

$$\frac{V_{out}}{\frac{1}{j\omega C}} = \frac{V_{ant}}{j\omega L + R + \frac{1}{j\omega C}}$$

The frequency response function

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{ant}(j\omega)} = \frac{\frac{1}{j\omega C}}{j\omega L + R + \frac{1}{j\omega C}} = \frac{1}{- \omega^2 LC + j\omega RC + 1}$$

$$s = j\omega$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

(b) Eq. (2.2.4)  $f = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \text{ Hz}$  or  $\omega_{res} = \frac{1}{\sqrt{LC}} \text{ rad/s}$

Eq. (2.2.6)  $BW_{dB} = \frac{1}{2\pi} \frac{R}{L} \text{ Hz}$

Eq. (2.2.7)  $Q = 2\pi f_{res} \frac{L}{R} = \frac{1}{\sqrt{LC}} \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{f}{BW_{dB}}$

Values read off from the plot:

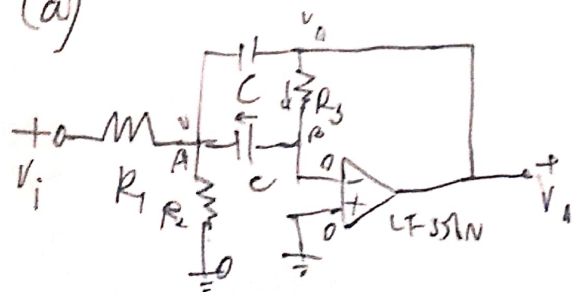
	Peak Freq (kHz)	3dB BW (kHz)	Quality Factor
C = 100pF	513.65	3.312	155.09
C = 30pF	937.17	14.685	63.86

Values computed from the equations

	Peak Freq (kHz)	3dB BW (kHz)	Quality Factor
C = 100pF	513.67	3.316	154.92
C = 30pF	937.83	14.921	62.85

Plot see attached pages

3.2 (a)



At node 1

$$\frac{V_i - V}{R_1} + \frac{V_o - V}{\frac{1}{j\omega C}} + \frac{V_o}{R_3} = \frac{V}{R_2}$$

$$V_o - V = \frac{V_o}{R_3} (R_3 + \frac{1}{j\omega C}) = V_o + \frac{V_o}{j\omega R_3 C}$$

$$\frac{V_i}{R_1} + \frac{V_o}{j\omega R_1 R_3 C} + j\omega C V_o + \frac{2V_o}{R_3} = -\frac{V_o}{j\omega R_2 R_3 C}$$

$$\frac{V_i}{R_1} = -V_o \left( \frac{1}{j\omega R_1 R_3 C} + \frac{2}{R_3} + j\omega C + \frac{1}{j\omega R_2 R_3 C} \right)$$

$$H_{IF}(j\omega) = \frac{V_o}{V_i} = \frac{1}{\left( \frac{1}{j\omega R_1 R_3 C} + \frac{2}{R_3} + j\omega C + \frac{1}{j\omega R_2 R_3 C} \right)}$$

$$= \frac{1}{\frac{R_1}{j\omega R_2 R_3 C} + \frac{2R_1}{R_3} + j\omega R_1 C + \frac{1}{j\omega R_3 C}} = \frac{-j\omega R_1 R_3 C}{R_1 + 2j\omega R_1 R_2 C - \omega^2 R_1 R_2 R_3^2 C^2 + R_3}$$

$$= \frac{-j\omega R_2 R_3 C / (R_1 + R_2)}{-\omega^2 \frac{R_1 R_2 R_3^2 C^2}{R_1 + R_2} + j\omega \frac{R_1 R_2 C}{R_1 + R_2} + 1} = \frac{-R_2 R_3 C / (R_1 + R_2) s}{\frac{R_1 R_2 R_3^2 C^2}{R_1 + R_2} s^2 + \frac{2R_1 R_2 C}{R_1 + R_2} s + 1}$$

$$\left[ \begin{aligned} a_2 &= \frac{R_1 R_2 R_3^2 C^2}{R_1 + R_2} & a_3 &= \frac{2R_1 R_2 C}{R_1 + R_2} & H_{IF}(j\omega) &= \frac{-\frac{R_2 R_3 C}{R_1 + R_2} s}{\frac{R_1 R_2 R_3^2 C^2}{R_1 + R_2} s^2 + \frac{2R_1 R_2 C}{R_1 + R_2} s + 1} \end{aligned} \right]$$

