

# A Novel Surrogate Polytope Method for Day-ahead Virtual Power Plant Scheduling with Joint Probabilistic Constraints

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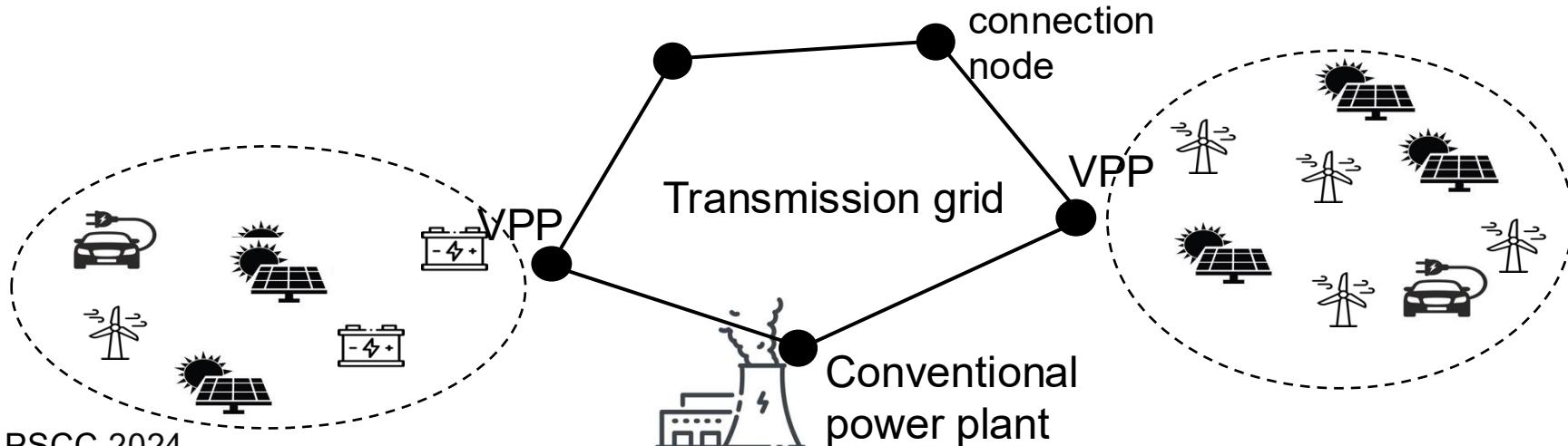
# Background

There is an increasing number of distributed energy resources (DERs) in the context of the worldwide decarbonization.

These DERs contain valuable power flexibility that can be and need to be exploited.

The power size of individual DERs is too small, and directly scheduling those numerous DERs is computationally hard.

Aggregating DERs into a virtual power plant (VPP) can improve the operational efficiency and visibility.



# Background

VPPs can be divided into:

1. Commercial VPPs (CVPP): focusing on financial activities, and DERs are not restricted to a local area; no network constraints considered.
2. Technical VPPs (TVPP): focusing on DERs in a local area and considering the distribution network constraints.

The presented paper focusses on TVPPs.



# Research question

In a standard market engagement process [1], a VPP schedules the aggregated power (VPP power) for the next day; in real-time operation, this VPP needs to fulfil the scheduled aggregated power by adjusting its managed DERs.

But many DERs' power capability is **uncertain** at the day-ahead (DA) phase...

Reliably scheduling a VPP becomes crucial (securing the VPP power availability with a high probability), especially for safety-critical applications.

*Research question: how to reliably schedule the VPP power at the DA phase?*



# Research gap

Chance constraints (CCs) provide such probability guarantee. Existing CC methods apply CCs to constraints individually (ICCs) for computational tractability. However,

- 1) Power system applications may require that constraints are satisfied jointly (JCC), which **may not be guaranteed by standard ICCs**.
- 2) Although one can use Bonferroni approximation-based ICC to approximate the JCC by setting a much lower violation probability, **the result could be overly conservative again**, especially the probability is set as  $\varepsilon/N$ .



# Contribution

- 1) This paper proposed a tractable JCC approach to schedule the VPP power at the day-ahead phase, which is more reliable than the normal ICC and less conservative than the Bonferroni-based ICCs.
- 2) A lower-order surrogate polytope is used to approximate the VPP dispatch region, which 1) leads to a special structure for an exact convex JCC reformulation and 2) reduces the complexity of solving the JCC.
- 3) Verified by extensive case studies using real-world scenarios.



# Problem formulation – DER modelling

The VPP operational constraints are composed of:

## 1) DER constraints

### Renewable generators

$$\forall t \in [T], \forall i_{\text{RG}} \in [N^{\text{RG}}],$$

$$0 \leq p_{t,i_{\text{RG}}}^{\text{RG},\psi} \leq \bar{p}_{t,i_{\text{RG}}}^{\text{RG},\psi}$$

$$(p_{t,i_{\text{RG}}}^{\text{RG},\psi}, q_{t,i_{\text{RG}}}^{\text{RG},\psi}) \in \mathcal{PQ}_{t,i_{\text{RG}}}^{\text{RG},\psi}$$

### Controllable loads

$$\forall t \in [T], \forall i_{\text{CL}} \in [N^{\text{CL}}],$$

$$\sigma_{\min} \tilde{p}_{t,i_{\text{CL}}}^{\text{CL},\psi} \leq p_{t,i_{\text{CL}}}^{\text{CL},\psi} \leq \tilde{p}_{t,i_{\text{CL}}}^{\text{CL},\psi}$$

$$q_{t,i_{\text{CL}}}^{\text{CL},\psi} = \alpha \cdot p_{t,i_{\text{CL}}}^{\text{CL},\psi}$$

