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1. $\langle \phi_k, \phi_l \rangle = \int_a^{a+T_0} \phi_k(t)\phi_l^*(t)\,dt = \int_a^{a+T_0} e^{j\omega_0 kt} e^{-j\omega_0 lt}\,dt$

$= \int_a^{a+T_0} e^{j\omega_0 t(k-l)}\,dt = \left.\frac{e^{j\omega_0(k-l)t}}{j\omega_0(k-l)}\right|_{t=a}^{a+\frac{2\pi}{\omega_0}}$

$= \frac{1}{j\omega_0(k-l)}\left(e^{j\omega_0(k-l)(a+\frac{2\pi}{\omega_0})} - e^{j\omega_0(k-l)a}\right) = \frac{e^{j\omega_0(k-l)a}\left(e^{j\omega_0(k-l)\frac{2\pi}{\omega_0}}-1\right)}{j\omega_0(k-l)}$

$= \frac{1}{j\omega_0(k-l)} e^{j\omega_0(k-l)a}\left(e^{2\pi j(k-l)}-1\right)$

$m=\frac{l}{k}$, $n=1-m$

If $l=k$, $m=1$, $\langle \phi_k, \phi_l \rangle = \lim_{m\to 1} \frac{1}{j\omega_0(1-m)} e^{j\omega_0(1-m)a}\left(e^{2\pi j(1-m)}-1\right)$

$= \lim_{n\to 0} \frac{1}{j\omega_0 n}\left(e^{\omega_0 jn(a+\frac{2\pi}{\omega_0})} - e^{j\omega_0 na}\right)$

$= \frac{1}{j\omega_0}\left(j\omega_0(a+\frac{2\pi}{\omega_0})e^{\omega_0 jn(a+\frac{2\pi}{\omega_0})} - j\omega_0 a e^{j\omega_0 na}\right) = \alpha \neq 0$

If $l\neq k$, $e^{2\pi j(k-l)}=1$, so $\langle \phi_k, \phi_l \rangle = 0$

Therefore, the set of complex exponentials $\{e^{j\omega_0 kt}: k=0,\pm1,\pm2,\ldots\}$ is orthogonal on any interval over a period T_0, where $T_0=\frac{2\pi}{\omega_0}$

2. $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k(\cos(k\omega_0 t) + j\sin(k\omega_0 t))$

If $k=0$, $x(t) = \sum_{k=0} a_0\cdot 1 = a_0$

$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = B[0]$

If $k\neq 0$, $a_k = \frac{1}{T}\int_0^T x(t)e^{-j\omega_0 t}\,dt = \frac{1}{T}\int_0^T x(t)[\cos(k\omega_0 t) - j\sin(k\omega_0 t)]\,dt$

$= \frac{1}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt - \frac{j}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$

$= \frac{1}{2}\frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt - \frac{j}{2}\frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$

$= \frac{1}{2}B(k) - \frac{j}{2}A(k)$

(continued)

Therefore, $x(t) = B[0] + \sum_{k=1}^{\infty} B[k]\cos(k\omega_0 t) + A[k]\sin(k\omega_0 t)$

where
$$\begin{cases} B[0] = \frac{1}{T}\int_0^T x(t)dt \\ B[k] = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt \\ A[k] = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt \end{cases}$$

3. $x(t) = \sin(3\pi t) + \cos(4\pi t)$

$$\sin(3\pi t) = \frac{e^{j3\pi t} - e^{-j3\pi t}}{2j}, \quad \cos(4\pi t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2}$$

$$x(t) = \frac{1}{2j}(e^{3\pi jt} - e^{-3\pi jt}) + \frac{1}{2}(e^{4\pi jt} + e^{-4\pi jt})$$

$$= \frac{1}{2j}e^{3\pi jt} - \frac{1}{2j}e^{-3\pi jt} + \frac{1}{2}e^{4\pi jt} + \frac{1}{2}e^{-4\pi jt}$$

$$= c_3 e^{3j\omega_0 t} + c_{-3}e^{-3j\omega_0 t} + c_4 e^{j4\omega_0 t} + c_{-4}e^{j4\omega_0 t}$$

FS coefficients: $\boxed{c_3 = \frac{1}{2j},\ c_{-3} = -\frac{1}{2j},\ c_4 = \frac{1}{2},\ c_{-4} = \frac{1}{2} \quad \text{for other value of } k\ c_k = 0}$

4. (a) $T_0 = 4$ $\quad c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$

$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$

$$= \frac{1}{4}\int_0^4 \delta(t-4n)e^{-jk\omega_0 t}dt = \frac{1}{4}e^{-jk\omega_0 4}$$

$$= \frac{e^{-2\pi jk}}{4} = \frac{1}{4} \qquad c_0 = \frac{1}{T_0}\int_{T_0}x(t)dt = \frac{1}{4}\int_0^4\delta(t-4n)dt = \frac{1}{4}$$

$$x(t) = \sum_{k=-\infty}^{\infty}\frac{1}{4}e^{\frac{\pi jk}{2}t} = \frac{1}{4} + \frac{1}{2}\sum_{k=1}^{\infty}\cos\left(\frac{\pi k}{2}t\right)$$

