刘翊华 Liu Yihua 518021910498 VE216 Homework 5

1. (a) $H(jw) = \frac{2jw+1}{-jw^3-5w^2+8jw+4} = \frac{-2w+j}{[w-2j]^2(w-j)} = \frac{2jw+1}{(jw+2)^2(jw+1)}$

$G(v) = \frac{2v+1}{(v+2)^2(v+1)} = \frac{A_{11}}{v+2} + \frac{A_{12}}{(v+2)^2} + \frac{A_{21}}{v+1}$

$A_1 = \frac{d}{dv}[(v+2)^2 G(v)]\Big|_{v=-2} = \frac{d}{dv}\left(\frac{2v+1}{v+1}\right)\Big|_{v=-2} = \frac{d}{dv}\left(2-\frac{1}{v+1}\right)\Big|_{v=-2}$

$= \frac{d}{dv}\left(-\frac{1}{v+1}\right)\Big|_{v=-2} = \frac{1}{(v+1)^2}\Big|_{v=-2} = 1$

$A_{12} = [(v+2)^2 G(v)]\Big|_{v=-2} = \frac{2v+1}{v+1}\Big|_{v=-2} = 3$

$A_{21} = [(v+1)G(v)]\Big|_{v=-1} = \frac{2v+1}{(v+2)^2}\Big|_{v=-1} = -1$

$$H(jw) = \frac{1}{jw+2} + \frac{3}{(jw+2)^2} - \frac{1}{jw+1}$$

MATLAB code:

```
Input: Coefficients of numerator and denominator polynomial
Output: Residues of PFE, poles of PFE, and direct term
b = [2 1];
a = [1 5 8 4];
[r, p, k] = residue (b, a)
```

Output:

```
r =
    1.0000
    3.0000
   -1.0000
p =
   -2.0000
   -2.0000
   -1.0000
k =
    []
```

In the case of repeated pole(s), p is ordered in A_{ik} k first, Hence, MATLAB gives $\frac{1}{jw+2} + \frac{3}{(jw+2)^2} - \frac{1}{jw+1}$ which is exactly our result.

(b) $e^{-at}u(t) \leftrightarrow \frac{1}{jw+a}$

$te^{-at}u(t) \leftrightarrow \frac{1}{(jw+a)^2}$

$H(jw) = \frac{1}{jw+2} + \frac{3}{(jw+1)^2} - \frac{1}{jw+1}$

Do Fourier Transform, the unit impulse response h(t) of this system is

$h(t) = e^{-2t}u(t) + 3te^{-2t}u(t) - e^{-t}u(t)$

$= \boxed{(e^{-2t} + 3te^{-2t} - e^{-t})u(t)}$

MATLAB plot

```
fplot((exp(-2*t)+3*t*exp(-2*t)-exp(-t))*heaviside(t),[2 8])
```

Graph see attached pages.

Use MATLAB impulse to generate a same plot Graph see attached pages

By comparing the two graphs, we can verify our results.

2. (a) $x(t) = \cos w_0 t$

$x_s(t) = x(t)p(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT) = \sum_{n=-\infty}^{\infty}x(nT)\delta(t-nT)$

$X_p(w) = \frac{1}{2\pi}X(w)*P(w) = \frac{1}{T}\sum_{k=-\infty}^{\infty}X(w-nw_s)$

$\cos w_0 t \leftrightarrow \pi\delta(w-w_0) + \pi\delta(w+w_0)$

$X(w-kw_s) = \pi\delta(w-w_0-nw_s) + \pi\delta(w+w_0-nw_s) \quad w_s = \frac{2\pi}{T} = 6\pi$

$= \pi\delta(w-w_0-6n\pi) + \pi\delta(w+w_0-6n\pi)$

$X_p(w) = 3\pi\sum_{k=-\infty}^{\infty}\left(\delta(w-w_0-6n\pi) + \delta(w+w_0-6n\pi)\right)$

The sketches are included in attached pages.

