf{Children}(x) \setminus \mathcal{R} \cup \{r\}$;
14 let $\mathcal{F} = \emptyset$ ;                    // result set of $(s, f)$ pairs to be computed
15 foreach $c \in C$ do
       // Combine bounds from all other children:
16     let $[l', u'] = [\Theta_{c' \in C \setminus \{c\}} l_{c'},\ \Theta_{c' \in C \setminus \{c\}} u_{c'}]$;
17     if $\Theta$ is $\wedge$ then
18         let $l^\star_c = l^\star$; let $u^\star_c = u_c \wedge (u^\star \vee \neg u')$;
19     else // $\Theta$ is $\vee$
20         let $l^\star_c = l_c \vee (l^\star \wedge \neg l')$; let $u^\star_c = u^\star$;
21     if $c$ is not $r$ then let $\mathcal{F} = \mathcal{F} \cup$ DeriveFixes$(c, \mathcal{S}[c], l^\star_c, u^\star_c)$;
22     else let $\mathcal{F} = \mathcal{F} \cup$ DistributeFixes(MinFix$(l^\star_c, u^\star_c), C)$ ;
23 return $\mathcal{F}$;
```

their respective repair bounds in the table of Example 7; in general, we will need to raise these lower bounds and/or lower these upper bounds in a way such that any repairs on x_2 and x_3 within these bounds ensure that x_1's target bound is met. Let us focus on setting the target bound for x_2. As argued above, we would like it to be as loose as possible. Thankfully, because $x_1 \Leftrightarrow x2 \vee x3$, x_3 can help "cover" some of x_1. Specifically, no matter how we end up repairing x_3, we know it is lower-bounded by $A{=}C \wedge D{<}7$ (denote this formula by l'). Hence, x_3 will certainly cover the $P^\star \wedge l'$ part of $P^\star$, leaving x_2 responsible to cover only $P^\star \wedge \neg l'$. This observation motivates us to set the lower target bound for x_2 by raising its lower repair bound (denote it by l_c) to $l_c \vee (P^\star \wedge \neg l')$ (Line 20) instead of all the way up to $l_c \vee P^\star$. On the other hand, x_3 does not help with setting the upper target bound for x_2. We have to set x_2's upper target bound to $P^\star$, because if x_2 "overshoots" $P^\star$, $\vee$-ing it with any x_3 formula will not bring it down. In sum, we set the target bound for x_2 as $[l_c \vee (P^\star \wedge \neg l'), P^\star] = [P^\star \neg(A{=}C \wedge D{<}7), P^\star]$. A symmetric argument leads to setting the target bound for x_3 as $[(A{=}C \wedge D{<}7) \vee P^\star, P^\star]$ (in this case x_2 offers no help to x_3 because it is lower-bounded only by false). The intuition behind pushing the target bound through $\wedge$ is analogous to that described above for $\vee$ but instead boils down to lowering upper bounds as little as possible (as opposed to raising lower bounds). Completing the rest of Example 7, we show the target bounds derived by DeriveFixes for P given repair sites $\{x_4, x_{10}, x_{12}\}$ in Table 1.

Node(s)	target lower bound	target upper bound
x_1 (P)	$P^\star$	$P^\star$
x_2	$P^\star \wedge \neg(A{=}C \wedge D{<}7)$	$P^\star$
x_4	$P^\star \wedge \neg(A{=}C \wedge D{<}7)$	$P^\star \vee \neg(D{\neq}E \vee D{>}F)$
x_5, x_8, x_9	same as in original predicate	
x_3	$P^\star \vee (A{=}C \wedge D{<}7)$	$P^\star$
x_6	same as in original predicate	
x_7	$P^\star \vee (A{=}C \wedge D{<}7)$	$P^\star \vee \neg(A{=}C)$
$x_{10} \wedge x_{12}$	$P^\star \wedge \neg(D{<}7)$	$P^\star \vee \neg(A{=}C)$
x_{11}	same as in original predicate	

Table 1: Target lower and upper bounds in Example 5

Another aspect of DeriveFixes worth mentioning is its handling of the case when multiple repair sites have the same $\wedge$ or $\vee$ parent (which is common because many queries in practice are conjunctive; therefore, their trees have only two levels- the root and the leaves). Since $\wedge$ and $\vee$ are commutative, all such sites can be combined into effectively one site (r in Algorithm 3) to be fixed. In Example 5 above, x_{10} and x_{12} are handled in this manner. Once we obtain a fix for r using MinFix (in conjunctive normal form for $\wedge$ or disjunctive normal form for $\vee$), DistributeFixes distributes the r's clauses to the repair sites (Line 22) based on syntactic similarities between them.

The following is the main result of this section, which affirms that so long as a candidate set $\mathcal{S}$ of repair sets passes the repair bound check in Section 5.1, there must exist a correct repair for $\mathcal{F}$ and DeriveFixes will find it. This lemma and Lemma 5.3 together imply that our repair bound check is *exact.*

Lemma 5.4 (Existence of Correct Repair). *Suppose $P^\star \in$ CreateBounds$(P, \mathcal{S})$. DeriveFixes$(P, \mathcal{S}, P^\star, P^\star)$ returns $\mathcal{F}$ such that applying $(\mathcal{S}, \mathcal{F})$ to P yields a formula equivalent to $P^\star$.*

In the remainder of this section, we first focus on MinFix, which DeriveFixes uses to find the smallest formula within a target bound. We end with a discussion of complexity, optimality, and, when we cannot guarantee optimality, techniques to mitigate suboptimality.

Finding Smallest Formula with a Bound. Given a target bound $[l^\star, u^\star]$ for a repair site, MinFix needs to find a formula g with the smallest size possible such that $g \in [l^\star, u^\star]$. This goal is intimately related to the *Boolean minimization* problem, which has been well studied and known to be hard [16]. Many practically effective tools have been developed over the years, so our strategy is to leverage these tools for Qr-Hint. There are two technical challenges: 1) Boolean minimization is formulated in terms of expressions involving independent Boolean variables, while our formulae involve atomic predicates whose truth values are not independent. 2) Our minimization problem is given a bound as opposed to a single expression that Boolean minimization typically expects.

To address (1), we run a heuristic procedure using Z3 to identify a set $\mathcal{A}$ of "unique" atomic predicates that appear in $l^\star$ and $u^\star$; those that are logically equivalent to others or can be expressed easily in terms of others (e.g., with a negation) are excluded. This procedure does not need to detect or remove intricate dependencies (such that $A{>}C$ follows from $A{>}B$ and $C{\leq}B$); any such dependencies will still be caught later. Then, we map each predicate in $\mathcal{A}$ to a unique Boolean variable and convert $l^\star$ and $u^\star$ into Boolean expressions involving these variables.

To address (2), we note that many practical Boolean minimization tools accept the specification of Boolean expressions as truth tables with possible *don't-care* output entries. Our idea is to use *don't-cares* to encode the constraint implied by the target bound. Specifically, we generate a truth table whose rows correspond to truth assignments of the Boolean variables for $\mathcal{A}$. If a particular assignment is not feasible (which is testable in Z3) due to interacting atomic predicates, we mark the output for the row as *don't-care*. For each feasible assignment, if $l^\star$ and $u^\star$ evaluate to the same truth value, we designate the output for that row to be this value. If $l^\star$ evaluates to false and $u^\star$ evaluates to true, we mark the output as *don't-care*—reflecting the flexibility offered by the bound. (Note that because $l^\star \implies u^\star$, the case where $l^\star$ and $u^\star$ evaluate to true and false respectively cannot occur.)

The current implementation of Qr-Hint uses *ESPRESSO* [14] as the primitive MinBoolExp for finding a minimum-size Boolean expression given a truth table with *don't-cares*.

Complexity and Optimality. In our analysis below, let κ denote the combined size of formulae P and $P^\star$. DeriveFixes's main cost comes from calls to MinFix and Z3. The number of times that MinFix is invoked is $|\mathcal{S}|$, which is $\mathrm{O}(\kappa)$ but is usually a small constant in practice. MinFix runs in time exponential in the number of Boolean variables, which is capped at κ. To construct the input truth table for MinBoolExp, MinFix will also call Z3 $O(2^\kappa)$ times. Each Z3 call may take time exponential in the length of its input, though in practice, we time out with an inconclusive answer. Finally, as discussed at the beginning of Section 5, the number of calls to DeriveFixes by RepairWhere can be worst-case exponential in κ, but in practice it will be $\mathrm{O}(\kappa^3)$. Regardless, the overall complexity of RepairWhere is exponential in the complexity of the WHERE predicates. Although this worst-case complexity seems daunting, we have found that Qr-Hint delivers acceptable performance in practice: thankfully, κ is often small, and the structures of P and $P^\star$ and the interdependencies among their atomic predicates tend to be much simpler than, e.g., our Example 5.

The optimality result is presented earlier as Lemma 5.2. Intuitively, the guarantees (which still depend on the primitives Z3 and MinBoolExp) stem from two observations: 1) if repair is limited to a single site, the target bound computed by DeriveFixes is indeed the best one can do; and 2) if all sites share the same parent, DeriveFixes would effectively process them as a single site. However, target bounds for non-combinable repair sites cannot be set optimally in an independent manner; the approach taken by DeriveFixes, which essentially assumes that siblings receive the least amount of help possible from each other when pushing down target bounds, cannot guarantee a minimum-size repair. Indeed, our running example Example 5 with repair sites $\{x_4, x_{10}, x_{12}\}$ is an instance where DeriveFixes fails to set target bounds optimally, because x_4 has a different parent from x_{10} and x_{12}. To mitigate this problem, we have developed a more sophisticated algorithm (called DeriveFixesOPT) for finding fixes for multiple sites holistically. A full discussion of DeriveFixesOPT is beyond the scope of this paper (details in [28]). DeriveFixesOPT increases the complexity by another factor of $2^{|\mathcal{S}|}$. It is heuristic in nature (as it prioritizes repair sites by how constrained they are) and cannot guarantee optimality beyond Lemma 5.2. However, it does well in practice and better than DeriveFixes. Since $|\mathcal{S}|$ is small in practice, the complexity overhead is a good price to pay.

Example 8. *In Example 5, for repair sites $\{x_4, x_{10}, x_{12}\}$, DeriveFixes returns fixes $x_4 \mapsto A{=}B \vee (A{=}C \wedge D{>}10) \vee (A{=}C \wedge D{<}7)$; $x_{10} \mapsto (A{=}B \wedge D{\neq}E) \vee (A{=}B \wedge D{>}F)$; $x_{12} \mapsto (A{=}C \wedge D{>}10) \vee (A{=}C \wedge E{<}5)$.*

On the other hand, DeriveFixesOPT finds the optimal fixes $x_4 \mapsto A{=}B$; $x_{10} \mapsto D{>}10$; $x_{12} \mapsto E{<}5$.

6 GROUP BY STAGE

We check the GROUP BY equivalence assuming $Q^\star$, Q have equivalent FROM and WHERE clauses. We focus on ensuring $\mathsf{FWG}(Q) \equiv \mathsf{FWG}(Q)$, regardless of the order and the number of expressions involved in their GROUP BY clauses.

In the following, we consider the case where both Q and $Q^\star$ have grouping and/or aggregation. Suppose we have unified the WHERE conditions and GROUP BY expressions in the two queries according to the table mapping $\mathfrak{m}$. Let P denote the resulting formula for $Q^\star$'s WHERE condition (which at this point is logically equivalent to Q's), and let $\vec{o}$ and $\vec{o^\star}$ denote the resulting lists of GROUP BY expressions for Q and $Q^\star$, respectively. Note that the ordering of the GROUP BY expressions is unimportant. Also, if a query involves aggregation but has no GROUP BY, we consider the list of GROUP BY expressions to be an empty list. Same column references across P, $\vec{o}$, and $\vec{o^\star}$ are treated as same variables. Our goal is to compute a subset Δ^- of GROUP BY expressions to be removed from Q, as well as a set Δ^+ of additional GROUP BY expressions to be added to Q, such that the resulting query will always produce the same grouping of intermediate result tuples (produced by FROM-WHERE) as $Q^\star$. In practice, we may not want to reveal Δ^+, but instead simply hint that Q misses some GROUP BY expressions. We may repeat the hinting process several times until GROUP BY is completely fixed.

Repairing grouping is trickier than it seems because seemingly very different GROUP BY lists can produce equivalent grouping, as illustrated by the following example.

Example 6.1. Consider two queries over tables R(A, B) and S(C, D):

```
SELECT B FROM R, S WHERE B=C GROUP BY B, D; -- Q★
SELECT C FROM R, S WHERE B=C GROUP BY C+D, C; -- Q
```

The two queries are equivalent, even though none of the pairs of GROUP BY expressions are equivalent when examined in isolation.

To address this challenge, instead of comparing pairs from $\vec{o^\star}$ and $\vec{o}$ in isolation, we holistically consider these lists as well as the WHERE condition, and go back to the definition of GROUP BY as computing a partitioning of intermediate result tuples. Formally, the viability check for this stage is that $\vec{o}$ and $\vec{o^\star}$ achieve the same partitioning, or more precisely:

$$V_3 : \text{Check if } \forall t_1, t_2 \in \mathsf{FW}(Q^\star) : \bigwedge_i (o_i[t_1] = o_i[t_2]) \Leftrightarrow \bigwedge_i (o_i^\star[t_1] = o_i^\star[t_2])$$

Here, t_1 and t_2 denote intermediate result tuples, which are known to satisfy P; we use $o[t]$ to denote evaluating e over t.[7] This approach underlines our algorithm FixGrouping (Algorithm 4).

Example 9. *Consider the two queries in Example 6.1. The table mapping is trivial and we simply use column names to name variables.*

[7] Formally, we treat t as an assignment of variables (column references) in e to variables representing corresponding column values in t. Hence, $e[t]$ is an expression obtained from e by replacing each variable (column reference) v with variable $t(v)$.

