VE216
Introduction to Signals and Systems

HOMEWORK 1

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1.
(a) The mathematical representation for this signal is

$$x(t) = |\sin t|.$$

(b) The energy of this signal is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \mathrm{d}t = \int_{-\infty}^{\infty} \sin^2 t \mathrm{d}t = (\frac{t}{2} - \frac{\sin 2t}{4})\bigg|_{-\infty}^{\infty} = \infty.$$

Since E is infinite, we would like to calculate the power P:

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \mathrm{d}t = \lim_{T\to\infty} (\frac{1}{2} - \frac{\sin 2T}{4T}) = \frac{1}{2}.$$

Since P is finite and nonzero, it is an power signal rather than energy signal.

(c) For $t \in [-\pi, 0]$, $x(t) = -\sin t$, $y(t) = -\int_{-\infty}^{t} \sin \tau \mathrm{d}\tau = \cos t + 1$. For $t \in [0, 2\pi]$, $x(t) = \sin t$, $y(t) = \int_{-\infty}^{t} \sin \tau \mathrm{d}\tau = \cos 0 + 1 + \int_{0}^{t} \sin \tau \mathrm{d}\tau - \cos t + 1 = 3 - \cos t$.

Hence,

$$y(t) = \begin{cases} \cos t + 1, t \in [-\pi, 0] & (1) \\ 3 - \cos t, t \in [0, \pi] & (2) \end{cases}$$

The sketch is shown below:

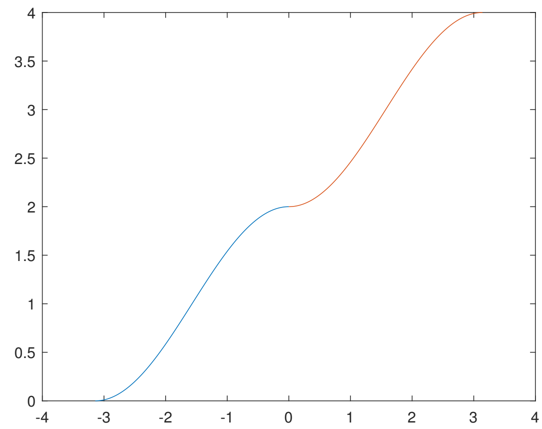


Figure 1. 1(c).

2.

(a) The average power is

$$
\begin{aligned}
P &= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|e^{-2t}u(t)|^2\mathrm{d}t \\
&= \lim_{T\to\infty}\frac{1}{2T}(\int_{-T}^{0}|e^{-2t}\cdot 0|^2\mathrm{d}t+\int_{0}^{T}|e^{-2t}\cdot 1|^2\mathrm{d}t) \\
&= \lim_{T\to\infty}\frac{1}{2T}\int_{0}^{T}e^{-4t}\mathrm{d}t \\
&= \lim_{T\to\infty}-\frac{1}{8T}(e^{-4T}-1) \\
&= 0.
\end{aligned}
$$

The energy is

$$E=\int_{-\infty}^{\infty}|e^{-2t}u(t)|^2\mathrm{d}t=\int_{-\infty}^{0}|e^{-2t}\cdot 0|^2\mathrm{d}t+\int_{0}^{\infty}|e^{-2t}\cdot 1|^2\mathrm{d}t=\int_{0}^{\infty}e^{-4t}\mathrm{d}t=\frac{1}{4}.$$

(b) Using $|e^{j(\omega_0 t+\varphi)}|=1$, we have the average power is

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|e^{j(2t+\frac{\pi}{4})}|^2\mathrm{d}t=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}1\mathrm{d}t=1.$$

The energy is

$$E=\int_{-\infty}^{\infty}|e^{j(2t+\frac{\pi}{4})}|^2\mathrm{d}t=\int_{-\infty}^{\infty}1\mathrm{d}t=\infty.$$

(c) The average power is

$$P=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|\cos t|^2\mathrm{d}t=\lim_{T\to\infty}(\frac{1}{2T}+\frac{\sin 2T}{4T})=\frac{1}{2}.$$

The energy is

$$E=\int_{-\infty}^{\infty}|\cos t|^2\mathrm{d}t=(\frac{t}{2}+\frac{\sin 2t}{4})\Big|_{-\infty}^{\infty}=\infty.$$