(b) For $\boxed{w_0 = \pi \text{ and } 5\pi}$ $x_p(t)$ is identical

By Sampling theorem. $w_s \geq 2w_{max} \Rightarrow w_{max} < 3\pi$

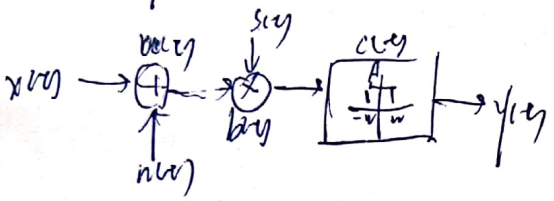
Thus, for $\boxed{w_0 = 3\pi \text{ and } 5\pi}$ we will NOT be able to recover the input sinusoidal signal after bypass filtering $x_p(t)$.

3. Denote the response of the unknown deterministic system as $P(\omega)$;
then $Q(\omega) = P(\omega) X_p(\omega)$ cos ωt can be seen as a function of ω, too
No matter what is $P(\omega)$ since $X_p(t)$ is a train of impulses is periodic as a function of ω
$Q(\omega)$ is constant for t, and $Q(\omega)$ should also be periodic

~~$P(n\omega_0)\cdot\delta(\omega - n\omega_0) = P(n\omega_0)\,\delta(\omega - n\omega_0)$~~

However, 0303(b) are not like this,
so the following two plots 0303(b) is NOT possible for the variation of Q as a function of ω, but (c) is possible

4. Denote the systems as follows

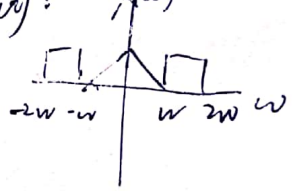


then $a(\omega) = x(\omega) + n(\omega)$
$b(\omega) = a(\omega) s(\omega)$

$$s(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

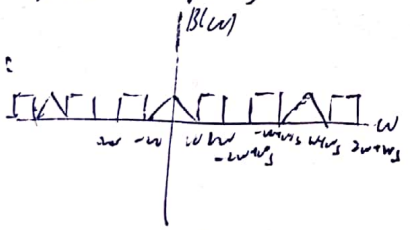
$$B(\omega) = \frac{1}{T} \sum_{n=-\infty}^{+\infty} A(\omega - n\omega_s), \quad \omega_s = \frac{2\pi}{T}$$

$P(\omega)$:



$$Y(\omega) = B(\omega) C(\omega) = X(\omega)$$

$B(\omega)$:

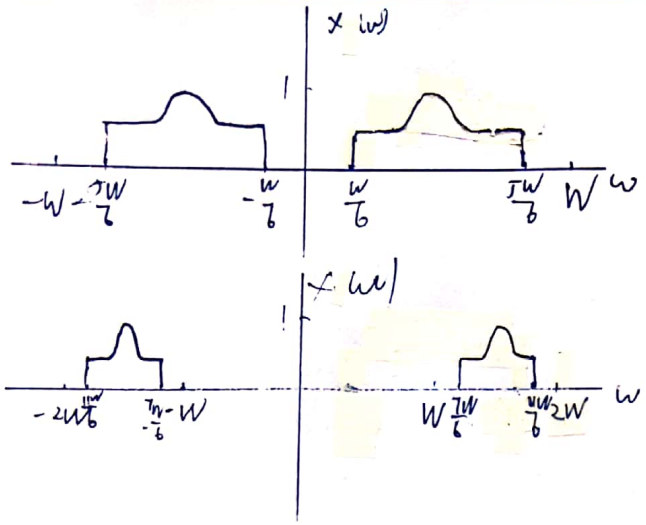


we have
$$-2w + \omega_s \geq w$$
$$\Rightarrow \omega_s = \frac{2\pi}{T} \geq 3w$$
$$\Rightarrow T \leq \frac{2\pi}{3w}$$

Suppose the amplitude of $X(\omega)$ and $b(\omega)$ is A_0,
then
$$\frac{A}{T} = A_0 \quad A \leq \frac{2\pi A_0}{3w}$$

Thus the maximum values for T is $\frac{2\pi}{3w}$
for A is $\frac{2\pi A_0}{3w}$

5. $\omega_{max} = \frac{5\pi}{6}$ $\omega_s = \frac{2\pi}{T} = 2w$ $\omega_s > 2\omega_{max}$, so $X(\omega)$ can simply be identical to $X_0(\omega)$
Alternatively, $X(\omega)$ can also be just one side part of $X_0(\omega)$
The sketches are:



6. (a) $x(t) = x_1(t-\frac{\pi}{2}) + x_2(t)$

Doing Fourier transform

$X(\omega) = X_1(\omega)e^{-j\omega\frac{\pi}{2}} + X_2(\omega)$, $X_1(\omega)=0$ for $|\omega|>2W$ and $X_2(\omega)=0$ for $|\omega|>W$

So the ranges for W_c is $|W_c| \geq 2W$ and $|W_c| \leq \frac{2\pi}{T} - 2W$

$X_p(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$

$\frac{A}{T} = 1$

then $A = T$

$-2W + \frac{2\pi}{T} \geq 2W$

$\frac{2\pi}{T} \geq 4W$

$T \leq \frac{\pi}{2W}$

Thus the maximum value of T is $\frac{\pi}{2W}$

the maximum value of A is $\frac{\pi}{2W}$

(b) $x(t) = x_1(t)x_2(t)$

$X(\omega) = \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$

By the property of convolution the range for W_c is $|W_c| \geq 3W$ and $|W_c| \leq \frac{2\pi}{T} - 3W$

$\frac{A}{T} = 1$ $A = T$

$-3W + \frac{2\pi}{T} \geq 3W$

$\frac{2\pi}{T} \geq 6W$

$T \leq \frac{\pi}{3W}$

The maximum value of T is $\frac{\pi}{3W}$, the maximum value of A is $\frac{\pi}{3W}$

(c) $x(t) = \frac{dx_0(t)}{dt}$

$X(\omega) = j\omega X_2(\omega)$ The range for W_c is $|W_c| \geq W$ and $|W_c| \leq \frac{\pi}{T} - W$

$\frac{A}{T} = 1$ $A = T$

$-W + \frac{\pi}{T} \geq W$

$\frac{\pi}{T} \geq 2W$

$T \leq \frac{\pi}{2W}$

The maximum value of T is $\frac{\pi}{2W}$ the maximum value of A is $\frac{\pi}{2W}$

(d) $x \cdot y = x_2(t)\cos(2Wt)$

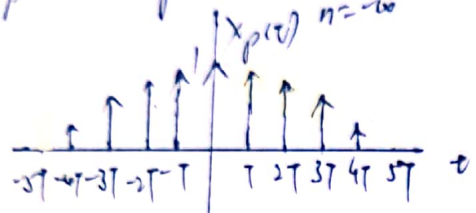
Fourier transform.

$X(\omega) = \frac{X_2(\omega - 2W) + X_2(\omega + 2W)}{2}$

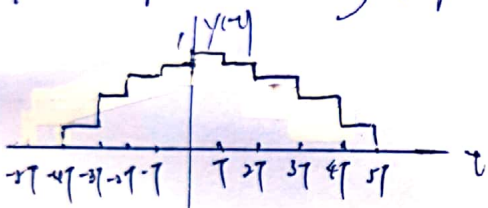
The range of cutoff frequency W_c is $\pm W$ and $\pm 3W$ | $|\omega_c| \geq 3W$ and $|\omega_c| \leq \frac{4\pi}{T} - 3W$

No applicable T and A values such that $x_r(t) = x(t)$

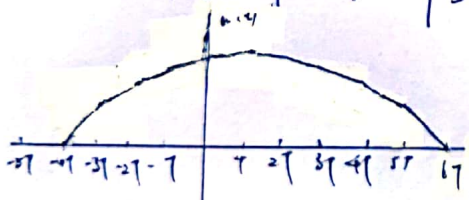
7. $x_p(t) = x(t) p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT)$



$y(t) = x_p(t) * h(t) = \int_{-\infty}^{\infty} x_p(\tau) h(t-\tau)\, d\tau$



$w(t) = y(t) * \frac{1}{T} h(t) = \frac{1}{T} \int_{-\infty}^{\infty} y(\tau) h(t-\tau) d\tau$



8. $v(t) = y(t)\cos(\omega_d t)$

$y(t) = x(t)\cos(\omega_c t)$

$w(t) = x(t)\cos(\omega_c t)\cos(\omega_d t)$

$= \frac{1}{2} x(t)\left(\cos((\omega_c + \omega_d)t) + \cos((\omega_c - \omega_d)t)\right)$

$= \frac{1}{2} x(t)\left(\cos((\omega_c + \omega_d)t) + \cos(\Delta\omega\, t)\right)$