### Energy storages

$$\forall t \in [T], \forall i_{\text{ES}} \in [N^{\text{ES}}]$$

$$(1) \quad p_{t,i_{\text{ES}}}^{\text{ES},\psi} = \hat{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} + \check{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} \quad (3)$$

$$(2) \quad \underline{p}_{i_{\text{ES}}}^{\text{ES},\psi} \leq \check{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} \leq 0 \quad (4)$$

$$0 \leq \hat{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} \leq \bar{p}_{i_{\text{ES}}}^{\text{ES},\psi} \quad (5)$$

$$e_{t,i_{\text{ES}}} = e_{t-1,i_{\text{ES}}} + \sum_{\psi} (\hat{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} \hat{\eta} + \check{p}_{t,i_{\text{ES}}}^{\text{ES},\psi} / \check{\eta}) \cdot \Delta t \quad (6)$$

$$(7) \quad \underline{e}_{i_{\text{ES}}} \leq e_{t,i_{\text{ES}}} \leq \bar{e}_{i_{\text{ES}}}$$

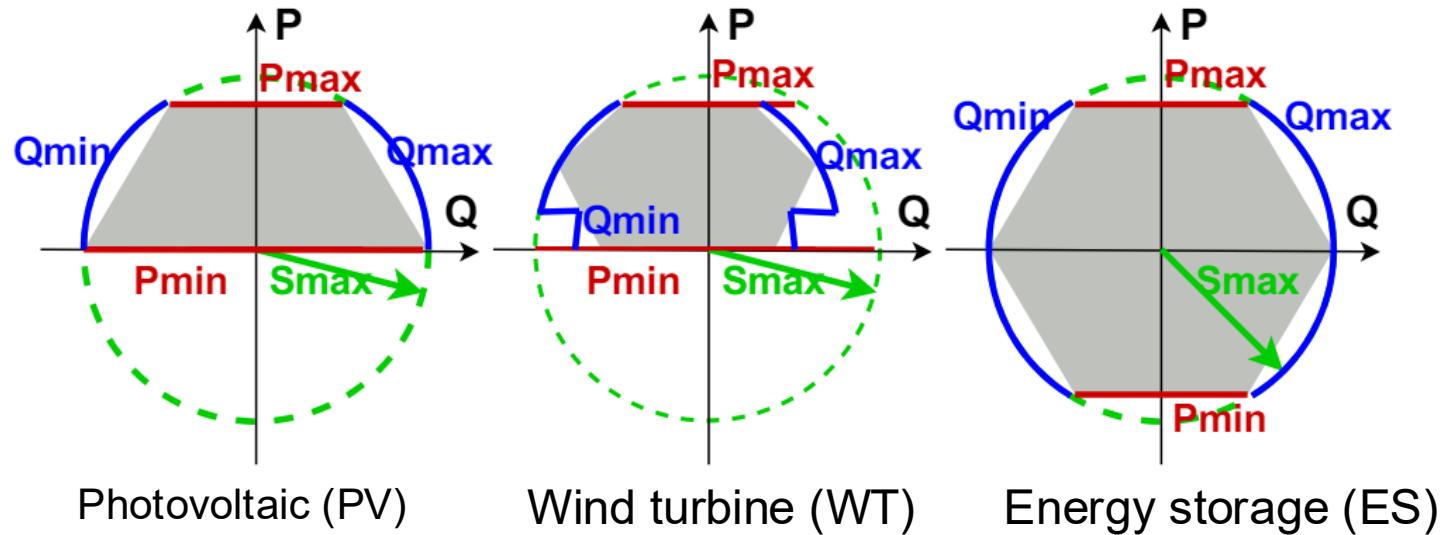
$$(10) \quad (p_{t,i_{\text{ES}}}^{\text{ES},\psi}, q_{t,i_{\text{ES}}}^{\text{ES},\psi}) \in \mathcal{PQ}_{t,i_{\text{ES}}}^{\text{ES},\psi} \quad (8)$$



# Problem formulation – DER modelling

The VPP operational constraints are composed of:

## 1) DER constraints (PQ capability chart)



# Problem formulation – Network modelling

The VPP operational constraints are composed of:

## 2) network constraints and VPP-DER power coupling

Vector representation of the DER power and non-controllable loads (NLs)

$$\boldsymbol{x}_t := [p_{t,i_d}^{d,\psi}, q_{t,i_d}^{d,\psi}, \tilde{p}_{t,i_{\text{NL}}}^{\text{NL},\psi}, \alpha \tilde{p}_{t,i_{\text{NL}}}^{\text{NL},\psi}] \quad (11)$$

$$i_d \in [N^d], d \in \{\text{RG, ES, CL}\}, i_{\text{NL}} \in [N^{\text{NL}}], \psi \in \Psi$$

Linear three-phase unbalanced distribution network model

$$\forall t \in [T], \quad \boldsymbol{i}_t = \boldsymbol{J}_t \boldsymbol{x}_t + \boldsymbol{j}_t^0 \quad (12) \quad \text{Line currents}$$

$$\boldsymbol{v}_t = \boldsymbol{K}_t \boldsymbol{x}_t + \boldsymbol{k}_t^0 \quad (13) \quad \text{Nodal voltages}$$

$$p_t^{\text{VPP}} = \boldsymbol{g}_t^\top \boldsymbol{x}_t + g_t^0 \quad (14) \quad \text{VPP-DER power coupling}$$

$$\boldsymbol{i}_{\min} \leq \boldsymbol{i}_t \leq \boldsymbol{i}_{\max} \quad (15) \quad \text{Thermal limits}$$

$$\boldsymbol{v}_{\min} \leq \boldsymbol{v}_t \leq \boldsymbol{v}_{\max} \quad (16) \quad \text{Voltage limits}$$