$R_1 = 10^3 \Omega$   $R_2 = 10^4 \Omega$   $R_3 = 10^2 \Omega$   $C = 1.5 \times 10^{-8} F$

$$\begin{aligned} a_1 &= -1.607 \times 10^{-6} \\ a_2 &= 2.411 \times 10^{-13} \\ a_3 &= 3.214 \times 10^{-6} \end{aligned}$$

(b)  $H(s) = H_0 \frac{\beta s}{s^2 + \beta s + \omega_0^2}$   $H(j\omega) = H_0 \frac{1}{\frac{\omega}{\omega_0} + j \frac{(\omega/\omega_0)^2 - 1}{\beta/\omega_0}}$   $\approx H_0 \frac{1}{1 + j \frac{\omega}{\omega_0} \frac{\beta}{R/2}}$   $\beta/\omega_0 \approx \beta$  when  $\beta \ll \omega_0$

Formulas:

The peak value of the frequency response function  $H_0$ , the 3-dB bandwidth  $\beta$

$$H_{IF}(s) = \frac{-\frac{1}{R_3 C} s}{s^2 + \frac{2}{R_3 C} s + \frac{R_1 + R_2}{R_1 R_2 R_3^2 C^2}} = -\frac{R_3}{2R_1} \frac{\frac{2}{R_3 C} s}{s^2 + \frac{2}{R_3 C} s + \frac{R_1 + R_2}{R_1 R_2 R_3^2 C^2}}$$

$$\left[ \begin{aligned} H_0 &= -\frac{R_3}{2R_1} & \beta &= \frac{2}{R_3 C} & \omega_0 &= \sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3^2 C^2}} \end{aligned} \right]$$

The center of the passband in frequency

$$f = \frac{\omega_0}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3^2 C^2}}$$

using eq. (2.5.2) through (2.5.5)  
that  $\beta \ll \omega_0$  so we can use  $\beta/\omega_0 \approx \beta$

eq. (2.5.4) shows  $H(j\omega)$  achieves a maximum value of  $H_0$  at  $\omega_0$ , i.e. the peak value of the frequency response function



Numerical values: Substituting given values in formulas above

$\boxed{H_0 = -5}$        $\boxed{\beta = 1.33 \times 10^5 \text{ Hz}}$        $\boxed{f = 1.025 \times 10^5 \text{ Hz}}$   
 peak value of  $H(f_{IF})$       3dB bandwidth of IF      center of the passband

(c) The graph see attached pages

3.3 a) Carrier frequency  $f_c = 1600 \text{ kHz}$

IF filter centered at  $f_{IF} = 100 \text{ kHz}$        $f_c > f_{IF}$

LO frequency  $f_{L0} = f_c - f_{IF} = 1500 \text{ kHz}$

$f_{L0} = f_c + f_{IF} = 1700 \text{ kHz}$

For  $f_{L0} = 1500 \text{ kHz}$  the frequency band centered at  $f_{img}$  known as image band

$f_{img} = f_c + 2f_{IF} = 1800 \text{ kHz}$

For  $f_{L0} = 1700 \text{ kHz}$ ,  $f_{img} = |f_{IF} - f_{L0}| = |2f_{IF} - f_c| = 1400 \text{ kHz}$

b)  $f_c = 530 \text{ kHz}$        $f_{IF} = 100 \text{ kHz}$

LO frequency  $f_{L0} = f_c - f_{IF} = 430 \text{ kHz}$ , image band center  $f_{img} = |2f_{IF} - f_c| = 330 \text{ kHz}$

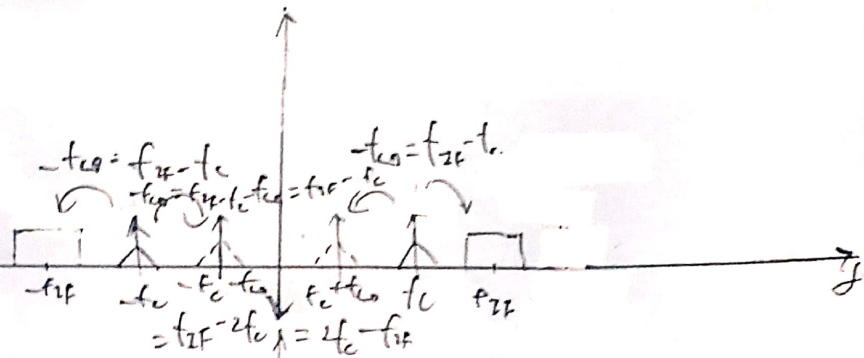
$f_{L0} = f_c + f_{IF} = 630 \text{ kHz}$ , image band center  $f_{img} = f_c + 2f_{IF} = 730 \text{ kHz}$