Since $\omega_M + \Delta\omega < \omega_{co} < 2\omega_c + \Delta\omega - \omega_M$

and $\frac{1}{2} x(t)\cos((\omega_c + \omega_d)t)$ is in $\omega_c + \omega_d - \omega_M \leq |\omega| \leq \omega_c + \omega_d + \omega_M$

However, after filtering the spectrum $\omega \leq \omega_{co} < 2\omega_c + \Delta\omega - \omega_M = \omega_d + \omega_c - \omega_M$

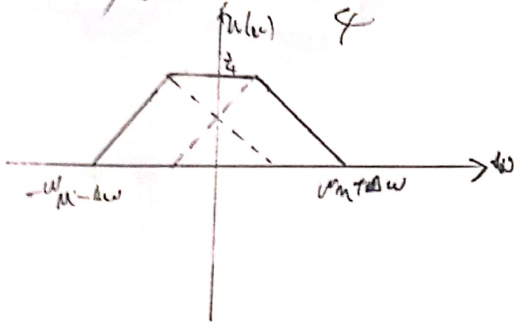
So this part is eliminated, $v(t) = \frac{1}{2} x(t)\cos(\Delta\omega\, t)$

Thus, the output of the lowpass filter in the demodulator is proportional to $x(t)\cos(\Delta\omega\, t)$

(b) $w(t) = \frac{1}{2} x(t) \cos(\Delta w \cdot t)$

Fourier transform:

$W(\omega) = \frac{X(\omega-\Delta\omega)+X(\omega+\Delta\omega)}{4}$



9. $x(t) = \text{sinc}\left(\frac{t}{\pi}\right)$

$\frac{1}{2}x(t) = \frac{1}{2}\text{sinc}\left(\frac{t}{\pi}\right) \quad \omega_0 = 2$

$\frac{1}{2}X(\omega) = \text{rect}\left(\frac{\omega}{2}\right)$

$X(\omega) = \pi\,\text{rect}\left(\frac{\omega}{2}\right)$

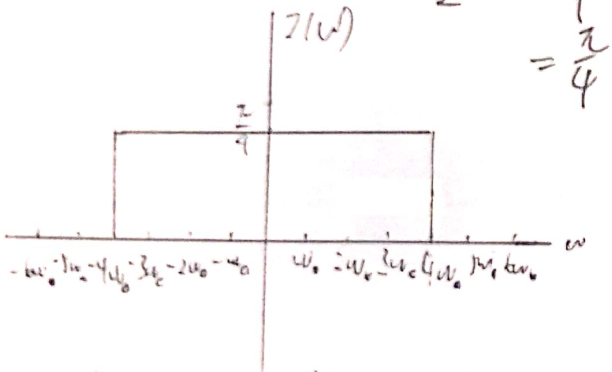
$y(t) = x(t)\cos(\omega_0 t)$

$Y(\omega) = \frac{X(\omega-\omega_0)+X(\omega+\omega_0)}{2} = \frac{\pi}{2}\left(\text{rect}\left(\frac{\omega-\omega_0}{2}\right)+\text{rect}\left(\frac{\omega+\omega_0}{2}\right)\right)$

$z(t) = y(t)\cos(\omega_0 t)$

$Z(\omega) = \frac{Y(\omega-\omega_0)+Y(\omega+\omega_0)}{2} = \frac{\pi}{4}\left(\text{rect}\left(\frac{\omega-\omega_0}{2}-\omega_0\right)+\text{rect}\left(\frac{\omega+\omega_0}{2}-\omega_0\right)+\text{rect}\left(\frac{\omega-\omega_0}{2}+\omega_0\right)+\text{rect}\left(\frac{\omega+\omega_0}{2}+\omega_0\right)\right)$

$= \frac{\pi}{4}\left(\text{rect}\left(\frac{\omega-3\omega_0}{2}\right)+\text{rect}\left(\frac{\omega-\omega_0}{2}\right)+\text{rect}\left(\frac{\omega+\omega_0}{2}\right)+\text{rect}\left(\frac{\omega+3\omega_0}{2}\right)\right)$



10. $X(\omega) = \left(1-\left|\frac{\omega}{2\pi}\right|\right)\text{rect}\left(\frac{\omega}{4\pi}\right)$

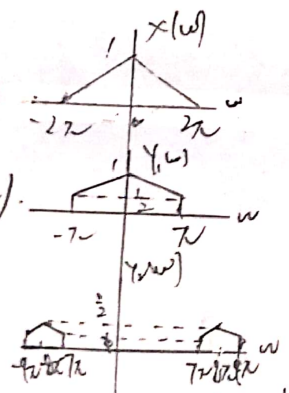
Sketch of $X(\omega)$:

$Y_1(\omega) = X(\omega)H(\omega)$

Sketch of $Y_1(\omega)$.