(d) The average power is

$$
\begin{aligned}
P &= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{+N}|(\frac{1}{2})^n u[n]|^2 \\
&= \lim_{N\to\infty}\frac{1}{2N+1}(\sum_{n=-N}^{-1}|(\frac{1}{2})^n\cdot 0|^2+\sum_{n=0}^{+N}|(\frac{1}{2})^n\cdot 1|^2) \\
&= \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=0}^{+N}(\frac{1}{4})^n \\
&= \lim_{N\to\infty}\frac{4}{3(2N+1)}(1-\frac{1}{4^N}) \\
&= 0.
\end{aligned}
$$

The energy is

$$E=\sum_{n=-\infty}^{+\infty}|(\frac{1}{2})^n u[n]|^2=\sum_{n=-\infty}^{-1}|(\frac{1}{2})^n\cdot 0|^2+\sum_{n=0}^{\infty}|(\frac{1}{2})^n\cdot 1|^2=\sum_{n=0}^{\infty}(\frac{1}{4})^n=\frac{4}{3}.$$

(e) The average power is

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |e^{j(\frac{\pi}{2n}+\frac{pi}{8})}|^2 = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} 1 = 1.$$

The energy is

$$E = \sum_{n=-\infty}^{+\infty} |e^{j(\frac{\pi}{2n}+\frac{\pi}{8})}|^2 = \sum_{n=-\infty}^{+\infty} 1 = \infty.$$

(f) Since $x_3[n] = \cos\frac{\pi}{4}n$ is a periodic signal with the period $T = \frac{2\pi}{\omega_0} = 8$, we have $\sum_{n=N}^{N+T-1} |\cos\frac{\pi}{4}n| = 2+2\sqrt{2}$. Thus, the energy in a period is

$$E_{\text{period}} = \sum_{n=0}^{T-1} |\cos\frac{\pi}{4}n|^2 = 12 + 8\sqrt{2}$$

and the average power in a period

$$P_{\text{period}} = \frac{1}{T} E_{\text{period}} = \frac{3}{2} + \sqrt{2}.$$

Thus, the average power is

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{+N} |\cos\frac{\pi}{4}n|^2 = \frac{3}{2} + \sqrt{2}.$$

The power is

$$E = \sum_{n=-\infty}^{+\infty} |\cos\frac{\pi}{4}n|^2 = \infty.$$

3.
The average value is

$$A = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\mathrm{d}t = \lim_{T\to\infty} \frac{1}{2T}(\int_{-T}^{0} 0 \cdot \mathrm{d}t + \int_{0}^{T} e^{-t}\mathrm{d}t) = \lim_{T\to\infty} -\frac{e^{-T}}{2T} = 0.$$

The average power is

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\mathrm{d}t = \lim_{T\to\infty} \frac{1}{2T}(\int_{-T}^{0} 0 \cdot \mathrm{d}t + \int_{0}^{T} |e^{-t}|^2\mathrm{d}t) = \lim_{T\to\infty} -\frac{e^{-2T}}{4T} = 0.$$

The energy is

$$E = \int_{-\infty}^{\infty} |x(t)|^2\mathrm{d}t = \int_{-\infty}^{0} 0 \cdot \mathrm{d}t + \int_{0}^{\infty} |e^{-t}|^2\mathrm{d}t = \int_{0}^{\infty} e^{-2t}\mathrm{d}t = \frac{1}{2}.$$

4.
(a) A mathematical representation fot $x(t)$ is

$$x(t) = (t+2)(u(\frac{t}{2}+1) - u(\frac{t}{2})) + 2(u(t) - u(t-2)) + 2(t-1)(u(t-1) - u(t-2)).$$

(b) To sketch $s(t) = x(-2t+1)/2$, we first do time shifting:

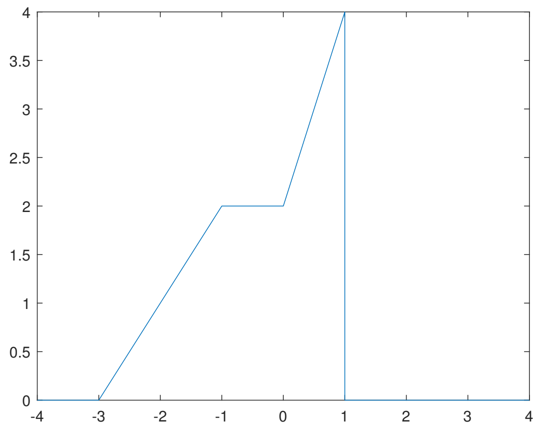


Figure 2. $x(t+1) = (t+3)(u(\frac{t}{2}+\frac{3}{2}) - u(\frac{t}{2}+\frac{1}{2})) + 2(u(t+1) - u(t-1)) + 2t(u(t) - u(t-1))$

and then do time scaling:

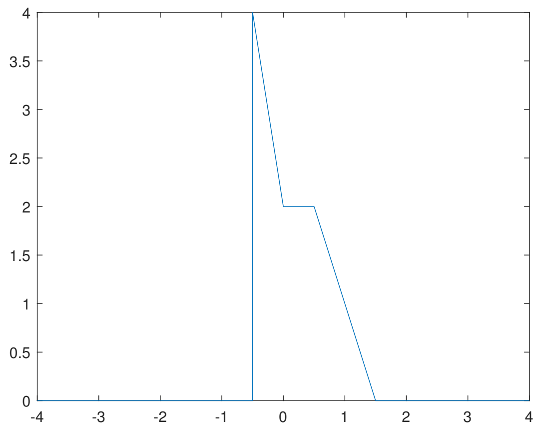


Figure 3. $x(-2t+1) = (-2t+3)(u(-t+\frac{3}{2}) - u(-t+\frac{1}{2})) + 2(u(-2t+1) - u(-2t-1)) - 4t(u(-2t) - u(-2t-1))$.

Finally, we do amplitude scaling and finally sketch $s(t)$:

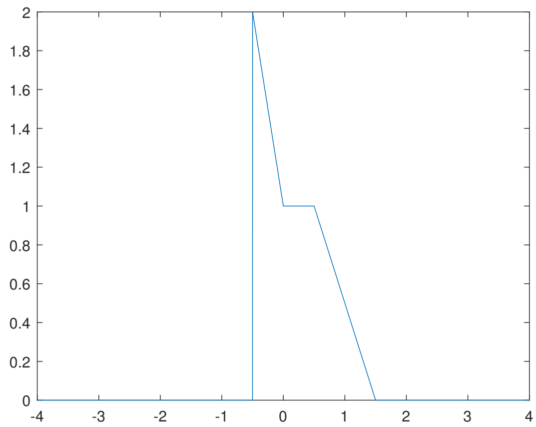


Figure 4. $s(t) = x(-2t+1)/2 = (-t+\frac{3}{2})(u(-t+\frac{3}{2}) - u(-t+\frac{1}{2})) + (u(-2t+1) - u(-2t-1)) - 2t(u(-2t) - u(-2t-1))$.

(c) We can decompose $x(t)$ into its even and odd components by $x(t) = x_e(t) + x_o(t)$ where the even component is

$$\begin{aligned} x_e(t) =& \frac{1}{2}[x(t) + x(-t)] \\ &= (\frac{1}{2}t+1)(u(t+2) - u(t)) + u(t) - u(t-2) + (t-1)(u(t-1) - u(t-2)) \\ &+ (-\frac{1}{2}t+1)(u(-t+2) - u(-t)) + u(-t) - u(-t-2) + (-t-1)(u(-t-1) - u(-t-2)) \end{aligned}$$

and the odd component is

$$\begin{aligned} x_o(t) =& \frac{1}{2}[x(t) - x(-t)] \\ &= (\frac{1}{2}t+1)(u(t+2) - u(t)) + u(t) - u(t-2) + (t-1)(u(t-1) - u(t-2)) \\ &- (-\frac{1}{2}t+1)(u(-t+2) - u(-t)) - u(-t) + u(-t-2) - (-t-1)(u(-t-1) - u(-t-2)). \end{aligned}$$

Their sketches are shown below respectively:

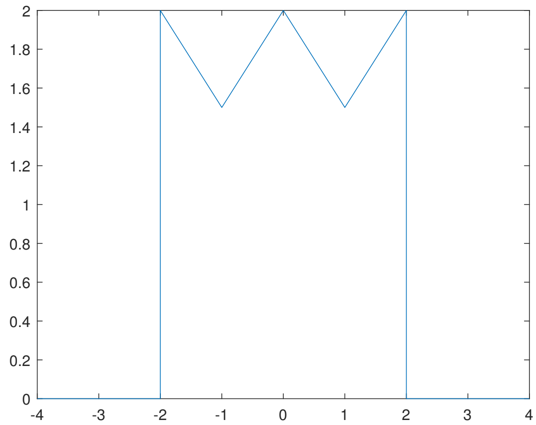


Figure 5. The sketch of the even component.