(b) $T_0 = 5$ $\quad c_k = \frac{1}{T_0}\int_{T_0}x(t)e^{-jk\omega_0 t}dt$

$\omega_0 = \frac{2\pi}{5}$

$$= \frac{1}{5}\int_0^5 \text{rect}\left(\frac{t-5n-3}{6}\right)e^{-jk\omega_0 t}dt$$

$c_0 = \frac{1}{T_0}\int_{T_0}x(t)dt$

$= \frac{1}{5}\left(\int_0^1 2\,dt + \int_1^5 (dt)\right)$

$= \frac{6}{5}$

$$= \frac{1}{5}\left(\int_0^1 2e^{-jk\omega_0 t}dt + \int_1^5 e^{-jk\omega_0 t}dt\right)$$

$$= \frac{1}{-5jk\omega_0}\left(2e^{-jk\omega_0 t}\Big|_0^1 + e^{-jk\omega_0 t}\Big|_1^5\right)$$

$$= -\frac{1}{5jk\omega_0}\left(e^{-jk\omega_0} - 2 + e^{-5jk\omega_0}\right)$$

$$= -\frac{1}{2\pi jk}\left(e^{-2\pi jk} + e^{-\frac{2\pi}{5}jk} - 2\right) = \frac{j}{2\pi k}\left(e^{-\frac{2\pi}{5}jk} - 1\right)$$

$$= \frac{j}{2\pi k}\left(\cos\frac{2\pi}{5}k - j\sin\frac{2\pi}{5}k - 1\right) = \frac{1}{2\pi k}\sin\frac{2\pi}{5}k + \frac{j}{2\pi k}\left(\cos\frac{2\pi}{5}k - 1\right) = A_k + jB_k$$

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} A_k\cos k\omega t - B_k\sin k\omega t$$

$$\boxed{= \frac{6}{5} + \sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\left(\sin\frac{2\pi}{5}k\cos\frac{2\pi}{5}kt - \left(\cos\frac{2\pi}{5}k - 1\right)\sin\frac{2\pi}{5}kt\right)\right)}$$

(c) $T_0 = 2$ $\quad c_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\omega_0 k t} dt$

$\omega_0 = \pi$ $\quad = \frac{1}{2}\int_0^1 e^{-t} e^{-jk\omega_0 t} dt$

$c_0 = \frac{1}{T_0}\int_0^{T_0} x(t) dt$ $\quad = \frac{1}{2}\int_0^1 e^{-(1+jk\pi)t} dt = -\frac{1}{2(1+jk\pi)} e^{-(1+jk\pi)t}\Big|_0^1$

$= \frac{1}{2}\int_0^1 e^{-t} dt$

$= \frac{1}{2}(1-e^{-1})$ $\quad = \frac{1}{2(1+jk\pi)}\left(1-e^{-(1+jk\pi)}\right) = \frac{1}{2(1+jk\pi)}\left(1-\frac{(-1)^k}{e}\right) = \frac{1-(-1)^k/e}{2(1+\pi^2k^2)}(1-j\pi k)$

$$\boxed{x(t) = \frac{1}{2} - \frac{1}{2e} + \sum_{k=1}^{\infty} \frac{1-\frac{(-1)^k}{e}}{\pi^2k^2+1}\left(\cos(k\pi t) + \pi k \sin(k\pi t)\right)}$$

5.(a) We first derive a_{-k}^* $\quad$ ($x(t)$ is real iff $a_k = a_{-k}^*$)

$$a_{-k}^* = \begin{cases} 2 & k=0 \\ -j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} \neq a_k$$

So $x(t)$ is $\boxed{\text{NOT real}}$

(b) $x(t) = x(-t)$ iff $x(t)$ is even

or $a_k = a_{-k}$

$$a_{-k} = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|-k|} & \text{otherwise} \end{cases} = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} = a_k$$

So $x(t)$ is $\boxed{\text{even}}$

(c) Fourier coefficient of $\frac{dx(t)}{dt}$ is $b_k = jk\frac{2\pi}{T}a_k$

$$b_k = \begin{cases} jk\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}jk\cdot j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} = \begin{cases} jk\frac{4\pi}{T} & k=0 \\ -\frac{2\pi}{T}k(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases}$$

$$b_{-k} = \begin{cases} j(-k)\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}k(\frac{1}{2})^{|-k|} & \text{otherwise} \end{cases} = \begin{cases} -jk\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}k(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} \neq b_k$$

So $\frac{dx(t)}{dt}$ is $\boxed{\text{NOT even}}$

6. See attached page.

7. (a) $x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$

If $x(t)$ is odd harmonic,

$$x(t+\tfrac{T}{2}) = \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} e^{jk\frac{2\pi}{T}\frac{T}{2}}$$

$$= \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} e^{jk\pi}$$

$$= \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} (-1)^k$$

$$= -\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

Thus, $\boxed{x(t) = -x(t+\tfrac{T}{2})}$

(b) FS coefficient of $x(t)$:

$$a_k = \frac{1}{T}\left(\int_0^{\frac{T}{2}} x(t) e^{-jk\omega_0 t} dt + \int_{\frac{T}{2}}^{T} x(t) e^{-jk\omega_0 t} dt\right)$$

Let $t = t + \frac{T}{2}$ for the second part,

$$a_k = \frac{1}{T} \int_0^T \left(x(t) + x(t+\tfrac{T}{2}) e^{-j\pi k}\right) e^{-jk\omega_0 t} dt$$

$\forall k$ is even $x(t) + x(t+\frac{T}{2}) = 0$ i.e. $x(t) = -x(t+\frac{T}{2})$

Therefore, $x(t)$ is $\boxed{\text{odd harmonic}}$.

