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1. (a) $H(j\omega) = \dfrac{2j\omega+1}{-j\omega^3-5\omega^2+8j\omega+4} = \dfrac{-2\omega+j}{(\omega-2j)^2(\omega-j)} = \dfrac{2j\omega+1}{(j\omega+2)^2(j\omega+1)}$

$$G(v) = \frac{2v+1}{(v+2)^2(v+1)} = \frac{A_{11}}{v+2} + \frac{A_{12}}{(v+2)^2} + \frac{A_{21}}{v+1}$$

$$A_{11} = \frac{d}{dv}\left[(v+2)^2G(v)\right]\Big|_{v=-2} = \frac{d}{dv}\left(\frac{2v+1}{v+1}\right)\Big|_{v=-2} = \frac{d}{dv}\left(2-\frac{1}{v+1}\right)\Big|_{v=-2}$$

$$= \frac{d}{dv}\left(-\frac{1}{v+1}\right)\Big|_{v=-2} = \frac{1}{(v+1)^2}\Big|_{v=-2} = 1$$

$$A_{12} = \left[(v+2)^2G(v)\right]\Big|_{v=-2} = \frac{2v+1}{v+1}\Big|_{v=-2} = 3$$

$$A_{21} = \left[(v+1)G(v)\right]\Big|_{v=-1} = \frac{2v+1}{(v+2)^2}\Big|_{v=-1} = -1$$

$$\boxed{H(j\omega) = \frac{1}{j\omega+2} + \frac{3}{(j\omega+2)^2} - \frac{1}{j\omega+1}}$$

MATLAB code:

Input: Coefficients of numerator and denominator polynomial
Output: Residues of PFE, poles of PFE, and direct term

```
b = [2 1];
a = [1 5 8 4];
[r, p, k] = residue (b, a)
```

Output:

```
r =

    1.0000
    3.0000
   -1.0000

p =

   -2.0000
   -2.0000
   -1.0000

k =

    []
```

In the case of repeated pole(s), p is ordered in A_{ik}, k first.
Hence, MATLAB gives $\dfrac{1}{j\omega+2} + \dfrac{3}{(j\omega+2)^2} - \dfrac{1}{j\omega+1}$

which is exactly our result.

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(1b) $e^{-at}u(t) \leftrightarrow \frac{1}{j\omega+a}$

$te^{-at}u(t) \leftrightarrow \frac{1}{(j\omega+a)^2}$

$$H(j\omega) = \frac{1}{j\omega+2} + \frac{3}{(j\omega+1)^2} - \frac{1}{j\omega+1}$$

Do Fourier Transform, the unit impulse response h(t) of this system is

$$h(t) = e^{-2t}u(t) + 3te^{-2t}u(t) - e^{-t}u(t)$$

$$= \boxed{(e^{-2t} + 3te^{-2t} - e^{-t})u(t)}$$

MATLAB plot

```
fplot((exp(-2*t)+3*t*exp(-2*t)-exp(-t))*heaviside(t),[2 8])
```

Graph see attached pages.

Use MATLAB impulse to generate a same plot. Graph see attached pages

By comparing the two graphs, we can verify our result.

2. (a) $x(t) = \cos\omega_0 t$

$$x_p(t) = x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)$$

$$X_p(\omega) = \frac{1}{2\pi}X(\omega)*P(\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty}X(\omega-n\omega_s)$$

$$\cos\omega_0 t \leftrightarrow \pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$$

$$X(\omega-k\omega_s) = \pi\delta(\omega-\omega_0-n\omega_s)+\pi\delta(\omega+\omega_0-n\omega_s) \qquad \omega_s = \frac{2\pi}{T} = 6\pi$$

$$= \pi\delta(\omega-\omega_0-6n\pi)+\pi\delta(\omega+\omega_0-6n\pi)$$

$$X_p(\omega) = 3\pi\sum_{k=-\infty}^{\infty}\left(\delta(\omega-\omega_0-6n\pi)+\delta(\omega+\omega_0-6n\pi)\right)$$

The sketches are included in attached pages.

