Liu Yihua 刘翊华 518021910298 VE216 Homework 6

1. $\left(2\cdot10^6 s^3 + 10^5 s^2 + 60 s + 1\right) Y(s) = \left(8\cdot10^6 s - 10^4\right) X(s)$

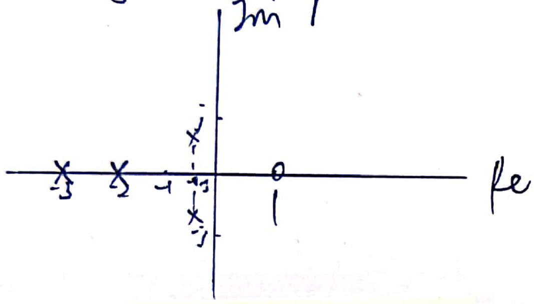
$$H(s) = \frac{Y(s)}{X(s)} = \frac{8\cdot10^6 s - 10^4}{2\cdot10^6 s^3 + 10^5 s^2 + 60 s + 1}$$

Since the system is causal, solving $2\cdot10^6 s^3 + 10^5 s^2 + 60 s + 1 = 0$
ROC is RHP

all the poles have a negative real part or in LHP

Thus, the system is $\boxed{\text{STABLE}}$

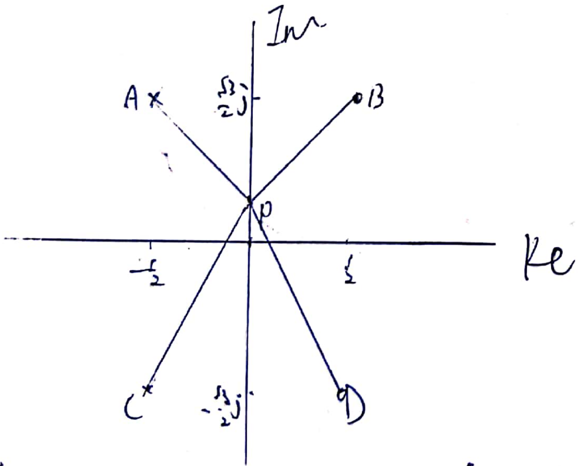
2. It has 5 zeros or poles



$Re\{s\} > -1/2$
$Re\{s\} < -3$
$-3 < Re\{s\} < -2$
$-2 < Re\{s\} < -\frac{1}{2}$

$\boxed{4}$ signals have a Laplace transform expressed in the problem

3.



poles: $\frac{-1 \pm \sqrt{3}j}{2}$

zeros: $\frac{1 \pm \sqrt{3}j}{2}$

$Re\{s\} > -\frac{1}{2}$

Fourier transform: $X(j\omega) = \frac{(j\omega)^2 - j\omega + 1}{(j\omega)^2 + j\omega + 1}$

$$= \frac{\left(j\omega + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)\right)\left(j\omega + \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)\right)}{\left(j\omega + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)\right)\left(j\omega + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)\right)}$$

Using geometric evaluation from the pole-zero plot,
The magnitude of the Fourier transform is

$$|X(j\omega)|^2 = \frac{\left(\omega^2 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)^2\right)\left(\omega^2 + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)^2\right)}{\left(\omega^2 + \left(\frac{1}{2} - \frac{\sqrt{3}}{2}j\right)^2\right)\left(\omega^2 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)^2\right)} = 1$$

$$\boxed{|X(j\omega)| = \frac{|PC||PD|}{|PA||PB|} = 1}$$

4. $sY(s) = 2X(s)$

$sX(s) = -2Y(s) + 1$

$$\boxed{\begin{aligned} Y(s) &= \frac{2}{s^2+4} = \frac{2}{(s-2j)(s+2j)} \\ X(s) &= \frac{s}{s^2+4} = \frac{s}{(s-2j)(s+2j)} \end{aligned}}$$

Since $x(t)$ and $y(t)$ are right-sided signals, $X(s)$, $Y(s)$ are rational, the real part of zeros or poles are 0,

ROC of $X(s)$ is $\boxed{\mathrm{Re}\{s\} > 0}$

ROC of $Y(s)$ is $\boxed{\mathrm{Re}\{s\} > 0}$

5. (a) Laplace transform.