Algorithm 4: FixGrouping$(P, \vec{o}, \vec{o^\star})$

```
Input  : a formula $P$ and two expression lists $\vec{o}$ and $\vec{o^\star}$
Output : a pair $(\Delta^-, \Delta^+)$, where $\Delta^- \subseteq [1..\dim(\vec{o})]$ is a subset of
         indices of $\vec{o}$ and $\Delta^+ \subseteq [1..\dim(\vec{o^\star})]$ is a subset of
         indices of $\vec{o^\star}$
let $\vec{v}$ denote the set of variables in $P$, $\vec{o}$, and $\vec{o^\star}$;
let $t_1, t_2$ be two assignments of $\vec{v}$ to new sets of variables $\vec{v}_1$ and $\vec{v}_2$;
let $G^\star$ denote the formula $\bigwedge_i (o^\star_i[t_1] = o^\star_i[t_2])$;
let $\Delta^- = \emptyset$;
foreach $o_i \in \vec{o}$ do
    if IsSatisfiable$(P[t_1] \wedge P[t_2] \wedge G^\star \wedge o_i[t_1] \neq o_i[t_2])$ then
        let $\Delta^- = \Delta^- \cup \{i\}$;
let $G$ denote the formula $\bigwedge_{i \notin \Delta^-} (o_i[t_1] = o_i[t_2])$;
let $\Delta^+ = \emptyset$;
foreach $o^\star_i \in \vec{o^\star}$ do
    if IsSatisfiable$(P[t_1] \wedge P[t_2] \wedge G \wedge o^\star_i[t_1] \neq o^\star_i[t_2])$ then
        let $\Delta^+ = \Delta^+ \cup \{i\}$;
        let $G = G \wedge o^\star_i[t_1] \neq o^\star_i[t_2]$;
return $(\Delta^-, \Delta^+)$;
```

We have: P is $B = C$, $\vec{o^\star} = [B, D]$, and $\vec{o} = [C + D, C]$. The logical statement that establishes the equivalence of grouping is

$$\begin{gathered}\forall (A_1, B_1, C_1, D_1), (A_2, B_2, C_2, D_2) : \\ (B_1{=}C_1 \wedge B_2{=}C_2) \quad \textit{// both } (A_1, B_1, C_1, D_1) \textit{ and } (A_2, B_2, C_2, D_2) \textit{ satisfy } P \\ \Rightarrow \begin{pmatrix} (B_1{=}B_2 \wedge D_1{=}D_2) \quad \textit{// } Q^\star\textit{'s grouping criterion} \\ \Leftrightarrow (C_1{+}D_1{=}C_2{+}D_2 \wedge C_1{=}C_2) \quad \textit{// } Q\textit{'s grouping criterion} \end{pmatrix}.\end{gathered}$$

Note that instead of referring to tuples t_1 and t_2, we simply refer to variables representing their column values in the above.

In FixGrouping, to find Δ^-, which are "wrong" expressions in $\vec{o}$, we check, for each o_i, whether it is possible that given $P[t_1] \wedge P[t_2]$, we can have $\bigwedge_i (o^\star_i[t_1] = o^\star_i[t_2])$ but not $o_i[t_1] = o_i[t_2]$. If yes, that means o_i is wrong with respect to $o^\star$, because while t_1 and t_2 should belong to the same group per $o^\star$, grouping by o_i alone would have forced them into separate groups instead. After identifying all wrong expressions in $\vec{o}$ and removing them, we are left with a partitioning potentially coarser than $o^\star$ but otherwise consistent with $o^\star$. We then find Δ^+ to be further added in a similar fashion.

Lemma 6.2. *We say that two lists of* GROUP BY *expressions are* equivalent *if they produce the same partitioning for the above query over any database instance. Let (Δ^-, Δ^+) = FixGrouping$(P, \vec{o}, \vec{o^\star})$. Assuming that subroutine IsSatisfiable returns no false positives, we have:*

Correctness: GROUP BY*-stage hint leads to a fixed working query Q_3 that 1) passes the viability check ($\vec{o}, \vec{o^\star}$ are equivalent), 2) satisfies FWG$(Q_3) \equiv$ FWG$(Q^\star)$; and 3) leads to eventual correctness.*

Further assuming that IsSatisfiable returns no false negatives, we have:

Strong Minimality of Δ^-: *Let $(\Delta^-_\circ, \Delta^+_\circ)$ denote the minimal Δ^- and Δ^+ respectively, then for any $(\Delta^-_\circ, \Delta^+_\circ)$ such that $\vec{o} \setminus \Delta^-_\circ \cup \Delta^+_\circ$ is equivalent to $\vec{o^\star}$, $\Delta^- \subseteq \Delta^-_\circ$.*

Weak Minimality of Δ^+: *If $\Delta^+ \neq \emptyset$, then there exists no $\Delta^-_\circ$ such that $\vec{o} \setminus \Delta^-_\circ$ is equivalent to $\vec{o^\star}$.*

The strong minimality of Δ^- means that we can hint each expression therein as a "must-fix." The weak minimality of Δ^+ works perfectly as we simply hint that the wrong query needs some additional GROUP BY expressions.

7 HAVING STAGE

At HAVING stage, we aim at further ensuring that FWGH$(G) \equiv$ FWGH$(G^\star)$ assuming that $Q^\star$ and Q unified by a table mapping and have equivalent FROM, WHERE, and GROUP BY. While HAVING can also be modeled as a logical formula, there are new challenges: 1) unlike WHERE, inputs to HAVING formulae are arrays of tuples $[t_1, ..., t_n]$ instead of single tuples, 2) we need to consider aggregate functions, and 3) we cannot test HAVING alone without considering WHERE's effect.

Example 10. *Consider two queries over* R(A, B) *and* S(C, D)*:*

```
SELECT A FROM R, S WHERE A=C AND A>4 GROUP BY A, B
  HAVING A > B + 3 AND 2*SUM(D) > 10; -- Q★
SELECT A FROM R, S WHERE A=C GROUP BY A, B, C
  HAVING C > B + 3 AND SUM(D * 2) > 10 AND A>4; -- Q
```

The two queries are equivalent because A=C *in* WHERE, *because* 2* *distributes over* SUM, *and because* A>4 *can be either in* WHERE *or* HAVING.

Our strategy is to construct two formulae $H^\star, H$ for the HAVING conditions of $Q^\star, Q$ respectively, such that equivalence of $H^\star$ and H implies FWGH$(G) \equiv$ FWGH$(G^\star)$. To this end, for each reference to a GROUP BY column in HAVING, we replace it with a variable from the same domain, and we translate HAVING expressions outside aggregate function calls in the same way as we handle WHERE: e.g., A>B+3 becomes $A{>}B{+}3$. For each reference to a column not in GROUP BY, we introduce an array variable to capture the fact that it refers to a collection of values from rows in the same group. Moreover, for each aggregate function call, we introduce a new array variable to represent the collection of input values if they are computed from an expression, and we use a universally quantified assertion to relate this variable to the source column values: e.g., for SUM(D*2) we introduce array-valued $\mathbf{D}_2$ to represent D*2 values, and we related it to the array-valued $\mathbf{D}$ representing D values by asserting $\forall i \in \mathbb{N} : \mathbf{D}_2[i] = \mathbf{D}[i] \times 2$. Such assertions, along with the WHERE condition and additional inference rules for aggregate functions, go into a context as discussed in Section 3 and illustrated in Example 3.

Example 11. *For Example 10,* HAVING *formulae for $Q^\star, Q$ are:*

$$\begin{array}{lr} (H^\star) & A{>}B{+}3 \wedge (2{\times}\mathsf{SUM}(\mathbf{D}){>}10) \\ (H) & C{>}B{+}3 \wedge \mathsf{SUM}(\mathbf{D}_2){>}10 \wedge A{>}4 \end{array}$$

We test their equivalence under the following context:

$$C : \left\{ \begin{array}{r} \mathbf{D}, \mathbf{D}_2 \textit{ have type } \mathit{Array}(\mathbb{Z}) \\ A = C \wedge A > 4 \\ \forall i \in \mathbb{N} : \mathbf{D}_2[i] = \mathbf{D}[i] \times 2 \\ \hline \mathsf{SUM} \textit{ has type } \mathit{Array}(\mathbb{Z}) \rightarrow \mathbb{Z} \\ \forall c \in \mathbb{Z}, \mathbf{X} \textit{ and } \mathbf{Y} \textit{ of type } \mathit{Array}(\mathbf{Z}) : \qquad\qquad\qquad\qquad \\ (\forall i \in \mathbb{N} : \mathbf{X}[i] \times c = \mathbf{Y}[i]) \Rightarrow \mathsf{SUM}(\mathbf{X}) \times c = \mathsf{SUM}(\mathbf{Y}) \end{array} \right\},$$

In the above, the assertions underneath the horizontal line are generic assertions encoding properties of aggregate functions useful for inferring equivalences. Only those relevant to Example 10 are listed here; for a complete list see [28].

The viability check for HAVING (Theorem 3.1, stage 4) is that H is logically equivalent to $H^\star$ under HAVING base context C, i.e.:

$$V_4 : \textit{Check if } H \Leftrightarrow H^\star \textit{ under } C$$

Note that this check implicitly applies to all groups. If a constraint solver fails to establish equivalence, we invoke the exact same procedures as for WHERE to find a repair.

Lemma 7.1. *HAVING-stage hint leads to a fixed working query Q_4 with HAVING condition that 1) passes the viability check; 2) satisfies FWGH$(Q_4) \equiv$ FWGH$(Q^\star)$; and 3) leads to eventual correctness.*

As with WHERE, the correctness of the above lemma relies only on the fact that Z3 inference is sound with respect to the logic exercised by H, $H^\star$, and C and that MinBoolExp always finds a Boolean formula equivalent to its given input. We could additionally guarantee optimality similar to Lemma 5.2 by making the same assumptions therein (completeness of Z3 inference and optimality of MinBoolExp) plus the additional assumption that the context C encodes all properties of aggregate functions relevant to inference.

8 SELECT STAGE

This stage aims at fixing SELECT as needed to ensure $Q \equiv Q^\star$, assuming that they already have equivalent FROM, WHERE, GROUP BY and HAVING. We test the equivalence between SELECT expressions with a context C dependent on the type of the query: if the queries are SPJ, we simply assert the WHERE condition in C; if the queries are SPJA, we use the same C defined by the HAVING-stage.

Let $\vec{o}$ and $\vec{o^\star}$ denote the resulting ordered lists of SELECT expressions for $Q, Q^\star$, respectively. The viability check (V_5) is that $\dim(\vec{o}) = \dim(\vec{o^\star})$ and $\vec{o}[i]$ is equivalent to $\vec{o^\star}[i]$ for $1 \leq i \leq \dim(\vec{o^\star})$, i.e. both SELECTs have the same number of expressions and expressions on the same index position are equivalent. If SELECT clauses are not equivalent between $Q^\star, Q$, our goal becomes to compute Δ^- of SELECT expression to be removed from Q at the corresponding index position and Δ^+ of expressions to be added to Q at the corresponding index position.

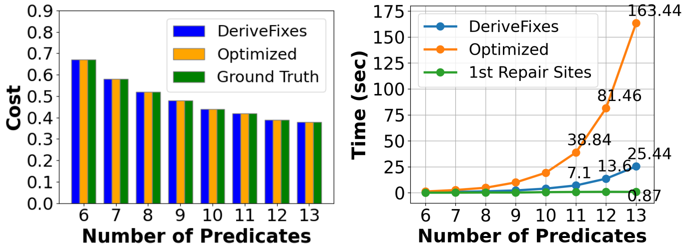
The algorithm checks the equivalence between $(\vec{o}[i], \vec{o^\star}[i])$ and add Δ^- and Δ^+ respectively if they are inequivalent. Finally, excessive expressions in Q or $Q^\star$ will also be added to Δ^- and Δ^+ respectively. After fixing SELECT, we guarantee $Q^\star \equiv Q$.

9 EXPERIMENTS

We test three aspects of Qr-Hint: coverage, accuracy, and running time. For coverage, we test the ability of Qr-Hint to fix wrong queries that arise in real-world classroom settings. For accuracy and running time, we focus on Algorithm 1, which is the bottleneck of Qr-Hint due to calls to DeriveFixes or DeriveFixesOPT. As fix minimization incurs exponential time, we examine 1) how the number of unique predicates affects running time, 2) how close the generated repairs are to the optimal if queries are not conjunctive, 3) a comparison between the running time and optimality of DeriveFixes and DeriveFixesOPT. In general, DeriveFixesOPT strives for smaller fixes and hence incurs longer running time than DeriveFixes.

Implementation/Test Environment. We implemented Qr-Hint in Python 3.10 using Apache Calcite [6] to parse SQL queries and Z3 SMT Solver [24] to test constraint satisfiability. We use ESPRESSO in PyEDA [26] for fix minimization. We run the experiments locally on a 64-bit Ubuntu 20.04 LTS server with 3.20GHz Intel Core i7-8700 CPU and 32GB 2666MHz DDR4.

Test Data Preparation. To prepare the first test dataset, denoted Students, we examined 2,000+ real student queries from an undergraduate database course in one semester at the first author's institution. These queries came from 4 introductory-level SQL questions (with 4 reference queries), and altogether they included 341 wrong queries. Out of these, 35 (11%) used SQL features not supported by Qr-Hint (see limitations at the end of Section 3). Hence, we end up with 306 supported wrong queries in Students. (At the time of writing, we are still exploring with the institutional review board the possibility of making this dataset publicly available.)



(a) Repair cost **(b) Running time**

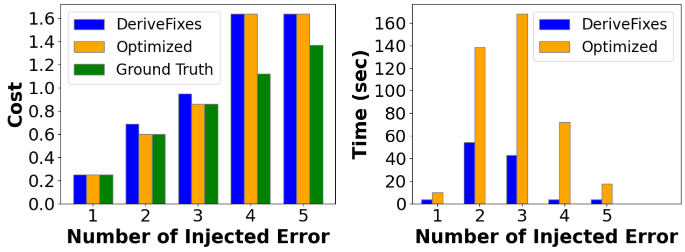
Figure 2: DeriveFixes vs. DeriveFixesOPT (Optimized) for conjunctive WHERE (TPCH)

To further expand coverage of errors, we cross-checked Students queries with the list of SQL issues indicative of semantic errors categorized by Brass et al. [12] (which did not publish a query dataset). Out of the 43 issues in [12], 18 involve SQL features not currently supported by Qr-Hint, but they only make up for a small minority (11.4%) of the observed instances as reported by [12]. Out of the 25 issues Qr-Hint should support, 17 are already represented in the 306 Students queries. To cover the remaining 8, we handcrafted two queries according to each issue and added to the dataset; we also handcrafted corresponding reference queries (free from any issue in [12]). We denote the resulting dataset Students+, with 322 queries having errors/issues.

Our second test dataset, denoted TPCH, is based on TPC-H [7] schema and queries, with synthetic errors injected. This dataset allows us to stress-test Qr-Hint with queries that are more complex than Students. Also, because errors are synthetic, we have the "ground-truth" repair sites and fixes, allowing us to easily assess the optimality of Qr-Hint fixes. Most WHERE conditions in TPC-H queries are conjunctive: we chose 7 TPC-H queries with conjunctions of 4,5,6,7,9,10,11 atomic predicates (TPC-H Query 4,3,10,9,5,8,21 respectively). Since we did not find a TPC-H query with exactly 8 predicates, we synthesized one by removing one predicate from TPC-H Query 5. For each query, we then introduced errors into two atomic predicates to make the wrong query, which remained conjunctive. Thus, each pair of wrong and reference queries has 6-13 unique atomic predicates. Furthermore, to test cases beyond conjunctive WHERE conditions, we chose TPC-H Query 7, whose WHERE contains multiple nested AND and OR, and created 5 wrong queries by injecting 1-5 errors by changing atomic predicates or logical operators. For fair comparison, we ensured that the number of unique atomic predicates is always 10 between the reference query and each wrong query.