(b) For $\omega_0 = \pi$ and 5π, is $x_p(t)$ identical

By sampling theorem, $\omega_s > 2\omega_{max} \Rightarrow \omega_{max} < 3\pi$

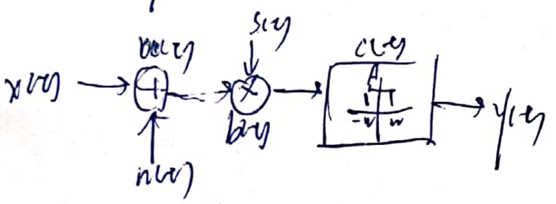
Thus, for $\boxed{\omega_0 = 3\pi \text{ and } 5\pi}$ we will NOT be able to recover the input sinusoidal signal after lowpass filtering $x_p(t)$.

3. Denote the response of the unknown deterministic system as $P(\omega)$, then $Q(\omega) = P(\omega) X_p(\omega)$. $\cos\omega t$ can be seen as a function of ω, too. No matter what is $P(\omega)$, since $X_p(t)$ is a train of impulses, is periodic as a function of ω. $Q(\omega)$ is constant ~~[illegible]~~ for t, and $Q(\omega)$ should also be ~~a train of impulses~~ periodic.

~~$P(\omega) \cdot \delta(\omega - n\omega_0) = P(n\omega_0)\,\delta(\omega - n\omega_0)$~~

However, a) & b) are not like this, so the following two plots a) & (b) ~~is~~ are $\boxed{\text{NOT}}$ possible for the variation of Q as a function of ω, but $\boxed{c}$ is $\boxed{\text{possible}}$.

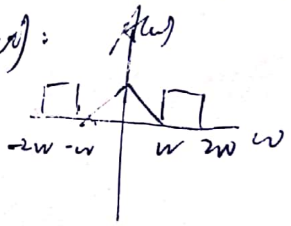
4. Denote the systems as follows



then $a(t) = x(t) + n(t)$

$b(t) = a(t)s(t)$

$P(\omega)$:



$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

$$B(\omega) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} A(\omega - n\omega_s), \quad \omega_s = \frac{2\pi}{T}$$

$$Y(\omega) = B(\omega) C(\omega) = X(\omega)$$

$B(\omega)$:

We have

$$-2\omega + \omega_s \geq \omega$$

$$\Rightarrow \omega_s = \frac{2\pi}{T} \geq 3\omega$$

$$\Rightarrow T \leq \frac{2\pi}{3\omega}$$

Suppose the amplitude of $X(\omega)$ and $N(\omega)$ is A_0, then

$$\frac{A}{T} = [illegible] \qquad A \leq \frac{2\pi [illegible]}{3\omega}$$

Thus the maximum values for T is $\frac{2\pi}{3\omega}$

for A is $\frac{2\pi [illegible]}{3\omega}$

5. $\omega_{max} = \frac{5\pi}{6}$ $\quad \omega_s = \frac{2\pi}{T} = 2W$ $\quad \omega_s > 2\omega_{max}$, so $X(\omega)$ can simply be identical to $X_r(\omega)$

Alternatively, $X(\omega)$ can also be just one side part of $X_r(\omega)$

The sketches are:

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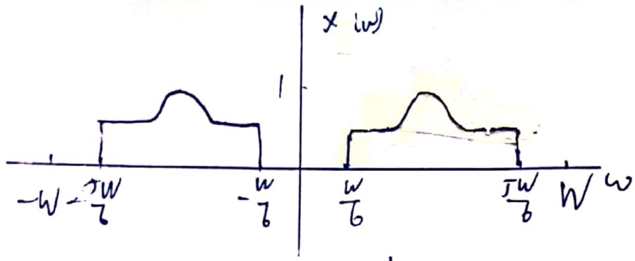
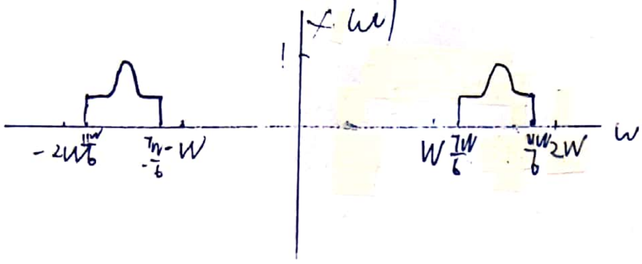