$\left(s^3 + (1+\alpha)s^2 + \alpha(\alpha+1)s + \alpha^2\right)Y(s) = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3 + (\alpha+1)s^2 + \alpha(\alpha+1)s + \alpha^2}$$

$g(t) = \frac{dh(t)}{dt} + h(t)$

$$G(s) = sH(s) + H(s) = (s+1)H(s) = \frac{s+1}{(s+1)(s^2+\alpha s+\alpha^2)} = \frac{1}{s^2+\alpha s+\alpha^2}$$

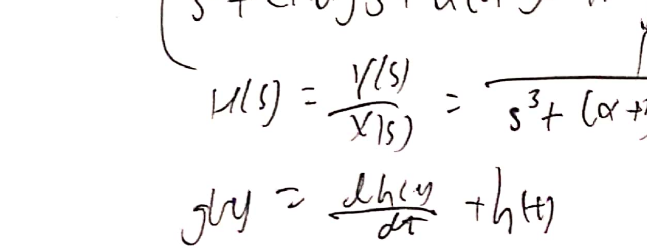
$G(s)$ has $\boxed{2 \text{ poles}}$.

(b) To make the system stable, the real part of the poles should be less than zero

poles are -1, $\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)\alpha$, $\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)\alpha$

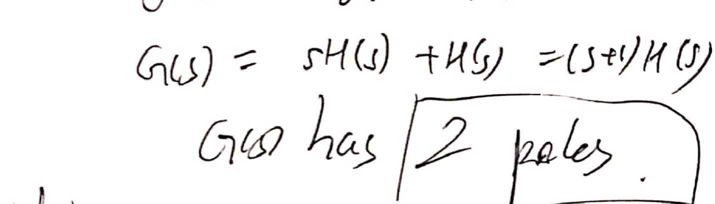
thus, $\boxed{\alpha > 0}$

6. (a)



(b)

(c)



7. $y(t) = H(s)e^{st}$

For $x(t) = e^{2t}$ $\quad y(t) = H(2)e^{2t} = \frac{1}{6}e^{2t} \Rightarrow H(2) = \frac{1}{6}$

LT of the DE

$$(s+2)H(s) = \frac{1}{s+4} + \frac{b}{s} = \frac{s+b(s+4)}{s(s+4)}$$

$$H(s) = \frac{s+b(s+4)}{s(s+2)(s+4)}$$

$$H(2) = \frac{6b+2}{48} = \frac{1}{6} \quad \Rightarrow b = 1$$

$$H(s) = \frac{2s+4}{s(s+4)(s+2)} = \boxed{\frac{2}{s(s+4)}}$$

8. The unit gain at DC indicates $H_1(0) = H_2(0) = 0$

Express $H_1(s) = \frac{\alpha(s-1)}{s+3}$, $H_2 = \frac{\beta(s-2)}{s+2}$

then $\frac{-\alpha}{3} = 1 \Rightarrow \alpha = -3$

$$\frac{-2\beta}{2} = -\beta = 1 \Rightarrow \beta = -1$$

Thus $H_1(s) = \frac{-3(s-1)}{s+3}$ $\quad H_2(s) = -\frac{s-2}{s+2}$

Since the two systems are connected in parallel

$$Y(s) = (H_1(s) + H_2(s))X(s) \qquad x(t) = u(t) \quad X(s) = \frac{1}{s}$$

$$Y(s) = -\frac{1}{s}\left(\frac{3(s-1)}{s+3} + \frac{s-2}{s+2}\right) = -4\left(\frac{s^2+s-3}{s(s+2)(s+3)}\right)$$

Doing PFE to inverse Laplace transform

$$Y(s) = \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+3}$$

$$A_1 = sY(s)\Big|_{s=0} = -4\frac{s^2+s-3}{(s+2)(s+3)}\Big|_{s=0} = -\frac{-12}{6} = 2$$

$$A_2 = (s+2)Y(s)\Big|_{s=-2} = -4\frac{s^2+s-3}{s(s+3)}\Big|_{s=-2} = \frac{4}{-2} = -2$$

$$A_3 = (s+3)Y(s)\Big|_{s=-3} = -4\frac{s^2+s-3}{s(s+2)}\Big|_{s=-3} = \frac{-12}{3} = -4$$

$$Y(s) = \frac{2}{s} - \frac{2}{s+2} - \frac{4}{s+3}$$

$$y(t) = 2(1 - e^{-2t} - 2e^{-3t})u(t)$$

9\. (a) $\boxed{X(s) = \frac{1}{s+1} \quad Re\{s\} > -1}$ $\boxed{H(s) = \frac{1}{s+2} \quad Re\{s\} > -2}$

