Student: [redacted] ID: [redacted] Scope: Units 1–6 Date: Nov. 10, 2020

1. (5%) When we say the players have common knowledge of rules, what are the rules?

所有玩家對 game 有基本的共識與了解，知道所有可行的策略及其對應之結果，在理性的狀況下追求利益最大化。

2. (5%) What are the characteristics of a prisoners' dilemma?

因犯困境為在同步賽局中，player A 與 B 因為無法共謀而選擇了一個對自己有優勢的決策，來避免一個很差的結果。

A \ B	認罪	不認罪
認罪	(10, 10)	(1, 25)
不認罪	(25, 1)	(3, 3)

3. (5%) What is the rollback equilibrium of a game?

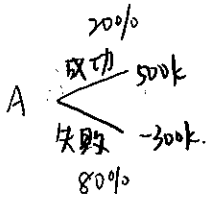
在 sequential move game 中利用 look forward and reason back 的方式從樹狀圖中由後往前推出 response，再從前往後找出 equilibrium 即是 rollback equilibrium。

path

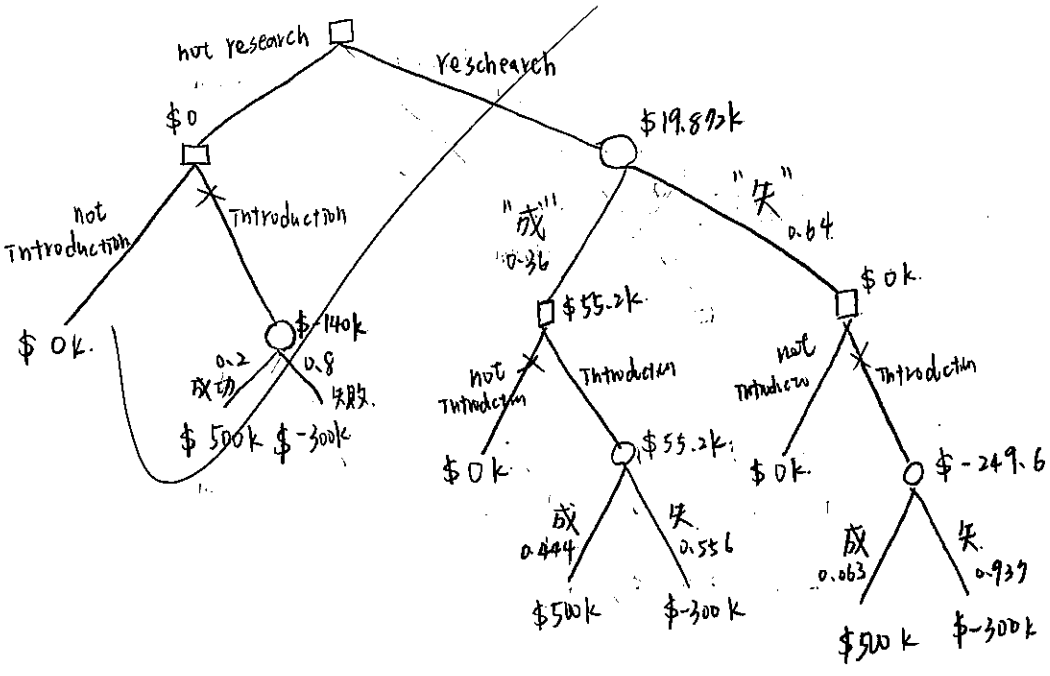
4. (5%) What are rationalizable strategies?

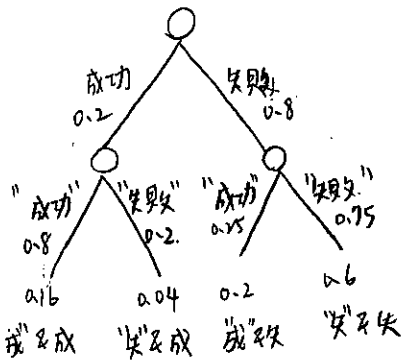
利用刪除 "never be a best response" 的方法後所得到的結果即為 rationalizable strategies。

5. (20%) Firm A is considering a final "GO" decision on a new product. If the product is introduced and it is successful, the profit is \$500,000 and if it is unsuccessful the loss is \$300,000. There is no profit or loss if the product is not introduced. The management believes that there is a 0.20 probability that the product will be successful. A market research firm specializing in new product introduction has offered its services to firm A. Its betting average in similar situations is as follows. (i) When advice was given on products that later proved to be successful, the market research firm gave "GO" advice 80% of the time. (ii) When advice was given on products that later proved to be unsuccessful, the market research firm gave "STOP" advice 75% of the time.
   (a) (10%) Construct the tree to model Firm A's decision.
   (b) (10%) Derive Firm A's value of imperfect information from the market research firm's service.



