HOMEWORK #6 SOLUTION

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1. Exercise 6.1 Illustrate local sensitivity analysis

Problem: Make an original example to illustrate the local-analysis concepts of this chapter.
Solution:

2. Exercise 6.2 Illustrate global sensitivity analysis

Problem: Using `AMPL`, make an original example, with at least three constraints, graphing the objective value of (P), as a single `b[i]` is varied from $-\infty$ to $+\infty$. As you work on this, bear in mind Theorem 6.3.
Solution:

3. Exercise 6.3 *"I feel that I know the change that is needed."* – Mahatma Gandhi

Problem: We are given $2m$ numbers satisfying $L_i \leq 0 \leq U_i$, $i = 1, 2, ..., m$. Let β be an optimal basis for all of the m problems

$$\begin{aligned} \min \quad & c'x \\ \text{s.t.} \quad & Ax = b + \Delta_i e^i; \\ & x \geq 0. \end{aligned} \tag{3.1}$$

for all Δ_i satisfying $L_i \leq \Delta_i \leq U_i$. Lets be clear on what this means: For each i individually, the basis β is optimal when the ith right-hand side component is changed from b_i to $b_i + \Delta_i$, as long as Δ_i is in the interval $[L_i, U_i]$.

The point of this problem is to be able to say something about *simultaneously* changing all of the b_i. Prove that we can simultaneously change b_i to

$$\tilde{b}_i := b_i + \lambda_i \left\{ \begin{array}{c} L_i \\ U_i \end{array} \right\}$$

where $\lambda_i \geq 0$, when $\sum_{i=1}^{m} \lambda_i \leq 1$. [Note that in the formula above, for each i we can $i = 1$ pick either L_i (a decrease) or U_i (an increase)].
Solution:

4. Exercise 6.4 Domain for objective variations

Problem: Prove Theorem 6.4: The domain of g is a convex set.
Proof:

5. Exercise 6.5 Concave piecewise-linear function

Problem: Prove Theorem 6.5: g is a concave piecewise-linear function on its domain.
Proof:
The function g is

$$\begin{aligned} g(c) := \min \quad & c'x \\ \text{s.t.} \quad & Ax = b; \\ & x \geq 0. \end{aligned} \tag{5.1}$$

So a basis β is feasible or not for (5.1), independent for c_β. So g can be written as

$$g(c) = \min \left\{ c'_\beta (A_\beta^{-1} b) : \beta \text{ is a feasible basis for (5.1)} \right\}$$

$\because$ $c'_\beta (A_\beta^{-1} b)$ are affine functions
$\therefore$ g is the pointwise minimum of a finite number of affine functions
$\therefore$ g is a concave piecewise-linear function

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