(b) $y(t) = x(t) * h(t)$

Using the convolution property $Y(s) = X(s)H(s)$ $\boxed{Y(s) = \frac{1}{(s+1)(s+2)} \quad Re\{s\} > -1}$

(c) $Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$

Inverse Laplace transform:

$$\boxed{y(t) = (e^{-t} - e^{-2t})u(t)}$$

(d) $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_0^{\infty} e^{-\tau} e^{t-\tau} u(t-\tau)\, d\tau$

$= \int_0^t e^{t-2\tau} d\tau = e^{t-2\tau}\Big|_0^t = e^{-t} - e^{-2t}$

$\boxed{y(t) = (e^{-t} - e^{-2t})u(t)}$, the result in part (c) is verified

10 Laplace transform of $x(t)$:

$$X(s) = \int_{-\infty}^{\infty} e^{-|t|} e^{-st} dt$$

$$= \int_{-\infty}^{0} e^{t} e^{-st} dt + \int_0^{\infty} e^{-t} e^{-st} dt$$

$$= \int_{-\infty}^{0} e^{(1-s)t} dt + \int_0^{\infty} e^{-(s+1)t} dt$$

$$= \frac{-1}{s-1} + \frac{1}{s+1} = -\frac{2}{s^2-1} = \frac{-2}{(s+1)(s-1)}$$

ROC: $-1 < Re\{s\} < 1$

$$Y(s) = X(s)H(s) = -\frac{2}{(s+1)(s-1)} \frac{s+1}{s^2+2s+2} = \frac{-2}{(s-1)(s^2+2s+2)}$$

$$= \frac{A}{s-1} + \frac{Bs+C}{s^2+2s+2}$$

$(s-1)Y(s)\Big|_{s=1} = -\frac{2}{s^2+2s+2}\Big|_{s=1} = -\frac{2}{5}$

$A(s^2+2s+2) + (Bs+C)(s-1) = (A+B)s^2 + (2A-B+C)s + (2A-C) = -2$

$A = -B$ $\quad B = \frac{2}{5}$ $\quad C = 2A+2 = -\frac{4}{5}+2 = \frac{6}{5}$

$$Y(s) = \frac{2}{5}\left(\frac{-1}{s-1} + \frac{s+3}{s^2+2s+2}\right) = \frac{2}{5}\left(\frac{s+1}{(s+1)^2+1^2} + \frac{2}{(s+1)^2+1^2} - \frac{1}{s-1}\right)$$

$$\boxed{y(t) = \frac{2}{5}u(t)(e^{-t}\cos t + 2e^{-t}\sin t) + \frac{2}{5}e^{t}u(-t)}$$

The sketch of the response $y(t)$ see attached pages

11.(a) $H_1(s) = \frac{1}{s^2+3s+2} = \frac{Y_1(s)}{X_1(s)} \Rightarrow Y_1(s) = \frac{X(s)}{s^2+3s+2}$

$H(s) = \frac{2s^2+4s-6}{s^2+3s+2} = \frac{Y(s)}{X(s)} \Rightarrow Y(s) = (2s^2+4s-6)Y_1(s)$

$$\boxed{y(t) = 2\frac{d^2y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)}$$

