Solutions to Chapter 2 Exercises

UNSOLVED EXERCISES

U1. Determine which of the following situations describe games and which describe decisions. In each case, indicate what specific features of the situation caused you to classify it as you did.

(a) A party nominee for president of the United States must choose whether to use private financing or public financing for her campaign.

- This can be considered as a game. Reason for the same is the option of financing may delay or help the candidate run against her opponent.
- For example, if her opponent had devoted to public finance. Therefore, the public viewed non-public finance as serving special interests, and then she may need to settle on public funding to avoid the looks of serving special interests.

(b) Frugal Fred receives a $20 gift card for downloadable music and must choose whether to purchase individual songs or whole albums.

- Fred is trying to optimize his purchase of songs by determining whether he would get more enjoyment out of purchasing songs, albums, or a mixture. However, these are only decisions, because they do not affect other individuals.

(c) Beautiful Belle receives 100 replies to her online dating profile and must choose whether to reply to each of them.

- One might be tempted to consider this to be merely a decision on Belle's part, unless you consider the suitors to be players in the game. Because Belle will presumably end up dating only a small number of the 100 suitors, the suitors' payoffs will be affected by Belle's choices. Furthermore, Belle may wish to be strategic in deciding how many to reply to, and when to respond to each. If she replies to several suitors early on, and turns out not to like any of them on a first date she may wish to return to others. However, the delay may cause the other suitors to lose interest.

(d) NBC chooses how to distribute its television shows online this season. The executives consider Amazon.com, iTunes, and/or NBC. com. The fee they might pay to Amazon or to iTunes is open to negotiation.

- This is a game between NBC and the potential distributors, in which the distributors may compete on price, and NBC may also choose to go it alone. For example, NBC initially distributed shows for a fee via Apple's iTunes Store. They simultaneously developed the NBC.com Web site in order to distribute shows with commercials included. In 2007, NBC balked at Apple's price and withdrew its shows from iTunes.

(e) China chooses a level of tariffs to apply to American imports.

- This is a game, because China's outcome is directly affected by how the United States responds to imports on its goods. A large Chinese tariff on U.S. goods could start a trade war, in which the United States reacts by placing a similar import tariff on Chinese goods.

U2. Consider the strategic games described below. In each case, state how you would classify the game according to the six dimensions outlined in the text. (i) Are moves sequential or simultaneous? (ii) Is the game zerosum or not? (iii) Is the game repeated? (iv) Is there imperfect information, and if so, is there incomplete (asymmetric) information? (v) Are the rules fxed or not? (vi) Are cooperative agreements possible or not? If you do not have enough information to classify a game in a particular dimension, explain why not.

(a) Garry and Ross are sales representatives for the same company.
Their manager informs them that of the two of them, whoever sells more this year wins a Cadillac.

(a)

(i)

Simultaneous, because each day the two sales representatives privately decide how hard to work that day. We could also model the game as each sales representative deciding how hard she will work to win the Cadillac right after the statement is given.

(ii)

The game is not zero-sum.

(iii)

Depending on how one models the game, it may or may not be repeated. See part (i) for options.

(iv)

There is imperfect information, because neither sales representative knows how hard the other will work.

(v)

The rules are fixed.

(vi)

Cooperative agreements are not possible, because each representative has an incentive to cheat.

(b) On the game show *The Price Is Right*, four contestants are asked to guess the price of a television set. Play starts with the leftmost player, and each player's guess must be different from the guesses of the previous players. The person who comes closest to the real price, without going over it, wins the television set.

(b)

(i)

Sequential, because the contestants are asked one at a time

(ii)

The game is not zero-sum.

(iii) The game is repeated for the losers, but not for the winner.

(iv)

This is a game of imperfect information for the first three contestants, because they do not know what the subsequent contestants will bid, but there is perfect information for the final contestant, who heard the first three bids. This is also a game of incomplete information, because no contestant can be certain of the product's price.

(v)

The rules are fixed.

(vi)

Cooperative agreements are not possible.

(c) Six thousand players each pay $10,000 to enter the World Series of Poker. Each starts the tournament with $10,000 in chips, and they play No-Limit Texas Hold 'Em (a type of poker) until someone wins all the chips. The top 600 players each receive prize money according to the order of finish, with the winner receiving more than $8,000,000.