9.1 Results and Discussion

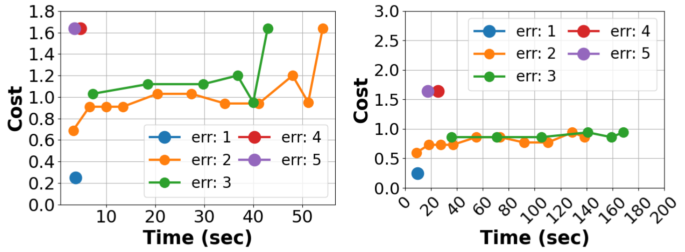
Student+. To test coverage and optimality of Qr-Hint, we ran Qr-Hint for the 322 Student+ queries with errors/issues, along with their reference queries, and examined all Qr-Hint fixes. For the 25 issues in [12] that Qr-Hint should support, we found that they were handled in three ways: 1) 11 of them were indeed errors,



(a) Repair cost

(b) Running time

Figure 3: DeriveFixes vs. DeriveFixesOPT (Optimized) for nested AND/OR (TPCH)



(a) DeriveFixes

(b) DeriveFixesOPT

Figure 4: Cost of repairs found during course of execution

and Qr-Hint correctly identified and fixed them all; 2) 3 of them were efficiency/stylistic issues where the queries were semantically still correct (e.g., logically correct WHERE containing some tautological conditions, such as A >= B OR A < B), and Qr-Hint did not flag any error; 3) the remaining 11 of them were also efficiency/stylistic issues (e.g., unnecessarily joining a primary key with its corresponding foreign key but only projecting the foreign key column), but Qr-Hint failed to detect query equivalence in this case and suggested some fixes. This last category is the only case where Qr-Hint showed suboptimal behavior, though its suggested fixes still lead to correct queries, and with the interesting side effect of resolving efficiency/stylistic issues. The detailed analysis can be found in [28]. It is worth noting that Qr-Hint perfectly handles all of the 10 most common issues in [12].

Qr-Hint's average running time per query on Student+ is 0.2 seconds, using DeriveFixes. However, note that most Student+ queries are rather simple, with conjunctive WHERE (which does not need DeriveFixesOPT for optimality) and at most 5 unique atomic predicates. Therefore, we further stress-tested Qr-Hint using TPCH.

TPCH, conjunctive WHERE with varying number of atomic predicates. Here, we study Qr-Hint's running time and optimality (as measured by repair cost, the lower the best) as we vary the number of atomic predicates involved in repairing WHERE. We compare versions of Qr-Hint using DeriveFixes vs. DeriveFixesOPT, both set to explore up to two repair sites. Figure 2a confirms that for conjunctive queries, both always return optimal repairs according to the ground truth, regardless of the size of WHERE. (Note that the repair cost is not proportional to the number of atomic predicates because it is normalized by the query sizes per Equation (1)). Figure 2b shows that as expected, both have running times exponential in the number of unique atomic predicates, but DeriveFixes runs much faster than DeriveFixesOPT. Furthermore, the plot labeled "1st Repair Sites" shows that it takes less than one second for Qr-Hint to find the first *viable* (not necessarily optimal) repair site, so there is additional room to trade optimality for faster running time.

TPCH, WHERE with nested AND/OR and varying number of injected errors. As shown in Figure 3a, when the optimal repair (according to the ground truth) involves only one repair site (a single error), both DeriveFixes and DeriveFixesOPT are able to find this optimal repair, confirming Lemma 5.2. When there are more errors (2-3), DeriveFixes returns suboptimal repairs while DeriveFixesOPT is still able to find optimal or near-optimal repairs (for the cases of 2 and 3 errors, respectively). However, with 4-5 errors—which are arguably not the cases Qr-Hint targets—both suffer from suboptimality because they are set to explore up to two repair sites; in fact, both decided that it was best to just repair the whole WHERE condition. Figure 3b shows that DeriveFixesOPT's better optimality comes at the expense of slower speed than DeriveFixes, however. Interestingly, with 4-5 errors, both run faster than with 2-3 errors, because the large numbers of errors severely limit the number of possibilities of single- and 2-site repairs, speaking to the effectiveness of CreateBounds in quickly spotting and bailing out of difficult situations.

Finally, Figure 4 shows all unpruned viable repairs found during Qr-Hint's course of execution, in terms of when they were found and how much they cost; there is one trace for each execution. Traces for 1 (blue), 4 (red), and 5 (purple) errors degenerate into single dots because Qr-Hint eventually finds only one solution as viable repair options are limited. Recall that we heuristically prioritize the viable repairs to consider, but there is no guarantee that a cheaper repair will always be found earlier. Hence, there are fluctuations in the repair costs over time, although the general trends are up, confirming the effectiveness of our heuristic. Furthermore, note that the lowest-cost repairs tend to surface early during execution. In closing, while the total and worst-case running times of Qr-Hint grow exponentially in query size, in practice the running times are reasonable considering that Qr-Hint is intended for education settings, where returning hints instantaneously may not be necessary or desirable for learning. With the observation that Qr-Hint often returns some low-cost repairs early, we can offer them as preliminary hints to get students thinking, while Qr-Hint continues to look for better repairs in the meantime.

10 USER STUDY

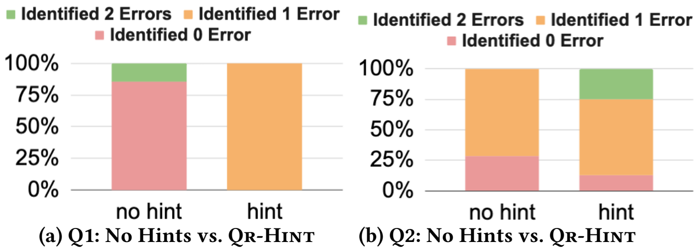
We conducted a small-scale user study to evaluate Qr-Hint: 1) whether students can understand what is wrong with the suggested hints, and 2) how the hints generated by Qr-Hint compare with ones provided by "*expert users*" (teaching assistants in our study).

Participants. We recruited 38 students who have taken/are taking a graduate or undergraduate database course. Except for an incentive of receiving a small gift card and practicing SQL, the participation was voluntary. In the end, we collected 15 complete and valid answers. A possible explanation for the low completion rate was the significant effort required to debug SQL queries with subtle mistakes (we observed that some participants took more than an hour to finish). We considered the possibility of recruiting participants from other sources (e.g., Amazon Mechanical Turk), but decided against it because they would not represent our targeted population (students). Furthermore, given the significant effort required from the participants as observed above, it would be hard to incentivize participants who are not actively learning SQL: a low reward would turn them away, while a high reward might encourage undesirable behaviors.

Preparation. To design the survey, we first performed an analysis of the Students queries to get a sense of what the common errors were. Overall, most errors came from WHERE and HAVING (130 out of 341 are wrong due to WHERE); students often missed join conditions for queries involving many tables. Other common errors include incorrect/redundant/missing tables in FROM, incorrect order and missing/redundant expressions in SELECT, and incorrect expressions in GROUP BY. We decided not to use the same queries from Students, as our participants had done the same/similar homework previously, which might bias the results. Nonetheless, based on these observations, we designed four SQL questions using a different schema, DBLP (details in [28]). For each question, we crafted a wrong solution containing one or more mistakes: two WHERE errors for Q_1, one GROUP BY error and one SELECT error for Q_2, one WHERE error for Q_3, and one each WHERE and HAVING errors in Q_4. Even though the queries are over a different schema, the errors above faithfully reflect real errors from Students, and they are consistent with the common errors found by others [2, 12].

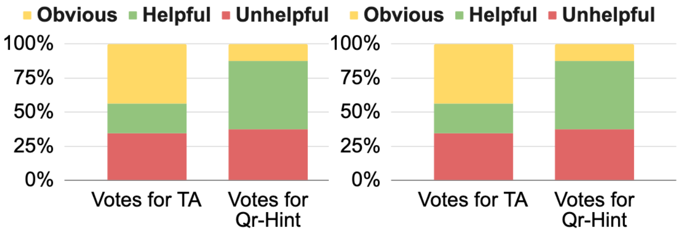
Then, we performed a small study with four graduate teaching assistants (TAs) to generate hints for these queries. Each TA was asked to pinpoint all mistakes in each query and offer hints, as if they were helping students debug wrong queries. To simulate an office-hour setting, we asked TAs to finish all four questions in one sitting, with no help from Qr-Hint. We collected all hints provided by the TAs as "expert" hints. Next, we ran Qr-Hint on all wrong queries to obtain repair sites and fixes. We removed fixes and only showed repair sites to the participants as hints. To prevent participants from recognizing the source of hints (experts vs. Qr-Hint) by their wording, we paraphrased all hints to use a common template "In [*SQL clause*], [*hint*]" and standard wording.

Tasks. Using the four queries, each participant saw and completed three questions. Students were required to complete questions on Q1 and Q2, and they completed one of Q3 and Q4 at random. For each question, students were given the database schema, problem statement in English, and the wrong SQL query, and were asked to explain what is wrong with the query. For creating *treatment* and *control* groups, students received hints from Qr-Hint for either Q1 or Q2 (not both) at random, and for the other one they were asked to detect errors without any hints provided; the order of the two questions with and without hints was also chosen at random. For the last question, participants received Q3 or Q4 at random, and we showed the union of hints (mixed together) generated by the TAs as well as by Qr-Hint, and asked participants to categorize each hint as one of the following: "*Unhelpful or incorrect*", "*Helpful but require thinking*", and "*Obvious and giving away the answer*". Participants were asked to finish all questions in one sitting. We recorded the time a participant spent on each question[8]. In our study, for Q1, 8 students answered it with no hints and 7 with hints from Qr-Hint. For Q2, these numbers are 7 and 8 respectively. For the third question, 7 received Q3 and 8 received Q4.



(a) Q1: No Hints vs. Qr-Hint (b) Q2: No Hints vs. Qr-Hint

Figure 5: User performance with/without Qr-Hint



(a) Q3 Vote Cast (b) Q4: Vote Cast

Figure 6: Hint categorization from participants for Q3, Q4

Result and Analysis. Our results for Q1 and Q2 show that participants were better at identifying at least one error in the query given the hints provided by Qr-Hint compared to no hints. As shown in Figure 5a and Figure 5b, 100% and 87.3% of the participants were able to identify at least one of the two errors in the wrong query in Q1 and Q2 respectively after receiving hints from Qr-Hint, as opposed to 14.3% and 71.4% who were able to do so without a hint. While there is a single participant who correctly identified both errors without any hint for Q1, this participant spent more than 20 minutes doing so, while most participants spent no more than 10 minutes on the same question without hints.

Q3 and Q4 are used to evaluate whether Qr-Hint provided hints that are comparable to the ones given by teaching assistants in terms of their quality. For Q3, there are four TA hints and one hint from Qr-Hint; and there are four TA hints and two hints generated by Qr-Hint for Q4. For all responses, we sum up the number of times participants vote for each of the three categories of hint ranks: "Obvious", "Unhelpful", and "Helpful". The results are shown in Figures 6a, 6b. In summary, the quality of TAs' hints varies greatly as perceived by participants. On the other hand, Qr-Hint is consistently perceived by participants as "helpful but require thinking", which might be best suited for classroom settings.

11 CONCLUSION AND FUTURE WORK

We presented Qr-Hint, a framework for automatically generating hints and suggestions for fixes for a wrong SQL query with respect to a reference query. We developed techniques to fix all clauses in a query and gave theoretical guarantees. There are multiple intriguing directions of future work, including the support of more complex constructs such as subqueries, outer-joins (NULL), and database constraints. There are many steps where the framework evaluates all possible options (e.g., repair sites), hence improving the scalability of the system is also a future work. It will also be interesting to develop techniques to avoid the limitations of SMT solvers in our framework. We are implementing a graphical user interface so that Qr-Hint can better assist students/TAs in database courses. Conducting a larger-scale user study to further understand the effectiveness of Qr-Hint is also important for future work.

[8] Q_1 without/with hints took 704s/460s on average; Q_2 took 756s/658s. Students completed the survey asynchronously, so the time recorded may not be accurate.

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A RELATED WORK SUPPLEMENT

Extended Discussion on Testing query equivalence. There are several classical results on query equivalence. Chandra et al. [17] show that equivalence testing of conjunctive queries is NP-complete. Aho et al. [4] propose tableau to represent the value of a query, which is used to give algorithms for checking equivalence of SPJ queries. Sagiv et al. [50] give a procedure for testing the equivalence of Select-Project-Join-Difference-Union (SPJDU) queries. Klug [40] presents algorithms for checking the equivalence of conjunctive queries with inequalities. The equivalence problem of some classes of queries under bag semantics has been proved to be undecidable [38, 39]. While the query equivalence problem in general is undecidable [1, 53], tools are developed to check the equivalence of various classes of queries with restrictions and assumptions. Cosette [21–23] transforms SQL queries to algebraic expressions and uses a decision procedure and rewrite rules to check if the resulting expressions of two queries are equivalent. EQUITAS [58] develops a symbolic representation of SQL queries in first-order logic, and uses satisfiability modulo theories (SMT) to check query equivalence. WeTune [56] builds a query equivalence verifier by utilizing the U-semiring structure [21]. These tools for query equivalence do not give hints on how to modify one query to become equivalent to another.

B FROM STAGE SUPPLEMENT

B.1 Finding Table Mapping

In this section, We describe our heuristics for determining a table mapping. Since looking at FROM alone does not have enough information for determining a mapping, we gather information from other clauses (i.e., WHERE, GROUP BY, SELECT) to help us decide. The general idea is to create a "table signature" for each table involved in self-join, build a bipartite graph where weights of edges represent the difference between two table signatures, and select a mapping by solving the minimum-cost bipartite matching problem.

We first describe our heuristics for creating a table signature for each table in FROM of a query Q:

1. Scanning Q's WHERE and HAVING, for each selected operator ($=, <, >, \leq, \geq$) and each attribute a in the table, we create a set of attributes that "interact" with a in some atomic predicates in WHERE and/or HAVING(note: we rewrite the predicates to make sure a is on the left-hand side of the operator in the case of inequality). If a does not appear in WHERE or HAVING or it does not appear in a predicate with the selected operator, the corresponding set will be empty. After creating each set, we expand it to the entire equivalence class of its current attributes. We then replace each attribute in the set with the name of the original table they belong to.
2. Scanning Q's GROUP BY, create a set of attributes from the table that appear in any GROUP BY expression.
3. Scanning Q's SELECT, for each attribute a in the table, create a set of indices such that this attribute appears in the SELECT expression at the indexed position.

In summary, let t denote a table in a query Q, a table signature is a 3-tuple $\sigma = (W, G, S)$, where W is a function $W(a, o) \mapsto T, a \in \text{Attributes}(t), o \in =, <, >, \leq, \geq$ (T is a set of tables), G is a set such that $G \subseteq \text{Attributes}(t)$, and S is a function $S(a) \mapsto I, a \in \text{Attributes}(t)$ (I is a set of integer indexes).

With table signatures, let $O = \{=, <, >, \leq, \geq\}$ denote the set of operators, we then define the following metric for calculating a normalized distance between two signatures $\sigma = (W, G, S), \sigma' = (W', G, S')$:

$$\begin{aligned}\text{Cost}(\sigma, \sigma') = & \frac{\sum_{a \in \text{Attributes}(t), o \in O} \text{dist}(W(a, o), W'(a, o))}{|\text{Attributes}(t)| \times |O|} \\ & + \text{dist}(G, G') \\ & + \frac{\sum_{a \in \text{Attributes}(t)} \text{dist}(S(a), S'(a))}{|\text{Attributes}(t)|}\end{aligned}$$

Here we define dist as the Jaccard similarity between two sets. Each component is a normalized Jaccard similarity between the corresponding sets, and we take the sum of three Jaccard similarities (i.e. for WHERE, GROUP BY, SELECT respectively) as our final distance metric. Note that when two sets are empty, we count their Jaccard similarity as 1.