(b) $\frac{1}{s}F(s) = Y_1(s)$ inverse Laplace transform

$$\boxed{\frac{dy_1(t)}{dt} = f(t)}$$

(c) $\frac{1}{s}E(s) = F(s)$ inverse Laplace transform

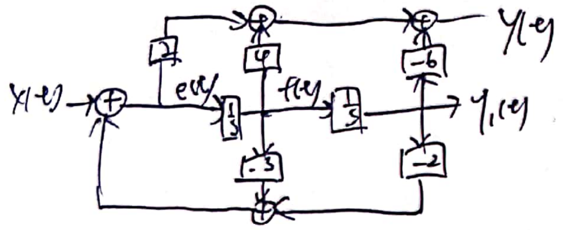
$\frac{1}{s^2}E(s) = Y(s)$

$$\boxed{\frac{d^2y_1(t)}{dt^2} = e(t)}$$

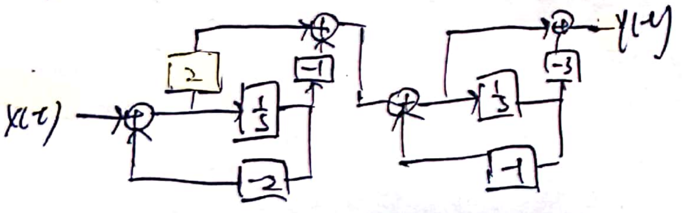
(d) $y(t) = 2\frac{d^2y_1(t)}{dt^2} + 4\frac{dy_1(t)}{dt} - 6y_1(t)$

$$= \boxed{2e(t) + 4f(t) - 6y_1(t)}$$

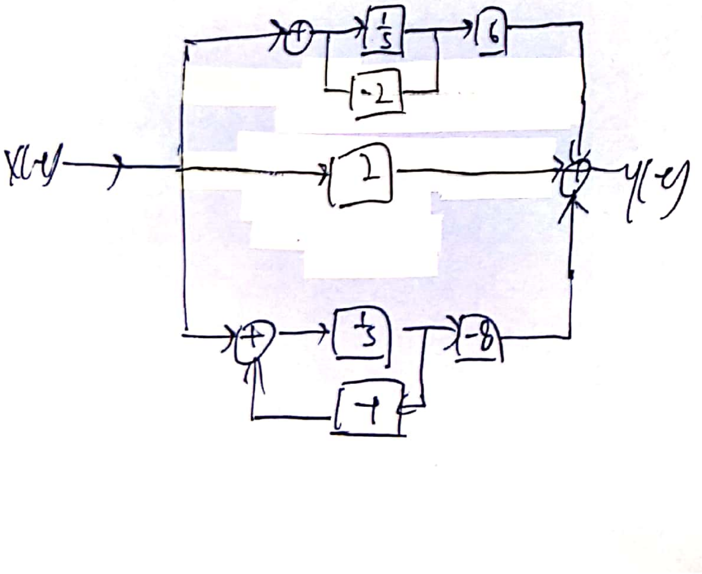
(e)

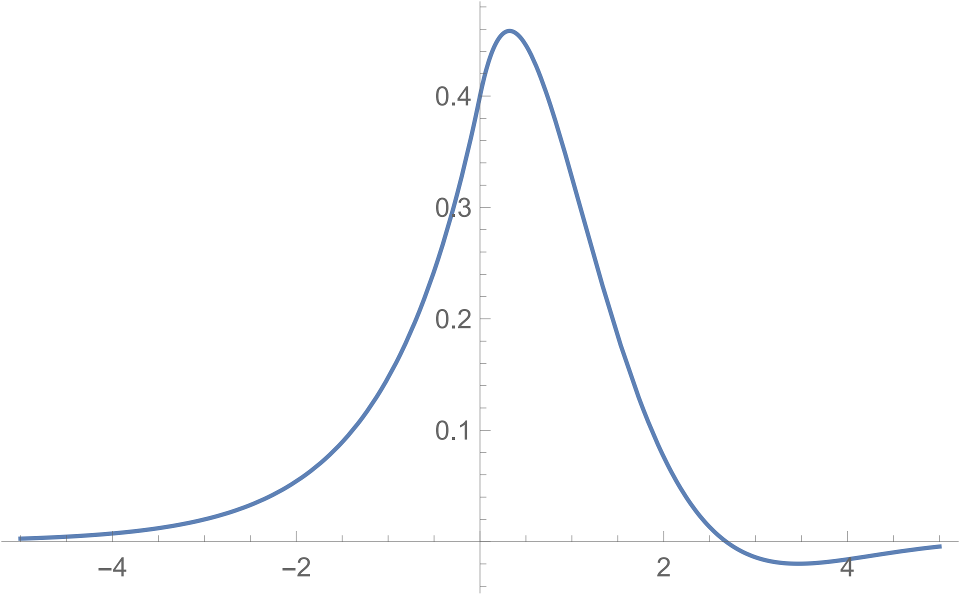


(f) Make a cascade combination,



(g)




0.4
0.3
0.2
0.1
−4
−2
2
4