# Problem formulation – Day-ahead VPP scheduling

When there is no uncertainty, the VPP scheduling problem can be written as:

$$\begin{aligned} \min \quad & -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \mathcal{C}(\mathbf{x}) \\ \text{s.t.} \quad & (1) - (16) \end{aligned}$$

The objective is to minimise the VPP operational cost, consisting of:

- 1) The VPP revenue by engaging on the day-ahead market:

$$\mathcal{J}(\mathbf{p}^{\text{VPP}}) = -\Delta t \sum_t \pi_t^{\text{DA}} p_t^{\text{VPP}}$$

- 2) The DER operational cost:

$$\begin{aligned} \mathcal{C}(\mathbf{x}) = & \omega^{\text{ES}} \Delta t \sum_{t,\psi,i^{\text{CL}}} (\hat{p}_{t,i^{\text{ES}}}^{\text{ES},\psi} - \check{p}_{t,i^{\text{ES}}}^{\text{ES},\psi}) \\ & + \omega^{\text{CL}} \Delta t \sum_{t,\psi,i^{\text{CL}}} (\tilde{p}_{t,i^{\text{CL}}}^{\text{CL},\psi} - p_{t,i^{\text{CL}}}^{\text{CL},\psi}) \end{aligned}$$



# Problem formulation – Day-ahead VPP scheduling (uncertainty)

The uncertain DERs make the VPP power capability at the DA phase uncertain, so the system operator wants to secure the VPP power that is implementable with a high probability:

$$\begin{aligned} \min \quad & -\mathcal{J}(p^{\text{VPP}}) + \mathcal{C}(x) \\ \text{s.t.} \quad & \mathbb{P}(p^{\text{VPP}} \text{ is implementable}) \geq 1 - \epsilon \end{aligned}$$

Note that the uncertain parameters can also affect the objective function, motivating the minimization of expectation or worst-case cost considering multi-stage recourse actions. The key focus here is the power availability.



# Problem formulation – Day-ahead VPP scheduling (uncertainty)

As the VPP power is defined through those individual DERs, an direct approach would be:

$$\begin{aligned} \min \quad & -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \mathcal{C}(\mathbf{x}) \\ \text{s.t.} \quad & \mathbb{P}((1), (9), (15), (16)) \geq 1 - \epsilon, \\ & (2), (3) - (8), (10) \end{aligned}$$

However, this formulation is hard to solve because:

- 1) The JCC may be huge in size, which hinders the tractability of reformulation.
- 2) The distribution of some DER uncertain parameters may be hard to predict at the DA phase (\*36 hours-ahead forecasting).

If normal ICCs are adopted, the scheduled VPP power may be less reliable;

If Bonferroni approximation is used, the scheduling may be overly conservative.



# Solve the JCC – surrogate polytope

The decision to be made at the DA stage is the VPP power, while the specific DER power can be dispatched in the real-world operation process with the gradual realisation of uncertain DERs' outputs. This motivates us to isolate the VPP power from the low-level DER power for the DA scheduling.

The VPP power availability can be expressed as:

$$\Omega = \{p^{\text{VPP}} \mid \exists x, (1) - (16) \text{ (i.e., } p^{\text{VPP}} \text{ is implementable)}\}$$

which can be inner-approximated by **a lower-order surrogate polytope with a preselected shape  $A$  and tuneable intercepts  $b$** :

$$\exists \Omega^P \subseteq \Omega, \quad \Omega^P = \{p^{\text{VPP}} \mid Ap^{\text{VPP}} \leq b\}$$

One typical (but not restricted to it) example is the virtual battery (VB):

$$\begin{aligned} \Omega^{\text{VB}} &= \{p^{\text{VPP}} \mid A^{\text{VB}} p^{\text{VPP}} \leq b^{\text{VB}}\} \\ A^{\text{VB}} &= [\mathbf{I}, -\mathbf{I}, \boldsymbol{\Lambda}, -\boldsymbol{\Lambda}]^\top \\ b^{\text{VB}} &= [\bar{p}_{t=1}^{\text{VB}}, \dots, \bar{p}_{t=T}^{\text{VB}}, -\underline{p}_{t=1}^{\text{VB}}, \dots, -\underline{p}_{t=T}^{\text{VB}}, \\ &\quad \bar{e}_{t=1}^{\text{VB}}, \dots, \bar{e}_{t=T}^{\text{VB}}, -\underline{e}_{t=1}^{\text{VB}}, \dots, -\underline{e}_{t=T}^{\text{VB}}] \end{aligned} \quad \boldsymbol{\Lambda} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{T \times T}$$



# Solve the JCC – surrogate polytope-based JCC

The constraint of the DA VPP scheduling under uncertainty

$$\begin{aligned} \min \quad & -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \mathcal{C}(\mathbf{x}) \\ \text{s.t.} \quad & \mathbb{P}(\mathbf{p}^{\text{VPP}} \text{ is implementable}) \geq 1 - \epsilon \end{aligned}$$

can **therefore be approximated** by the following form as (26):

$$\mathbb{P}(\mathbf{A}\mathbf{p}^{\text{VPP}} \leq \mathbf{b}) \geq 1 - \epsilon \Leftrightarrow F_{-\mathbf{b}}(-\mathbf{A}\mathbf{p}^{\text{VPP}}) \geq 1 - \epsilon \quad (26)$$

The equivalence to the right-hand side holds because  $\mathbf{A}$  is pre-selected which means  $\mathbf{b}$  is the only random part.  $F_{-\mathbf{b}}$  is the CDF function for the random vector  $-\mathbf{b} \in \mathbb{R}^{4T}$ .

