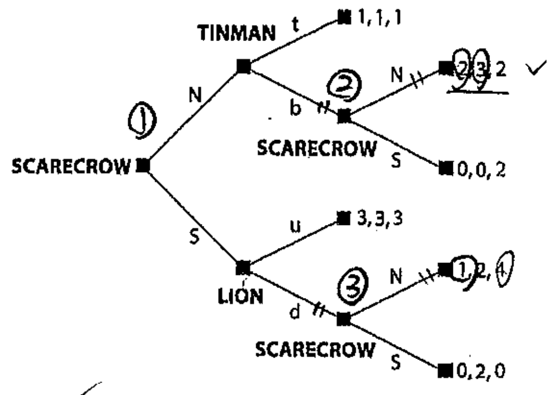
Student: [redacted] ID: [redacted] Scope: Units 1–6 Date: April 29, 2020

1. (10%) Consider the game to the right.



(a) (5%) How many pure strategies are available to each player?

(b) (5%) Describe the rollback equilibrium.

(a)

SCARECROW

N.N.N
N.N.S
N.S.N
N.S.S
S.N.N
S.N.S
S S N
S S S

⇒ 8種

TINMAN

{t, b} ⇒ 2種

LION

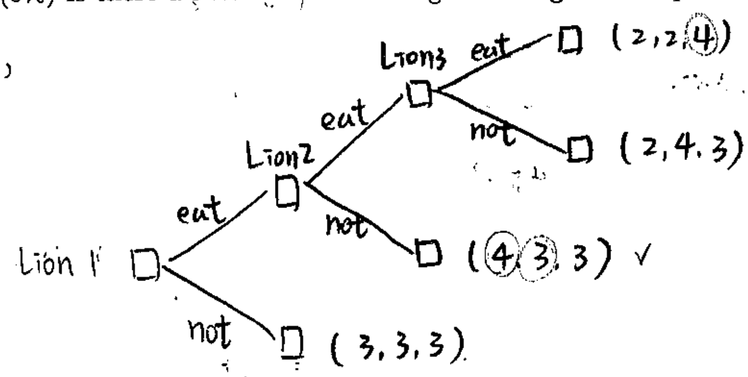
{u, d} ⇒ 2種

(b) 在 SCARECROW 的第②種情況中,他會選擇 N 因為 2 > 0.
在 SCARECROW 的第③種情況中,他會選擇 N 因為 1 > 0.
而此時 TINMAN 會選 b. 因為 3 > 1.
且 LION 會選 d. 因會 4 > 3
最後 SCARECROW 的第① 種情況中,他會選 N. 因為 2 > 1

N.E. = (2, 3, 2)

2. (20%) A slave has just been thrown to the lions in the Roman Colosseum. Three lions are chained own in a line, with Lion 1 closest to the slave. Each lion's chain is short enough that he can only reach the two players immediately adjacent to him. the game proceeds as follows. First, Lion 1 decides whether or not to eat the slave. If Lion 1 has eaten the slave, then Lion 2 decides whether or not to eat Lion 1 (who is then too heavy to defend himself). If Lion 1 has not eaten the slave, then Lion 2 has no choice: he cannot try to eat Lion 1, because a fight would kill both lions. Similarly, if Lion 2 has eaten Lion 1, then Lion 3 decides whether or not to eat Lion 2. Each Lion's preferences are fairly natural: best (4) is to eat and stay alive, next best (3) is to stay alive but go hungry, next (2) is to eat and be eaten, and worst (1) is to go hungry and be eaten.
(a) (10%) Draw the game tree, with payoffs, for this three-player game.
(b) (5%) What is the rollback equilibrium to this game?
(c) (5%) Is there a first-mover advantage to this game? Explain why or why not.

(a)



(b) Lion 3 先選擇 eat (4>3)，而 Lion 2 會選 not eat (3>2)。
最後 Lion 1 會選 eat (4>3). Ans: (4,3,3) #

(c) 有 first-mover advantage，因為當 Lion 3 選擇吃時效益最大，不可能不吃。
而 Lion 2 若選吃，他一定會被 Lion 3 吃掉，所以一定不吃。
此時 Lion 1 就可以吃 slave，因為 Lion 2 也不會吃他
若 Lion 1 選不吃，也只是 3隻一起餓，所以有 first-mover advantage.