(c)

(i)

Each hand of poker is a distinct simultaneous game, but the tournament requires repeated hands, so it is also sequential.

(ii)

The game is zero-sum.

(iii)

Although the tournament is not repeated, the individual hands are, so this game may be considered a repeated, simultaneous game.

(iv)

There is imperfect and incomplete (asymmetric) information: no player knows the cards that the other players hold.

(v)

The rules are fixed.

(vi)

Cooperative agreements are possible: there are multiple prizes, and a subset of players could collude to defeat other players. While possible, cooperation through collusion is forbidden in poker and results in expulsion from the game if discovered.

(d) Passengers on Desert Airlines are not assigned seats; passengers choose seats once they board. The airline assigns the order of boarding according to the time the passenger checks in, either on the Web site up to 24 hours before takeoff or in person at the airport.

(d)

(i)

Passengers do not know when others check in, so it is simultaneous.

(ii)

The game is zero-sum, because any seat I have, you cannot have.

(iii)

The game is repeated for frequent flyers.

(iv)

There is imperfect information about the actions of other flyers.

(v)

The rules are fixed and enforced by the airlines.

(vi)

One possible cooperative agreement would be for two frequent flyers to agree to alternate between the best seats they could obtain.

U3. "Any gain by the winner must harm the loser." Is this statement true or false? Explain your reasoning in one or two sentences.

This is only true for constant-sum games. There are numerous games in which both can win, for example, two investors purchasing stock in the same company can both win as the stock price increases. Two students working on a group project must choose between working and letting the other person finish the project.

If they both work, then the project will be of high quality, and both students will learn more and receive a higher grade. Finally, even though an employer may give up marginal profit to increase employee pay, the overall sales may dramatically increase as employees respond to incentives, thus increasing the employer's profit.

U4. Alice, Bob, and Confucius are bored during recess, so they decide to play a new game. Each of them puts a dollar in the pot, and each tosses a quarter. Alice wins if the coins land all heads or all tails. Bob wins if two heads and one tail land, and Confucius wins if one head and two tails land. The quarters are fair, and the winner receives a net payment of $2 ($3 2 $1 5 $2), and the losers lose their $1.

(a) What is the probability that Alice will win and the probability that she will lose?

Before answering the sub questions, it helps to calculate the probability of each player winning or losing. The probability of winning and losing for Confucius is identical to the probability for Bob.

(a)

For Alice, the probability of having all heads is $0.5 \cdot 0.5 \cdot 0.5 = 0.125$, and the probability of all tails is the same, so Alice will win with probability 0.25, and accordingly lose with probability 0.75.

(b) What is Alice's expected payoff?

(b)

Alice's expected payoff is

$$0.25 \times 2 + 0.75 \times -1 = -0.25$$

(c) What is the probability that Confucius will win and the probability that he will lose?

(c)

For Confucius, there are three ways to have two heads and one tail land: the first quarter could be the tail, the second quarter could be the tail, or the third quarter could be the tail. The probability of having a tail in one position and two heads in the others is $0.5 \cdot 0.5 \cdot 0.5 = 0.125$, but there are three positions that could be the tail, so we must multiply this by three: $3 \cdot 0.125 = 0.375$. Thus Bob's probability of winning is 0.375, and of losing is 0.625.

(d) What is Confucius' expected payoff?

(d)

The payoff of Confucius is $0.375 \cdot 2 + 0.625 \cdot -1 = 0.125$.

(e) Is this a zero-sum game? Please explain your answer.

(e)

Yes, this is a zero-sum game, because any money won comes directly from the two losers.

U5. "When one player surprises another, this indicates that the players did not have common knowledge of the rules." Give an example that illustrates this statement, and give a counterexample that shows that the statement is not always true.

One possible example is that in many card games, when a player has only one remaining card, she must declare it, and if she does not and another player catches her, then she receives a penalty, usually more cards. If she was not informed of this rule, then someone could surprise her by catching her with one card.

However, players may know all the rules and still surprise another, because their actions are not observable, or they may have information that others do not. For example, in poker, you may surprise your opponent who has a full house by revealing a straight flush. Both of you knew the rules, but your opponent did not know which cards you were holding.