

Lin Yihua 2021/10/21 10:58 VE216 Homework 6

$$1. (2 \cdot 10^6 s^3 + 10^5 s^2 + 60 s + 1) Y(s) = (8 \cdot 10^6 s - 10^4) X(s)$$

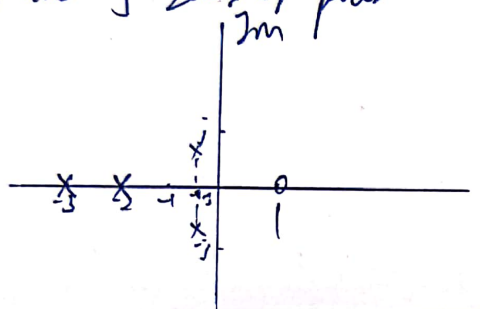
$$H(s) = \frac{Y(s)}{X(s)} = \frac{8 \cdot 10^6 s - 10^4}{2 \cdot 10^6 s^3 + 10^5 s^2 + 60 s + 1}$$

Since the system is causal, solving $2 \cdot 10^6 s^3 + 10^5 s^2 + 60 s + 1 = 0$
Rac is RHP

all the poles have a negative real part or in LHP

Thus, the system is **STABLE**

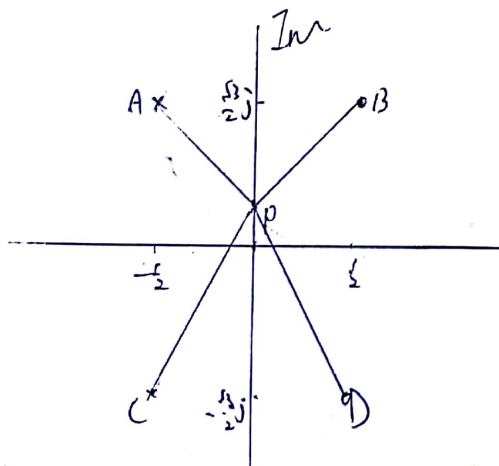
2. It has 5 zeros or poles



$$\begin{aligned} \operatorname{Re}\{s\} &> -\frac{1}{2} \\ \operatorname{Re}\{s\} &< -3 \\ -3 < \operatorname{Re}\{s\} < -2 \\ -2 < \operatorname{Re}\{s\} < -\frac{1}{2} \end{aligned}$$

4. signals have a Laplace transform expressed in the problem

3.



$$\begin{aligned} \text{poles: } & \frac{-1 \pm \sqrt{3}j}{2} \\ \text{zeros: } & \frac{1 \pm \sqrt{3}j}{2} \end{aligned}$$

$$\operatorname{Re}\{s\} > -\frac{1}{2}$$

$$\begin{aligned} \text{Fourier transform: } X(j\omega) &= \frac{(j\omega)^2 - j\omega + 1}{(j\omega)^2 + j\omega + 1} \\ &= \frac{(j\omega + (-\frac{1}{2} - \frac{\sqrt{3}}{2}j))(j\omega + (-\frac{1}{2} + \frac{\sqrt{3}}{2}j))}{(j\omega + (\frac{1}{2} - \frac{\sqrt{3}}{2}j))(j\omega + (\frac{1}{2} + \frac{\sqrt{3}}{2}j))} \end{aligned}$$

Using geometric evaluation from the pole-zero plot,

The magnitude of the Fourier transform is

$$|X(j\omega)| = \frac{(\omega^2 + (\frac{1}{2} + \frac{\sqrt{3}}{2}j)^2)(\omega^2 + (\frac{1}{2} - \frac{\sqrt{3}}{2}j)^2)}{(\omega^2 + (\frac{1}{2} - \frac{\sqrt{3}}{2}j)^2)(\omega^2 + (\frac{1}{2} + \frac{\sqrt{3}}{2}j)^2)} = 1$$

$$|X(j\omega)| = \frac{|P_c| |P_p|}{|P_A| |P_B|} = 1$$

4. $sY(s) = 2X(s)$

$sX(s) = -2Y(s) + 1$

$$\begin{cases} Y(s) = \frac{2}{s^2 + 4} = \frac{2}{(s-j2)(s+j2)} \\ X(s) = \frac{s}{s^2 + 4} = \frac{s}{(s-j2)(s+j2)} \end{cases}$$

Since $x(t)$ and $y(t)$ are right-sided signals, the real part of zeros or poles are < 0 , $X(s), Y(s)$ are rational

ROC of $X(s)$ is $\text{Re}\{s\} > 0$

ROC of $Y(s)$ is $\text{Re}\{s\} > 0$

5. (a) Laplace transform.