8. See attached page.

9.(a) $T_0 = 4 \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$k=0 \quad c_k = \frac{1}{4}\int_0^4 x(t)\,dt$

$= \frac{1}{4}\int_2^4 (-\frac{1}{2}t+1)\,dt = -\frac{1}{4}$

Using differentiation property $d_k = jk\omega_0 c_k$

$y(t) = \frac{d\,x(t)}{dt}$

For $0 \le t \le 4$, $x(t) = (-\frac{1}{2}t+1)\,(u(t-2)-u(t-4))$

For all t $\quad x(t) = \sum_{k=-\infty}^{\infty} (-\frac{1}{2}t+1+2k)\,(u(t-2-4k)-u(t-4-4k))$

then $y(t) = \frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty} \left((-\frac{1}{2}t+1+2k)(\delta(t-2-4k)-\delta(t-4-4k)) + \frac{1}{2}(u(t-4k-4)-u(t-4k-2))\right)$

The first part for $0 \le t \le 4$, when $t=4$ $\quad 1-\frac{t}{2} = -1$

$t=2 \quad 1-\frac{t}{2} = 0$

Sketch:

$T_0' = 4$

$d_k = -\frac{1}{4}\int_0^4 \delta(t-4)e^{-j\frac{2\pi}{4}kt}\,dt - \frac{1}{8}\int_2^4 e^{-j\frac{\pi}{2}kt}\,dt$

$= \frac{1}{4}\int_0^4 \delta(t-4)e^{-j2\pi k}\,dt + \frac{1}{4\pi k}\, e^{-\frac{\pi}{2}jkt}\Big|_2^4$

$= \frac{1}{4} + \frac{1}{4\pi kj}(e^{-2\pi kj} - e^{-\pi kj})$

$= \frac{1}{4} - \frac{j}{4\pi k}(1-(-1)^k)$

$k \ne 0, \quad c_k = \frac{d_k}{jk\omega_0} = \frac{-j(1-(-1)^k)}{4\pi k\, jk\frac{\pi}{2}} + \frac{1}{4jk\frac{\pi}{2}} = -\frac{j}{2\pi k} - \frac{1-(-1)^k}{2\pi^2k^2}$

$k=0 \quad c_0 = \frac{1}{T}\int_T x(t)\,dt = -\frac{1}{4}$

$$\boxed{x(t) = -\frac{1}{4} + \sum_{k=1}^{\infty}\frac{1}{\pi k}\left(\sin(\frac{\pi}{2}kt) - \frac{1-(-1)^k}{\pi k}\cos(\frac{\pi}{2}kt)\right)}$$

(b) Let $y(t) = \cos(\pi t) = \sum_{k=-\infty}^{\infty} b_k e^{j\frac{\pi}{2}kt} = \frac{e^{j\pi t}+e^{-j\pi t}}{2} \Rightarrow b_k = \begin{cases}\frac{1}{2}, & k=\pm 2\\ 0, & k \ne \pm 2\end{cases}$

then $z(t) = x(t)\,y(t)$, using multiplication property, $z(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$

$d_k = \sum_{l=-\infty}^{\infty} b_l c_{k-l} = b_2 c_{k-2} + b_{-2}c_{k+2} = \frac{1}{2}(c_{k-2}+c_{k+2})$

$k=2 \quad d_2 = \frac{1}{2}(c_0 + c_4) = \frac{1}{2}(-\frac{1}{4} - \frac{j}{8\pi}) = -\frac{1}{8} - \frac{j}{16\pi}$

$k=-2 \quad d_{-2} = \frac{1}{2}(c_{-4}+c_0) = \frac{1}{2}(-\frac{1}{4} + \frac{j}{8\pi}) = -\frac{1}{8} + \frac{j}{16\pi}$

$k \ne \pm 2 \quad d_k = \frac{1}{2}\left(-\frac{j}{2\pi(k+2)} - \frac{1-(-1)^{k+2}}{2\pi^2(k+2)^2} - \frac{j}{2\pi(k-2)} - \frac{1-(-1)^{k-2}}{2\pi^2(k-2)^2}\right)$ (continued)