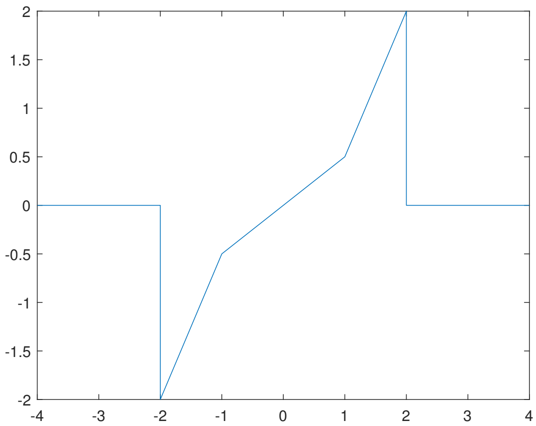
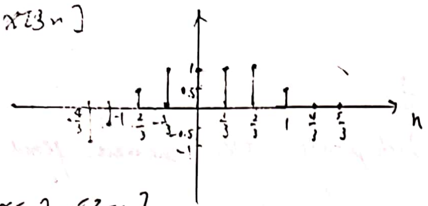
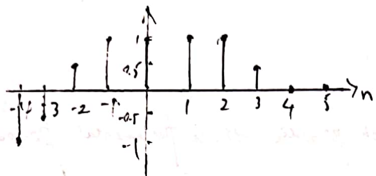


Figure 6. The sketch of the odd component.

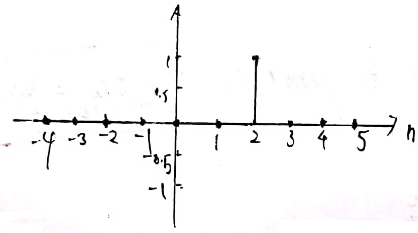
5. (a) $x[3n]$



(b) $x[n]\,u[3-n]$



(c) $x[n-2]\,\delta[n-2]$



6.

(a) Since $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods $T_1, T_2 > 0$

$x_1(t) = x_1(t+T_1)$, $x_2(t) = x_2(t+T_2)$

If $\frac{T_1}{T_2}$ is rational, suppose $\frac{T_1}{T_2} = \frac{k_2}{k_1}$ where k_1, k_2 are integers, then $k_1T_1 = k_2T_2$

Thus we can find $T = k_1T_1 = k_2T_2$ where T satisfies

$x_1(t+T) = x_1(t)$, $x_2(t+T) = x_2(t)$

thus $x_1(t+T) + x_2(t+T) = x_1(t) + x_2(t)$

Hence $x(t) = x_1(t) + x_2(t)$ is periodic

(b) If $\frac{T_1}{T_2}$ is rational, suppose $\frac{T_1}{T_2} = \frac{k_2}{k_1}$ where k_1, k_2 are integers, then $k_1T_1 = k_2T_2$

then we can find $T = k_1T_1 = k_2T_2 = \mathrm{lcm}(T_1, T_2)$ that

$x_1(t+T) = x_1(t)$, $x_2(t+T) = x_2(t)$

$x_1(t+T)\,x_2(t+T) = x_1(t)\,x_2(t)$

Hence $x(t) = x_1(t)\,x_2(t)$ is periodic.

(c) $x(t) = x_1(t) + x_2(t)$

$x_1(t) = \sin\left(\frac{\pi t}{3}\right)\cos\left(\frac{\pi t}{4}\right)$

$\sin\frac{\pi t}{3}$ and $\cos\frac{\pi t}{4}$ are both periodic, their fundamental period

$w_1 = \frac{\pi}{3}$ $\quad w_2 = \frac{\pi}{4}$ $\qquad T_1 = \text{lcm}(6, 8) = 24$

$T_{11} = 6$ $\quad T_{12} = 8$

$x_2(t) = \sin\left(\frac{\pi t}{5}\right)\sin\left(\frac{\pi t}{2}\right)$

$\sin\frac{\pi t}{5}$ and $\sin\frac{\pi t}{2}$ are both periodic, their fundamental period

$w_1 = \frac{\pi}{5}$ $\quad w_2 = \frac{\pi}{2}$ $\qquad T_2 = \text{lcm}(10, 4) = 20$

$T_{21} = 10$ $\quad T_{22} = 4$

Hence, $x(t)$ is periodic and its period is $\boxed{T = \text{lcm}(T_1, T_2) = 120}$

$x(t) = \sin\frac{\sqrt{3}\pi t}{3} + \sin\frac{\pi t}{5}$

$w_1 = \frac{\sqrt{3}\pi}{3}$ $\quad w_2 = \frac{\pi}{5}$

$T_1 = 2\sqrt{3}$ $\quad T_2 = 10$

We first want to prove

when $x_1(t)$ and $x_2(t)$ are periodic, $x(t) = x_1(t) + x_2(t)$ is periodic if and only if $\frac{T_1}{T_2}$ is rational
(T_1) $\quad (T_2)$