Furthermore, (26) is **an inner approximation** because:

$$\begin{aligned} F_{-\mathbf{b}}(-\mathbf{A}\hat{\mathbf{p}}^{\text{VPP}}) \geq 1 - \epsilon &\Leftrightarrow \mathbb{P}(\mathbf{A}\hat{\mathbf{p}}^{\text{VPP}} \leq \mathbf{b}) \geq 1 - \epsilon \\ &\Leftrightarrow \mathbb{P}(\hat{\mathbf{p}}^{\text{VPP}} \in \Omega^{\text{P}}) \geq 1 - \epsilon \\ &\Rightarrow \mathbb{P}(\hat{\mathbf{p}}^{\text{VPP}} \in \Omega) \geq 1 - \epsilon \\ &\Rightarrow \mathbb{P}(\mathbf{p}^{\text{VPP}} \text{ is implementable}) \geq 1 - \epsilon \end{aligned}$$



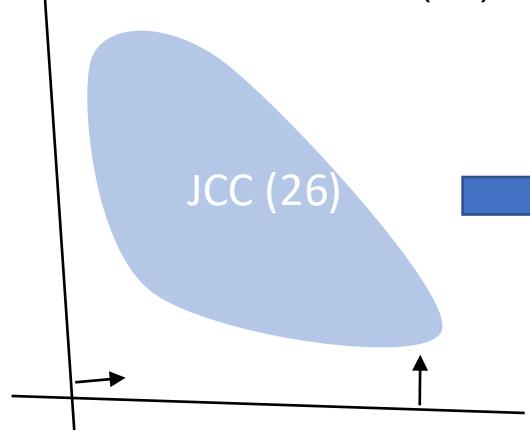
# Solving the surrogate-polytope JCC

If we model the random vector  $\mathbf{b}$  by a log-concave distribution, e.g., multivariate Gaussian, we have:

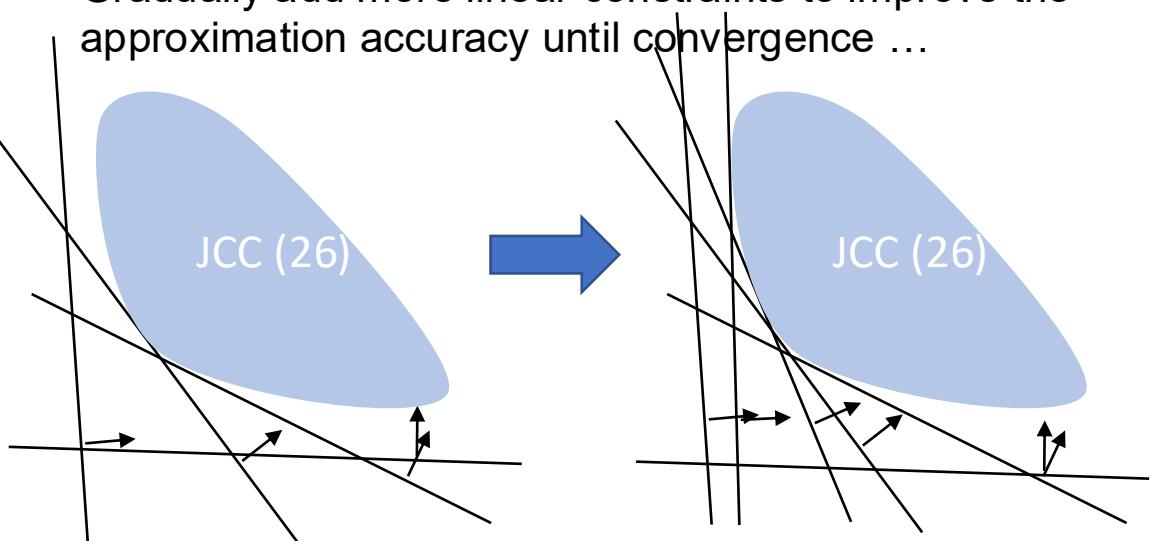
*Theorem 10.2.1 [2]: the CDF function  $F_{-\mathbf{b}}$  in (26) is log-concave, and subsequently (26) is a convex constraint.*

We can exactly solve (26) through the supporting-hyperplane method:

Start with an unreliable linear approximation of the JCC (26)



Gradually add more linear constraints to improve the approximation accuracy until convergence ...