6. (a) $x(t) = x_1(t-\frac{\pi}{2}) + x_2(t)$

Doing Fourier Transform

$X(\omega) = X_1(\omega)e^{-j\omega\frac{\pi}{2}} + X_2(\omega)$, $x_1(\omega)=0$ for $|\omega|>2W$ and $X_2(\omega)=0$ for $|\omega|>W$

So the ranges for W_c is $\boxed{|W_c| \geq 2W}$ and $\boxed{|W_c| \leq \frac{2\pi}{T} - 2W}$

$X_p(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

$\frac{A}{T}=1$

then $\boxed{A = T}$

$-2W + \frac{2\pi}{T} \geq 2W$

$\frac{2\pi}{T} \geq 4W$

$T \leq \frac{\pi}{2W}$

Thus the maximum value of T is $\boxed{\frac{\pi}{2W}}$

the maximum value of A is $\boxed{\frac{\pi}{2W}}$

(b) $x(t) = x_1(t)x_2(t)$

$X(\omega) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$

By the property of convolution the range for W_c is $\boxed{|W_c| \geq 3W}$ and $\boxed{|W_c| \leq \frac{2\pi}{T} - 3W}$

$\frac{A}{T}=1$ $\boxed{A = T}$

$-3W + \frac{2\pi}{T} \geq 3W$

$\frac{2\pi}{T} \geq 6W$

$T \leq \frac{\pi}{3W}$

The maximum value of T is $\boxed{\frac{\pi}{3W}}$, the maximum value of A is $\boxed{\frac{\pi}{3W}}$

(c) $x(t) = \frac{d x_2(t)}{dt}$

$X(\omega) = j\omega X_2(\omega)$ The range for W_c is $\boxed{|W_c| \geq W}$ and $\boxed{|W_c| \leq \frac{2\pi}{T} - W}$

$\frac{A}{T}=1$ $\boxed{A = T}$

$-W + \frac{2\pi}{T} \geq W$

$\frac{2\pi}{T} \geq 2W$

$T \leq \frac{\pi}{W}$

The maximum value of T is $\boxed{\frac{\pi}{W}}$ the maximum value of A is $\boxed{\frac{\pi}{W}}$

(d) $x(t) = x_2(t)\cos(2Wt)$

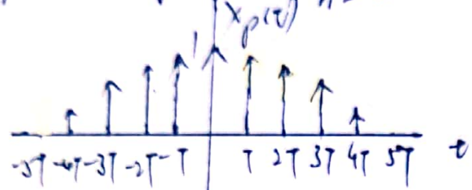
Fourier transform

$$X(\omega) = \frac{X_2(\omega - 2W) + X_2(\omega + 2W)}{2}$$

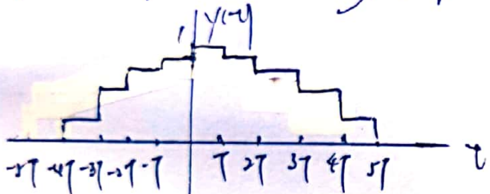
The range of cutoff frequency W_c is $\cancel{\pm W \text{ and } \pm 3W}$ $\boxed{|\omega_c| > 3W \text{ and } |\omega_c| \le \frac{\omega_s}{T} - 3W}$

