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4.1 (a) $RC\frac{dV_{out}(t)}{dt}+V_{out}(t)=V_{in}(t)$

Let $V_{out}(t)=y_{step}(t)$, $V_{in}(t)=u(t)$

$$y_{step}(t)=(1-e^{-t/RC})u(t)=\begin{cases}1-e^{-t/RC} & t\geq 0\\ 0 & t<0\end{cases}$$

$t>0$ $\quad RC\frac{1}{RC}e^{-\frac{t}{RC}}+1-e^{-t/RC}=1$

$t<0$

$$0=0$$

thus, the solution corresponding to $V_{in}(t)=u(t)$ is the step response

(b) $y_{step}=(1-e^{-1000t})u(t)$

Plot see attached pages

4.2 Using (3.e.4) $\frac{dy_{step}(t)}{dt}=\frac{d}{dt}\int_{-\infty}^{t}h(\tau)d\tau=h(t)$

$$\frac{dy_{step}(t)}{dt}=\frac{d}{dt}(1-e^{-t/RC})u(t)=\delta(t)(1-e^{-t/RC})+\frac{1}{RC}e^{-t/RC}u(t)$$

$\delta(t)(1-e^{-t/RC})=0$ $\quad\boxed{h(t)=\frac{1}{RC}e^{-t/RC}u(t)}$

4.3 (a) Using time-invariance with the step response

$$y_{step}=(1-e^{-t/RC})u(t)$$

The output of $u(t)-u(t-\Delta)$ is

$$(1-e^{-t/RC})u(t)-(1-e^{-(t-\Delta)/RC})u(t-\Delta)$$

Using linearity

The output of $V_{in}(t)$ is

$$V_{out}(t)=y_{b,\Delta}(t)=\frac{b}{\Delta}[(1-e^{-t/RC})u(t)-(1-e^{-(t-\Delta)/RC})u(t-\Delta)]$$

(b) When $b=1$

$$\lim_{\Delta\to 0} y_{b,\Delta}(t) = \lim_{\Delta\to 0} \frac{1}{\Delta}\left[(1-e^{-t/RC})u(t) - (1-e^{-(t-\Delta)/RC})u(t-\Delta)\right]$$

Using L'Hôpital's rule

$$= \lim_{\Delta\to 0}\left(-\frac{1}{RC}e^{-(t-\Delta)/RC}u(t-\Delta) - (1-e^{-(t-\Delta)/RC})\delta(t-\Delta)\right)$$

$$= -\frac{1}{RC}e^{-t/RC}u(t) - (1-e^{-t/RC})\delta(t)$$

$$= -\frac{1}{RC}e^{-t/RC}u(t)$$

(c) (i) $$y_{b,\Delta}(t) = \frac{10^{-4}}{10^{-3}}\left[(1-e^{-1000t})u(t) - (1-e^{-1000(t-10^{-3})})u(t-10^{-3})\right]$$

$$= \frac{1}{10}\left[(1-e^{-1000t})u(t) - (1-e^{-1000(t-10^{-3})})u(t-10^{-3})\right]$$

(ii) $$y_{b,\Delta}(t) = \frac{1}{5}\left[(1-e^{-1000t})u(t) - (1-e^{-1000(t-0.5\times 10^{-3})})u(t-0.5\times 10^{-3})\right]$$

(iii) $$y_{b,\Delta}(t) = (1-e^{-1000t})u(t) - (1-e^{-1000(t-0.1\times10^{-3})})u(t-0.1\times 10^{-3})]$$

MATLAB plots see attached pages

$$h(t) = \frac{1}{RC}e^{-t/RC}u(t) = 1000e^{-1000t}u(t)$$

$$bh(t) = 10^{-4}h(t) = \frac{1}{10}e^{-1000t}u(t)$$

It can be observed from the three plots that

the plots for cases (i) - (iii) approach the plot of $bh(t)$ as Δ decreases.

From the three graphs we can verify our answers for parts (i) - (iii) are correct by the partial results provided in hint.

4.4 $$V_{out} = V_{in}(t) * h(t)$$

$$= \int_{-\infty}^{\infty} [u(\tau) - u(\tau-0.01)] + 200(\tau-0.011)[u(\tau-0.011) - u(\tau-0.016)]$$
$$1000\, e^{-1000(t-\tau)} u(t-\tau)\, d\tau$$

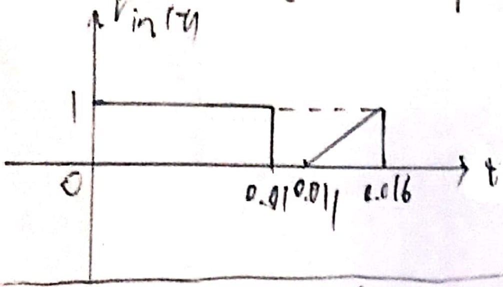
$$= \int_0^{0.01} 1000\, e^{-1000(t-\tau)} u(t-\tau)\, d\tau + \int_{0.011}^{0.016} 2\times10^5\, e^{-1000(t-\tau)} u(t-\tau)(\tau-0.011)\, d\tau$$