$y_2(t) = y_1(t)\cos(8\pi t) \quad Y_2(\omega) = \frac{Y_1(\omega-8\pi)+Y_1(\omega+8\pi)}{2}$

Sketch of $Y_2(\omega)$.

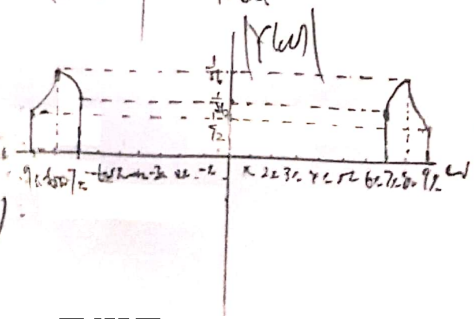


Passing through an integrator system.

$Y(\omega) = \frac{1}{j\omega}Y_2(\omega) + \pi Y_2(0)\delta(\omega) = \frac{1}{j\omega}Y_2(\omega)$

$= \frac{1}{2j\omega}\left(Y_1(\omega-8\pi)+Y_1(\omega+8\pi)\right)$

Sketch of $Y(\omega)$.



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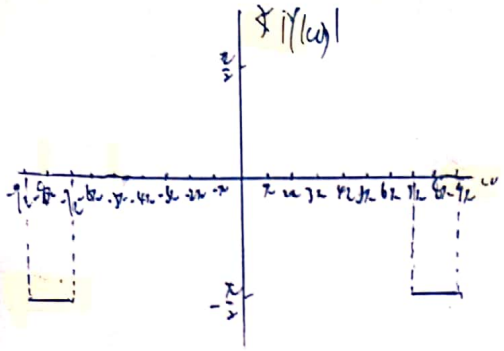


11(a) The maximum absolute value of $x(t) = \cos \omega_m t$ is 1, so the modulation index $m = \frac{1}{A}$

$$y(t) = [A + x(t)] \cos(\omega_c t + \theta_c)$$
$$= [A + \cos(\omega_m t)] \cos(\omega_c t + \theta_c)$$
$$= A\cos(\omega_c t + \theta_c) + \cos(\omega_m t)\cos(\omega_c t + \theta_c)$$
$$= A\cos(\omega_c t + \theta_c) + \frac{1}{2}\cos((\omega_c + \omega_m)t + \theta_c) + \frac{1}{2}\cos((\omega_c - \omega_m)t + \theta_c)$$

Consider the power of a sinusoid of frequency ω_0,

$$P_x = \frac{1}{T}\int_0^T \cos^2(\omega_0 t)\,dt$$
$$= \frac{\omega_0}{2\pi}\int_0^{\frac{2\pi}{\omega_0}} \frac{1+\cos(2\omega_0 t)}{2}\,dt$$
$$= \frac{\omega_0}{2\pi}\cdot\frac{1}{2}\cdot\frac{2\pi}{\omega_0} + \frac{\omega_0}{2\pi}\cdot\frac{1}{2}\cdot\frac{\sin(2\omega_0 t)}{2\omega_0}\Big|_0^{\frac{2\pi}{\omega_0}}$$
$$= \frac{1}{2}$$

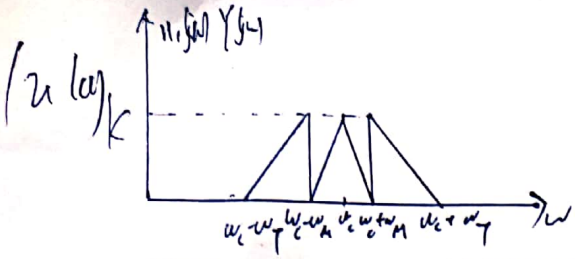
$$P_y = \frac{1}{T}\int_T y^2(t)\,dt$$
$$= \frac{A^2}{2} + \frac{1}{8} + \frac{1}{8} = \frac{A^2}{2} + \frac{1}{4} = \frac{1}{2m^2} + \frac{1}{4}$$

$$\boxed{P_y = \frac{1}{2m^2} + \frac{1}{4}}$$

(b) Denote the efficiency as ε, then $\varepsilon = \frac{\frac{1}{2}}{\frac{1}{2}\left(\frac{1}{m^2}+\frac{1}{2}\right)} = \frac{m^2}{m^2+2}$

Thus the efficiency of the modulated signal as a function of the modulation index m is $\boxed{\frac{m^2}{m^2+2}}$

The Sketch see the attached page.