No applicable T and A values such that $x_r(t) = x(t)$

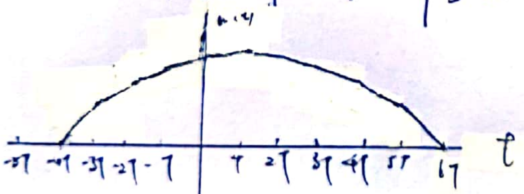
7. $x_p(t) = x(t)\,p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$



$y(t) = x_p(t) * h(t) = \int_{-\infty}^{\infty} x_p(\tau)\, h(t-\tau)\, d\tau$



$w(t) = y(t) * \frac{1}{T} h(t) = \frac{1}{T}\int_{-\infty}^{\infty} y(\tau)\, h(t-\tau)\, d\tau$



8. $w(t) = y(t)\cos(\omega_d t)$

$y(t) = x(t)\cos(\omega_c t)$

$w(t) = x(t)\cos(\omega_c t)\cos(\omega_d t)$

$= \frac{1}{2}x(t)\left(\cos((\omega_c + \omega_d)t) + \cos((\omega_c - \omega_d)t)\right)$

$= \frac{1}{2}x(t)\left(\cos((\omega_c + \omega_d)t) + \cos(\Delta\omega\, t)\right)$

Since $\omega_M + \Delta\omega < \omega_{co} < 2\omega_c + \Delta\omega - \omega_M$

and $\frac{1}{2}x(t)\cos((\omega_c + \omega_d)t)$ is in $\omega_c + \omega_d - \omega_M \le |\omega| \le \omega_c + \omega_d + \omega_M$

However, after filtering the spectrum $\omega \le \omega_{co} < 2\omega_c + \Delta\omega - \omega_M = \omega_d + \omega_c - \omega_M$

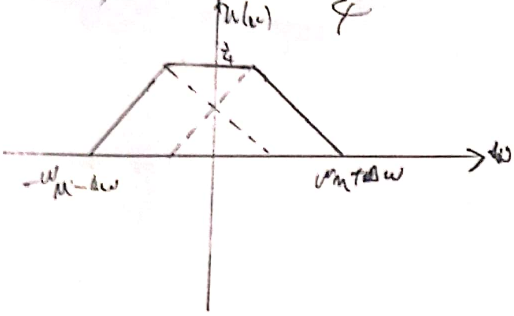
So this part is eliminated, $w(t) = \frac{1}{2}x(t)\cos(\Delta\omega\, t)$

Thus, the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega\, t)$

(b) $w(t) = \frac{1}{2} x(t)\cos(\Delta\omega \cdot t)$

Fourier transform:

$$W(\omega) = \frac{X(\omega-\Delta\omega)+X(\omega+\Delta\omega)}{4}$$



9. $x(t) = \mathrm{sinc}(\frac{t}{\pi})$

$\frac{1}{\pi}x(t) = \frac{1}{\pi}\mathrm{sinc}(\frac{t}{\pi})$ $\omega_0 = 2$

$\frac{1}{\pi}X(\omega) = \mathrm{rect}(\frac{\omega}{2})$

$X(\omega) = \pi\,\mathrm{rect}(\frac{\omega}{2})$

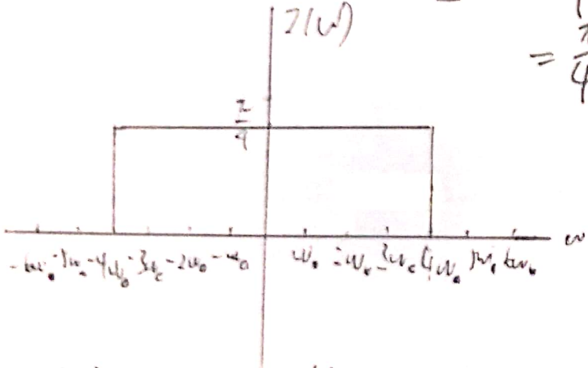
$y(t) = x(t)\cos(\omega_0 t)$

$$Y(\omega) = \frac{X(\omega-\omega_0)+X(\omega+\omega_0)}{2} = \frac{\pi}{2}\left(\mathrm{rect}\left(\frac{\omega-\omega_0}{2}\right) + \mathrm{rect}\left(\frac{\omega+\omega_0}{2}\right)\right)$$