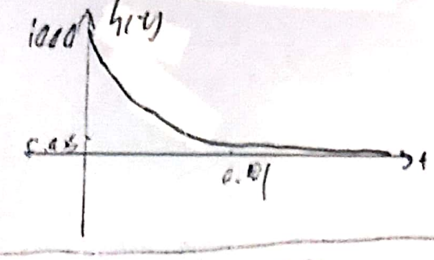
$$= \int_{t-0.01}^{t} 1000\, e^{-1000\tau}\, d\tau + \int_{t-0.016}^{t-0.011} 2\times10^5\, e^{-1000\tau}(t-\tau-0.011)\, d\tau$$

$$= -e^{-1000\tau}\Big|_{t-0.01}^{t} - (t-\tau-0.011)\,200\, e^{-1000\tau}\Big|_{t-0.016}^{t-0.011}$$

$$= e^{-1000(t-0.01)} - e^{-1000t} + \quad 0.8\times e^{-1000(t-0.016)} - \quad 0.2\times e^{-1000(t-0.0}$$

$$V_{out} = e^{-1000t}\left(e^{10}-1 + 0.8\times e^{16} - 0.2\times e^{11}\right)$$





$$\boxed{\begin{aligned} V_{out} &= \left(e^{10} e^{-1000t} - 1\right) u(t-0.01) + \left(1 - e^{-1000t}\right) u(t) \\ &+ \left(0.8 e^{-1000(t-0.016)} - 0.4 e^{-1000(t-0.011)} - 200(t-0.012)\right) u(t-0.016) \\ &+ \left(200(t-0.012) + 0.2 e^{-1000(t-0.011)}\right) u(t-0.011) \end{aligned}}$$

$t<0$ conv $=0$

$0 \le t < 0.01$ $\quad$ conv $= \int_0^t 1000 e^{-1000\tau}\, d\tau = 1 - e^{-1000t}$

$0.01 \le t < 0.011$ $\quad$ Conv $= \int_{t-0.01}^{t} 1000 e^{-1000\tau}\, d\tau = e^{10} e^{-1000t} - e^{-1000t}$

$0.011 \le t < 0.016$ $\quad$ conv $= \int_{t-0.01}^{t} 1000 e^{-1000\tau}\, d\tau + 200 \int_{0.011}^{t} 2\times10^5 (\tau - 0.011)/1000\, e^{-1000(t-\tau)}\, d\tau$

$$= e^{-1000t}\left(e^{10}-1\right) + 200(\tau - 0.012)\, e^{-1000(t-\tau)} \Big|_{0.011}^{t}$$

$$= e^{-1000t}\left(e^{10}-1\right) + 200(t-0.012) + 0.2 e^{-1000(t-0.011)}$$

$t \ge 0.016$ $\quad$ Conv $= \int_{t-0.01}^{t} 1000 e^{-1000\tau}\, d\tau + 200 \int_{0.011}^{0.016} (\tau - 0.011)\, 1000\, e^{-1000(t-\tau)}\, d\tau$

$$= e^{-1000t}\left(e^{10}-1\right) + 200(\tau - 0.012)\, e^{-1000(t-\tau)} \Big|_{0.011}^{0.016}$$

$$= e^{-1000t}\left(e^{10}-1\right) + 200\times(-0.001)\, e^{-1000(t-0.011)} + 200\times 0.004\, e^{-1000(t-0.016)}$$

$$= e^{-1000t}\left(e^{10}-1\right) + \left(0.8 e^{16} - 0.2 e^{11}\right) e^{-1000t}$$

We can verify the correctness of our answer with the provided partial results.

$V_{out}(2ms) = 1 - e^{-1000\times0.002} = 0.865V$, $V_{out}(12ms) = e^{-1000\times0.012}(e^{10}-1) + 0.2 e^{-1000\times0.001} = 0.21V$,

$V_{out}(18ms) = e^{-1000\times0.018}\left(e^{10}-1) + (0.8)e^{16} - 0.2e^{11}\right) \times e^{1000\times0.018} = 0.11V$

4.5 Using the phasor circuit analysis learned in VE215,

$$\frac{V_{out}}{\frac{1}{j\omega C}} = \frac{V_{in}}{R + \frac{1}{j\omega C}}$$

Frequency transfer function $\quad H(j\omega) = \dfrac{V_{out}}{V_{in}} = \dfrac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \boxed{\dfrac{1}{j\omega RC + 1}}$

4.b $H(j2\pi f_c) = \frac{1}{j2\pi f_c RC+1}$

$$|H(j2\pi f_c)| = \left|\frac{1}{j2\pi f_c RC+1}\right| = \frac{1}{\sqrt{4\pi^2 f_c^2 R^2 C^2 + 1}} = \frac{1}{\sqrt{4\pi^2 \times 10^{-6} f_c^2 + 1}}$$

f_c (Hz)	$\lvert H(j2\pi f_c)\rvert$	$\angle(H(j2\pi f_c))$ (deg)	τ_d (ms)
50	0.9540	-17.4406	0.9689
200	0.6227	-51.4881	0.7151
500	0.3033	-72.3432	0.4019
1k	0.1572	-80.9569	0.2249
5k	0.0318	-88.1768	0.0490

$\angle(H(j2\pi f_c)) = -\arctan(2\pi f_c RC) = -\arctan(2\pi \times 10^{-3} f_c)$ (in degree)

$$\tau_d = \frac{-\angle H(j2\pi f_c)}{2\pi f_c} = \frac{\arctan(2\pi f_c RC)}{2\pi f_c} = \frac{\arctan(2\pi \times 10^{-3} f_c)}{2\pi f_c} \text{ (in radian)}$$