# Solving the surrogate-polytope JCC

If we model the random vector  $\mathbf{b}$  by a log-concave distribution, e.g., multivariate Gaussian, we have:

*Theorem 10.2.1 [2]: the CDF function  $F_{-\mathbf{b}}$  in (26) is log-concave, and subsequently (26) is a convex constraint.*

Finally, the DA VPP scheduling problem with the JCC (26) becomes:

$$\begin{aligned} \min \quad & -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \mathcal{C}(\mathbf{x}) \\ \text{s.t.} \quad & F_{-\mathbf{b}}(-A\mathbf{p}^{\text{VPP}}) \geq 1 - \epsilon \quad (26) \end{aligned}$$



$$\begin{aligned} \min \quad & -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \boxed{\mathcal{C}(\mathbf{x})} \\ \text{s.t.} \quad & \mathcal{K}^M \end{aligned} \quad \mathcal{K}^M = \left\{ \begin{array}{l} \mathbf{a}_i^\top \mathbf{p}^{\text{VPP}} \leq F_{b_i}^{-1}(\epsilon), \forall i \in [\text{Dim}(\mathbf{b})], \\ \nabla_{p^{\text{agg}}} F_{-\mathbf{b}}(-A\mathbf{p}^{\text{VPP},Bj})^\top (\mathbf{p}^{\text{VPP}} - \mathbf{p}^{\text{VPP},Bj}) \geq 0, \\ \forall j \in [M] \end{array} \right\}$$

But the DER power  $\mathbf{x}$  is excluded from  $\mathcal{K}^M$   
How to evaluate  $\mathcal{C}(\mathbf{x})$  ?



# Aggregated cost mapping

For the DER cost  $\mathcal{C}(x)$ , we need a reasonable cost mapping (followed by [3]):

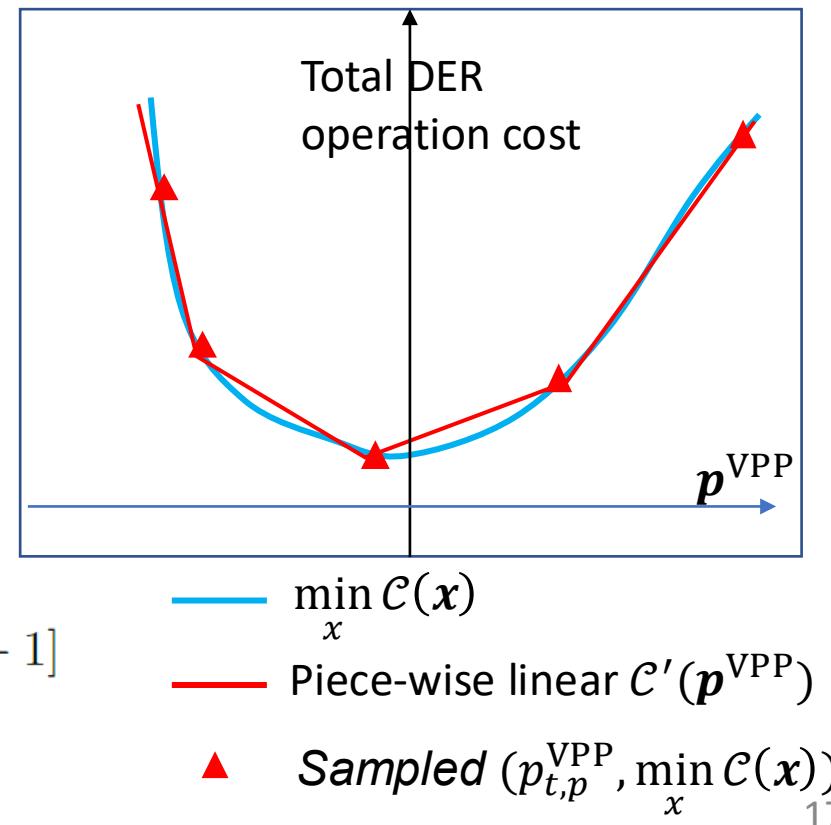
*For a given  $x \rightarrow p^{\text{VPP}}$ , we map the minimised total DER cost  $\min_x \mathcal{C}(x) \rightarrow \mathcal{C}'(p^{\text{VPP}})$*

A piece-wise linear approximation is used to get the aggregated DER cost mapping.  
A set of points are sampled:

$$\begin{aligned} C'_{t,p} &= \min_{x, p_t^{\text{VPP}}} \mathcal{C}(x) \\ \text{s.t. } & (1) - (16), \\ & p_t^{\text{VPP}} = p_{t,p} \end{aligned}$$

Based on these samples, we can fit the coefficients  $a_{t,p}$  and  $b_{t,p}$ . The fitted **piece-wise linear approximation** can be expressed as:

$$\begin{aligned} \mathcal{C}'(p^{\text{VPP}}) &= \min_{c_t} \sum_{t \in [T]} c_t \\ \text{s.t. } & a_{t,p} p_t^{\text{VPP}} + b_{t,p} \leq c_t, \quad p \in [P-1] \end{aligned}$$



# Aggregated cost mapping

Finally, the DA VPP scheduling becomes:

$$\min -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \mathcal{C}(\mathbf{x})$$

$$\text{s.t. } \mathbb{P}(\mathbf{p}^{\text{VPP}} \text{ is implementable}) \geq 1 - \epsilon$$



$$\min_{\mathbf{p}^{\text{VPP}}} -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \boxed{\mathcal{C}'(\mathbf{p}^{\text{VPP}})} \quad (33a)$$

$$\text{s.t. } \boxed{\mathcal{K}^M} \quad (33b)$$



$$\min_{\mathbf{p}^{\text{VPP}}, c_t} -\mathcal{J}(\mathbf{p}^{\text{VPP}}) + \sum_{t \in [T]} c_t$$

$$\text{s.t. } \mathcal{K}^M,$$

$$a_{t,p} p_t^{\text{VPP}} + b_{t,p} \leq c_t, \quad p \in [P-1]$$



# DER power dispatch

After determining the scheduled VPP power  $p^{\text{VPP}}$  at the DA phase, we need to retrieve the DER power  $x$  in real-time operation. The formulation is similar to the VPP DA scheduling, with the decision variable  $p^{\text{VPP}}$  replaced with the DA scheduling  $p^{\text{VPP}*}$ .

$$\min_{\boldsymbol{x}} \quad -\mathcal{J}(\boldsymbol{p}^{\text{VPP}*}) + \mathcal{C}(\boldsymbol{x}) \quad (39\text{a})$$

$$\text{s.t.} \quad (1) - (16) \quad (39\text{b})$$

\*Note that, in the real-world VPP dispatch process, the uncertainty still exists (although gradually revealed). Here, we do not address the disaggregation process under uncertainty, which can be explored in future work.