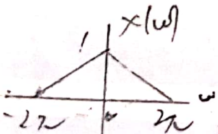
$z(t) = y(t)\cos(\omega_0 t)$

$$Z(\omega) = \frac{Y(\omega-\omega_0)+Y(\omega+\omega_0)}{2} = \frac{\pi}{4}\left(\mathrm{rect}\left(\frac{\omega-\omega_0}{2}-\omega_0\right)+\mathrm{rect}\left(\frac{\omega+\omega_0}{2}-\omega_0\right)+\mathrm{rect}\left(\frac{\omega-\omega_0}{2}+\omega_0\right)+\mathrm{rect}\left(\frac{\omega+\omega_0}{2}+\omega_0\right)\right)$$

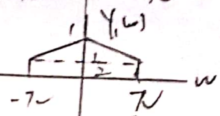
$$= \frac{\pi}{4}\left(\mathrm{rect}\left(\frac{\omega-3\omega_0}{2}\right) + \mathrm{rect}\left(\frac{\omega-\omega_0}{2}\right) + \mathrm{rect}\left(\frac{\omega+\omega_0}{2}\right) + \mathrm{rect}\left(\frac{\omega+3\omega_0}{2}\right)\right)$$



10. $X(\omega) = \left(1-\left|\frac{\omega}{2\pi}\right|\right)\mathrm{rect}\left(\frac{\omega}{4\pi}\right)$ Sketch of $X(\omega)$:

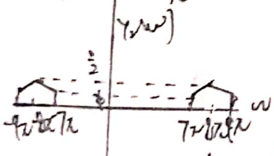


$Y_1(\omega) = X(\omega)H(\omega)$ Sketch of $Y_1(\omega)$.



$y_2(t) = y_1(t)\cos(8\pi t)$ $Y_2(\omega) = \frac{Y_1(\omega-8\pi)+Y_1(\omega+8\pi)}{2}$

Sketch of $Y_2(\omega)$.



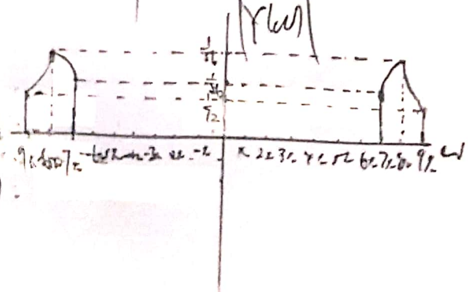
passing through an integrator system.

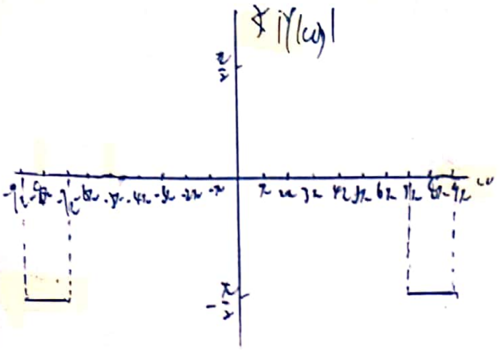
$$Y(\omega) = \frac{1}{j\omega}Y_2(\omega) + \pi Y_2(0)\delta(\omega) = \frac{1}{j\omega}Y_2(\omega)$$

$$= \frac{1}{2j\omega}(Y_1(\omega-8\pi)+Y_1(\omega+8\pi))$$

Sketch of $Y(\omega)$.

