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1. $\langle \phi_k, \phi_l \rangle = \int_a^{a+T_0} \phi_k(t)\phi_l^*(t)\,dt = \int_a^{a+T_0} e^{j\omega_0 kt} e^{-j\omega_0 lt}\,dt$

$$= \int_a^{a+T_0} e^{j\omega_0 t(k-l)}\,dt = \left.\frac{e^{j\omega_0(k-l)t}}{j\omega_0(k-l)}\right|_{t=a}^{a+\frac{2\pi}{\omega_0}}$$

$$= \frac{1}{j\omega_0(k-l)}\left(e^{j\omega_0(k-l)(a+\frac{2\pi}{\omega_0})} - e^{j\omega_0(k-l)a}\right) = \frac{e^{j\omega_0(k-l)a}}{j\omega_0(k-l)}\left(e^{j\omega_0(k-l)\frac{2\pi}{\omega_0}}-1\right)$$

$$= \frac{1}{j\omega_0(k-l)} e^{j\omega_0(k-l)a}\left(e^{2\pi j(k-l)}-1\right)$$

$m = \frac{l}{k}$, $n = 1-m$

If $l = k$, $m=1$, $\langle \phi_k, \phi_l \rangle = \lim_{m\to 1} \frac{1}{j\omega_0(1-m)} e^{j\omega_0(1-m)a}\left(e^{2\pi j(1-m)}-1\right)$

$$= \lim_{n\to 0} \frac{1}{j\omega_0 n}\left(e^{\omega_0 jn(a+\frac{2\pi}{\omega_0})} - e^{j\omega_0 na}\right)$$

$$= \frac{1}{j\omega_0}\left(j\omega_0(a+\tfrac{2\pi}{\omega_0}) e^{\omega_0 jn(a+\frac{2\pi}{\omega_0})} - j\omega_0 a\, e^{j\omega_0 na}\right) = d \neq 0$$

If $l \neq k$, $e^{2\pi j(k-l)} = 1$, so $\langle \phi_k, \phi_l \rangle = 0$

Therefore, the set of complex exponentials $\{e^{j\omega_0 kt} : k = 0, \pm 1, \pm 2, \ldots\}$ is orthogonal on any interval over a period T_0, where $T_0 = \frac{2\pi}{\omega_0}$

2. $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k\left(\cos(k\omega_0 t) + j\sin(k\omega_0 t)\right)$

If $k=0$, $x(t) = \sum_{k=0} a_0 \cdot 1 = a_0$

$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = B[0]$

If $k \neq 0$, $a_k = \frac{1}{T}\int_0^T x(t) e^{-j\omega_0 t}\,dt = \frac{1}{T}\int_0^T x(t)\left[\cos(k\omega_0 t) - j\sin(k\omega_0 t)\right]dt$

$$= \frac{1}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt - \frac{j}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{1}{2}\frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)\,dt - \frac{j}{2}\frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)\,dt$$

$$= \frac{1}{2}B(k) - \frac{j}{2}A(k)$$

(continued)

Therefore, $x(t) = B[0] + \sum_{k=1}^{\infty} B[k]\cos(k\omega_0 t) + A[k]\sin(k\omega_0 t)$

where
$$\begin{cases} B[0] = \frac{1}{T}\int_0^T x(t)dt \\ B[k] = \frac{2}{T}\int_0^T x(t)\cos(k\omega_0 t)dt \\ A[k] = \frac{2}{T}\int_0^T x(t)\sin(k\omega_0 t)dt \end{cases}$$

3. $x(t) = \sin(3\pi t) + \cos(4\pi t)$

$$\sin(3\pi t) = \frac{e^{j3\pi t} - e^{-j3\pi t}}{2j}, \quad \cos(4\pi t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2}$$

$$x(t) = \frac{1}{2j}(e^{3\pi jt} - e^{-3\pi jt}) + \frac{1}{2}(e^{4\pi jt} + e^{-4\pi jt})$$

$$= \frac{1}{2j}e^{3\pi jt} - \frac{1}{2j}e^{-3\pi jt} + \frac{1}{2}e^{4\pi jt} + \frac{1}{2}e^{-4\pi jt}$$

$$= c_3 e^{3j\omega_0 t} + c_{-3}e^{-3j\omega_0 t} + c_4 e^{j4\omega_0 t} + c_{-4}e^{j4\omega_0 t}$$