We have proved if $\frac{T_1}{T_2}$ is rational, $x(t)$ is periodic,

We now prove if $x(t)$ is periodic, $\frac{T_1}{T_2}$ is rational:

If $x(t)$ is periodic, $x(t) = x_1(t) + x_2(t) = x_1(t+T) + x_2(t+T)$

This equation is true only when $x_1(t+T) = x_1(t)$ and $x_2(t+T) = x_2(t)$

then T can be written as $T = k_1 T_1 = k_2 T_2$ for $k_1, k_2 \in N^+$

i.e., $\frac{T_1}{T_2} = \frac{k_2}{k_1}$, $\frac{T_1}{T_2}$ is rational. $\square$

Therefore, if $\frac{T_1}{T_2}$ is not rational, $x(t)$ is not periodic.

In this problem $\frac{T_1}{T_2} = \frac{\sqrt{3}}{5}$ which is irrational,

So $\boxed{x(t) \text{ is not periodic}}$

7. $z(t) = x(t)\,y(t) = \left(\cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3}\right)\sin \pi t$

$$= \sin\pi t\cos\frac{2\pi t}{3} + 2\sin\pi t\sin\frac{16\pi t}{3}$$

$$= \frac{1}{2}\left(\sin\frac{5\pi t}{3} + \sin\frac{\pi t}{3}\right) + \cos\frac{13\pi t}{3} - \cos\frac{19\pi t}{3}$$

$\omega_1 = \frac{5\pi}{3}$ $\quad \omega_2 = \frac{\pi}{3}$ $\quad \omega_3 = \frac{13\pi}{3}$ $\quad \omega_4 = \frac{19\pi}{3}$

$T_1 = \frac{6}{5}$ $\quad T_2 = 6$ $\quad T_3 = \frac{6}{13}$ $\quad T_4 = \frac{6}{19}$

$\forall\ i,j \in \{1,2,3,4\},\ i \neq j$, $\frac{T_i}{T_j}$ is rational, so $z(t)$ is periodic.

Transform trigonometric functions to complex exponential functions

$$\sin(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \qquad \cos(\omega t) = \frac{e^{j\omega t} + e^{-j\omega t}}{2}$$

$$z(t) = \frac{1}{4j}\left(e^{j\frac{5\pi t}{3}} - e^{-j\frac{5\pi t}{3}} + e^{j\frac{\pi t}{3}} - e^{-j\frac{\pi t}{3}}\right) + \frac{1}{2}\left(e^{j\frac{13\pi t}{3}} + e^{-j\frac{13\pi t}{3}} - e^{j\frac{19\pi t}{3}} - e^{-j\frac{19\pi t}{3}}\right)$$

$$= \frac{1}{4j}e^{j\cdot 1\cdot\frac{2\pi}{6}t} + \frac{1}{4j}e^{j\cdot 5\cdot\frac{2\pi}{6}t} - \frac{1}{4j}e^{j(-5)\frac{2\pi}{6}t} - \frac{1}{4j}e^{j(-1)\frac{2\pi}{6}t}$$
$$- \frac{1}{2}e^{j(-19)\frac{2\pi}{6}t} + \frac{1}{2}e^{j(-13)\frac{2\pi}{6}t} + \frac{1}{2}e^{j\cdot 13\cdot\frac{2\pi}{6}t} - \frac{1}{2}e^{j\cdot 19\cdot\frac{2\pi}{6}t}$$

We can conclude $z(t) = \sum_{k=\pm 19,\pm 13,\pm 1,\pm 5} c_k e^{jk\frac{2\pi}{6}t}$

Comparing with $z(t) = \sum_k c_k e^{jk(2\pi/T)t}$. $\boxed{T = 6}$

$$\boxed{c_k = \begin{cases} -\frac{1}{4j} & k = -1, -5 \\ \frac{1}{4j} & k = 1, 5 \\ -\frac{1}{2} & k = \pm 19 \\ \frac{1}{2} & k = \pm 13 \end{cases}}$$

8.

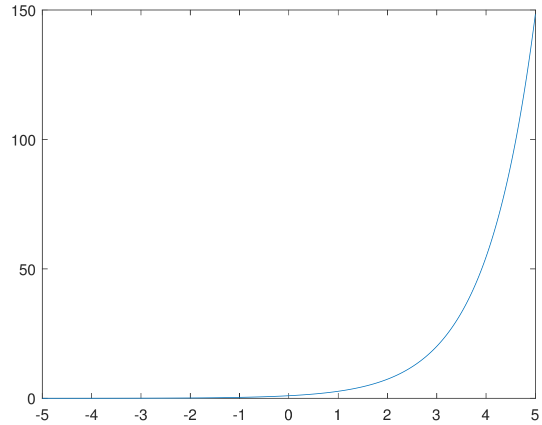


Figure 7. 8(a).