With such a distance metric, we then build a bipartite graph where

- Each node in partition 1 represents a table in $Q^{\star}$, and each node in partition 2 represents a table in Q.
- Each node in partition 1 is connected with at least one counterpart node that refers to the same table in partition 2. The weight of the edge between two nodes is the absolute difference between their signatures.

Once we have built the bipartite graph, we can use a linear program to solve the minimum-weight perfect matching problem. We now use an example to demonstrate the heuristic.

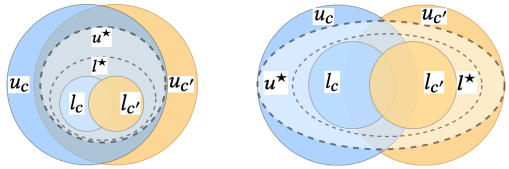
Example 12. *Continuing with Example 1, the initial signatures for S1, S2, s1, s2 are shown in Example 4. After replacing attribute names with table names, the final signatures are the following:*

		S1 in $Q^{\star}$	*S2 in $Q^{\star}$*	*s1 in Q*	*s2 in Q*
WHERE &	*bar:*	$=\{$*Frequents*$\}$	$=\{$*Frequents*$\}$	*None*	*None*
HAVING	*beer:*	$=\{$*Likes, Serves*$\}$	$=\{$*Likes, Serves*$\}$	$=\{$*Likes, Serves*$\}$	$=\{$*Likes, Serves*$\}$
	price:	$\leq\{$*Serves*$\}$	$\geq\{$*Serves*$\}$	$>\{$*Serves*$\}$	$<\{$*Serves*$\}$
GROUP BY		$\{$*bar, beer*$\}$	$\{$*beer*$\}$	$\{$*beer*$\}$	$\{$*beer*$\}$
SELECT	*bar:*	$\{2\}$	$\emptyset$	$\emptyset$	$\{2\}$
	beer:	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
	price:	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

The weight of the edge between S1 and s1 is calculated as followed: $\frac{4+5+3}{15} + \frac{1}{2} + \frac{0+1+1}{3} \approx 1.97$. *Comparing the* WHERE *and* HAVING *in S1 and s1's signatures, the Jaccard distance between $W(\text{bar}, =)$ and $W'(\text{bar}, =)$ is 0 as they do not have common element. Since the other 4 operators do not involve bar, their corresponding sets are all empty and thus yield a Jaccard distance of 1 between S1 and s1. Thus the total Jaccard distance for bar is 4. Similarly, we obtain 5, 3, respectively for beer and price, and the sum of these Jaccard distances is normalized. The Jaccard distance between the* GROUP BY *is simply $\frac{1}{2}$ as there is only one common element. For* SELECT, *beer and price have the same sets between S1 and s1, so the normalized Jaccard distance is $\frac{2}{3}$.*

Following the same fashion, the weight of the rest of edges are below:

- $S1 \mapsto s2 : \frac{4+5+3}{15} + \frac{1}{2} + \frac{1+1+1}{3} = 2.3$
- $S2 \mapsto s1 : \frac{4+5+3}{15} + 1 + \frac{1+1+1}{3} = 2.8$
- $S2 \mapsto s2 : \frac{4+5+3}{15} + 1 + \frac{0+1+1}{3} \approx 2.47$



(a) The bounds for ∧ **(b) The bounds for ∨**

Figure 8: Venn Diagrams for visualizing relationship among formulae in Algorithm 3

With the signatures, the corresponding bipartite graph is shown below:

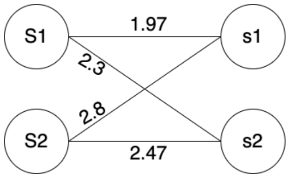


Figure 7: Constructed bipartite graph for Example 12

By negating the weight of each edge, we then convert the problem into solving the minimum-weight perfect matching on the graph, Qr-Hint eventually choose the following mapping: S1 ↦ s2, S2 ↦ s1, which has the minimum overall weight (−2.3+−2.8 = −5.1) among all matchings, and it is the best one for suggesting fixes in the downstream stages.

B.2 Proof for Lemma 4.2

Proof of Lemma 4.2. Without loss of generality, suppose that $Q^\star$ returns a non-empty result over some database instance D. We construct a new database instance D' as follows:

1. For each unique table $T \in \mathsf{Tables}(Q^\star)$, duplicate each row in T a number of times equal to a unique prime p_T (such that $p_{T_1} \neq p_{T_2}$ for any $T_1 \neq T_2$).
2. For each table $T \notin \mathsf{Tables}(Q^\star)$, make T empty.

We have

$$|Q^\star(D')|/|Q^\star(D)| = \prod_{\text{unique } T \in \mathsf{Tables}(Q^\star)} p_T^{|\mathsf{Aliases}(Q^\star,T)|}.$$

For any Q equivalent to $Q^\star$, it must be the case that $\mathsf{Tables}(Q) \subseteq \mathsf{Tables}(Q^\star)$ (disregarding counts); otherwise $Q(D')$ would be empty by construction of D'. Furthermore, note that for Q to be equivalent to $Q^\star$, we must have $|Q(D')|/|Q(D)| = |Q^\star(D')|/|Q^\star(D)|$, so

$$\prod_{\text{unique } T \in \mathsf{Tables}(Q)} p_T^{|\mathsf{Aliases}(Q,T)|} = \prod_{\text{unique } T \in \mathsf{Tables}(Q^\star)} p_T^{|\mathsf{Aliases}(Q^\star,T)|}.$$

It then follows from the Prime Factorization Theorem [29] that $\mathsf{Tables}(Q)$ and $\mathsf{Tables}(Q)$ contain the same *set* of tables, and that for each unique $T \in \mathsf{Tables}(Q)$ (or $\mathsf{Tables}(Q)$), $|\mathsf{Aliases}(Q,T)| = |\mathsf{Aliases}(Q^\star,T)|$. In other words, $\mathsf{Tables}(Q) \overset{\square}{=} \mathsf{Tables}(Q^\star)$. □

C WHERE STAGE SUPPLEMENT

In this section, we give a complete story for the derivation of fixes (i.e. Algorithm 3) and its optimization, as well as give proof to Lemma 5.1, Lemma 5.2, Lemma 5.3 and Lemma 5.4.

C.1 DeriveFixes Revisited

C.1.1 Target Bound Derivation. We first completely map out the story for lines 15–22 of Algorithm 3, which contain the strategy for pushing down target bound.

For illustration and without loss of generality, we again assume that all ∧, ∨ nodes have only two children c, c'. While addressing a particular node x, we use the same notation as in Section 5, where $[l_c, u_c]$, $[l_{c'}, u_{c'}]$ represent the repair bound of the child c, c' respectively, and $[l^\star, u^\star]$ denotes the x's target bound, and $[l_c^\star, u_c^\star]$, $[l_{c'}^\star, u_{c'}^\star]$ denote the target bound of c, c' respectively.

The principle that guides the formulation of the target bound is simple: we want to recursively expand the range of the target bound as we traverse down to each repair site so that their target bounds will potentially contain small fixes. We use an example to demonstrate this principle.

Example 13. *Let $l \equiv (p_1 \vee p_2) \wedge p_3$ and $u \equiv p_1 \vee p_2$ be the lower and upper bounds, where p_1, p_2, p_3 are arbitrary predicates. The minimal formula within $[l, u]$ is $p_1 \vee p_2$. However, let $u' \equiv p_1 \vee p_2 \vee p_3$ be the new upper bound, the minimal formula that falls within $[l, u']$ becomes p_3. p_3 is smaller than $p_1 \vee p_2$ in terms of the size of their syntax trees. Here we expand the range of $[l, u]$ by relaxing the upper bound (i.e. adding a disjunct to the upper bound).*

Symmetrically, we can create a case when range expansion is done by restricting the lower bound (i.e. adding a conjunct to the lower bound). Let $l \equiv p_1 \wedge p_2$ and $u \equiv (p_1 \wedge p_2) \vee p_3$ be the lower and upper bounds. If we restrict l further by constructing $l' \equiv p_1 \wedge p_2 \wedge p_3$, the minimal formula moves from $p_1 \wedge p_2$ to p_3 for bounds $[l, u]$ and $[l', u]$ respectively.

With such a principle, the next question becomes how to expand the range of target bounds $[l^\star, u^\star]$ in the context of Algorithm 3. Situations differ based on the logical operator at each node.

When x is rooted at ∧, the formula has the form $c \wedge c'$. Assuming both c, c' contain some repair sites, we want to have $p_c, p_{c'}$ satisfying the following constraints after fixes are applied:

1. $p_c \wedge p_{c'} \in [l^\star, u^\star]$, i.e., combining $p_c, p_{c'}$ using ∧ forms a new formula that falls within the target bounds at x.
2. $p_c \in [l_c, u_c]$ and $p_{c'} \in [l_{c'}, u_{c'}]$.
3. The target bounds to be pushed down to c, c' are contained in their repair bounds, i.e., (i) $[l_c^\star u_c^\star] \in [l_c, u_c]$, and (ii) $[l_{c'}^\star, u_{c'}^\star] \in [l_c, u_{c'}]$.

Given the above constraints, the relationship among target bounds and repair bounds are depicted in Figure 8a: $l_c \wedge l_{c'} \Rightarrow l^\star \Rightarrow u^\star \Rightarrow u_c \wedge u_{c'}$.

We now make the following observation that helps us determine how we expand the range of $[l^\star, u^\star]$ to form $[l_c^\star, u_c^\star]$ and $[l_{c'}^\star, u_{c'}^\star]$.

- $p_1 \wedge p_2 \Leftrightarrow (p_1 \vee \neg p_2) \wedge p_2 \equiv p_1 \wedge (p_2 \vee \neg p_1)$

Such observation indicates that when pushing new target bounds to c, we can expand the current target bound by excluding the semantics of c' (and vice versa), and such expansion can be done by relaxing the upper bound (i.e. $u^\star$) since we are adding a disjunct. However, given the repair bound of c', which formula within $[l_{c'}, u_{c'}]$ should we pick to ensure $p_c, p_{c'}$ satisfy the above constraints? To answer this question, we make another observation as follows:

- $p_1 \wedge p_2 \Leftrightarrow (p_1 \vee \neg p_3) \wedge p_2$, if $p_2 \Rightarrow p_3$.
- $p_1 \wedge p_2 \Leftrightarrow p_1 \wedge (p_2 \vee \neg p_3)$, if $p_1 \Rightarrow p_3$.

```
Algorithm 5: MapAtomPreds($\mathcal{P}$)
Input  : a set $\mathcal{P}$ of predicates
Output : a list of atomic predicates $\vec{a} = [a_1, a_2, \ldots, a_{\dim(\vec{a})}]$
         denoted by Boolean variables $\vec{\mathrm{a}} = [\mathrm{a}_1, \mathrm{a}_2, \ldots, \mathrm{a}_{\dim(\vec{\mathrm{a}})}]$,
         and a mapping $\phi$ such that for any Boolean
         subexpression $s$ in any predicate in $\mathcal{P}$, $\phi(s)$ returns a
         Boolean function (with variables in $\vec{\mathrm{a}}$) that is equivalent
         to $s$ after replacing each variable $\mathrm{a}_i$ with predicate $a_i$
1  let $\vec{a} = [\,]$, $\vec{\mathrm{a}} = [\,]$, and $\phi$ = empty mapping;
2  foreach predicate in $\mathcal{P}$ and each atomic predicate $t$ therein do
3      foreach $a_i \in \vec{a}$ do // see if $t$ is expressible by a chosen predicate
4          if IsEquiv($t, a_i$) then
5              let $\phi(t) = \mathrm{a}_i$; break;
6          else if IsEquiv($t, \neg a_i$) then
7              let $\phi(t) = \neg\mathrm{a}_i$; break;
8      if $\phi(t)$ is undefined then // a "new" atomic predicate found
9          $\vec{a}$.append($t$); $\vec{\mathrm{a}}$.append($\mathrm{a}_{\dim(\vec{a})}$); let $\phi(t) = \mathrm{a}_{\dim(\vec{\mathrm{a}})}$;
10 foreach predicate in $\mathcal{P}$ and each Boolean subexpression $s$ therein do
11     if $\phi(s)$ is undefined then
12         let $\phi(s)$ be the Boolean function obtained from $s$ by
             replacing each atomic predicate $t$ with $\phi(t)$;
13 return $\vec{a}, \vec{\mathrm{a}}, \phi$;
```

```
Algorithm 6: MinFix($l^\star, u^\star$)
Input  : predicates $l^\star$ and $u^\star$ together defining a target bound
         $[l^\star, u^\star]$
Output : a predicate bounded by $[l^\star, u^\star]$ that is as simple as
         possible
1  let $\vec{a}, \vec{\mathrm{a}}, \phi$ = MapAtomPreds($\{l^\star, u^\star\}$);
2  let $g_l = \phi(l^\star)$ and $g_u = \phi(u^\star)$;        // both are Boolean functions of $\vec{\mathrm{a}}$
3  let $g^\star$ = BuildTruthTable($\vec{a}, \vec{\mathrm{a}}, g_l, g_u$);
4  let $g$ = MinBoolExp($g^\star$);
5  return predicate obtained from $g$ by replacing each variable $\mathrm{a}_i$ with
     atomic predicate $a_i$;

   SUBROUTINE BuildTruthTable($\vec{a}, \vec{\mathrm{a}}, g_l, g_u$)
   Output : a partial Boolean function $g^\star$ of $\vec{\mathrm{a}}$, consistent with the
            bounds defined by $g_l$ and $g_u$, represented as a mapping
            $\{0,1\}^{\dim(\vec{\mathrm{a}})} \to \{*, 0, 1\}$
1  let $g^\star$ = empty mapping;
2  foreach assignment $\vec{v} \in \{0,1\}^{\dim(\vec{\mathrm{a}})}$ of $\vec{\mathrm{a}}$ do
3      let $x = \bigwedge_{i \in [1..\dim(\vec{a})]} x_i$, where $x_i$ is $a_i$ if $\vec{v}$ assigns $\mathrm{a}_i$ to 1, or
         $\neg a_i$ if $\vec{v}$ assigns $\mathrm{a}_i$ to 0;
4      if IsUnSatisfiable($x$) then // input setting not possible
5          let $g^\star(\vec{v}) = *$;
6      else if $g_l(\vec{v}) = g_u(\vec{v})$ then // both true or both false
7          let $g^\star(\vec{v}) = g_l(\vec{v})$;
8      else // $g_l(\vec{v}) = 0$ and $g_u(\vec{v}) = 1$, because $l \Rightarrow u$
9          let $g^\star(\vec{v}) = *$;
10 return $g^\star$;
```

This implies that we can guarantee the formula at the current $\wedge$ node falls within $[l^\star, u^\star]$ as long as we relax $u^\star$ with a formula that is implied by all possible formulas within the repair bounds (because at this time, we do not know what formula p_c and $p_{c'}$ will eventually be, so we need to make a safe assumption that they could be any formula within the repair bound), and it is clear that only the repair upper bounds (i.e. $u_c, u_{c'}$) satisfy such constraints. Thus, we obtain $u_c^\star = u^\star \vee \neg u_{c'}$ and $u_{c'}^\star = u^\star \vee \neg u_c$ as the new target upper bounds for c and c' respectively.