FS coefficients:
$$\boxed{c_3 = \frac{1}{2j}, \quad c_{-3} = -\frac{1}{2j}, \quad c_4 = \frac{1}{2}, \quad c_{-4} = \frac{1}{2}}$$
for other value of k $c_k = 0$

4. (a) $T_0 = 4$ $\quad c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$

$\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$
$$= \frac{1}{4}\int_0^4 \delta(t-4n)e^{-jk\omega_0 t}dt = \frac{1}{4}e^{-jk\omega_0 4}$$

$$= \frac{e^{-2\pi jk}}{4} = \frac{1}{4} \qquad c_0 = \frac{1}{T_0}\int_{T_0} x(t)dt = \frac{1}{4}\int_0^4 \delta(t-4n)dt = \frac{1}{4}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{1}{4}e^{\frac{\pi jk}{2}t} = \frac{1}{4} + \frac{1}{2}\sum_{k=1}^{\infty}\cos\left(\frac{\pi k}{2}t\right)$$

(b) $T_0 = 5$ $\quad c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$

$\omega_0 = \frac{2\pi}{5}$
$$= \frac{1}{5}\int_0^5 rect\left(\frac{t-5n-3}{6}\right)e^{-jk\omega_0 t}dt$$

$c_0 = \frac{1}{T_0}\int_{T_0} x(t)dt$
$$= \frac{1}{5}\left(\int_0^1 2e^{-jk\omega_0 t}dt + \int_1^5 e^{-jk\omega_0 t}dt\right)$$

$= \frac{1}{5}\left(\int_0^1 2\,dt + \int_1^5 1\,dt\right)$
$$= \frac{1}{-5jk\omega_0}\left(2e^{-jk\omega_0 t}\Big|_0^1 + e^{-jk\omega_0 t}\Big|_1^5\right)$$

$= \frac{6}{5}$
$$= -\frac{1}{5jk\omega_0}\left(e^{-jk\omega_0} - 2 + e^{-5jk\omega_0}\right)$$

$$= -\frac{1}{2\pi jk}\left(e^{-2\pi jk} + e^{-\frac{2\pi}{5}jk} - 2\right) = \frac{j}{2\pi k}\left(e^{-\frac{2\pi}{5}jk} - 1\right)$$

$$= \frac{j}{2\pi k}\left(\cos\frac{2\pi}{5}k - j\sin\frac{2\pi}{5}k - 1\right) = \frac{1}{2\pi k}\sin\frac{2\pi}{5}k + \frac{j}{2\pi k}\left(\cos\frac{2\pi}{5}k - 1\right) = A_k + jB_k$$

$$x(t) = c_0 + 2\sum_{k=1}^{\infty} A_k\cos k\omega t - B_k\sin k\omega t$$

$$\boxed{= \frac{6}{5} + \sum_{k=1}^{\infty}\left(\frac{1}{\pi k}\left(\sin\frac{2\pi}{5}k\cos\frac{2\pi}{5}kt - \left(\cos\frac{2\pi}{5}k - 1\right)\sin\frac{2\pi}{5}kt\right)\right)}$$

(c) $T_0 = 2$ $\quad a_k = \frac{1}{T_0}\int_0^{T_0} x(t) e^{-j\omega_0 k t} dt$

$\omega_0 = \pi$ $\quad = \frac{1}{2}\int_0^1 e^{-t} e^{-jk\omega_0 t} dt$

$a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)dt$ $\quad = \frac{1}{2}\int_0^1 e^{-(1+jk\pi)t} dt = -\frac{1}{2(1+jk\pi)} e^{-(1+jk\pi)t}\Big|_0^1$