3. (20%) Intel and AMD compete with one another in the mid-range chip category. Assume that global demand for mid-range chips depends on the quantity that the two firms make, so that the price (in dollars) for mid-range chips is given by $P = 210 - Q$, where $Q = q_{\text{Intel}} + q_{\text{AMD}}$ and where the quantities are measured in millions. Each mid-range chip costs Intel \$60 to produce and costs AMD \$48 to produce.
(a) (10%) If Intel is the monopolist, what are Intel's optimal quantity and profit?
(b) (10%) Consider a sequential-move version of the game with Intel moving first and AMD moving second. What are the equilibrium quantities and profits of Intel and AMD?

(a) $Q = q_{\text{Intel}}$

$$\begin{aligned}\pi_{\text{Intel}} &= (P - 60)\, q_{\text{Intel}} \\ &= (210 - q_{\text{Intel}} - 60)\, q_{\text{Intel}} \\ &= (150 - q_{\text{Intel}})\, q_{\text{Intel}} \\ &= 150 q_{\text{Intel}} - q_{\text{Intel}}^2\end{aligned}$$

$$\frac{\partial \pi_{\text{Intel}}}{\partial q_{\text{Intel}}} = 150 - 2 q_{\text{Intel}} = 0$$

$$\Rightarrow \underline{q_{\text{Intel}} = 75} \#$$

$$\begin{aligned}\pi_{\text{Intel}} &= 150 \times 75 - 75^2 \\ &= \underline{5625} \#\end{aligned}$$

(b) $Q = q_{\text{Intel}} + q_{\text{AMD}}.$

$$\begin{aligned}\pi_{\text{AMD}} &= (P - 48)\, q_{\text{AMD}} \\ &= (210 - q_{\text{Intel}} - q_{\text{AMD}} - 48)\, q_{\text{AMD}} \\ &= 162 q_{\text{AMD}} - q_{\text{AMD}} q_{\text{Intel}} - q_{\text{AMD}}^2\end{aligned}$$

$$\frac{\partial \pi_{\text{AMD}}}{\partial q_{\text{AMD}}} = 162 - q_{\text{Intel}} - 2 q_{\text{AMD}} = 0$$

$$q_{\text{AMD}} = 81 - \frac{1}{2} q_{\text{Intel}}$$

$$\begin{aligned}\pi_{\text{Intel}} &= (P - 60)\, q_{\text{Intel}} \\ &= (210 - q_{\text{Intel}} - q_{\text{AMD}} - 60)\, q_{\text{Intel}} \\ &= 150 q_{\text{Intel}} - q_{\text{Intel}}^2 - q_{\text{AMD}} q_{\text{Intel}} \\ &= 150 q_{\text{Intel}} - q_{\text{Intel}}^2 - \left(81 - \tfrac{1}{2} q_{\text{Intel}}\right) q_{\text{Intel}} \\ &= \underline{150 q_{\text{Intel}} - q_{\text{Intel}}^2} - 81 q_{\text{Intel}} + \underline{\tfrac{1}{2} q_{\text{Intel}}^2} \\ &= 69 q_{\text{Intel}} - \tfrac{1}{2} q_{\text{Intel}}^2\end{aligned}$$

$$\frac{\partial \pi_{\text{Intel}}}{\partial q_{\text{Intel}}} = 69 - q_{\text{Intel}} = 0$$

$$\Rightarrow \underline{q_{\text{Intel}} = 69} \#$$

$$\pi_{\text{Intel}} = 69 \cdot 69 - \tfrac{1}{2} \cdot 69^2 = \underline{2380.5} \#$$

$$\begin{aligned}q_{\text{AMD}} &= 81 - \tfrac{1}{2} \cdot 69 \\ &= \underline{46.5} \#\end{aligned}$$

$$\begin{aligned}\pi_{\text{AMD}} &= 162 \times 46.5 - 46.5 \times 69 - 46.5^2 \\ &= \underline{2162.25} \#\end{aligned}$$

4. (20%) Consider the game in which three pirates divide 100 coins among themselves. The pirates are rational and do not cooperate with the others, and base their decisions on the following priorities: (i) survive, (ii) maximize their profits, and (iii) throw the other pirates overboard. The pirates are randomly ordered. Pirate 1 announces the allocation of 100 coins among the three pirates. Then, the pirates on board vote as to whether they agree with the allocation. If more than 50% of them agree, then the game is over, and the coins are allocated among the pirates according to pirate 1's allocation. Otherwise, pirate 1 is thrown to the sea (and killed). The process then repeats with pirate 2. Finally, if pirate 2 is thrown to the sea, pirate 3 obtains 100 coins and the game is over. >50%
(a) (10%) What would be pirate 1's equilibrium allocation? >50% >50%.
(b) (10%) Now, suppose we change the voting rule "more than 50% of them agree" to be "no less than 50% of them agree" in the above game. Please analyze three pirates' actions under the new voting rule.