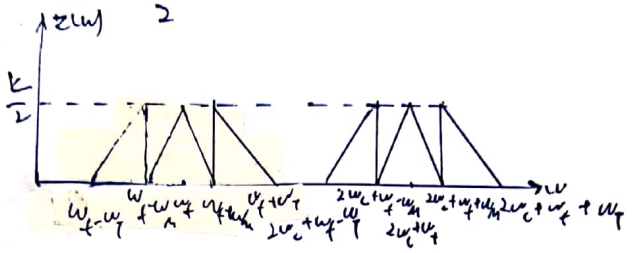
Denote $Y_2(jw) = H_1(jw)Y(jw)$

then $z(t) = y_2(t)\cos(w_c + w_f)t$

F.T. $Z(jw) = \frac{Y_2(j(w+w_c+w_f)) + Y_2(j(w-w_c-w_f))}{2}$

Sketch of $Z(w)$:

for $w>0$



(b)

$2w_c + w_f - w_f \geq w_f + w_m$

$2w_c + 2w_f - w_m \geq w_c + w_f$

$\Rightarrow$

$w_f \geq w_m$

$w_f \leq 2w_c - w_m$

$w_f \leq 2w_f - w_m$

(c)

$G = \frac{2}{K}$

$\alpha = w_f - w_m$

$\beta = w_f + w_m$

VE216
Introduction to Signals and Systems

HOMEWORK 5 ATTACHED PAGES

April 24, 2020

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1. (b) The graph of my answer as a function of t is

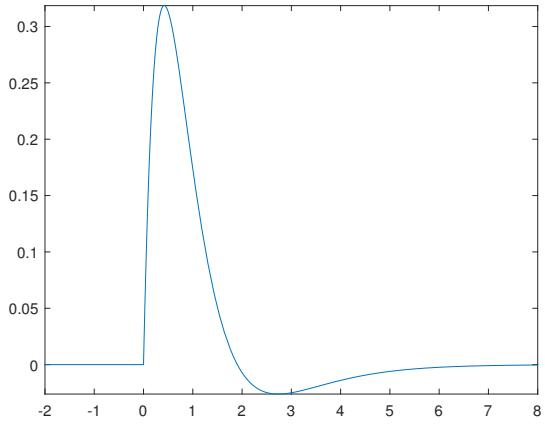

Figure 1. 1(b)-1.

The graph generated by MATLAB impulse is

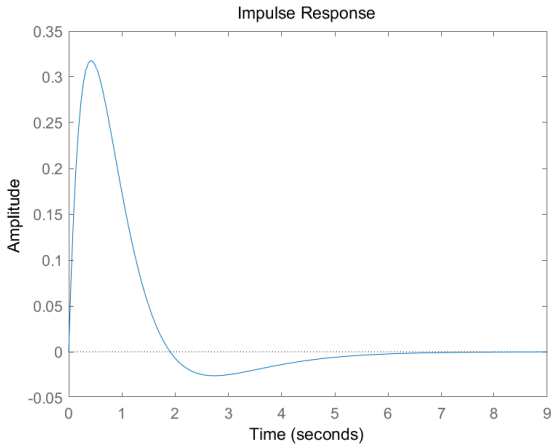

Figure 2. 1(b)-2.