$= \frac{1}{2}\int_0^1 e^{-t} dt$

$= \frac{1}{2}(1-e^{-1})$ $\quad = \frac{1}{2(1+jk\pi)}(1-e^{-(1+jk\pi)}) = \frac{1}{2(1+jk\pi)}(1-\frac{(-1)^k}{e}) = \frac{1-(-1)^k/e}{2(1+\pi^2k^2)}(1-j\pi k)$

$$\boxed{x(t) = \frac{1}{2} - \frac{1}{2e} + \sum_{k=1}^{\infty} \frac{1-\frac{(-1)^k}{e}}{\pi^2 k^2+1}(\cos(k\pi t) + \pi k \sin(k\pi t))}$$

5.(a) We first derive a^*_{-k} ($x(t)$ is real iff $a_k = a^*_{-k}$)

$$a^*_{-k} = \begin{cases} 2 & k=0 \\ -j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} \neq a_k$$

So $x(t)$ is $\boxed{\text{NOT real}}$

(b) $x(t) = x(-t)$ iff $x(t)$ is even

or $a_k = a_{-k}$

$$a_{-k} = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|-k|} & \text{otherwise} \end{cases} = \begin{cases} 2 & k=0 \\ j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} = a_k$$

So $x(t)$ is $\boxed{\text{even}}$

(c) Fourier coefficient of $\frac{dx(t)}{dt}$ is $b_k = jk\frac{2\pi}{T}a_k$

$$b_k = \begin{cases} jk\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}jk\cdot j(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} = \begin{cases} jk\frac{4\pi}{T} & k=0 \\ -\frac{2\pi}{T}k(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases}$$

$$b_{-k} = \begin{cases} j(-k)\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}k(\frac{1}{2})^{|-k|} & \text{otherwise} \end{cases} = \begin{cases} -jk\frac{4\pi}{T} & k=0 \\ \frac{2\pi}{T}k(\frac{1}{2})^{|k|} & \text{otherwise} \end{cases} \neq b_k$$

So $\frac{dx(t)}{dt}$ is $\boxed{\text{NOT even}}$

6. See attached page.

7. (a) $x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$

If $x(t)$ is odd harmonic,

$$x(t+\tfrac{T}{2}) = \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} e^{jk\frac{2\pi}{T}\frac{T}{2}}$$

$$= \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} e^{jk\pi}$$

$$= \sum_{k \text{ is odd}} a_k e^{jk\frac{2\pi}{T}t} (-1)^k$$

$$= -\sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T}t}$$

Thus, $\boxed{x(t) = -x(t+\tfrac{T}{2})}$

(b) FS coefficient of $x(t)$:

$$a_k = \frac{1}{T}\left(\int_0^{\frac{T}{2}} x(t) e^{-jk\omega_0 t} dt + \int_{\frac{T}{2}}^{T} x(t) e^{-jk\omega_0 t} dt\right)$$

Let $t = t + \frac{T}{2}$ for the second part,

$$a_k = \frac{1}{T}\int_0^{T} \left(x(t) + x(t+\tfrac{T}{2}) e^{-j\pi k}\right) e^{-jk\omega_0 t} dt$$

$\forall k$ is even $x(t) + x(t+\frac{T}{2}) = 0$ i.e. $x(t) = -x(t+\frac{T}{2})$

Therefore, $x(t)$ is $\boxed{\text{odd harmonic}}$.

8. See attached page.

9.(a) $T_0 = 4 \quad \omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$k=0 \quad c_k = \frac{1}{4}\int_0^4 x(t)\,dt$