$d_3 = -\frac{1}{4\pi}(\frac{-j}{5} - j + \frac{2}{\pi \cdot 25} + \frac{2}{\pi})$

$$\boxed{d_k = \begin{cases} -\frac{1}{4\pi}\left(\frac{j}{k+2} + \frac{j}{k-2} + \frac{1-(-1)^k}{\pi(k+2)^2} + \frac{1-(-1)^k}{\pi(k-2)^2}\right) & k \neq \pm 2 \\ -\frac{1}{8} - \frac{j}{16\pi} & k = 2 \\ -\frac{1}{8} + \frac{j}{16\pi} & k = -2 \end{cases}}$$

$$\boxed{\begin{aligned} d_2 &= -\frac{1}{8} - \frac{j}{16\pi} \\ d_{-3} &= \frac{3j}{10} - \frac{13}{25\pi^2} \end{aligned}}$$

(c) Using time transformations $a_k = c_k e^{jk\omega_0 b} = c_k e^{-2jk\omega_0} = c_k e^{-jk\pi} = (-1)^k c_k$

$\omega_0 = \frac{\pi}{2}$ $\quad b = -2$

$a = \frac{1}{3}$ $\quad \omega_1 = a\omega_0 = \frac{\pi}{6}$

Using amplitude transformations $b_k = \begin{cases} b' + a' a_0 & k=0 \\ a' a_k & k \neq 0 \end{cases} = \begin{cases} 5 + 4a_0 & k=0 \\ 4c_k(-1)^k & k \neq 0 \end{cases}$

$c_0 = -\frac{1}{4}$ $\quad b' = 5$

$\omega_2 = \omega_1 = \frac{\pi}{6}$ $\quad a' = 4$

$a_0 = c_0 = -\frac{1}{4}$

$$= \begin{cases} 4 & k=0 \\ \frac{-2(-1)^k}{\pi k}\left(j + \frac{1-(-1)^k}{\pi k}\right) & k \neq 0 \end{cases}$$

The Fourier series expansion of $y(t)$ is

$$\boxed{y(t) = 4 + \sum_{\substack{k=-\infty \\ k \neq 0}}^{\infty} \frac{-2(-1)^k}{\pi k}\left(\frac{1-(-1)^k}{\pi k} + j\right) e^{\frac{j\pi k}{6} t}}$$

10. (a) $x(t) = Ri(t) + L\frac{di(t)}{dt} + y(t)$

$$i(t) = C\frac{dy(t)}{dt}$$

$$x(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t)$$

$$R = 1\Omega,\ L = 1H,\ C = 1F$$

$$x(t) = \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t)$$

(b) Use Fourier transform

$$X(j\omega) = (j\omega)^2 Y(j\omega) + (j\omega)Y(j\omega) + Y(j\omega)$$

$$= (-\omega^2 + j\omega + 1)\ Y(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{-\omega^2 + j\omega + 1}$$

The system's response $\boxed{y(t) = H(j\omega)x(t) = \frac{e^{j\omega t}}{-\omega^2 + j\omega + 1}}$

(c) The system's transfer function $\boxed{H(s) = \frac{1}{s^2+s+1}}$

(d) $|H(j\omega)| = \left|\frac{-\omega^2 - j\omega + 1}{(\omega-1)^2 + \omega^2}\right| = \frac{1}{\sqrt{\omega^4 - \omega^2 + 1}}$

Plot see attached page

(e) See attached page

(f) $x(t) = 1 + \sin t + \sin(4t) = 1 + \frac{e^{jt} - e^{-jt}}{2j} + \frac{e^{j4t} - e^{-j4t}}{2j}$, $\omega_0 = \frac{2\pi}{T} = 1$

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ where $a_k = \begin{cases} 1 & k=0 \\ \frac{1}{2j} & k=1,4 \\ -\frac{1}{2j} & k=-1,-4 \\ 0, & \text{otherwise} \end{cases}$

power density spectrum see attached page $|a_k|^2 = \begin{cases} 1 & k=0 \\ \frac{1}{4} & k=\pm1, \pm4 \\ 0 & \text{otherwise} \end{cases}$

(g) $P = \sum_{k=-\infty}^{\infty} |a_k|^2 = 4 \times \frac{1}{4} + 1 = 2$

$$\frac{1}{T}\int_T |x(t)|^2 dt = \frac{1}{2\pi}\int_0^{2\pi} (1+\sin t + \sin 4t)^2\, dt = 2$$

So $P = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

we've verified the Parseval's relation.

(h) $H(j\omega) = \frac{1}{-\omega^2 + j\omega + 1}$

$\omega = 4 \quad H(4j) = \frac{1}{4j - 15} \quad \omega = -4 \quad H(-4j) = \frac{-1}{4j + 15}$

$\omega = 1 \quad H(j) = -j \quad \omega = -1 \quad H(-j) = j$

$\omega = 0 \quad H(0) = 1$

$$y(t) = 1 + \frac{e^{jt}}{2j}(-j) - \frac{e^{-jt}}{2j} j + \frac{e^{j4t}}{2j}\frac{1}{4j-15} + \frac{e^{-j4t}}{2j}\frac{1}{4j+15}$$

$$\boxed{y(t) = 1 - \frac{e^{jt}}{2} - \frac{e^{-jt}}{2} - \frac{e^{j4t}}{8+30j} + \frac{e^{-j4t}}{-8+30j}}$$

power density spectrum see attached page

(i) $\omega = \pm 4\omega_0$ component of $x(t)$ is attenuated

This RLC circuit is a $\boxed{\text{bandpass}}$ filter

11. $\omega_0 = \frac{2\pi}{T} = 12$

$y(t) = \sum_{k=-\infty}^{\infty} H(j12k) e^{j12kt}$ $\quad |12k| < 100 \Rightarrow |k| \leq 8 \quad H(j12k) = 1 = a_k$