$f_c (\text{kHz})$	$f_{L0} (\text{kHz})$	$f_{L02} (\text{kHz})$	$f_{img1} (\text{kHz})$	$f_{img2} (\text{kHz})$
1600	1500	1700	1800	1400
530	430	630	330	730

3.  $\psi$  Equivalency of Figs 2.4.2

that  $\frac{1}{2}f_{IF} < f_c < f_{IF}$

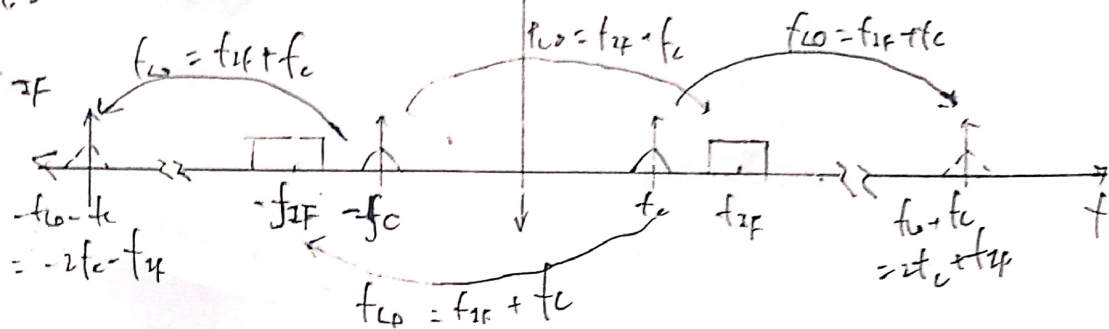
$$f_{LO} = f_c - f_{IF}$$



Equivalency of Figs 2.4.3

that  $\frac{1}{2}f_{IF} < f_c < f_{IF}$

$$f_{LO} = f_c + f_{IF}$$



# VE216

## Introduction to Signals and Systems

PRELAB 2 ATTACHED PAGES

June 6, 2020

Yihua Liu 518021910998

3.1 (b)

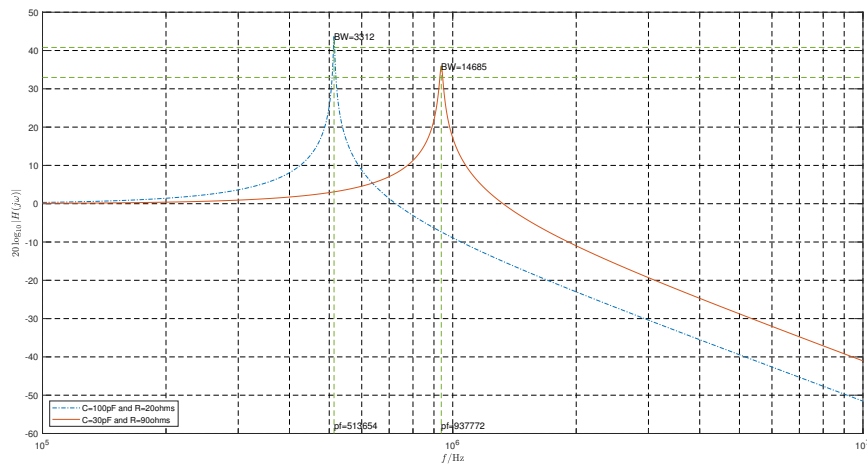


Figure 1. 3.1(b).