## Remarks

*Remark 1: Other log-concave functions can be applied as well, such as log-concave Gaussian mixtures, whose derivative can be calculated as well.*

*Remark 2: The supporting hyper-plane method requires a non-degenerate distribution, which can be ensured by adding  $\epsilon I$  to the covariance matrix, with  $\epsilon$  being a small positive number.*

*Remark 3: To hedge the ambiguity towards the exact form of distributions, especially under limited historical data, our recent paper showed that the JCC can be inner-approximated by the Wasserstein distributionally robust JCC (WDRCJJ), which is exactly-reformulated as a tractable set of MILP constraints:*

*Zhou, Yihong, Chaimaa Essayeh, and Thomas Morstyn. "Aggregated Feasible Active Power Region for Distributed Energy Resources with a Distributionally Robust Joint Probabilistic Guarantee." IEEE Transactions on Power Systems (2024).*



# Case study settings

We consider an IEEE-123 distribution network.

We consider three network settings with  
1) low renewable penetration (LowRG);  
2) high solar penetration (HighPV), and  
3) high wind penetration (HighWT).

We use real-world solar and wind forecasting error scenarios, which are non-Gaussian.  
The network loads are also uncertain and are synthesized from Gaussian distributions.

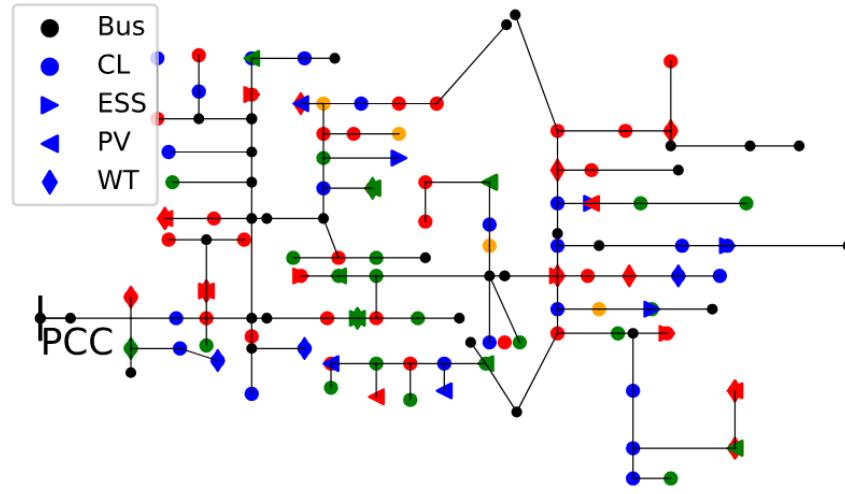


Figure 3. DER locations on IEEE-123 distribution network: red for phase a or ab, green for phase b or bc, and blue for the remaining phases.

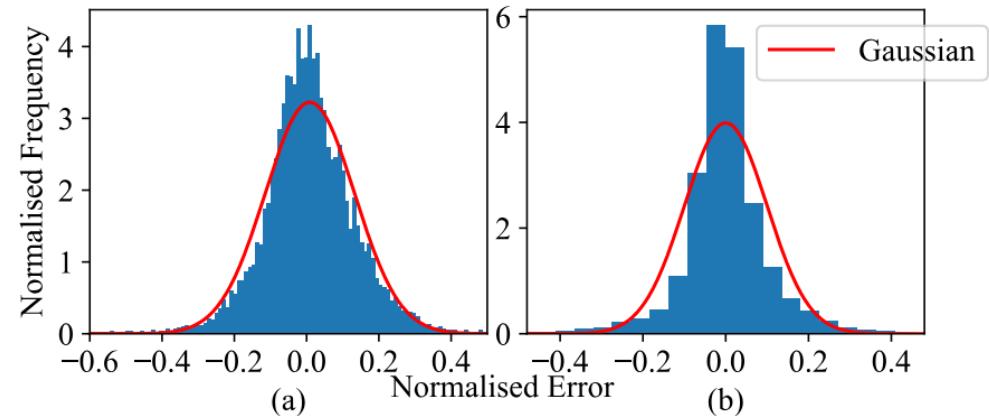


Figure 4. Histogram of the real-world (a) wind and (b) solar forecasting error.



# Evaluation metrics

We consider the following two metrics:

- 1) Out-of-sample reliability: The proportion of scenarios where the DA VPP schedule  $p^{\text{VPP}*}$  are successfully dispatched to DERs without violating DER constraints and network constraints (checked by the non-linear Z-Bus power flow simulation)
- 2) Out-of-sample reliability: The average cost (DER operation cost minus VPP revenue) over the successfully dispatched out-of-sample scenarios.

Benchmarks are based on:

$$\begin{aligned} \min \quad & -\mathcal{J}(p^{\text{VPP}}) + \mathcal{C}(x) \\ \text{s.t.} \quad & \mathbb{P}((1), (9), (15), (16)) \geq 1 - \epsilon, \end{aligned}$$

with two ways to replace the JCC: (2), (3) – (8), (10)

- 1) (**B1**) Standard individual chance constraints (ICCs) with a risk level  $\epsilon$  the same as our JCC approach;
- 2) (**B2**) Bonferroni-based ICCs (ICCs with  $\epsilon/N$ ), with  $N$  being the number of the ICCs.