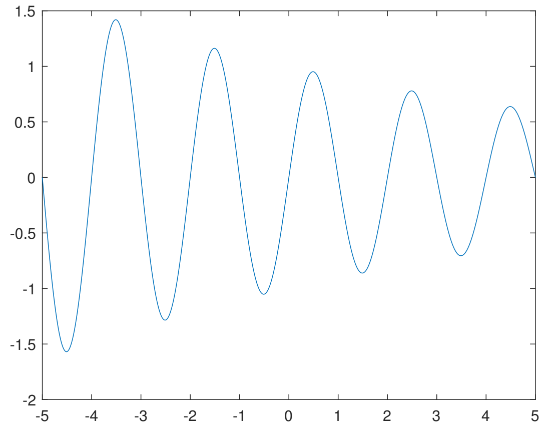


Figure 8. 8(b).

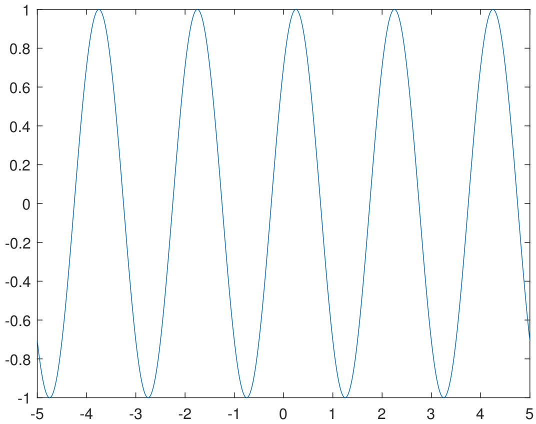
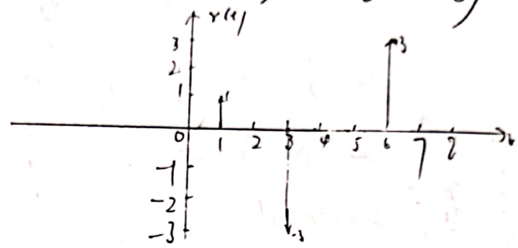


Figure 9. 8(c).

9. (a) $x(t) = u(t-1) - 3u(t-4) + 3u(t-6)$

(b) $\frac{dx(t)}{dt} = \delta(t-1) - 3\delta(t-4) + 3\delta(t-6)$

Sketch:



10. (a) $y(t)$ depends on both future and past. so it's

NOT memoryless, NOT Time Invariant, Linear, NOT causal, Stable

$y(t) = a x_1(t-2) + b x_2(t-2) + a x_1(2-t) + b x_2(2-t) = a y_1(t) + b y_2(t) \Rightarrow$ linear

$\exists M_x$ s.t. $|x(t)| \le M_x < \infty\ \forall t \Rightarrow \exists M_y$ s.t. $|y(t)| \le M_y < \infty\ \forall t \Rightarrow$ Stable. Depend on past & future $\Rightarrow$ NOT memoryless & NOT causal

$x(t-t_0) = x(t-t_0-2) + x(2-t+t_0)$ $y_d(t) = x(t-t_0-2) + x(2-t-t_0) \neq y(t-t_0) \Rightarrow$ NOT time variant

(b) NOT memoryless NOT Time Invariant Linear. NOT causal Stable

Depend on past & future. $\Rightarrow$ not memoryless & NOT causal, $y(t) = a x_1(\frac{t}{3}) + b x_2(\frac{t}{3}) = a y_1(t) + b y_2(t) \Rightarrow$ linear; $y(t-t_0) = x(\frac{t-t_0}{3}) \neq y_d(t) = x(\frac{1}{3}t - t_0)$

$\exists M_x$ s.t. $|x(t)| \le M_x < \infty\ \forall t \Rightarrow \exists M_y$ s.t. $|y(t)| \le M_y < \infty\ \forall t \Rightarrow$ Stable $\Rightarrow$ NOT time variant

Depend on past & future $\Rightarrow$ NOT memoryless & not causal

(c) NOT memoryless, Time Invariant, NOT Linear, causal, Stable

$\forall t$ $y(t) = \cos(a x_1(t) + b x_2(t)) \neq a y_1(t) + b y_2(t) \Rightarrow$ NOT linear, $y(t) \in [-1, 1]\ \forall t \Rightarrow$ Stable