(continued)





11(a) The maximum absolute value of $x(t) = \cos\omega_m t$ is 1, so the modulation index $m = \frac{1}{A}$

$$y(t) = [A + x(t)]\cos(\omega_c t + \theta_c)$$
$$= [A + \cos(\omega_m t)]\cos(\omega_c t + \theta_c)$$
$$= A\cos(\omega_c t + \theta_c) + \cos(\omega_m t)\cos(\omega_c t + \theta_c)$$
$$= A\cos(\omega_c t + \theta_c) + \frac{1}{2}\cdot\cos((\omega_c + \omega_m)t + \theta_c) + \frac{1}{2}\cos((\omega_c - \omega_m)t + \theta_c)$$

Consider the power of a sinusoid of frequency ω_0,

$$P = \frac{1}{T}\int_0^T \cos^2(\omega_0 t)\,dt$$
$$= \frac{\omega_0}{2\pi}\int_0^{\frac{2\pi}{\omega_0}} \frac{1 + \cos(2\omega_0 t)}{2}\,dt$$
$$= \frac{\omega_0}{2\pi}\cdot\frac{1}{2}\cdot\frac{2\pi}{\omega_0} + \frac{\omega_0}{2\pi}\cdot\frac{1}{2}\cdot\frac{\sin(2\omega_0 t)}{2\omega_0}\Bigg|_0^{\frac{2\pi}{\omega_0}}$$
$$= \frac{1}{2}$$

$$P_y = \frac{1}{T}\int_T y^2(t)\,dt$$
$$= \frac{A^2}{2} + \frac{1}{8} + \frac{1}{8} = \frac{A^2}{2} + \frac{1}{2} = \frac{1}{2m^2} + \frac{1}{4}$$

$$\boxed{P_y = \frac{1}{2m^2} + \frac{1}{4}}$$

(b) Denote the efficiency as ε, then $\varepsilon = \dfrac{\frac{1}{2}}{\frac{1}{2}\left(\frac{1}{m^2} + \frac{1}{2}\right)} = \dfrac{m^2}{m^2 + 2}$

Thus the efficiency of the modulated signal as a function of the modulation index m is $\boxed{\dfrac{m^2}{m^2+2}}$

The sketch see the attached page.

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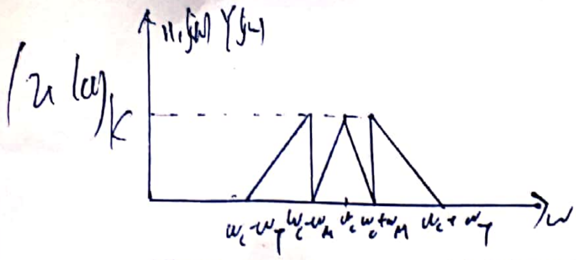


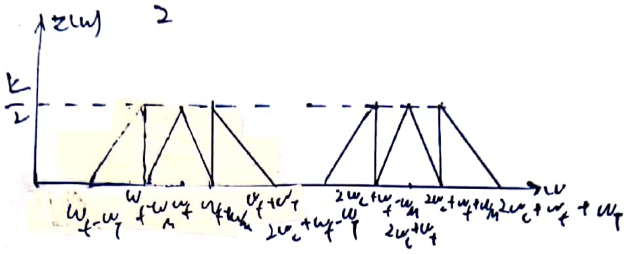
Denote $Y_2(j\omega) = H_1(j\omega)Y(j\omega)$

then $z(t) = y_2(t)\cos((\omega_c+\omega_f)t)$

F.T. $Z(j\omega) = \dfrac{Y_2(j(\omega+\omega_c+\omega_f)) + Y_2(j(\omega-\omega_c-\omega_f))}{2}$

Sketch of $Z(\omega)$:

for $\omega > 0$



(b)

$2\omega_c+\omega_f-\omega_T \geq \omega_f+\omega_M$

$\omega_c+2\omega_f-\omega_M \geq \omega_c+\omega_T$

$\Rightarrow \boxed{\begin{array}{c}\omega_T \geq \omega_M \\ \omega_T \leq 2\omega_c-\omega_M \\ \omega_T \leq 2\omega_f-\omega_M\end{array}}$

(c)

$\boxed{\begin{array}{c}G = \frac{2}{K} \\ \alpha = \omega_f-\omega_M \\ \beta = \omega_f+\omega_M\end{array}}$