$$(s^3 + (\alpha+2)s^2 + \alpha(\alpha+1)s + \alpha^2)Y(s) = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (\alpha+2)s^2 + \alpha(\alpha+1)s + \alpha^2}$$

$$ghy = \frac{dh(s)}{ds} + h(s)$$

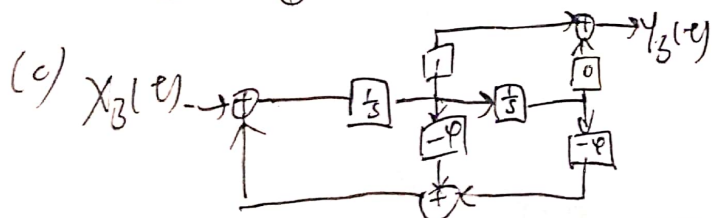
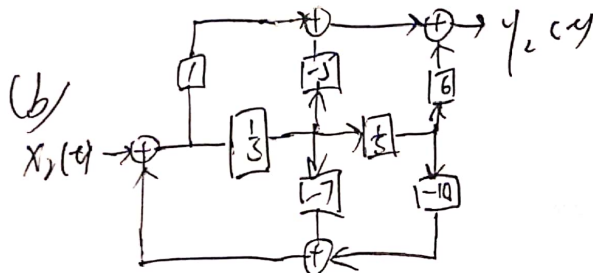
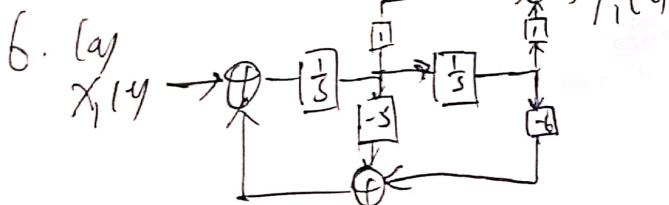
$$G(s) = sH(s) + H(s) = (s+1)H(s) = \frac{s+1}{(s+1)(s^2 + \alpha s + \alpha^2)} = \frac{1}{s^2 + \alpha s + \alpha^2}$$

$G(s)$ has 2 poles.

(b) To make the system stable, the real part of the poles should

poles are $-1, (-\frac{1}{2} + \frac{\sqrt{3}}{2}j)\alpha, (-\frac{1}{2} - \frac{\sqrt{3}}{2}j)\alpha$ be less than zero

thus, $\alpha > 0$



7. $y(t) = H(t) e^{2t}$

For $x(t) = e^{2t}$ $y(t) = H(t) e^{2t} = \frac{1}{6} e^{2t} \Rightarrow H(2) = \frac{1}{6}$

LT of the DE

$$(s+2)H(s) = \frac{1}{s+4} + \frac{1}{s} = \frac{s+b(s+4)}{s(s+4)}$$

$$H(s) = \frac{s+b(s+4)}{s(s+2)(s+4)}$$

$$H(2) = \frac{6b+2}{48} = \frac{1}{6} \Rightarrow b=1$$

$$H(s) = \frac{2s+4}{s(s+2)(s+4)} = \boxed{\frac{2}{s(s+4)}}$$

8. The unit gain at DC indicates $H_1(0) = H_2(0) = 1$

Express $H_1(s) = \frac{\alpha(s-1)}{s+3}$, $H_2(s) = \frac{\beta(s-1)}{s+2}$

then $\frac{-\alpha}{3} = 1 \Rightarrow \alpha = -3$

$$\frac{-\beta}{2} = -1 \Rightarrow \beta = 1$$

Thus $H_1(s) = \frac{-3(s-1)}{s+3}$, $H_2(s) = \frac{s-1}{s+2}$

Since the two systems are connected in parallel

$$Y(s) = [H_1(s) + H_2(s)] X(s) \quad x(t) = u(t) \quad X(s) = \frac{1}{s}$$

$$Y(s) = -\frac{1}{s} \left(\frac{3s-1}{s+3} + \frac{s-1}{s+2} \right) = -\frac{1}{s} \left(\frac{s^2+s-3}{s(s+1)(s+3)} \right)$$

Doing PFE to inverse Laplace transform

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+3}$$

$$A_1 = sY(s) \Big|_{s=0} = -\frac{1}{s} \frac{s^2+s-3}{(s+1)(s+3)} \Big|_{s=0} = -\frac{-3}{6} = \frac{1}{2}$$

$$A_2 = (s+2)Y(s) \Big|_{s=-2} = -\frac{1}{s} \frac{s^2+s-3}{s(s+3)} \Big|_{s=-2} = \frac{4}{-2} = -2$$

$$A_3 = (s+3)Y(s) \Big|_{s=-3} = -\frac{1}{s} \frac{s^2+s-3}{s(s+1)} \Big|_{s=-3} = \frac{-12}{-3} = 4$$

$$Y(s) = \frac{1}{2s} - \frac{2}{s+2} + \frac{4}{s+3}$$

$$y(t) = \frac{1}{2}(1 - e^{-2t} - 2e^{-3t})u(t)$$

9. (a) $X(s) = \frac{1}{s+1}$ $H(s) = \frac{1}{s+2}$
 $\text{Re}\{s\} > -1$ $\text{Re}\{s\} > -2$