Depend only on present $\rightarrow$ memoryless & causal $y(t-t_0) = \cos(x(t-t_0)) = y_d(t) \Rightarrow$ time invariant

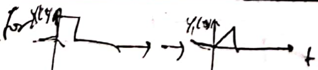
(d) NOT memoryless, NOT Time Invariant, Linear NOT causal, NOT Stable

$y(t) = \int_{-\infty}^{t/2} (a x_1(\tau) + b x_2(\tau)) d\tau = a y_1(t) + b y_2(t) \Rightarrow$ linear, $y(t-t_0) = \int_{-\infty}^{\frac{t-t_0}{2}} x(\tau - t_0) d\tau \neq y_d(t) \Rightarrow$ NOT time invariant

(e) NOT memoryless, NOT Time Invariant, Linear, Causal, NOT Stable

$y(t) = a\frac{dx_1(t)}{dt} + b\frac{dx_2(t)}{dt} = a y_1(t) + b y_2(t) \Rightarrow$ linear, depend on past and current $\Rightarrow$ causal, Not memoryless; for incontinuous function, $\frac{dx}{dt}$ may reach infinity; $y(t-t_0) = \frac{a\, x(t-t_0)}{d(t-t_0)} \neq y_d(t) \Rightarrow$ NOT time invariant

11. for



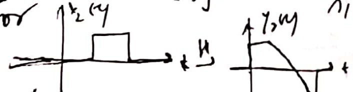
This system is NOT causal, NOT time invariant and NOT memoryless;

depends only on present values

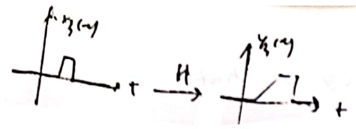
Comparing $x_1 \xrightarrow{H} y(t)$ and $x_2 \xrightarrow{H} y(t)$, $x_1(t) \xrightarrow{H} y_1(t)$.

$x_1(t-1) = x_2(t) \xrightarrow{H} y_2(t) \neq y_1(t-1)$, so it's not time invariant

For

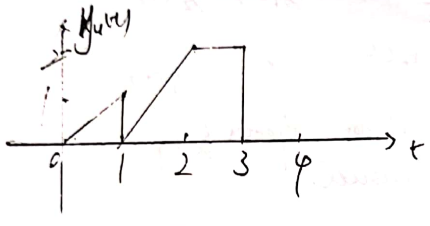


Using result from 12, for $t \in [0, 1]$, $x_2(t) = 0$ but $y_2(t) \neq 0$, so it is not causal

For 

For $t \in (2,3)$ $x_3(t) = 0$ but $y_3(t) \neq 0$, so it is not memoryless (3)

(d)



12. We denote the two odd signals as $x_1(t), x_2(t)$

with property $x_1(-t) = -x_1(t)$

$x_2(-t) = -x_2(t)$

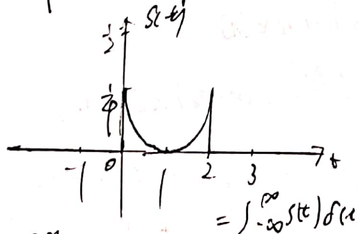
The product denoted as $x(t) = x_1(t) \cdot x_2(t) = (-x_1(t))(-x_2(t))$

$= x_1(-t) \cdot x_2(-t)$

$= x(-t)$, which is even

Therefore, the product of two odd signals is an even signal.

13. (a)



(b) $\int_{-\infty}^{\infty} s(t)\,x(t)\,dt = \int_{-\infty}^{\infty} s(t)\delta(t-\frac{1}{2})dt + \int_{-\infty}^{\infty} s(t)\delta(t-2)dt - \int_{-\infty}^{\infty} s(t)\delta(3t-4)dt$

$= s(\frac{1}{2}) + s(2) - \frac{1}{3}s(\frac{4}{3})$

$= \frac{1}{16} + \frac{1}{4} - \frac{1}{108} = \boxed{\frac{131}{432}}$

14. For signal $x_1(t)$ we construct $x_2(t)$ so that

$$\begin{cases} x_2(t) = x_1(t), & t < t_0 \\ x_2(t) \neq x_1(t), & t > t_0 \end{cases}$$

and their corresponding output is $y_1(t)$ and $y_2(t)$

Consider another input signal $x_3(t) = x_1(t) - x_2(t)$

we have $x_1(t) - x_2(t) \to y_1(t) - y_2(t)$

when $t < t_0$

$x_1(t) = x_2(t) \Rightarrow y_1(t) = y_2(t)$

when $t > t_0$, output is not affected by input.

thus, the system is casual.

i.e. for any time t_0 and any input $x(t)$ such that $x(t) = 0$ for $t < t_0$, the corresponding output $y(t)$ must also be zero for $t < t_0$.