# Out-of-sample reliability

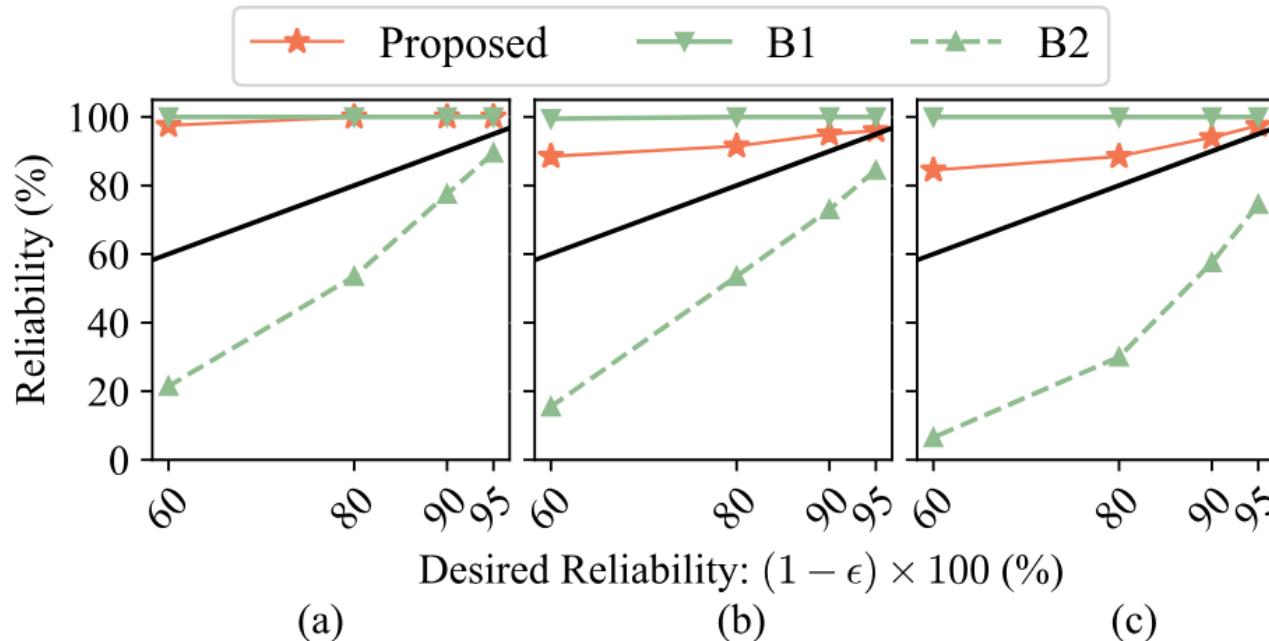


Figure 5. Out-of-sample reliability evaluation for (a) ‘LowRG’, (b) ‘HighPV’, and (c) ‘HighWT’. The black lines indicate the desired reliability levels.

We see that both our proposed method and B1 can meet the reliability requirements for all the case studies, but B2 cannot. This result suggests that ICC (B2) can lead to unreliable solutions, highlighting the need for JCC.



# Out-of-sample cost

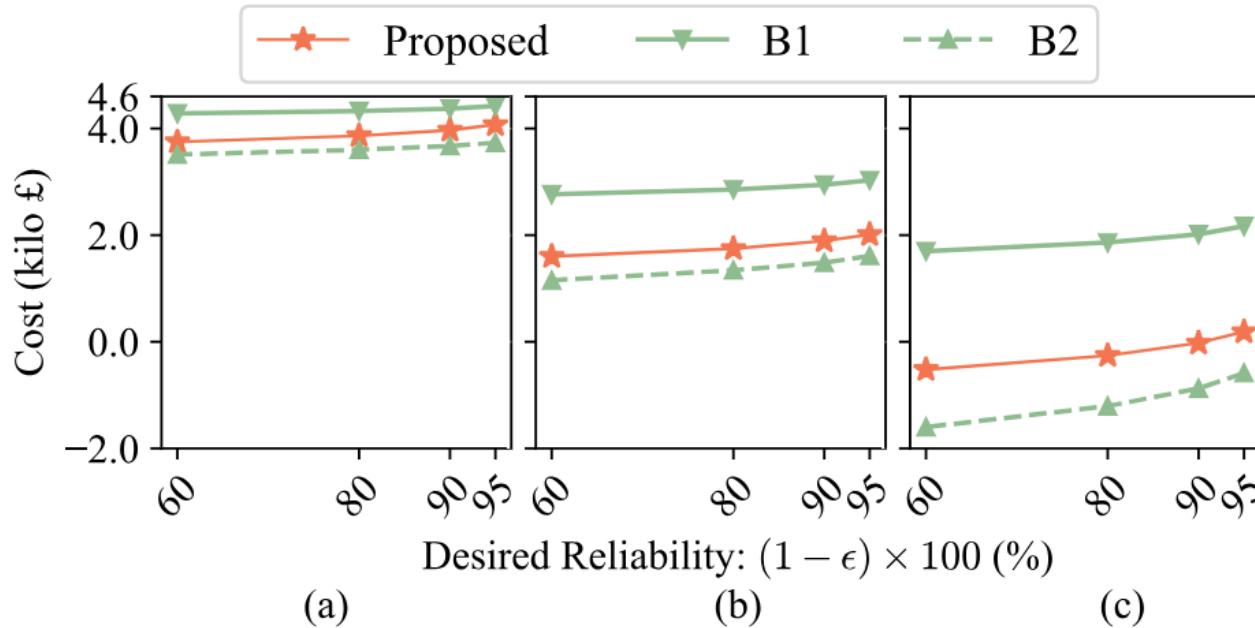


Figure 6. Out-of-sample DA cost evaluation for (a) ‘LowRG’, (b) ‘HighPV’, and (c) ‘HighWT’.