$\quad |12k| > 100 \Rightarrow |k| \geq 9 \quad H(j12k) = 0 = a_k$

$= x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j12kt}$

hence for $\boxed{k \leq -9 \text{ or } k \geq 9}$ it's guaranteed that a_k must be zero

12. see attached page

13. If the FS coefficients of $x(t)$ is periodic with period N,

$a_k = a_{k-N}, \quad x(t) = x(t) e^{j(\frac{2\pi}{T})Nt}$

It is possible only when $x(t) = 0 \ \forall t$ or $\frac{2\pi}{T}N = 2\pi k$

therefore $x(t)$ can be expressed by $x(t) = \sum_{k=-\infty}^{\infty} g[k] \delta(t - \frac{kT}{N})$

14. Since $\sin^5 x = \frac{\sin 5x - 5\sin x + 15\sin x - 5\sin 3x}{16}$

$\sin^5(3t) = \frac{\sin 15t + 10\sin 3t - 5\sin 9t}{16}$

$y(t) = 7\sin(3t) + a7\frac{\sin 15t + 10\sin 3t - 5\sin 9t}{16}$

$= \frac{49}{16}\sin(3t) + \frac{7}{160}\sin(15t) - \frac{7}{32}\sin(9t)$ (continued)

$\omega_0 = 3 \quad \omega_3 = 9, \omega_5 = 15$

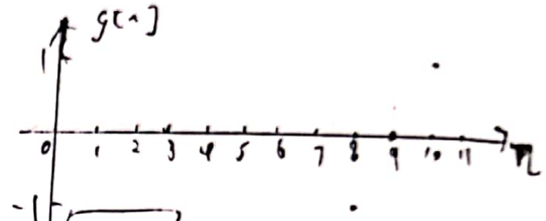
Total power $P = \frac{(\frac{119}{16})^2}{2} + \frac{(\frac{1}{10})^2}{2} + \frac{(\frac{?}{32})^2}{2}$

power in fundamental $P_f = \frac{(\frac{119}{16})^2}{2}$

$THD = (1 - \frac{P_f}{P}) \times 100\% = 8.988 \times 10^{-4} \times 100\% = 0.09\%$

THD for this amplifier $\boxed{THD = 0.09\%}$

Graph:



15. (a) The graph see the attached page

The fundamental period of $g[n]$ is $\boxed{10}$.

(b) $a_k = \frac{1}{10} \sum_{n=0}^{9} x[n] e^{-j\frac{\pi}{5}nk}$

$= \frac{1}{10}(1 + e^{-j\frac{\pi}{5}} + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{3\pi}{5}k} + e^{-j\frac{4\pi}{5}k} + e^{-j\pi k} + e^{-j\frac{6\pi}{5}k} + e^{-j\frac{7\pi}{5}k})$

the Fourier coefficient of $g[n]$ denoted as b_k is

$b_k = \frac{1}{10}(e^{-j\frac{\pi}{5} 0 k} - e^{-jk\frac{2\pi}{5} 8}) = \frac{1}{10}(1 - e^{-j\frac{8\pi}{5}k})$

Thus the Fourier series coefficients of $g[n]$ is $\boxed{b_k = \frac{1}{10}(1 - e^{-j\frac{8\pi}{5}k})}$

(c) $g[n] = x[n] - x[n-1] \overset{FS}{\longleftrightarrow} b_k = (1 - e^{-jk\frac{2\pi}{N}}) a_k$

$N = 10$

So $\boxed{a_k = \frac{1 - e^{-j\frac{8}{5}\pi k}}{10(1 - e^{-j\frac{\pi}{5}k})}}$.

16. (a) $w_0 = 4\pi \quad x(t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} = \sum_{k=-\infty}^{\infty} a_k e^{j4\pi kt}$

So $a_1 = \frac{1}{2} \quad a_{-1} = \frac{1}{2}$

The FS coefficients of $x(t)$ is $\boxed{a_1 = \frac{1}{2},\ a_{-1} = \frac{1}{2},\ a_k = 0, k \neq \pm 1}$

(b) $w_0 = 4\pi \quad y(t) = \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} = \sum_{k=-\infty}^{\infty} b_k e^{j4\pi kt}$

So $b_1 = \frac{1}{2j} \quad b_{-1} = -\frac{1}{2j}$

The FS coefficients of $y(t)$ is $\boxed{b_1 = \frac{1}{2j},\ b_{-1} = -\frac{1}{2j},\ b_k = 0, k \neq \pm 1}$