$= \frac{1}{4}\int_2^4 (-\frac{1}{2}t+1)\,dt = -\frac{1}{4}$

Using differentiation property $d_k = jk\omega_0 c_k$

$y(t) = \frac{d\,x(t)}{dt}$

For $0 \le t \le 4$, $x(t) = (-\frac{1}{2}t+1)(u(t-2)-u(t-4))$

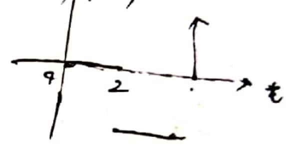
For all t $\quad x(t) = \sum_{k=-\infty}^{\infty} (-\frac{1}{2}t+1+2k)(u(t-2-4k)-u(t-4-4k))$

then $y(t) = \frac{dx(t)}{dt} = \sum_{k=-\infty}^{\infty}\left((-\frac{1}{2}t+1+2k)(\delta(t-2-4k)-\delta(t-4-4k)) + \frac{1}{2}(u(t-4k-4)-u(t-4k-2))\right)$

The first part for $0 \le t \le 4$, when $t=4 \quad 1-\frac{t}{2} = -1$

$t=2 \quad 1-\frac{t}{2} = 0$

Sketch:



$T_0 = 4$

$d_k = -\frac{1}{4}\int_0^4 \delta(t-4)e^{-j\frac{2\pi}{4}kt}\,dt - \frac{1}{8}\int_2^4 e^{-j\frac{\pi}{2}kt}\,dt$

$= -\frac{1}{4}\int_0^4 \delta(t-4)e^{-j2\pi k}\,dt + \frac{1}{4\pi k}e^{-\frac{\pi}{2}jkt}\Big|_2^4$

$= \frac{1}{4} + \frac{1}{4\pi kj}(e^{-2\pi kj} - e^{-\pi kj})$

$= \frac{1}{4} - \frac{j}{4\pi k}(1-(-1)^k)$

$k \ne 0$, $\quad c_k = \frac{d_k}{jk\omega_0} = \frac{-j(1-(-1)^k)}{4\pi k\, jk\frac{\pi}{2}} + \frac{1}{4jk\frac{\pi}{2}} = -\frac{j}{2\pi k} - \frac{1-(-1)^k}{2\pi^2k^2}$

$k=0 \quad c_0 = \frac{1}{T}\int_T x(t)\,dt = -\frac{1}{4}$

$$\boxed{x(t) = -\frac{1}{4} + \sum_{k=1}^{\infty}\frac{1}{\pi k}\left(\sin(\frac{\pi}{2}kt) - \frac{1-(-1)^k}{\pi k}\cos(\frac{\pi}{2}kt)\right)}$$

(b) Let $y(t) = \cos(\pi t) = \sum_{k=-\infty}^{\infty} b_k e^{j\frac{\pi}{2}kt} = \frac{e^{j\pi t}+e^{-j\pi t}}{2} \Rightarrow b_k = \begin{cases}\frac{1}{2}, & k=\pm 2\\ 0, & k \ne \pm 2\end{cases}$

then $z(t) = x(t)y(t)$, using multiplication property, $z(t) = \sum_{k=-\infty}^{\infty} d_k e^{jk\omega_0 t}$

$d_k = \sum_{l=-\infty}^{\infty} b_l c_{k-l} = b_2 c_{k-2} + b_{-2}c_{k+2} = \frac{1}{2}(c_{k-2}+c_{k+2})$

$k=2 \quad d_2 = \frac{1}{2}(c_0+c_4) = \frac{1}{2}(-\frac{1}{4} - \frac{j}{8\pi}) = -\frac{1}{8} - \frac{j}{16\pi}$

$k=-2 \quad d_{-2} = \frac{1}{2}(c_{-4}+c_0) = \frac{1}{2}(-\frac{1}{4} + \frac{j}{8\pi}) = -\frac{1}{8} + \frac{j}{16\pi}$

$k \ne \pm 2 \quad d_k = \frac{1}{2}\left(-\frac{j}{2\pi(k+2)} - \frac{1-(-1)^{k+2}}{2\pi^2(k+2)^2} - \frac{j}{2\pi(k-2)} - \frac{1-(-1)^{k-2}}{2\pi^2(k-2)^2}\right)$ (continued)