However, at this point, both new target upper bounds fail to satisfy the second constraint outlined earlier (i.e. fall out of the repair bounds) as $u^\star \vee \neg u_{c'} \not\Rightarrow u_c$ and $u^\star \vee \neg u_c \not\Rightarrow u_{c'}$. Consequently, we have to add u_c and $u_{c'}$ as a conjunct to restrict $u_c^\star$ and $u_{c'}^\star$ respectively so that they stay in the corresponding repair bounds. Now all constraints are satisfied, and we do not change the lower bound based on the observation that relaxing lower bounds and upper bounds simultaneously does not necessarily expand the range.

When x is rooted at $\vee$, the formula has the form $c \vee c'$. Similar to an $\wedge$ node, assuming both c, c' contain repair sites, we want to have $p_c, p_{c'}$ satisfying the following constraints:

(1) $p_c \vee p_{c'} \in [l^\star, u^\star]$, i.e., combining $p_c, p_{c'}$ (after fixes are applied) falls within the target bounds at the root $\vee$ node.
(2) $p_c \in [l_c, u_c]$ and $t_1 \in [l_{c'}, u_{c'}]$.
(3) The target bounds to be pushed down to $p_c, p_{c'}$ are contained in their repair bounds, i.e., (i) $[l_c^\star, u_c^\star] \in [l_c, u_c]$, and (ii) $[l_{c'}^\star, u_{c'}^\star] \in [l_{c'}, u_{c'}]$.

With symmetric observations and reasoning, instead of relaxing the upper bound as for $\wedge$ node, here we further restrict the current target lower bound (i.e. $l^\star$) based on the following observation:

- $p_1 \vee p_2 \Leftrightarrow (p_1 \wedge \neg p_3) \vee p_2$, if $p_3 \Rightarrow p_2$.
- $p_1 \vee p_2 \Leftrightarrow p_1 \vee (p_2 \wedge \neg p_3)$, if $p_3 \Rightarrow p_1$.

We can guarantee the formula at the current $\vee$ node falls within $[l^\star, u^\star]$ as long as we restrict $l^\star$ with a formula that implies all possible formulas within the repair bounds, and it is clear that only the repair lower bounds (i.e. $l_c, l_{c'}$) satisfy such constraints. Furthermore, to keep the new target lower bounds within the repair bounds, we have to add l_c and $l_{c'}$ as disjunct to $l_c^\star$ and $l_{c'}^\star$ respectively, thus forming the final target lower bounds for c and c'. Note that we do not change the upper bounds based on the observation, as restricting upper bounds and lower bounds simultaneously does not necessarily expand the range.

When x is rooted at $\neg$, it has only one child, and we thus push down the target bounds by setting them to be $[\neg u^\star, \neg l^\star]$ as negation inverts the direction of implication.

C.1.2 Fix Minimization.

At each repair site, Algorithm 3 calls MinFix to compute the minimal fix given the target bound. The pseudocode of MinFix is shown in Algorithm 6. It takes a lower bound $l^\star$ and an upper bound $u^\star$ as inputs, builds a desired truth table, and utilizes the ESPRESSO [14] to find a formula t in disjunctive normal form, having the minimum number of minterms among all formulas that fall within the bounds.

However, since the truth table and Quine-McCluskey's method only work with Boolean variables instead of atomic predicates, MinFix leverages a subroutine MapAtomPreds (Algorithm 5, line 1 in Algorithm 6) to

(1) scan through both $l^\star$ and $u^\star$ and extracts all semantically unique atomic predicates.
(2) determine a mapping that maps each semantically unique atomic predicate in both $l^\star$, $u^\star$ to a set of unique Boolean variables that represent the atomic predicates (e.g. $a = b$ is

semantically equivalent to $a + 1 = b + 1$, thus they will be mapped to the same Boolean variable).

(3) construct a mapping ϕ which maps any subexpression in a predicate (formula) to a Boolean function so that $\phi(l^\star)$ and $\phi(u^\star)$ return the Boolean functions that represent the truth table of $l^\star$ and $u^\star$ respectively.

Given the Boolean functions for $l^\star$ and $u^\star$ (line 2), MinFix then calls a subroutine BuildTruthTable to construct the Boolean function representing the truth table of the minimal formula $t \in [l^\star, u^\star]$ (line 3 in MinFix) by going through all possible truth value assignments for $l^\star$ and $u^\star$ (line 2 in BuildTruthTable) with the following criteria:

- If the conjunction of all atomic predicates in a row is not satisfiable (e.g. $a = b$ and $a > b$ cannot both be true simultaneously), we mark the truth value for t by "$*$" (don't-care) since such situation can never happen (line 4-5 in BuildTruthTable).
- If the conjunction of all atomic predicates in a row is satisfiable (line 6-9 in BuildTruthTable):
  (1) if $l^\star$ and $u^\star$ are evaluated to the same truth value (i.e. both true or both false), then the same truth value will also be assigned to t.
  (2) if $l^\star$ and $u^\star$ are evaluated to different truth values (i.e. false for $l^\star$ and true for $u^\star$), then a don't-care is assigned to t. The purpose of such a don't-care assignment is to allow more flexibility for Quine-McCluskey's method to minimize t as much as possible. Note that the situation where $l^\star$ is evaluated to true and $u^\star$ is evaluated to false can never happen due to $l^\star \Rightarrow u^\star$.

After obtaining the Boolean function (i.e. truth table) for t, MinFix feeds it to a subroutine MinBoolExp which minimizes a given Boolean function f with possible "don't-care outputs" (denoted $*$). Boolean minimization in general is NP-complete, but good heuristics exist that often find near-optimal solutions for even a large number of variables. Our implementation uses the standard Quine-McCluskey algorithm [44], but better alternatives such as *ESPRESSO* [14] can also be used. We demonstrate how MinFix works using the following example.

$a \geq b$	$f = e$	$a = b$	$a > b$	$l^\star$	$u^\star$	t
0	0	0	0	0	0	0
0	0	0	1	*	*	*
0	0	1	0	*	*	*
0	0	1	1	*	*	*
0	1	0	0	0	1	*
0	1	0	1	0	1	*
0	1	1	0	*	*	*
0	1	1	1	*	*	*
1	0	0	0	*	*	*
1	0	0	1	0	1	*
1	0	1	0	1	1	1
1	0	1	1	*	*	*
1	1	0	0	*	*	*
1	1	0	1	1	1	1
1	1	1	0	1	1	1
1	1	1	1	*	*	*

Figure 9: Compact Truth tables for $l^\star$, $u^\star$ and t in Example 14.

EXAMPLE 14. *Consider the following bounds for a minimal formula t: $l^\star \equiv (a \geq b \wedge f = e) \vee a = b$, and $u^\star \equiv a = b \vee e = f \vee a > b$. Thus, we can construct a truth table for each formula shown in Figure 9. Based on the truth tables, it is clear that contradiction occurs when*

- *$a = b$ and $a > b$ are both true.*
- *Either $a = b$ or $a > b$ is true but $a \geq b$ is false.*
- *$a \geq b$ is true but both $a = b$ and $a > b$ are false.*

Running the Quine-McCluskey method on the final truth table for t yields $t \equiv a \geq b$, and $t \in [l^\star, u^\star]$.

Given MinFix, the fixes computed for the repair sites in Example 8 are the following:

- $x_4 : a = b \vee (a = c \wedge d > 10) \vee (a = c \wedge d < 7)$
- $x_{10}, x_{12} : (N_{10}, N_{12}) : (a = b \wedge d \neq e) \vee (a = b \wedge d > f) \vee (a = c \wedge d > 10) \vee (a = c \wedge e < 5)$

While the sizes of these fixes are still quite large, it is verifiable that the resulting formula is equivalent to $P^\star$ in Example 5 after inserting the fixes into P.

C.2 Optimization for Algorithm 3

The suboptimality of multiple fixes comes from the independent derivation of the target formula bounds (Algorithm 3). Using the same notations as in Algorithm 3, consider the target bounds for the children of an $\vee$ node with only two children and their Venn Diagram in Figure 8b:

$$[l_c^\star, u_c^\star] = [l_c \vee (l^\star \wedge \neg l_{c'}), u^\star], \; [l_{c'}^\star, u_{c'}^\star] = [l_{c'} \vee (l^\star \wedge \neg l_c), u^\star]$$

Here we use the Venn diagram to illustrate the source of suboptimality. The regions representing $l_c^\star$ and $l_{c'}^\star$ overlap (same for $u_c^\star, u_{c'}^\star$). Though their union matches exactly the region of $l^\star$ ($u^\star$ respectively), such overlap indicates an overlap in their semantics, meaning semantic redundancy exists in the target bounds of Children(x). This implies that any combination of $p_c \in [l_c^\star, u_c^\star]$ and $p_{c'} \in [l_{c'}^\star, u_{c'}^\star]$ have semantic overlaps, causing suboptimality in the subsequent derivation of fixes. To illustrate this, reconsider Example 8. The predicates $a = c$, $d > 10$, and $d < 7$ appear in the fix for x_4 unnecessarily.

To reduce such suboptimality, we propose to use a new procedure **instead of Algorithm 3** to consider all repair sites simultaneously by leveraging the fact that Algorithm 3 returns optimal fix for a single repair site (Lemma 5.2). The overall routine is shown in Algorithm 7.

The general intuition of Algorithm 7 is to start with $l^\star$ and $u^\star$ being the reference formula and updates them in the same manner as Algorithm 3 until the lowest common ancestor (LCA) of all repair sites. Treating the LCA as a single repair site, the corresponding optimal fix lies within its $[l^\star, u^\star]$, and Algorithm 7 aims to make up such optimal fix by collectively deriving fixes for all actual repair sites. We next describe the major steps of Algorithm 7. For a concise and clear illustration, we denote the optimal single fix at the LCA with T.

Build consistency table. We first replace each repair site with a unique Boolean variable, forming a new Boolean predicate P' at the LCA, and the goal is to make $P' \equiv T$. Because two Boolean formulas are equivalent if they share the same truth table, we then want to observe under what value assignments T and P' are consistent (i.e., evaluated to the same truth value) before determining how to construct each individual fix. Therefore, Algorithm 7 first generates a "**consistency table**" (line 4) where the inputs are all unique Boolean variables and atomic predicates from T, P', and the formulas being evaluated are $l^\star, u^\star, T, P'$, here $l^\star, u^\star$ are present for the derivation of T which follows the same procedures as in MinFix (Algorithm 6).

```
Algorithm 7: MinFixMult($x, \mathcal{S}, l^\star, u^\star$)
  Input  : a formula $x$, a set $\mathcal{S}$ of disjoint subtrees (repair sites) of $x$,
           and a target bound $[l^\star, u^\star]$ for $x$ to achieve by fixes
  Output : a repair $(\mathcal{S}, F)$, where $F$ maps each site in $\mathcal{S}$ to a formula
  let $\mathcal{U}$ denote the set of atomic formulas in $x$ that belong to none of
    the subtrees in $\mathcal{S}$;
  let $\vec{a}, \vec{\mathrm{a}}, \phi$ = MapAtomPreds($\mathcal{U} \cup \{l^\star, u^\star\}$);
  let $g_l = \phi(l^\star)$ and $g_u = \phi(u^\star)$;        // both are Boolean functions of $\vec{\mathrm{a}}$
  let $g^\star$ = BuildTruthTable($\vec{a}, \vec{\mathrm{a}}, g_l, g_u$);
  // Compute feasibility of truth values for repair sites:
  let $\vec{s} = [s_1, s_2, \ldots]$ be the list of subexpressions in $\mathcal{S}$, denoted by
    the list of Boolean variables $\vec{\mathrm{s}} = [\mathrm{s}_1, \mathrm{s}_2, \ldots]$;
  let $g_x$ be a Boolean function with variables $\vec{\mathrm{a}} \parallel \vec{\mathrm{s}}$, obtained from $x$
    by replacing each subexpression $s_i$ with variable $\mathrm{s}_i$, and replacing
    each atomic formula $t \in \mathcal{U}$ by $\phi(t)$;
  let $\mathfrak{C}$ = InitFeasibility($\vec{\mathrm{a}}, \vec{\mathrm{s}}, g_x, g^\star$);
  // Fix one site at a time, and incrementally update feasibility:
  let $F$ = empty mapping, and $\mathcal{I} = [1..\dim(\vec{\mathrm{s}})]$;
  while $\mathcal{I} \neq \emptyset$ do
      let $d, g^\star_d$ = PickSite($\vec{\mathrm{a}}, \vec{\mathrm{s}}, \mathfrak{C}, \mathcal{I}$);
      let $g_d$ = MinBoolExp($g^\star_d$);
      let $F(s_d)$ = formula obtained from $g_d$ by replacing each
        variable $\mathrm{a}_i$ with atomic formula $a_i$;
      let $\mathfrak{C}$ = UpdateFeasibility($\vec{\mathrm{a}}, \vec{\mathrm{s}}, \mathfrak{C}, d, g_d$);
      let $\mathcal{I} = \mathcal{I} \setminus \{d\}$;
  return $(\mathcal{S}, F)$;
```

```
Algorithm 8: Helper functions for MinFixMult

  SUBROUTINE InitFeasibility($\vec{\mathrm{a}}, \vec{\mathrm{s}}, g_x, g^\star$)
  Output : $\mathfrak{C} : \{0,1\}^{\dim(\vec{\mathrm{a}})} \to \{*\} \cup \mathbb{P}(\{0,1\}^{\dim(\vec{\mathrm{s}})})$, a mapping
           such that for each assignment $\vec{v}$ of variables in $\vec{\mathrm{a}}$, $\mathfrak{C}(\vec{v})$
           returns the set of feasible truth value settings for $\vec{\mathrm{s}}$ (such
           that $g_x$ is consistent with $g^\star(\vec{v})$), or $*$ if $\vec{v}$ is impossible
           or irrelevant (i.e., $g^\star(\vec{v}) = *$)
  let $\mathfrak{C}$ = empty mapping;
  foreach assignment $\vec{v} \in \{0,1\}^{\dim(\vec{\mathrm{a}})}$ of $\vec{\mathrm{a}}$ do
      if $g^\star(\vec{v}) = *$ then // impossible or irrelevant
          let $\mathfrak{C}(\vec{v}) = *$; continue;
      let $\mathfrak{C}(\vec{v}) = \emptyset$;
      foreach assignment $\vec{u} \in \{0,1\}^{\dim(\vec{\mathrm{s}})}$ of $\vec{\mathrm{s}}$ do
          if $g_x(\vec{v} \parallel \vec{u}) = g^\star(\vec{v})$ then let $\mathfrak{C}(\vec{v}) = \mathfrak{C}(\vec{v}) \cup \{\vec{u}\}$;
  return $\mathfrak{C}$;

  SUBROUTINE UpdateFeasibility($\vec{\mathrm{a}}, \vec{\mathrm{s}}, \mathfrak{C}, d, g_d$)
  Output : $\mathfrak{C}'$, a more constrained version of $\mathfrak{C}$ that reflects the
           effect of wiring variable $\mathrm{s}_d$ to the Boolean function $g_d$
  let $\mathfrak{C}'$ = empty mapping;
  foreach assignment $\vec{v} \in \{0,1\}^{\dim(\vec{\mathrm{a}})}$ of $\vec{\mathrm{a}}$ do
      if $\mathfrak{C}(\vec{v}) = *$ then // impossible or irrelevant
          let $\mathfrak{C}'(\vec{v}) = *$;
      else // only include settings consistent with $g_d$
          let $\mathfrak{C}'(\vec{v}) = \{\vec{u} \in \mathfrak{C}(\vec{v}) \mid u_d = g_d(\vec{v})\}$;
  return $\mathfrak{C}'$;

  SUBROUTINE PickSite($\vec{\mathrm{a}}, \vec{\mathrm{s}}, \mathfrak{C}, \mathcal{I}$)
  Output : index $d \in \mathcal{I}$ as the next site to fix, and a partial Boolean
           function $g^\star_d$ of $\vec{\mathrm{a}}$, represented as a mapping
           $\{0,1\}^{\dim(\vec{\mathrm{a}})} \to \{*, 0, 1\}$, specified in accordance with the
           feasibility map $\mathfrak{C}$
  foreach $i \in \mathcal{I}$ do let $c_i = 0$; // init accumulators for priority calculation
  foreach assignment $\vec{v} \in \{0,1\}^{\dim(\vec{\mathrm{a}})}$ of $\vec{\mathrm{a}}$ do
      if $\mathfrak{C}(\vec{v}) = *$ then continue;        // impossible or irrelevant
      foreach $i \in \mathcal{I}$ do
          let $r = |\{\vec{u} \in \mathfrak{C}(\vec{v}) \mid u_i = 1\}| \,/\, |\mathfrak{C}(\vec{v})|$;
          let $c_i = c_i + |r - 0.5|$;        // prioritize uneven splits
```

$a = 1$	$b = 2$	$c = 3$	r_1, r_2
0	0	0	00,01
0	0	1	00,01
0	1	0	00
0	1	1	01,10,11
1	0	0	10,11
1	0	1	10,11
1	1	0	01,10,11
1	1	1	01,10,11

Figure 11: Constraint table for Example 15

Example 15. *Consider the following formulae: $P^\star \equiv a = 1 \vee (b = 2 \wedge c = 3)$, $P \equiv c = 3 \vee (b = 2 \wedge a = 1)$.*

Let the repair sites in P be $c = 3$ and $a = 1$ with Boolean variables r_1, r_2, and their lowest common ancestor be the top-level $\vee$. We can obtain $l^\star \iff u^\star \iff P^\star$. Therefore, the consistency table can be constructed as shown in Figure 10.