(c) The FS coefficients of $z(t)$ is c_k, then

$$c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

$$c_0 = a_{-1}b_1 + a_1 b_{-1} = \frac{1}{4j} - \frac{1}{4j} = 0$$

$$c_1 = a_{-1}b_2 + a_1 b_0 = 0, \quad c_{-1} = a_{-1}b_0 + a_1 b_{-2} = 0$$

$$c_2 = a_{-1}b_3 + a_1 b_1 = \frac{1}{4j} \quad c_{-2} = a_{-1}b_{-1} + a_1 b_{-3} = -\frac{1}{4j}$$

Thus the FS coefficients of $z(t) = x(t)y(t)$ is $\boxed{c_2 = \frac{1}{4j},\ c_{-2} = -\frac{1}{4j},\ c_k = 0,\ k \neq \pm 2}$

(d) $z(t) = x(t)\,y(t) = \cos(4\pi t)\sin(4\pi t) = \frac{1}{2}\sin(8\pi t)$

$$z(t) = \frac{e^{j2(4\pi)t} - e^{-j2(4\pi)t}}{4j} \qquad \omega_0 = 4\pi$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \text{ so } c_2 = \frac{1}{4j},\ c_{-2} = -\frac{1}{4j}$$

So the FS coefficient of $z(t)$ is $\boxed{c_2 = \frac{1}{4j},\ c_{-2} = -\frac{1}{4j},\ c_k = 0,\ k \neq \pm 2}$

It is the $\boxed{\text{same}}$ as the result with that of part (c).

VE216
Introduction to Signals and Systems

HOMEWORK 3 ATTACHED PAGES

April 7, 2020

Yihua Liu 518021910998

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6. The Fourier series representation of 4(a) is

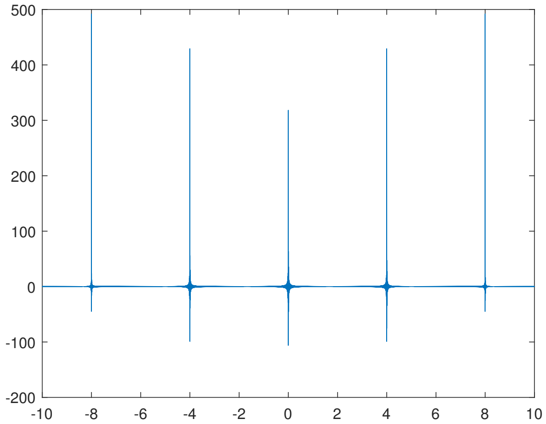

Figure 1. 6-1.

The Fourier series representation of 4(b) is

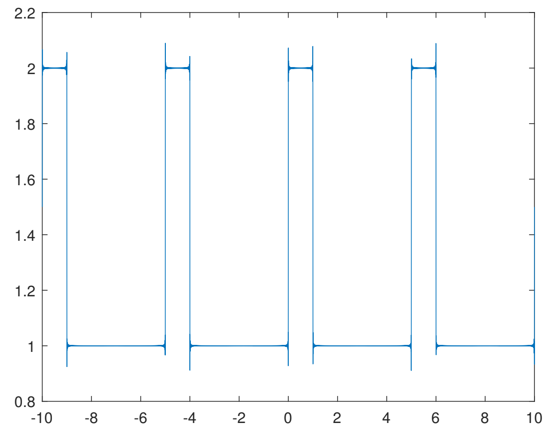

Figure 2. 6-2.

The Fourier series representation of 4(c) is

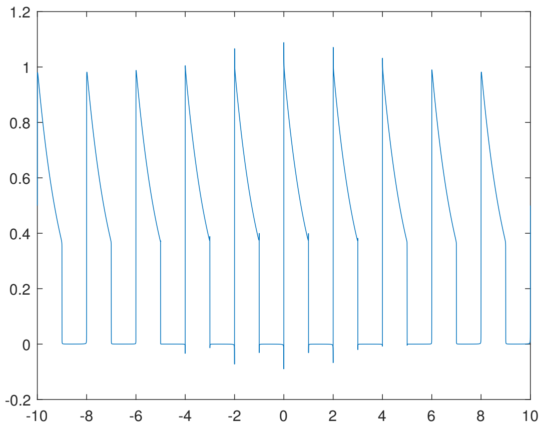

Figure 3. 6-3.

8.
(a) The graph of $S_N(t)$ with $N = 5$ for $t \in [0.5, 4.5]$ is

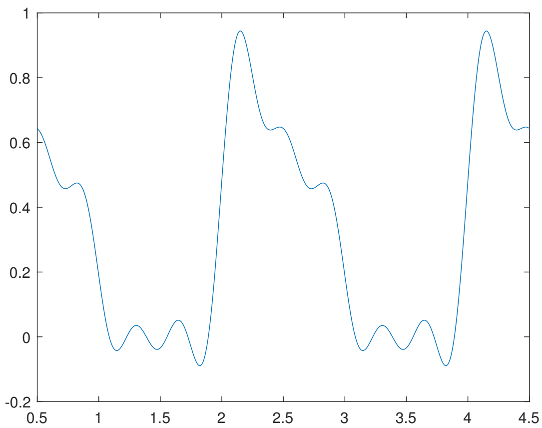

Figure 4. 8(a)-1.