(a) 會分 (100, 0, 0)

這樣分一定會通過，因為若 pirate2 投反對票，下一關一定會被 pirate3 丟下船。為了活命 pirate 2 會贊成 pirate 1 的分配。

(b) 會分 (99, 0, 1)

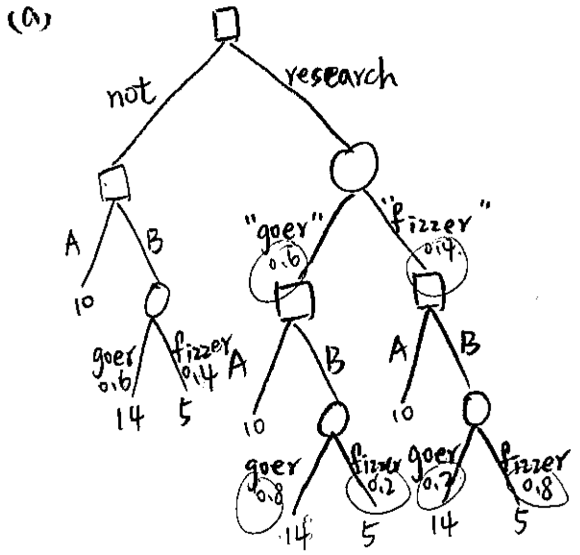
這樣分 pirate3 會同意，因為若他投反對，接下來 pirate 2 分 (100, 0) 自己投就通過 (≥50%)。所以 pirate 3 必須投同意 pirate 1 的分配。

5. (10%) Two players, Tom and Jerry, play the following game with a jar containing 11 coins. The players take turns; Tom goes first. Each time it is a player's turn, he takes between 1 and 4 coins out of the jar. The player whose move empties the jar wins. If both players play optimally, who will win the game? Does this game have a first-mover advantage? Explain your reasoning.

Tom會贏，因為Tom先拿1個，瓶中還10個，Tom可以依照Jerry所拿的數量決定他要拿幾個，倒數第二輪剩5個讓Jerry沒辦法拿完，Tom就贏了，故有first-mover advantage.

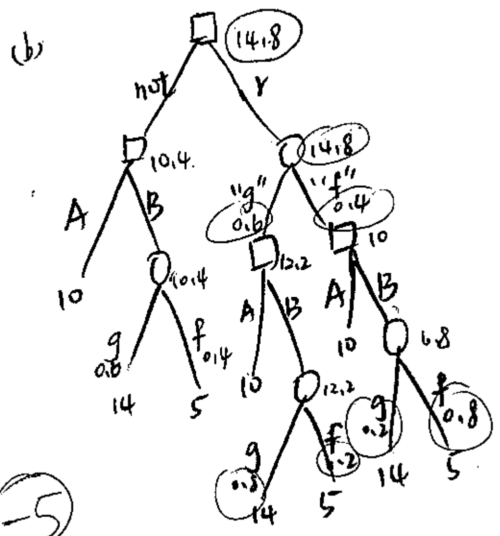
6. (20%) Consider a risk-neutral auto manager who has to decide whether to adopt color A or color B as the mainstream for new sedans produced next year. It is believed that car color has a significant influence on sales. Color A is traditional and generally acceptable from the analysis of historic data, whereas color B is dynamic and risky, compared to color A. Choosing color A gives the net profit of 10 million dollars. If color B is chosen and turns out to be a goer, the net profit is expected to be 14 million. But, if color B is chosen and turns out to be a fizzer, the sales is expected to be 5 million. The manager believes that color B has the success probability 0.6. Now, the manager has an option of employing the market-research firm, AMC, to test for the acceptance of color B. AMC's service is believed to be 80% reliable.
(a) (10%) Construct the tree to model the manager's decision.
(b) (10%) Derive the manager's value of imperfect information from AMC's service.

(a)



X

(b)



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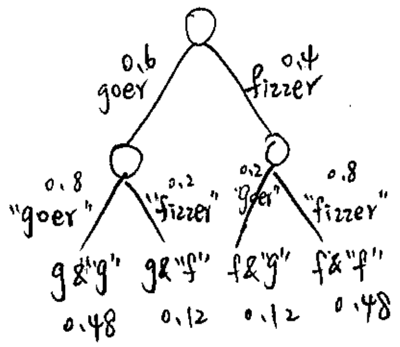

執行AMC會有14.8 million的收益
比不執行(收益為10)時多出了4.8 million.
當調查為"goer"時選B色.
為"fizzer"時選A色.

X

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