While B2 achieves the lowest cost for all cases, its unreliability compromises its suitability. Our proposed method, being the second lowest in terms of cost, closely approaches the lowest costs achieved by B2. **We conclude that our proposed method not only offers reliable solutions but also exhibits less conservativeness.**



# Scalability test

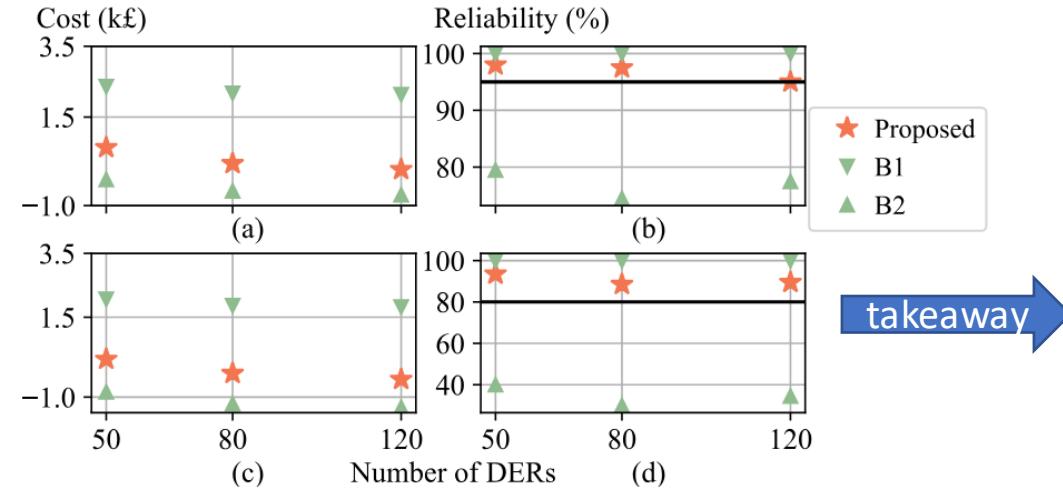


Figure 7. First column: out-of-sample cost under different numbers of DERs with desired reliability levels (a)  $1 - \epsilon = 0.95$  and (c)  $1 - \epsilon = 0.8$ ; Second column: out-of-sample reliability with desired reliability levels (b)  $1 - \epsilon = 0.95$  and (d)  $1 - \epsilon = 0.8$ , where the black horizontal lines indicate the desired reliability levels.

The previous conclusion holds for various numbers of DERs: our proposed method is reliable and less conservative

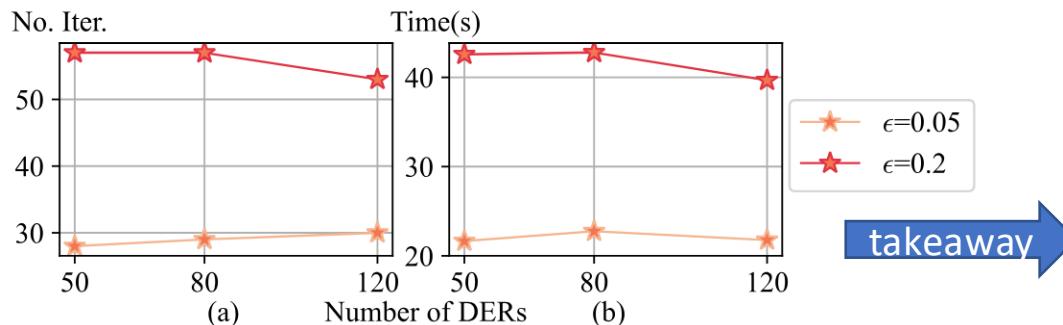


Figure 8. (a) The number of iterations (No. Iter.) in applying the proposed supporting hyperplane algorithm to solve the proposed surrogate JCC under different numbers of DERs; (b) The computing time in applying the proposed supporting hyperplane algorithm to solve the proposed surrogate JCC.

Increasing the number of DERs may increase the problem complexity, but the complexity of the supporting hyperplane method is independent of it.

# Limitations and future work

The following can be further explored:

- 1) Investigating the disaggregation process subject to gradually revealed uncertainty, given the VPP power that is available with  $1 - \varepsilon$  joint probability scheduled by our proposed method;
- 2) Comparison and synergy with other data-driven learning-based methods, such as reinforcement learning.



# References

- [1] Liu, H., Qiu, J. and Zhao, J., 2022. A data-driven scheduling model of virtual power plant using Wasserstein distributionally robust optimization. *International Journal of Electrical Power & Energy Systems*, 137, p.107801.
- [2] A. Prékopa, Stochastic programming. Springer Science & Business Media, 2013, vol. 324.
- [3] S. Wang, W. Wu, Q. Chen, J. Yu, and P. Wang, “Stochastic flexibility evaluation for virtual power plant by aggregating distributed energy resources,” CSEE Journal of Power and Energy Systems, 2022.



**Thank you!  
Questions?**