MATLAB Code:

```

1 R=20;L=960e-6;C=100e-12;
2 freq=logspace(5,7,10000);
3 H1=20*log10(abs(1./(-L*C.*(2*pi.*freq).^2+R*C*1i*2*pi.*freq+1)));
4 p=semilogx(freq,H1);
5 p.LineStyle='-.';
6 grid on;
7 hold on;
8 R=90;C=30e-12;
9 H2=20*log10(abs(1./(-L*C.*(2*pi.*freq).^2+R*C*1i*2*pi.*freq+1)));
10 semilogx(freq,H2);
11 [m1,index1]=max(H1);
12 pf1=freq(index1);
13 l1=line([pf1,pf1],[-60,m1],'linestyle','—');
14 l1.Color='#77AC30';
15 text(pf1,-58, strcat('pf=', num2str(pf1,6)));
16 [m2,index2]=max(H2);
17 pf2=freq(index2);

```

```

18 l2=line([pf2,pf2],[-60,m2],'linestyle','—');
19 l2.Color='#77AC30';
20 text(pf2,-58,strcat('pf=',num2str(pf2,6)));
21 db1=m1-3;
22 l3=line([1e5,1e7],[db1,db1],'linestyle','—');
23 l3.Color='#77AC30';
24 H11=H1(1:index1-1);
25 [~,index11]=min(abs(H11-db1));
26 H12=H1(index1+1:length(H1));
27 [~,index12]=min(abs(H12-db1));
28 bw1=-freq(index11)+freq(index12+index1);
29 text(freq(index1),m1,strcat('BW=',num2str(bw1,5)));
30 db2=m2-3;
31 l4=line([1e5,1e7],[db2,db2],'linestyle','—');
32 l4.Color='#77AC30';
33 H21=H2(1:index2-1);
34 [~,index21]=min(abs(H21-db2));
35 H22=H2(index2+1:length(H2));
36 [~,index22]=min(abs(H22-db2));
37 bw2=-freq(index21)+freq(index22+index2);
38 text(freq(index2),m2,strcat('BW=',num2str(bw2,5)));
39 qf1=pf1/bw1;
40 qf2=pf2/bw2;
41 hold off;
42 ax=gca;
43 ax.GridLineStyle='—';
44 ax.GridColor='k';
45 ax.GridAlpha=1;
46 ax.MinorGridLineStyle='—';
47 ax.MinorGridColor='k';
48 ax.MinorGridAlpha=1;
49 axis([1e5 1e7 -60 50]);
50 legend('C=100pF and R=20ohms','C=30pF and R=90ohms','Location','southwest');
51 xlabel('$$$f/\mathrm{Hz}$$$', 'Interpreter','latex');
52 ylabel('$$$20\log_{10}|H(j\omega)|$$$', 'Interpreter','latex');

```

3.2 (c)

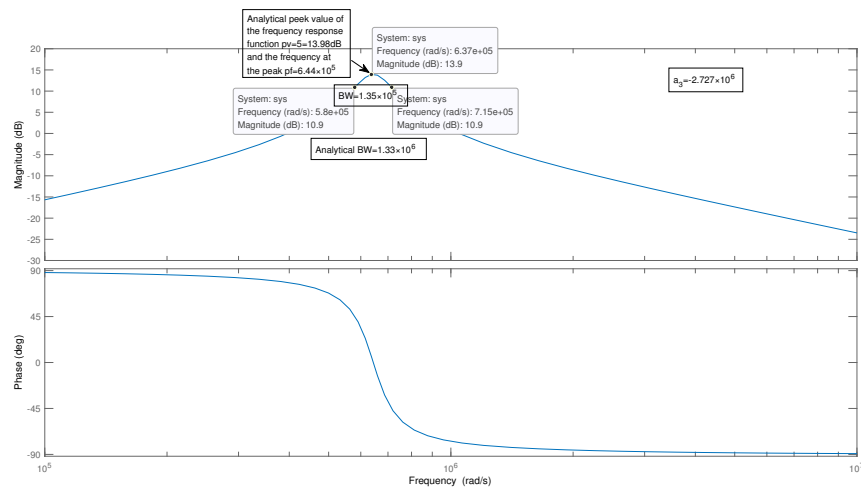


Figure 2. 3.2(c)

From the figure we can see that the analytical value of the peak value of the frequency response function and the frequency at the peak are very closed to the "exact" values read off my plot, however, the analytical value of  $BW_{3dB}$  is not very closed to the "exact" value read off my plot.

MATLAB Code:

```

1 R1=1e3;
2 R3=1e4;
3 R2=120;
4 C=1.5e-9;
5 sys=tf([R2*R3*C/(R1+R2) 0],[R1*R2*R3*C^2/(R1+R2) 2*R1*R2*C/(R1+R2)
6     1]);
7 fb=bandwidth(sys);
8 [mag, phase, wout]=bode(sys);
9 [gpeak, fpeak]=getPeakGain(sys);

```