$r1$	$r2$	$a = 1$	$b = 2$	$c = 3$	$l^\star$	$u^\star$	T	P'
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	1	1	0
0	0	1	0	0	1	1	1	0
0	0	1	0	1	1	1	1	0
0	0	1	1	0	1	1	1	0
0	0	1	1	1	1	1	1	0
0	1	0	0	0	0	0	0	0
0	1	0	0	1	0	0	0	0
0	1	0	1	0	0	0	0	1
0	1	0	1	1	1	1	1	1
0	1	1	0	0	1	1	1	0
0	1	1	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	1
1	0	0	0	1	0	0	0	1
1	0	0	1	0	0	0	0	1
1	0	0	1	1	1	1	1	1
1	0	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1	1
1	0	1	1	0	1	1	1	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	0	0	0	1
1	1	0	0	1	0	0	0	1
1	1	0	1	0	0	0	0	1
1	1	0	1	1	1	1	1	1
1	1	1	0	0	1	1	1	1
1	1	1	0	1	1	1	1	1
1	1	1	1	0	1	1	1	1
1	1	1	1	1	1	1	1	1

Figure 10: The consistency table for Example 15

The consistency table gives a direct view of the occasions where T and P' are consistent (e.g. blue highlights, evaluated to the same truth value) and inconsistent (e.g. red highlights, evaluated to different truth values). This helps us determine how to construct each fix in later steps as we want to avoid any inconsistency between T and P' to achieve $T \Leftrightarrow P'$.

Build constraint table. After acquiring the consistency table, the next question becomes how to use all available atomic predicates to construct fixes for each repair site while avoiding inconsistencies. For this purpose, we turn all atomic predicates as "constraints" for all Boolean variables that represent the repair sites, thus constructing a "**constraint table**" (line 7). A constraint table is a truth table where inputs are all the atomic predicates in the consistency table, and the output is the concatenation of truth values of Boolean variables. For each row (i.e. truth assignment of all atomic predicates), the constraint table aggregates and lists all truth assignments of all Boolean variables where T and P' are consistent, and these are the potential truth values to be assigned individually to each Boolean variable.

Example 16. *Consider the consistency table in Figure 10 from Example 15. The corresponding constraint table is shown in Figure 11.*

The blue-highlighted row reflects the highlighting in the consistency table, where only the truth assignments $(0, 1)$, $(1, 0)$ *or* $(1, 1)$ *for* (r_1, r_2) *produce consistent evaluations for* T *and* P*. This indicates that when constructing* r_1 *and* r_2 *using the available atomic predicates* $a = 1, b = 2$ *and* $c = 3$*, we must guarantee the truth assignment* $(1, 1, 1)$ *for* $(a = 1, b = 2, c = 3)$ *would cause* (r_1, r_2) *to be evaluated to either* $(0, 1)$, $(1, 0)$ *or* $(1, 1)$ *respectively. All other rows in the constraint table are constructed in the same manner and carry the same implication.*

Compute minimal fixes. The final step is to compute a fix for each repair site according to the constraint table. While a constraint table lists all possible simultaneous truth assignments for all repair sites (e.g. last column of Figure 11), we cannot make independent choices for the truth value of each repair site as dependencies exist. For example, in the highlighted row in Figure 11, if r_1 is assigned 0, then r_2 can only be assigned 1 for consistency. On the other hand, if r_1 is assigned 1, then r_2 can be assigned either 0 or 1. At this point, it is unclear how truth values can be assigned to each repair site to obtain minimum fixes, we thus follow a greedy procedure (line 10-14):

1. Randomly pick a repair site r.
2. Iterate over each row in the last column of the constraint table and give r the most flexible assignment (i.e. if both 0 and 1 are available, assign a don't-care).
3. Use Quine-McCluskey's method to compute the minimal fix for the repair site.
4. Update the assigned don't-cares to a determined value by evaluating the fix with the corresponding truth value assignment.
5. Update the available options in the last column of the constraint table based on the truth assignment of r.

Example 17. *Continue from Example 16, the derivation process for* r_1 *and* r_2 *are shown in an extended constraint table in Figure 12. Given the previous procedure, we first give* r_1 *maximum flexibility for constructing its formula, which yields* $a = 1$*. We then update the don't-cares accordingly before computing* r_2*. As for* r_2*, we follow the same procedure and derive the truth assignment based on the truth values of* r_1*. Finally, the procedure yields* $c = 3$*. These are indeed the optimal fixes.*

$a = 1$	$b = 2$	$c = 3$	r_1, r_2	r_1	r_2
0	0	0	00,01	0	$* \rightarrow 0$
0	0	1	00,01	0	$* \rightarrow 1$
0	1	0	00	0	0
0	1	1	01,10,11	$* \rightarrow 0$	1
1	0	0	10,11	1	$* \rightarrow 0$
1	0	1	10,11	1	$* \rightarrow 1$
1	1	0	01,10,11	$* \rightarrow 1$	0
1	1	1	01,10,11	$* \rightarrow 1$	$* \rightarrow 1$

Figure 12: Extended constraint table for Example 17

In addition, running Qr-Hint*-optimized over Example 5 yields optimal fixes* $a = b$ *and* $d > 10 \land e < 5$ *for repair sites* x_4, (x_{10}, x_{12}).

C.3 Proof of Lemma 5.1

Proof of Lemma 5.1. We prove the correctness of WHERE-stage by proving the correctness of repair returned by Algorithm 1, i.e. applying the repair sites and fixes returned by Algorithm 1 yields a new formula P' such that $P^\star \Leftrightarrow P'$. The proof contains two steps:

Step 1: Algorithm 2 returns the correct repair sites. Assume that there exists a set of fixes $\mathcal{F}$ for a set of repair sites $\mathcal{S}$ in P. By Lemma 5.3, we can create a lower bound $P_\perp$ and an upper bound $P_\top$ such that $P' \in [P_\perp, P_\top]$, where P' is the formula obtained by applying fixes to repair sites. Since $P' \Leftrightarrow P^\star$, $P^\star \in [P_\perp, P_\top]$. Thus, if a set of fixes exists for a set of repair sites, $P^\star$ must fall within the corresponding repair bounds at the root of P. This validates the procedure for determining repair sites.

Step 2: Algorithm 3 returns the correct fixes. Given a lower bound $P_\perp$ and an upper bound $P_\top$ for P with respect to $\mathcal{S}$ such that $P^\star \in [P_\perp, P_\top]$, by Lemma 5.4, we can derive a set of fixes $\mathcal{F}$ for $\mathcal{S}$ through Algorithm 3. □

C.4 Proof for Lemma 5.2

Proof. We use the induction over the structure of P to prove the minimality of the fix f for a single repair site s in P.

Base case. The base case is simply when s is P (i.e. the entire P is the repair site). In such case, f is a minimal DNF of $P^\star$ returned by the Quine-McCluskey's method. Thus, removing any clause from f causes $f \not\Leftrightarrow P^\star$.

Induction Step. When s is not P, there are three possible cases.

Case 1. P is in the form of $c_1 \land ... \land c_n$, where c_i is the repair site. The target bounds for c_i are derived to be $[P^\star, \top \land (P^\star \lor \neg \bigwedge_{j=1,j\neq i}^{n} c_j)]$ where the repair bounds of $\bigwedge_{j=1,j\neq i}^{n} c_j$ is simply $[\bigwedge_{j=1,j\neq i}^{n} c_j, \bigwedge_{j=1,j\neq i}^{n} c_j]$ as it does not contain any repair site. Running Quine-McCluskey's method over the target bounds of c_i yields a formula f, which is guaranteed to be in minimal DNF. Since we know $P^\star \Leftrightarrow \bigwedge_{j=1,j\neq i}^{n} c_j \land f$ and f is in minimal DNF, removing any of the clauses in f would cause $\bigwedge_{j=1,j\neq i}^{n} c_j \land f \Rightarrow P^\star$ but not vice versa.

Case 2. P is in the form of $c_1 \vee ... \vee c_n$, where c_i is the repair site. The target bounds for c_i are derived to be $[\bot \vee (P^\star \wedge \neg \bigvee_{j=1, j\neq i}^{n} c_j), P^\star]$ where the repair bounds of $\bigvee_{j=1, j\neq i}^{n} c_j$ are simply $[\bigvee_{j=1, j\neq i}^{n} c_j, \bigvee_{j=1, j\neq i}^{n} c_j]$. Running Quine-McCluskey's method over the target bounds of c_i yields a formula f, which is guaranteed to be in minimal DNF. Since we know $P^\star \Leftrightarrow \bigvee_{j=1, j\neq i}^{n} c_j \vee f$ and f is in minimal DNF, removing any of the clauses in f would cause $\bigvee_{j=1, j\neq i}^{n} c_j \vee f \Rightarrow P^\star$ but not vice versa.

Case 3. P is in the form of $\neg c$. Here $P^\star \Leftrightarrow \neg f$. Since f is in minimal DNF, removing a clause results in $P^\star \not\Leftrightarrow \neg f$. □

C.5 Proof of Lemma 5.3

Proof of Lemma 5.3. We use induction over the structure of P to prove that for any subtree x in P (for which CreateBounds is invoked), $\textsf{CreateBounds}(x, \mathcal{S}[x])$ returns a correct bound for x: i.e., applying any repair to $\mathcal{S}[x]$ in x will result in a formula x' bounded by $\textsf{CreateBounds}(x, \mathcal{S}[x])$.

Base case. Suppose x is an atomic formula, CreateBounds returns $[x, x]$ which bounds x. When x is a repair site, CreateBounds returns $[\textsf{false}, \textsf{true}]$, which certainly bounds x or any Boolean expression with which we can replace x.

Induction step. Suppose x is not atomic and is not itself a repair site. Let $\Theta = \textsf{op}(x)$ denote the logical operator at the root of x. Every repair on x (with the given $\mathcal{S}[x]$) is obtained by (potentially) repairing each child of x, but without changing Θ. In other words, every repair x results in $x' = \Theta_{c \in \textsf{Children}(x)} c'$, where c' is the result of some repair of c at sites $\mathcal{S}[c]$. By the inductive hypothesis, $\forall c \in \textsf{Children}(x) : c' \in [l_c, u_c] = \textsf{CreateBounds}(c, \mathcal{S}[c])$.

There are two cases depending on Θ. In the case that Θ is $\wedge$ or $\vee$, since $\forall c \in \textsf{Children}(x) : l_c \Rightarrow c' \Rightarrow u_c$, we have $\Theta_{c \in \textsf{Children}(x)} l_c \Rightarrow \Theta_{c \in \textsf{Children}(x)} c' \Rightarrow \Theta_{c \in \textsf{Children}(x)} u_c$, which means x' is within the bound returned by Line 6 of CreateBounds. In the case that Θ is $\neg$, since $l_c \Rightarrow c' \Rightarrow u_c$, we have $\neg u_c \Rightarrow \neg c' \Rightarrow \neg l_c$, which means x' is within the bound returned by Line 10 of CreateBounds. □

C.6 Proof of Lemma 5.4

Proof of Lemma 5.4. We use induction over the structure of P to prove that for any subtree x in P for which $\textsf{DeriveFixes}(x, \mathcal{S}, l^\star, u^\star)$ is invoked:

- (H1) $l^\star \Rightarrow u^\star$, and the bound $[l^\star, u^\star]$ implies (is equivalent or tighter than) the bound returned by $\textsf{CreateBounds}(x, \mathcal{S})$.
- (H2) The repair returned by $\textsf{DeriveFixes}(x, \mathcal{S}, l^\star, u^\star)$ yields some $x' \in [l^\star, u^\star]$.

Note that applying (H2) to the root of P proves Lemma 5.4.

Proving (H1) top-down. The base case is when x is the root of P; we only invoke DeriveFixes if $P^\star \in \textsf{CreateBounds}(x, \mathcal{S})$, so obviously $[l^\star, u^\star]$ implies $\textsf{CreateBounds}(x, \mathcal{S})$. For the induction step, assuming that (H1) holds for x, we now show that (H1) for each child of x for which DeriveFixes is invoked. There are three cases.

Case 1. x has form $\neg c$. Let $[l, u] = \textsf{CreateBounds}(x, \mathcal{S})$. By the inductive hypothesis $l \Rightarrow l^\star \Rightarrow u^\star \Rightarrow u$. Therefore $\neg u \Rightarrow \neg u^\star \Rightarrow \neg l^\star \Rightarrow \neg l$. In other words, we call DeriveFixes on c with a bound (Line 3) that implies $[\neg u, \neg l]$, which is $\textsf{CreateBounds}(c, \mathcal{S}[c])$ by Line 10 of CreateBounds.