2. (a) The sketches of $X_p(\omega)$ for $-9\pi \leq \omega \leq 9\pi$ for $\omega_0 = \pi,\ 2\pi,\ 3\pi,$ and 5π are respectively

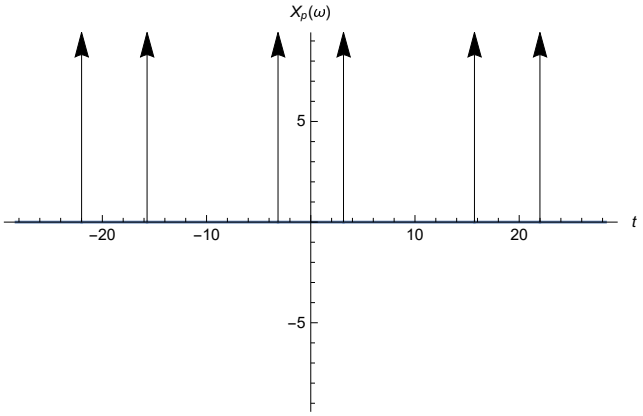

Figure 3. 2(a)i.

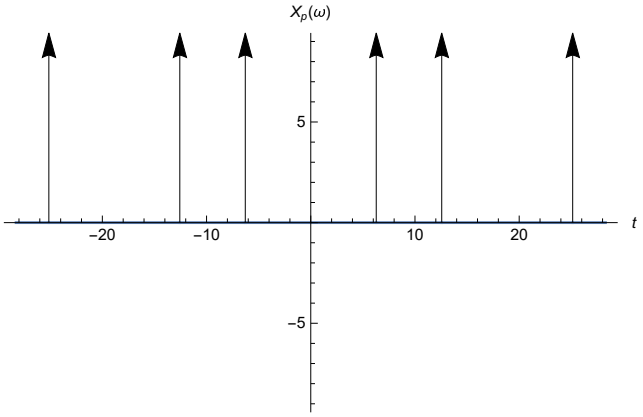

Figure 4. 2(a)ii.

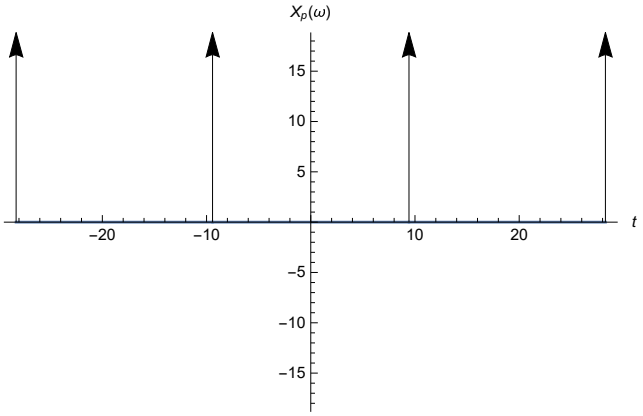

Figure 5. 2(a)iii.

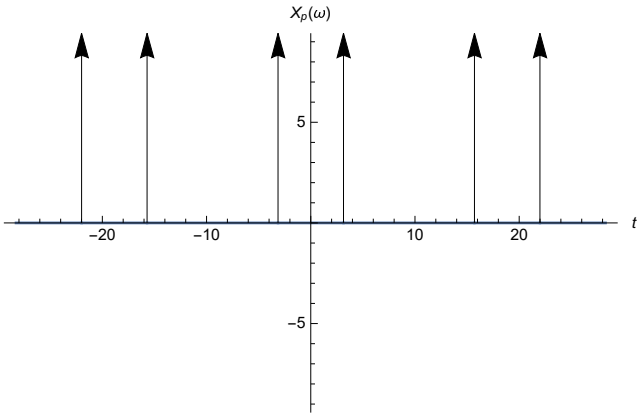

Figure 6. 2(a)iv.

11. (b) The sketch of the efficiency of the modulated signal as a function of the modulated index m is

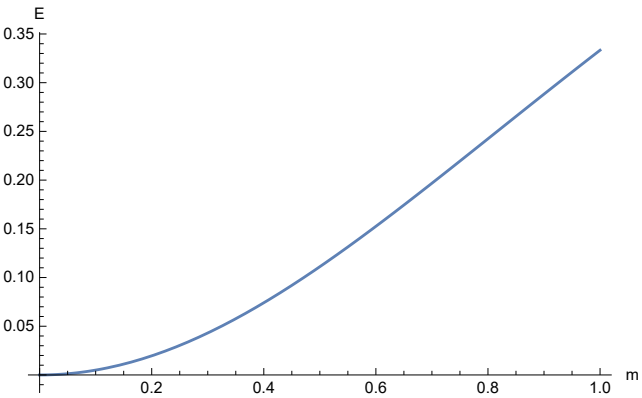

Figure 7. 11(b).