The graph of $S_N(t)$ with $N = 10$ for $t \in [0.5, 4.5]$ is

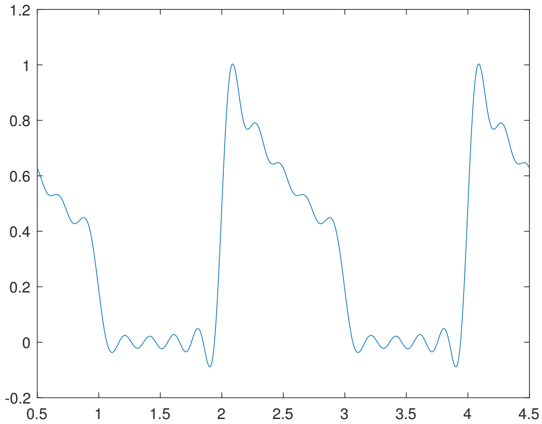

Figure 5. 8(a)-2.

The graph of $S_N(t)$ with $N = 15$ for $t \in [0.5, 4.5]$ is

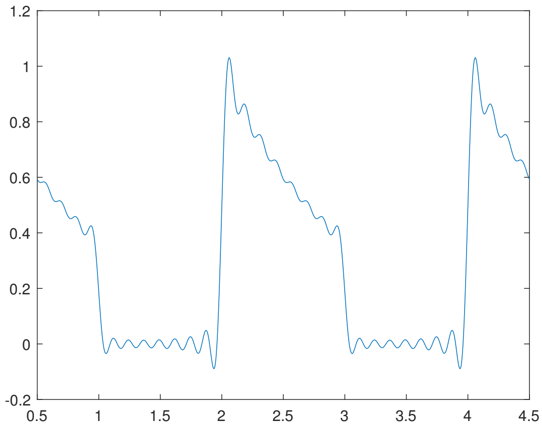

Figure 6. 8(a)-3.

The graph of $S_N(t)$ with $N = 19$ for $t \in [0.5, 4.5]$ is

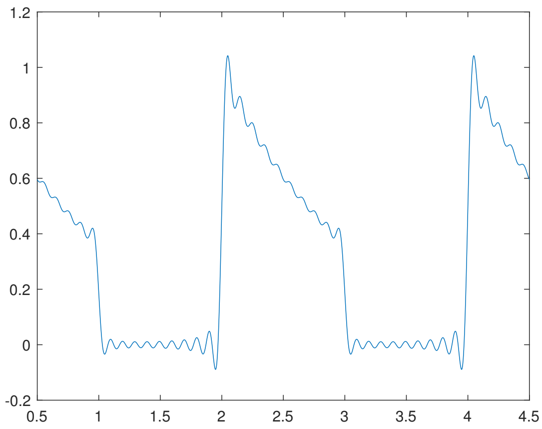

Figure 7. 8(a)-4.

From the graphs I see that the general shape and discontinuities of $S_N(t)$ does not change obviously as N increases, but the sawteeth of the curve increases as N increases proportionally. In general, the shape of the graph becomes closer to $x(t)$ as N increases.

(b) Use Matlab to calculate $S_N(0)$ and $S_N(1)$ for the N values in (a).

When $N = 5$, $S_N(0) = 0.4822$, $S_N(1) = 0.1891$.

When $N = 10$, $S_N(0) = 0.4902$, $S_N(1) = 0.1879$.

When $N = 15$, $S_N(0) = 0.4935$, $S_N(1) = 0.1861$.

When $N = 19$, $S_N(0) = 0.4949$, $S_N(1) = 0.1857$.

Hence, we can make an educated guess that $S_N(0) \to \frac{1}{2}$ and $S_N(1) \to \frac{1}{2e}$ when $N \to \infty$.

10.

(d) The graph of the magnitude of the system's frequency response $|H(j\omega)|$ as a function of ω is

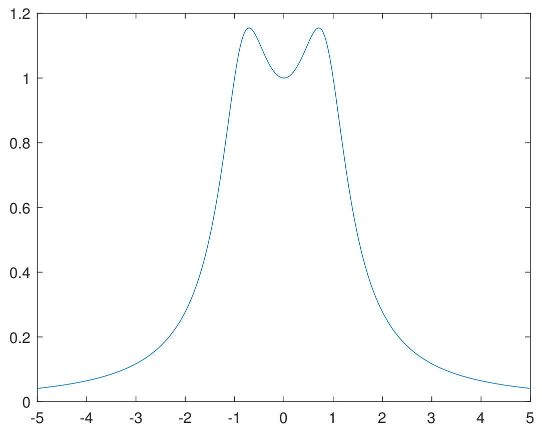

Figure 8. 10(d).