(a)

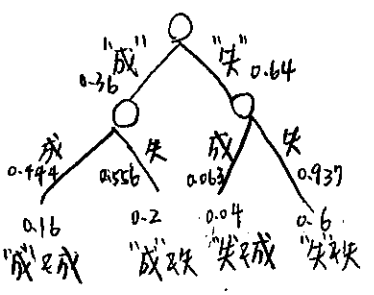


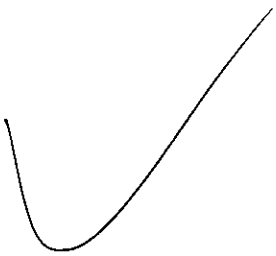


(b)

A's value of imperfect information

$= 19.872 - 0 = 19.872k.$





6. (25%) Two firms $i = 1, 2$, engage in quantity competition in the market. Each of them chooses the quantity it wants to produce: Firm 1 chooses q_1, $0 \le q_1 \le 30$, and firm 2 chooses q_2, $0 \le q_2 \le 30$. The resulting market price is $P(q_1, q_2) = 75 - q_1 - q_2$. In additional, the total cost $TC_i(q_1, q_2)$ of firm i depends on both firms' choices: $TC_1(q_1, q_2) = q_1^2(1 + q_2)$, $TC_2(q_1, q_2) = q_2^2(1 + q_1)$, Each firm maximizes its profit, i.e. the difference between its revenue (i.e. $P(q_1, q_2) \times q_i$) and its total cost $TC_i(q_1, q_2)$.

(a) (10%) Find the best response functions for both firms.

(b) (5%) Find the Nash equilibrium of the game. (Hint: These firms are symmetric.)

(b) (5%) Show that firm 1's strategy $q_1 = 20$ is strictly dominated.

(c) (5%) Is firm 2's strategy $q_2 = 1$ dominated?

(a) π_1 為 firm1 的 profit

$\pi_1 = (P - TC_1) q_1$ $P \cdot q_1 - TC_1$

$= [75 - q_1 - q_2 - q_1^2(1+q_2)] q_1$

$= [75 - q_1 - q_2 - q_1^2 - q_1^2 q_2] q_1$

$= 75q_1 - q_1^2 - q_1 q_2 - q_1^3 - q_1^3 q_2$

$\frac{d\pi_1}{dq_1} = 75 - 2q_1 - q_2 - 3q_2^2 - 3q_1^2 q_2 = 0$

$q_1 = \frac{75 - q_2 - 3q_2^2 - 3q_1^2 q_2}{2}$

π_2 為 firm2 的 profit

$\pi_2 = (P - TC_2) q_2$ $Pq_2 - TC_2$

$= [75 - q_1 - q_2 - q_2^2(1+q_1)] q_2$

$= [75 - q_1 - q_2 - q_2^2 - q_2^2 q_1] q_2$

$= 75q_2 - q_1 q_2 - q_2^2 - q_2^3 - q_2^3 q_1$

$\frac{d\pi_2}{dq_2} = 75 - q_1 - 2q_2 - 3q_2^2 - 3q_2^2 q_1 = 0$

$q_2 = \frac{75 - q_1 - 3q_2^2 - 3q_2^2 q_1}{2}$

(b≧1)

$\pi = (P - C) q$

$= (75 - 2q - q - q^2) q$

$= (75 - 2q - q - q^2) q$

$= (75 - 3q - q^2) q$

$= 150q - 6q^2 - 2q^3$

$\frac{d\pi}{dq} = 150 - 12q - 6q^2 = 0$

$q \doteq 3.6$ $(q_1, q_2) = (3.6, 3.6)$

(b≧2) $q_1 = 20$

$P = 75 - 20 - q_2 = 55 - q_2$

π

(c) Yes, r_5 dominated.

7. (35%) Two firms $i = 1, 2$, are competing in the market. They simultaneously choose their price levels — each firm's price can be low, $p_i = 1$ or high, $p_i = 3$. Product quality of firm 2 can be high or low, and is chosen by Nature. Product quality of firm 1 is known to be low. Total market demand is 2. Costs are zero. (If product quality of both firms is low and their prices are identical, they will split the market.) (If product quality of both firms is low and their prices are different, the cheaper firm will take the entire market.) (If product qualities differ, high quality firm will take the entire market, unless the low quality firm is cheaper, in the latter case they will split the market.) Firms maximize their profits. We assume that firm 1 is the row player and firm 2 is the column player. Please answer the following questions.

(a) (10%) Suppose that product quality of firm 2 turns out to be low and is known to both firms. Please construct the game table. Find all pure-strategy Nash equilibria.

(b) (10%) Suppose that product quality of firm 2 turns out to be high and is known to both firms. Please construct the game table. Find all pure-strategy Nash equilibria.

(c) (15%) Suppose that the product quality of firm 2 is low with probability 0.5 and high with probability 0.5. These probabilities are known to both firms ex ante. After Nature makes its choice, only firm 2 knows its product quality. Please construct the game table. Find all pure-strategy Nash equilibria. (Hint: You need to layout firm 2's strategies carefully.)

(a)

Firm 1 (quality low) \ Firm 2 (quality low)	price high	price low
price high	(1, 1)	(0, 2)
price low	(2, 0) ~~(0, 2)~~	~~(1, 1)~~ (3, 3)

N.E = (price low, price low)

outcome = (1,1), (3,3)

16

(b)

Firm 1 (quality low) \ Firm 2 (quality high)	price high	price low
price high	(0, 2)	~~(0, 2)~~ (1, 3)
price low	~~(1, 1)~~ (0, 2)	~~(0, 2)~~ (0, 6)

N.E = (price high, price low)

= (price low, price low)

outcome = (0,2)

(0,2)

8

(c)

Firm 1 (quality low) \ Firm 2 (quality high / quality low)	quality high price high	quality low price high	quality high price low	quality low price low
price high	(0, 2)	(1, 1)	(0, 2)	(0, 2)
price low	(1, 1)	(2, 0)	(0, 2)	(1, 1)

N-E = (price high, price low)

= (0, 2)

13

7

10

Total combination = 4