15. (a) $T[a_1 x_1[n] + a_2 x_2[n]] = n(a_1 x_1[n] + a_2 x_2[n])$

$= a_1 n x_1[n] + a_2 n x_2[n]$

$= a_1 T x_1[n] + a_2 T x_2[n]$

Thus the system is linear

(b) $x[n] \xrightarrow{T} y[n]$ is actually $x[n] \xrightarrow{T} n x[n]$

$x[n] \to \boxed{\text{delay}} \xrightarrow{x_d[n] = x[n-n_0]} \boxed{T} \to y_d[n] = n x[n-n_0]$

$x[n] \to \boxed{T} \xrightarrow{y[n]} \boxed{\text{delay}} \to y[n-n_0] = (n-n_0) x[n-n_0]$

$y[n-n_0] = (n-n_0) x[n-n_0] \neq n x[n-n_0]$

Hence the system is NOT time invariant.

(c) If $\exists M_x$ s.t. $|x[n]| \leq M_x < \infty$ $\forall n$ for example is 1

but $y[n] = n x[n] = n$ is not bounded as $n \to +\infty$

Therefore the system is NOT BIBO stable

(d) The output $y[n]$ depends only on the current input $x[n]$
not on previous or future values of the input signal,
So the system is memoryless.

(e) Since the output depends only on the current input $x[n]$
not on future input,
So the system is causal.

16. (a) If $x[n]$ is periodic, $x[n] = x[n+N]$ where N is its period

If N is even, $y_1[n] = x[2n] = x[2n+N] = y_1[n+N']$, $N' = \frac{N}{2}$

If N is odd, $y_1[n] = x[2n] = x[2n+2N] = y_1[n+N]$, $N' = N$

Therefore, $y_1[n]$ is periodic

(b) If $y_1[n]$ is periodic, satisfying $y_1[n] = y_1[n+N]$

i.e. $x[2n] = x[2n+2N]$

$x[2n]$ is periodic

However, we CANNOT conclude $x[n]$ is also periodic.

Take an counter example $x[n] = \begin{cases} n & n \text{ is even} \\ 0 & n \text{ is odd} \end{cases}$

then $y_1[n] = x[2n]$ is periodic

Thus if $y_1[n]$ is periodic, $x[n]$ may NOT be periodic.

(c) If $x[n]$ is period $x[n] = x[n+N]$

$y_2[n] = \begin{cases} x[\frac{n}{2}] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$ $y_2[n+2N] = y_2[n]$

thus $y_2[n]$ is periodic with period $N' = 2N$

(d) If $y_2[n]$ is periodic, $y_2[n] = y_2[n+N']$

$y_2[2n]$ is also periodic $y_2[2n] = y_2[2n+N'] = \begin{cases} x[n] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

Hence, $x[n]$ is periodic with period $N = 2N'$

17. (a) $E_x = E[x(t)] = \int_{-\infty}^{\infty} |x(t)|^2 dt$

For the energy of $x(-at+b)$

$$E[x(-at+b)] = \int_{-\infty}^{\infty} |x(-at+b)|^2 dt$$

Let $s=-at+b$, then $dt = -\frac{ds}{a}$

$a>0$: $E[x(-at+b)] = \int_{\infty}^{-\infty} \frac{-1}{a} |x(s)|^2 ds = \frac{1}{a}\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{E_x}{a}$

$a<0$: $E[x(-at+b)] = \int_{-\infty}^{\infty} \frac{-1}{a} |x(s)|^2 ds = \frac{-1}{a}\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{E_x}{-a}$

$$\boxed{E[x(-at+b)] = \frac{E_x}{|a|}}$$

(b) $P_x = P[x(t)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$

For the power of $x(-at+b)$

$$P[x(-at+b)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(-at+b)|^2 dt$$

Let $s=-at+b$, then $dt = -\frac{ds}{a}$, $t = \frac{b-s}{a}$

$$P[x(-at+b)] = -\frac{1}{a}\lim_{s\to\infty} \frac{a}{2(s-b)}\int_{\frac{s-b}{a}}^{\frac{b-s}{a}} |x(s)|^2 ds = \lim_{s\to\infty} \frac{1}{2(s+b)}\int_{\frac{-s+b}{a}}^{\frac{s-b}{a}} |x(s)|^2 ds$$

$$= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt = P[x(t)] = P_x$$

$$P[x(-at+b)] = P[x(t)]$$

$$\boxed{P[x(-at+b)] = P_x}$$