Case 2. x has form $c_1 \wedge ... \wedge c_n$. In this case, Algorithm 3 divides the formula as $p_c \wedge p_{c'}$ where $p_c = c_i$, $p_{c'} = \bigwedge_{j=1, j\neq i}^{n} c_j$. We shall show that for $l_i^\star$ and $u_i^\star$ defined under Line 17, $[l_i^\star, u_i^\star]$ implies $[l_i, u_i] = \textsf{CreateBounds}(c_i, \mathcal{S}[c_i])$ for all i; the cases for all other children are symmetric. Clearly, $l_i \Rightarrow l^\star = l_i^\star$ and $u_i^\star = u_i \wedge (u^\star \vee \neg u_i') \Rightarrow u_i$. Furthermore, note that:

$$\begin{aligned}
&l_i \Rightarrow u_i;\\
&l_i \Rightarrow u^\star \vee \neg u_i';\\
&l^\star \xRightarrow{\text{ind. hypo. and Line 6 of } \textsf{CreateBounds}} u_i \wedge u_i' \Rightarrow u_0;\\
&l^\star \xRightarrow{\text{ind. hypo.}} u^\star \Rightarrow u^\star \vee \neg u_i'.
\end{aligned}$$

Therefore, $l_i^\star = l^\star \Rightarrow u_i \wedge (u^\star \vee \neg u_i') = u_i^\star$.

Case 3. x has form $c_1 \vee ... \vee c_n$. In this case, Algorithm 3 divides the formula as $p_c \vee p_{c'}$ where $p_c = c_i$, $p_{c'} = \bigvee_{j=1, j\neq i}^{n} c_j$. We shall show that for $l_i^\star$ and $u_i^\star$ defined in the branch starting on Line 19, $[l_i^\star, u_i^\star]$ implies $[l_i, u_i] = \textsf{CreateBounds}(c_i, \mathcal{S}[c_i])$ for all i; the cases for all other children are symmetric. Clearly, $l_i \Rightarrow l_i \vee (l^\star \wedge \neg l_i') = l_i^\star$ and $u_i^\star = u^\star \Rightarrow u_i$. Furthermore, note that:

$$\begin{aligned}
l_i &\Rightarrow u_i;\\
l_i &\Rightarrow l_i \vee l_i' \xRightarrow{\text{ind. hypo. and Line 6 of } \textsf{CreateBounds}} u^\star;\\
l^\star \wedge \neg l_i' &\Rightarrow u_i;\\
l^\star \wedge \neg l_i' &\Rightarrow l^\star \xRightarrow{\text{ind. hypo.}} u^\star.
\end{aligned}$$

Therefore, $l_i^\star = l_i \vee (l^\star \wedge \neg l_i') \Rightarrow u^\star = u_i^\star$.

Proving (H2) bottom-up. For the base case, when $x \in \mathcal{S}$, assuming the correctness of MinFix, Lemma 5.3 and (H1) ensure that $\textsf{MinFix}(x, l^\star, u^\star)$ yields a repaired formula in $[l^\star, u^\star]$.

For the inductive step, assuming that (H2) holds for each child of x, we now show that (H2) holds for x. There are three cases.

Case 1. x has form $\neg c$. By the inductive hypothesis, DeriveFixes on c returns a repair that results in some c' such that $\neg u^\star \Rightarrow c' \Rightarrow \neg l^\star$. Clearly, the same repair, which is returned by $\textsf{DeriveFixes}(x, \mathcal{S}, l^\star, u^\star)$, changes x to $\neg c'$, which satisfies $l^\star \Rightarrow \neg c' \Rightarrow u^\star$.

Case 2. x has form $c_1 \wedge ... \wedge c_n$. By the inductive hypothesis, for all $1 \leq i \leq n$, DeriveFixes on c_i returns a repair that results in some c_i' such that $l^\star \Rightarrow c_i' \Rightarrow u_i \wedge (u^\star \vee \neg u_i')$. The repair returned by DeriveFixes on x results in $\bigwedge_{i=1}^{n} c_i'$. Clearly, $c_1' \wedge ... \wedge c_n' \Leftarrow l^\star \wedge l^\star \Leftrightarrow l^\star$.

Also,

$$
\begin{aligned}
\bigwedge_{i=1}^{n} c'_i &\Rightarrow \bigwedge_{i=1}^{n} \left(u_i \wedge (u^\star \vee \neg u'_i)\right) \\
&\Leftrightarrow \bigwedge_{i=1}^{n} u_i \wedge \left(u^\star \vee \left(\bigwedge_{i=1}^{n} \neg u'_i\right)\right) \\
&\Leftrightarrow \left(\bigwedge_{i=1}^{n} u_i \wedge u^\star\right) \vee \left(\bigwedge_{i=1}^{n} u_i \wedge \bigwedge_{i=1}^{n} \neg u'_i\right) \\
&\Leftrightarrow u^\star \vee \left(\bigwedge_{i=1}^{n} u_i \wedge \bigwedge_{i=1}^{n} \neg u'_i\right) \\
&\Leftrightarrow u^\star \vee \bot \\
&\Leftrightarrow u^\star.
\end{aligned}
$$

Case 3. x has form $c_1 \vee ... \vee c_n$. By the inductive hypothesis, for all $1 \leq i \leq n$, DeriveFixes on c_i returns a repair that results in some c'_i such that $l_i \vee (l^\star \wedge \neg l'_i) \Rightarrow c'_i \Rightarrow u^\star$. The repair returned by DeriveFixes on x results in $\bigvee_{i=1}^{n} c'_i$. Clearly, $c'_1 \vee ... \vee c'_n \Rightarrow u^\star \vee u^\star \Leftrightarrow u^\star$. Also,

$$
\begin{aligned}
\bigvee_{i=1}^{n} c'_i &\Leftarrow \bigvee_{i=1}^{n} (l_i \vee (l^\star \wedge \neg l'_i)) \\
&\Leftrightarrow \bigvee_{i=1}^{n} l_i \vee \left(l^\star \wedge \bigvee_{i=1}^{n} \neg l'_i\right) \\
&\Leftrightarrow \left(\bigvee_{i=1}^{n} l_i \vee l^\star\right) \wedge \left(\bigvee_{i=1}^{n} l_i \vee \bigvee_{i=1}^{n} \neg l'_i\right) \\
&\Leftrightarrow l^\star \wedge \top \\
&\Leftrightarrow l^\star.
\end{aligned}
$$

□

D GROUP BY STAGE SUPPLEMENT

D.1 Necessity of Fixing GROUP BY

We show that it is necessary to fix GROUP BY.

Lemma D.1. *Consider two single-block SQL queries Q_1 and Q_2, where Q_1 has no* GROUP BY *or aggregation, while Q_2 has* GROUP BY *and/or aggregation but no* HAVING*. Q_1 and Q_2 cannot be equivalent under bag semantics, assuming that no database constraints are present and there exists some database instance for which either Q_1 or Q_2 returns a non-empty result.*

Proof of Lemma D.1. Suppose for some instance D, both Q_1 and Q_2 return the same non-empty results. Pick any $T \in \text{Tables}(Q_1)$. Create a new instance D' by duplicating the contents of T in the same table (i.e., doubling the multiplicity of each tuple in T) while keeping all other tables unchanged. Since Q_1 has no GROUP BY or aggregation, the multiplicity of each tuple in $Q_1(D')$ will be increased by a factor of 2^c compared with that in $Q_1(D)$, where $c > 1$ is the count of T in $\text{Tables}(Q_1)$ (which is multiset). Hence, the size of $Q_1(D')$ is strictly larger than $Q_1(D)$. On the other hand, consider Q_2, which has GROUP BY and/or aggregation. Between $Q_2(D')$ and $Q_2(D)$, the grouping of intermediate join result tuples remains the same, except the number of duplicates within each group. Hence, the size of $Q_2(D')$ remains the same as $Q_2(D)$, which is the total number of groups. Therefore, $Q_1(D')$ and $Q_2(D')$ are different. □

D.2 Proof for Lemma 6.2

Proof of Lemma 6.2. **Correctness.** We prove the correctness of GROUP BY-stage by showing $\vec{o} \setminus \Delta^- \cup \Delta^+$ is equivalent to $\vec{o^\star}$. Assuming $\vec{o} \setminus \Delta^- \cup \Delta^+$ is not equivalent to $\vec{o^\star}$, then among all possible pairs of tuples, there must exist t_1, t_2 such that $\bigwedge_i (o_i[t_1] = o_i[t_2]) \not\Leftrightarrow \bigwedge_i (o^\star_i[t_1] = o^\star_i[t_2])$. This implies that $\text{IsSatisfiable}(P[t_1] \wedge P[t_2] \wedge G^\star \wedge o_i[t_1] \neq o_i[t_2])$ (line 6) and/or $\text{IsSatisfiable}(P[t_1] \wedge P[t_2] \wedge G \wedge o^\star_i[t_1] \neq o^\star_i[t_2])$ (line 11) must be satisfiable, which contradicts the fact that IsSatisfiable returns no false positive.

Strong Minimality of Δ^-. Assuming $\Delta^- \not\subseteq \Delta^-_\circ$ (i.e., there exists an $o_x \in \vec{o}$ such that $o_x \in \Delta^-$ but $o_x \notin \Delta^-_\circ$), and $\vec{o} \setminus \Delta^-_\circ \cup \Delta^+$ is equivalent to $\vec{o^\star}$. This implies that $o_x[t_1] = o_x[t_2]$ holds for all possible t_1, t_2, which contradicts the fact that p is evaluated to true (otherwise o_x should not be added to Δ^- according to the algorithm). Since IsSatisfiable does not return false positive, o_x does not exist and thus $\Delta^- \subseteq \Delta^-_\circ$.

Weak Minimality of Δ^+. Assuming $\Delta^+ = \{o^\star_x\}$ and $\vec{o} \setminus \Delta^-_\circ$ is equivalent to $\vec{o^\star}$ (denoted by $\vec{o} \setminus \Delta^-_\circ \equiv \vec{o^\star}$). Following Algorithm 4, $\Delta^- \subseteq \Delta^-_\circ$ due to strong minimality. This indicates that $\vec{o} \Rightarrow \vec{o^\star}$ (i.e. $P[t_1] \wedge P[t_2] \wedge G \Rightarrow P[t_1] \wedge P[t_2] \wedge G^\star$) but $\vec{o} \not\Leftarrow \vec{o^\star}$ (i.e. $P[t_1] \wedge P[t_2] \wedge G \not\Leftarrow P[t_1] \wedge P[t_2] \wedge G^\star$). However, $\Delta^+ = \{o^\star_x\}$ indicates that $\text{IsSatisfiable}(P[t_1] \wedge P[t_2] \wedge G \wedge o^\star_x[t_1] \neq o^\star_x[t_2])$ return true, where $o^\star_x$ is a conjunct in $G^\star$. This implies $\vec{o} \setminus \Delta^- \not\Rightarrow \vec{o^\star}$, thus further implying IsSatisfiable returns a false positive because $\vec{o} \setminus \Delta^-_\circ \equiv \vec{o^\star}$ was assumed at the beginning. Since IsSatisfiable never returns false positive, no such $o^\star_x$ can exist in Δ^+, and thus there does not exist a corresponding $\Delta^-_\circ$ such that $\vec{o} \setminus \Delta^-_\circ \equiv \vec{o^\star}$.

□

E HAVING STAGE SUPPLEMENT

We use the following base context as default for testing satisfiability for HAVING. For brevity, the following assumes all values are integers; if there are columns and literals of different domains, additional constraints will be added analogously. We note that these constraints are *not* intended to define the aggregation functions precisely; rather, they encode only a subset of their properties that allow SMT solvers to deduce useful equivalences reasonably efficiently.

Algorithm 9: FixSelect$(P, \vec{o}, \vec{o^\star})$

```
Input  : a formula $P$ and two expression lists $\vec{o}$ and $\vec{o^\star}$
Output : a pair $(\Delta^-, \Delta^+)$, where $\Delta^- \subseteq [1..\dim(\vec{o})]$ is a subset of
         indices of $\vec{o}$ and $\Delta^+ \subseteq [1..\dim(\vec{o^\star})]$ is a subset of
         indices of $\vec{o^\star}$
let $\Delta^- = \emptyset$;
let $\Delta^+ = \emptyset$;
let $n = \min(\dim(\vec{o}), \dim(\vec{o^\star}))$;
foreach $o_i \in \vec{o}, o^\star_i \in \vec{o^\star}, 1 \le i \le n$ do
    if $\mathit{IsSatisfiable}_C(o_i \neq o^\star_i)$ then
        let $\Delta^- = \Delta^- \cup \{i\}$;
        let $\Delta^+ = \Delta^+ \cup \{i\}$;
foreach $o_i \in \vec{o}, n < i \le \dim(\vec{o})$ do
    let $\Delta^- = \Delta^- \cup \{i\}$;
foreach $o^\star_i \in \vec{o^\star}, n < i \le \dim(\vec{o^\star})$ do
    let $\Delta^+ = \Delta^+ \cup \{i\}$;
return $(\Delta^-, \Delta^+)$;
```

$$C : \left\{ \begin{array}{r} \mathbf{X}, \mathbf{Y}, \mathbf{Z}, \textbf{Ones} \text{ have type } \mathsf{Array}(\mathbb{Z}) \\ i, c \text{ have type } \mathsf{Integer} \\ \mathsf{SUM}, \mathsf{AVG}, \mathsf{COUNT}, \mathsf{MAX}, \mathsf{MIN} \text{ have type } \mathsf{Array}(\mathbb{Z}) \rightarrow \mathbb{Z} \\ \forall \mathbf{X}, \mathbf{Y}, \mathbf{Z} : (\forall i : \mathbf{X}[i] + \mathbf{Y}[i] = \mathbf{Z}[i]) \Rightarrow \mathsf{SUM}(\mathbf{X}) + \mathsf{SUM}(\mathbf{Y}) = \mathsf{SUM}(\mathbf{Z}) \\ \forall \mathbf{X}, \mathbf{Y}, \mathbf{Z} : (\forall i : \mathbf{X}[i] - \mathbf{Y}[i] = \mathbf{Z}[i]) \Rightarrow \mathsf{SUM}(\mathbf{X}) - \mathsf{SUM}(\mathbf{Y}) = \mathsf{SUM}(\mathbf{Z}) \\ \forall \mathbf{X}, \mathbf{Y}, c : (\forall i : \mathbf{X}[i] \times c = \mathbf{Y}[i]) \Rightarrow \mathsf{SUM}(\mathbf{X}) \times c = \mathsf{SUM}(\mathbf{Y}) \\ \forall \mathbf{X}, \mathbf{Y}, c : (\forall i : \mathbf{X}[i] \div c = \mathbf{Y}[i]) \Rightarrow \mathsf{SUM}(\mathbf{X}) \div c = \mathsf{SUM}(\mathbf{Y}) \\ \text{// repeat the above 4 lines, replacing } \mathsf{SUM} \text{ by } \mathsf{AVG} \\ \forall i : \textbf{Ones}[i] = 1 \\ \forall \mathbf{X} : \mathsf{COUNT}(\mathbf{X}) = \mathsf{COUNT}(\textbf{Ones}) \\ \forall \mathbf{X} : \mathsf{SUM}(\mathbf{X}) = \mathsf{AVG}(\mathbf{X}) \times \mathsf{COUNT}(\textbf{Ones}) \\ \text{// above assumes no } \mathsf{NULL} \text{ values} \\ \forall \mathbf{X}, i : \mathsf{MAX}(\mathbf{X}) \ge \mathbf{X}[i] \\ \forall \mathbf{X}, i : \mathsf{MIN}(\mathbf{X}) \le \mathbf{X}[i] \end{array} \right\}$$

F SELECT STAGE SUPPLEMENT

The pseudocode for fixing SELECT is shown in Algorithm 9.