(e) The MATLAB code is shown below:

```
a=[1];
b=[1 1 1];
w=linspace(-5,5,10000);
h=freqs(a,b,w);
plot(w,abs(h));
```

The graph is

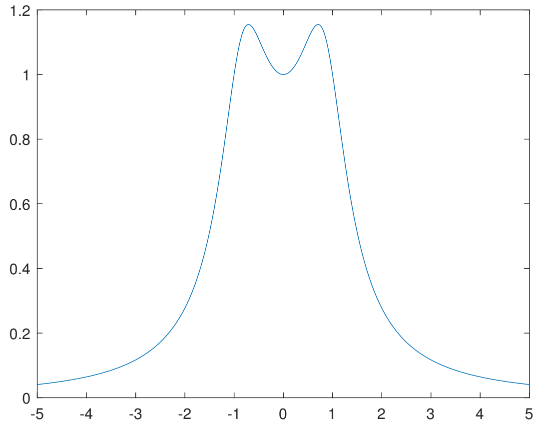

Figure 9. 10(e).

(f) The power density spectrum of $x(t)$ is

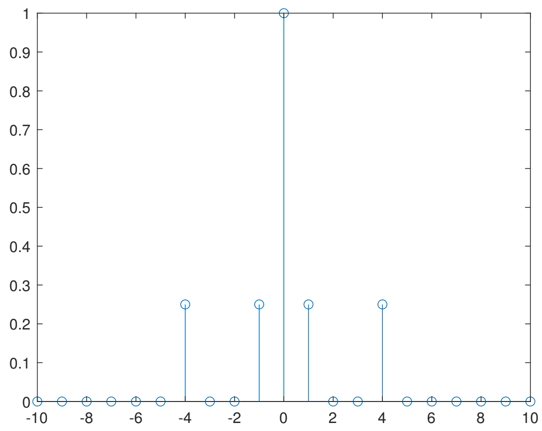

Figure 10. 10(f).

(h) The power density spectrum of $y(t)$ is

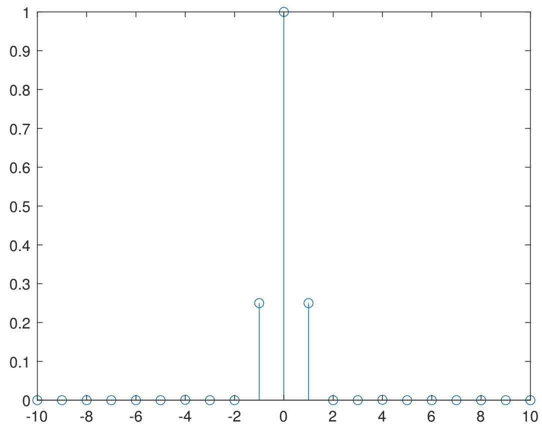

Figure 11. 10(h).

12.
(a) The graph of the fi
lter's magnitude response is

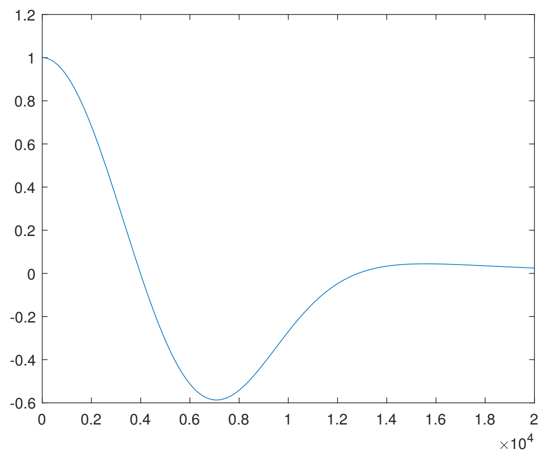

Figure 12. 12(a).

(b) The graph of the generated signal is

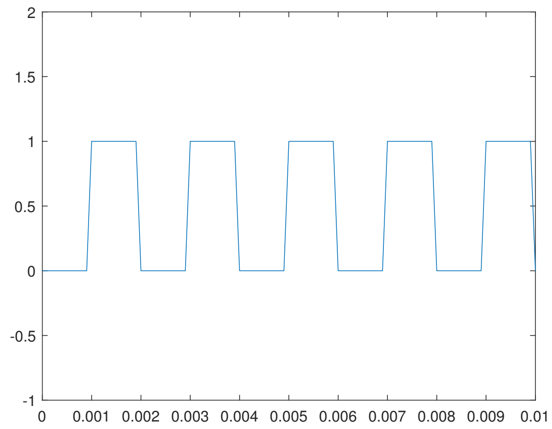

Figure 13. 12(b).

(c) The graph of the output signal is

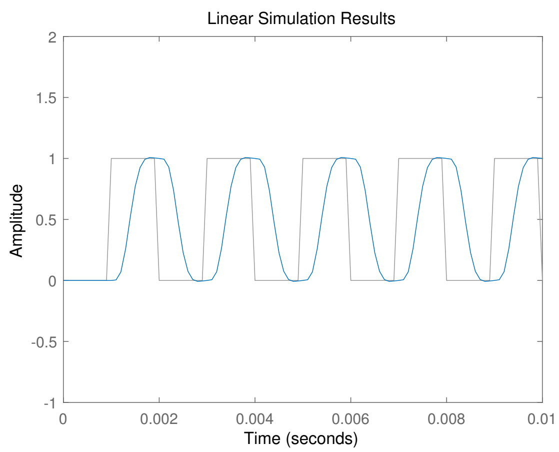

Figure 14. 12(c).