(b) $y(t) = x(t) * h(t)$

Using the convolution property $Y(s) = X(s)H(s)$ $\boxed{Y(s) = \frac{1}{(s+1)(s+2)} \quad \text{Re}\{s\} > -1}$

(c) $Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$

Inverse Laplace transform:

$\boxed{y(t) = (e^{-t} - e^{-2t})u(t)}$

(d) $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_0^{\infty} e^{-\tau} e^{-(t-\tau)} u(t-\tau) d\tau$
 $= \int_0^t e^{-\tau} d\tau = e^{-\tau} \Big|_0^t = e^{-t} - e^{-t}$

$\boxed{y(t) = (e^{-t} - e^{-2t})u(t)}$, the result in part (c) is verified

10 Laplace transform of $x(t)$:

$X(s) = \int_{-\infty}^{\infty} e^{-st} e^{-|t|} dt$
 $= \int_{-\infty}^0 e^{\tau} e^{-st} d\tau + \int_0^{\infty} e^{-\tau} e^{-st} d\tau$
 $= \int_{-\infty}^0 e^{(1-s)\tau} d\tau + \int_0^{\infty} e^{-(s+1)\tau} d\tau$
 $= \frac{-1}{s-1} + \frac{1}{s+1} = -\frac{2}{s^2-1} = \frac{-2}{(s+1)(s-1)}$
 ROC: $-1 < \text{Re}\{s\} < 1$

$Y(s) = X(s)H(s) = -\frac{2}{(s+1)(s-1)} \frac{s+1}{s^2+s+2} = \frac{-2}{(s-1)(s^2+s+2)}$
 $= \frac{A}{s-1} + \frac{Bs+C}{s^2+s+2}$

$(s-1)Y(s) \Big|_{s=1} = -\frac{2}{s^2+s+2} \Big|_{s=1} = -\frac{2}{5}$

$A(s^2+s+2) + (Bs+C)(s-1) = (A+B)s^2 + (A-B+C)s + (A-C) = -2$
 $A = -B$ $B = \frac{2}{5}$ $C = 2A - \frac{2}{5} + 2 = \frac{2}{5}$

$Y(s) = \frac{2}{5} \left(\frac{-1}{s-1} + \frac{s+1}{s^2+s+2} \right) = \frac{2}{5} \left(\frac{s+1}{(s+1)^2+1} + \frac{2}{(s+1)^2+1} - \frac{1}{s-1} \right)$

$\boxed{y(t) = \frac{2}{5} u(t) (e^{-t} \cos t + 2e^{-t} \sin t) + \frac{2}{5} e^t u(-t)}$

The sketch of the response $y(t)$ is attached pages

$$1. (a) H_1(s) = \frac{1}{s^2 + 3s + 2} = \frac{Y_1(s)}{X_1(s)} \Rightarrow Y_1(s) = \frac{X_1(s)}{s^2 + 3s + 2}$$

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2} = \frac{Y(s)}{X(s)} \Rightarrow Y(s) = (2s^2 + 4s - 6)Y_1(s)$$

$$Y(t) = 2 \frac{d^2 y_1(t)}{dt^2} + 4 \frac{dy_1(t)}{dt} - 6y_1(t)$$

(b) $\frac{1}{s} F(s) = Y_1(s)$ inverse Laplace transform

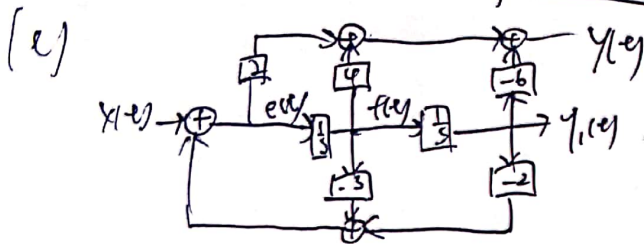
$$\frac{dy_1(t)}{dt} = f(t)$$

(c) $\frac{1}{s^2} E(s) = F(s)$ inverse Laplace transform

$$\frac{1}{s^2} E(s) = Y(s) \quad \frac{d^2 y_1(t)}{dt^2} = e(t)$$

(d) $y(t) = 2 \frac{d^2 y_1(t)}{dt^2} + 4 \frac{dy_1(t)}{dt} - 6y_1(t)$

$$= [2e(t) + 4f(t) - 6y_1(t)]$$



(f) Make a cascade combination,