The correctness and optimality are the following:

Lemma F.1. *We say that two lists of* SELECT *expression are equivalent if they produce the same set of columns in the same ordering. Let* $(\Delta^-, \Delta^+) = \mathit{FixSelect}(P, \vec{o}, \vec{o^\star})$. *Assuming that subroutine* IsSatisfiable$_C$ *returns no false positive, we have:*

Correctness: *Qr-Hint's* SELECT*-stage hint leads to a fixed working query* Q_5 *that 1) passes the viability check* ($\vec{o}$ *and* $\vec{o^\star}$ *are equivalent); 2) satisfies* $Q_5 \equiv Q^\star$. *This applies to both* SPJ *and* SPJA *queries.*

Strong minimality of (Δ^-, Δ^+) **for *SPJ*** *Let* $(\Delta^-_\circ, \Delta^+_\circ)$ *denote the minimal* (Δ^-, Δ^+) *respectively, then for any* $(\Delta^-_\circ, \Delta^+_\circ)$ *that make* $\vec{o}$ *and* $\vec{o^\star}$ *equivalent,* $\Delta^- \subseteq \Delta^-_\circ, \Delta^+ \subseteq \Delta^+_\circ$.

Proof of Lemma F.1. **Correctness.** We prove the correctness by showing $\vec{o} \setminus \Delta^- \cup \Delta^+$ is equivalent to $\vec{o^\star}$. Since IsSatisfiable$_C$ does not return false positive, $\vec{o}[i], \vec{o^\star}[i]$ are guaranteed to be added to Δ^-, Δ^+ respectively upon "satisfiable" or "unknown", and replacing $\vec{o}[i]$ with $\vec{o^\star}[i]$ guarantees the correctness of expression on position i. In addition, for any extra expressions in Δ^+, Δ^- that do not have a counterpart in the other list, they are removed to ensure the number of expressions is the same between the SELECT of $Q^\star, Q$.

Strong minimality of (Δ^+, Δ^-) **in *SPJ* queries.** Assuming $\Delta^- \subsetneq \Delta^-_\circ$ (i.e. there exists an $o_x \in \vec{o}$ s.t. $o_x \in \Delta^-$ but $o_x \notin \Delta^-_\circ$), and $\vec{o} \setminus Delta^-_\circ \cup \Delta^+$ is equivalent to $\vec{o}$. This implies that either of the following is true: 1) o_x is redundant in Q's SELECT and needs to be removed; 2) IsSatisfiable$_C(o_x \neq o^\star_x)$ returns "satisfiable". In the former case, o_x has to be removed and must be in $\Delta^-_\circ$ (otherwise $\Delta^-_\circ$ is not correct); in the latter case, $o_x \notin \Delta^-_\circ$ implies IsSatisfiable$_C$ returns a false positive, which contradicts with our assumption. Thus $\Delta^- \subseteq \Delta^-_\circ$. $\Delta^+ \subseteq \Delta^+_\circ$ follows a similar proof. □

G EXPERIMENTS AND USER STUDY

G.1 Coverage Dataset

The coverage dataset consists of 341 real wrong queries from students from an introductory database course at our institution, and Qr-Hint can successfully fix 306 of them. The remaining 35 queries are not supported by Qr-Hint due to the existence of set operations, outer joins, etc. The corresponding questions, solutions, and causes of errors are comprehensively shown in Table 4.

The coverage of Qr-Hint for the list of semantic errors proposed by Brass et al. [12] is shown in Table 5. In summary, 25 of the 43 errors can be caught by Qr-Hint, assuming the provided reference queries do not have any of the listed errors. Most importantly, the most frequent errors reported by [12] are supported by Qr-Hint. In addition, 17 out of the 25 supported errors are reflected in the coverage dataset. Note that a lot of errors in [12] are efficiency and stylistic errors instead of logical errors (e.g., unnecessity of expressions in different clauses).

G.2 Schema and Questions in User Study Survey

The following DBLP database schemas were used for the user study, we change the table name (`inproceedings` → `conference_paper`, `article` → `journal_paper`) in order to make them more intuitive for participants:

- `conference_paper: (pubkey, title, conference_name, year, area)`
- `journal_paper: (pubkey, title, journal_name, year)`
- `authorship: (pubkey, author)`

The area attribute in the conference_paper table can only be one of the following: "ML-AI", "Theory", "Database", "Systems" or "UNKNOWN".

The questions and the correct queries for the user study are shown in Table 2. The wrong queries and hints are shown in Table 3 (Hints from teaching assistants are in black, and hints from Qr-Hint are in blue). All hints are shown in the same order as they were shown to the participants.

Question Statement	Correct Query

Q_1: Find names of the authors, such that among the years when he/she published both conference paper and journal paper, 2 of the published papers are at least 20 years apart.	SELECT i1.author
	FROM conference_paper i1, conference_paper i2, journal_paper a1, journal_paper a2, authorship au1, authorship au2, authorship au3, authorship au4
	WHERE i1.pubkey = au1.pubkey AND i2.pubkey = au2.pubkey
	AND a1.pubkey = au3.pubkey AND a2.pubkey = au4.pubkey
	AND au1.author = au2.author AND au2.author = au3.author
	AND au3.author = au4.author AND i1.year + 20 >= i2.year
	AND i1.year = a1.year AND i2.year = a2.year
	GROUP BY i1.author

Q_2: For each author who has published conference papers in the database area, find the number of their conference paper collaborators in the database area by years before 2018 (ignore the years when they have 0 collaborators). Your output should be in the format of (author, year, number of collaborators in that year).	SELECT t2.author, t1.year, COUNT(DISTINCT t3.author)
	FROM conference_paper t1, authorship t2, authorship t3
	WHERE t1.pubkey = t2.pubkey AND t3.pubkey = t1.pubkey
	AND t3.author <> t2.author AND t1.year < 2018
	AND t1.area = 'Database'
	GROUP BY t2.author, t1.year

Q_3: Excluding publications in the year of 2015, find authors who publish conference papers in at least 2 areas.	SELECT t1.author
	FROM conference_paper t1, authorship t2, conference_paper t3, authorship t4
	WHERE t1.pubkey = t2.pubkey AND t2.author = t4.author
	AND t3.pubkey = t4.pubkey AND t1.year = t3.year
	AND t1.area <> t3.area AND t1.year <> 2015
	AND t1.area <> 'UNKNOWN' AND t3.area <> 'UNKNOWN'
	GROUP BY t1.author

Q_4: Among the authors who publish in the Systems-area conferences, find the ones that have no co-authors on such publications (i.e. the author does not have any collaborator for any conference paper in systems area).	SELECT t2.author
	FROM conference_paper t1, authorship t2, authorship t3
	WHERE t1.pubkey = t2.pubkey
	AND t2.pubkey = t3.pubkey AND t1.area = 'Systems'
	GROUP BY t2.author
	HAVING COUNT(DISTINCT t3.author) <= 1

Table 2: Question statements and correct queries in the user study.

Wrong Queries	Hints
Q_1: SELECT e.author FROM conference_paper a, authorship e, conference_paper b, authorship f, journal_paper c, authorship g, journal_paper d, authorship h WHERE a.pubkey = e.pubkey AND b.pubkey = g.pubkey AND c.pubkey = f.pubkey AND e.author = h.author AND d.pubkey = h.pubkey AND e.author = g.author AND f.author = h.author AND a.year + 20 > d.year GROUP BY e.author	1. In WHERE: You should change "a.year + 20 > d.year" to some other conditions.
Q_2: SELECT a.author, year, COUNT(*) FROM conference_paper, authorship, authorship a WHERE conference_paper.pubkey = a.pubkey AND authorship.pubkey = a.pubkey AND a.author <> authorship.author AND year < 2018 GROUP BY a.author, area, year, authorship.author HAVING area = 'Database' AND conference_paper.year < 2018	1. In GROUP BY: authorship.author is incorrect. 2. In SELECT: COUNT(*) is incorrect.
Q_3: SELECT b.author FROM conference_paper, authorship b, conference_paper a, authorship WHERE conference_paper.pubkey = authorship.pubkey AND a.year < 2015 OR a.year > 2015 AND b.author = authorship.author AND a.pubkey = b.pubkey AND conference_paper.year = a.year AND a.area <> conference_paper.area AND a.area <> 'UNKNOWN' AND conference_paper.area <> 'UNKNOWN' GROUP BY b.author	1. In WHERE, try to fix the whole condition by adding a pair of parentheses - in SQL AND takes higher precedence than OR (this fix alone should make the query correct) 2. In WHERE, you are missing a pair of parentheses around a.year < 2015 OR a.year > 2015. 3. GROUP BY is incorrect. 4. GROUP BY is incorrect without an aggregate function.

Q_4:
SELECT a.author
FROM authorship, conference_paper, authorship a
WHERE conference_paper.pubkey = a.pubkey AND a.pubkey = authorship.pubkey
GROUP BY a.author, conference_paper.area
HAVING conference_paper.area = 'System' AND COUNT(DISTINCT a.author) <= 1
	1. GROUP BY should not include t1.area.
	2. In HAVING, conference_paper.area = 'System' should not appear.
	3. In HAVING, try to fix conference_paper.area = 'System' (this plus another fix in HAVING will make the query right).
	4. In HAVING, conference_paper.area = 'System' should be = 'Systems'.
	5. In HAVING, try to fix COUNT(DISTINCT a.author) <= 1 (this plus another fix in HAVING will make the query right).
	6. In HAVING, COUNT(DISTINCT a.author) <= 1 is referring to the same author attribute as the GROUP BY.

Table 3: Wrong queries and the hints provided (QR-HINT hints are in blue).

Question (a)	Question	Find the names of all beers served at James Joyce Pub.			
	Solutions	SELECT beer FROM serves WHERE bar = ’James Joyce Pub’;			
	Error Statistics		Total Wrong Query	22	
			FROM	8	1. Wrong table; 2. Extra table (cross join with bar)
			WHERE	9	Wrong bar name or typo
			SELECT	5	SELECT * or bar alone instead of beer
Question (b)	Question	Find names and addresses of bars that serve Budweiser at a price higher than 2.20.			
	Solutions	SELECT name, address FROM bar, serves WHERE bar.name = serves.bar AND beer = ’Budweiser’ AND price > 2.20;			
	Error Statistics		Total Wrong Query	126	Note: 3 of them cannot be processed due to outer-join
			FROM	10	Missing either Bar table or Serves table
			WHERE	96	1. Missing join condition; 2. Use >= instead of >
			SELECT	17	1. Missing Columns; 2. Column order is wrong
Question (c)	Question	Find the names of drinkers who like Corona and frequent James Joyce Pub at least twice a week.			

	Solutions	SELECT likes.drinker
		FROM likes, frequents
		WHERE likes.beer = ’Corona’ AND likes.drinker = frequents.drinker
		AND frequents.bar = ’James Joyce Pub’ AND frequents.times_a_week >= 2;			

	Error Statistics		Total Wrong Query	143	Note: 20 of them cannot be processed due to usage of set operation, outer joins, complex subqueries
			FROM	11	1. Wrong table involved (serves); 2. Unnecessary drinker table (false positive, true error in SELECT/WHERE)
			WHERE	105	1. Missing join condition; 2. Using > instead of >=, or wrong number; 3. Missing condition on beer or bar
			SELECT	6	SELECT * instead of name
			GROUP BY	1	GROUP BY wrong columns
Question (d)	Question	Find the name of each drinker who likes at least two beers.			
	Solution 1	SELECT drinker FROM likes GROUP BY drinker HAVING COUNT(*) >= 2;			
	Solution 2	SELECT DISTINCT l1.drinker FROM likes l1, likes l2 WHERE l1.drinker = l2.drinker AND l1.beer <> l2.beer;			
	Error Statistics		Total Wrong Query	50	Note: 12 of them cannot be processed due to usage of set operations
		Solution 1	FROM	1	Wrong table
			GROUP BY	1	Group by 1
			HAVING	18	1. Using > instead of >=; 2. COUNT(DINSTINCT *)
			SELECT	4	Extra column COUNT
		Solution 2	FROM	5	Extra/wrong tables (likes / frequents)
			WHERE	2	Wrong conditions: l1.beer = l2.beer, l1.drinker <> l2.drinker
			SELECT	7	Missing DISTINCT

Table 4: Student Query Statistics

No.	Error	Frequency in [12]	In Coverage Dataset	Qr-Hint Support
1	Inconsistent condition	11.4%	Y	Logical errors Qr-Hint correctly gives hints
3	Constant output columns	3.2%	Y	
4	Duplicate output columns		Y	
5	Unused tuple variables	5.6%	Y	
12	LIKE without wildcard			
27	Missing join conditions	21.3%	Y	
31	Comparison between different domains		Y	
33	DISTINCT in SUM and AVG			
34	Wildcards without LIKE		Y	
37	Many duplicates	10.8%	Y	
38	DISTINCT that might remove important duplicates		Y	
2	Unnecessary DISTINCT	3.7%	Y	Efficiency/Stylistic issues Qr-Hint catch them if the reference queries are free from these errors
6	Unnecessary join	8.4%	Y	
7	Tuple variables are always identical	3.2%		
15	Unnecessary aggregation function			
16	Unnecessary DISTINCT in aggregation function		Y	
17	Unnecessary argument of COUNT		Y	
19	GROUP BY with singleton group	4.4%		
20	GROUP BY with only a single group			
22	GROUP BY can be replaced by DISTINCT		Y	
24	Unnecessary ORDER BY term	10.8%		
32	Strange HAVING			
8	Implied, tautological, or inconsistent subcondition	5.4%	Y	Efficiency/Stylistic issues Qr-Hint does not consider them as errors
21	Unnecessary GROUP BY attribute		Y	
25	Inefficient HAVING		Y	
9	Comparison with NULL			Not supported by Qr-Hint
10	NULL value in IN/ANY/ALL subquery			
11	Unnecessarily general comparison operator			
13	Unnecessarily complicated SELECT in EXISTS-subquery			
14	IN/EXISTS condition can be replaced by comparison			
18	Unnecessary GROUP BY in EXISTS subquery			
23	UNION can be replaced by OR			
26	Inefficient UNION			
28	Uncorrelated EXISTS subquery			
29	IN-Subquery with only one possible result value			
30	Condition in the subquery that can be moved up			
35	Condition on left table in left outer join condition			
36	Outer join can be replaced by inner join			
39	Subquery term that might return more than one tuple			
40	SELECT INTO that might return more than one tuple			
41	No indicator variable for argument that might be NULL			
42	Difficult type conversion			
43	Runtime error in datatype function. e.g. divided by 0			

Table 5: List of Semantic Errors categorized by Qr-Hint