$d_3 = -\frac{1}{4\pi}(\frac{-j}{5} - j + \frac{2}{\pi \cdot 25} + \frac{2}{\pi})$

$$\boxed{d_k = \begin{cases} -\frac{1}{4\pi}\left(\frac{j}{k+2} + \frac{j}{k-2} + \frac{1-(-1)^k}{\pi(k+2)^2} + \frac{1-(-1)^k}{\pi(k-2)^2}\right) & k \neq \pm 2 \\ -\frac{1}{8} - \frac{j}{16\pi} & k = 2 \\ -\frac{1}{8} + \frac{j}{16\pi} & k = -2 \end{cases}}$$

$$\boxed{\begin{aligned} d_2 &= -\frac{1}{8} - \frac{j}{16\pi} \\ d_{-3} &= \frac{3j}{10} - \frac{13}{25\pi^2} \end{aligned}}$$

(c) Using time transformations $\quad a_k = c_k e^{jkw_0 b} = c_k e^{-2jkw_0} = c_k e^{-jk\pi} = (-1)^k c_k$

$w_0 = \frac{\pi}{2} \quad b = -2$

$a = \frac{1}{3} \quad w_1 = aw_0 = \frac{\pi}{6}$

Using amplitude transformations $\quad b_k = \begin{cases} b' + a' a_0 & k=0 \\ a' a_k & k \neq 0 \end{cases} = \begin{cases} 5 + 4a_0 & k=0 \\ 4c_k(-1)^k & k \neq 0 \end{cases}$

$c_0 = -\frac{1}{4}\quad b' = 5$

$w_2 = w_1 = \frac{\pi}{6} \quad a' = 4$

$a_0 = c_0 = -\frac{1}{4}$

$$= \begin{cases} 4 & k=0 \\ \frac{-2(-1)^k}{\pi k}\left(j + \frac{1-(-1)^k}{\pi k}\right) & k \neq 0 \end{cases}$$

The Fourier series expansion of $y(t)$ is

$$\boxed{y(t) = 4 + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \frac{-2(-1)^k}{\pi k}\left(\frac{1-(-1)^k}{\pi k} + j\right) e^{\frac{j\pi k}{6}t}}$$

10. (a) $x(t) = Ri(t) + L\frac{di(t)}{dt} + y(t)$

$i(t) = C\frac{dy(t)}{dt}$

$x(t) = RC\frac{dy(t)}{dt} + LC\frac{d^2y(t)}{dt^2} + y(t)$

$R = 1\Omega,\ L = 1H,\ C = 1F$

$x(t) = \frac{d^2y(t)}{dt^2} + \frac{dy(t)}{dt} + y(t)$

(b) Use Fourier transform

$X(j\omega) = (j\omega)^2 Y(j\omega) + (j\omega)Y(j\omega) + Y(j\omega)$

$= (-\omega^2 + j\omega + 1)\ Y(j\omega)$

$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{-\omega^2 + j\omega + 1}$

The system's response $\boxed{y(t) = H(j\omega)x(t) = \frac{e^{j\omega t}}{-\omega^2 + j\omega + 1}}$

(c) The system's transfer function $\boxed{H(s) = \frac{1}{s^2+s+1}}$

(d) $|H(j\omega)| = \left|\frac{-\omega^2 - j\omega + 1}{(\omega-1)^2 + \omega^2}\right| = \frac{1}{\sqrt{\omega^4 - \omega^2 + 1}}$

Plot see attached page

(e) See attached page

(f) $x(t) = 1 + \sin t + \sin(4t) = 1 + \frac{e^{jt} - e^{jt}}{2j} + \frac{e^{j4t} - e^{j4t}}{4j}$, $\omega_0 = \frac{2\pi}{T} = 1$

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$ where $a_k = \begin{cases} 1 & k=0 \\ \frac{1}{2j} & k=1,4 \\ -\frac{1}{2j} & k=-1,-4 \\ 0, & \text{otherwise} \end{cases}$

power density spectrum see attached page $|a_k|^2 = \begin{cases} 1 & k=0 \\ \frac{1}{4} & k=\pm1, \pm4 \\ 0 & \text{otherwise} \end{cases}$

(g) $P = \sum_{k=-\infty}^{\infty} |a_k|^2 = 4\times\frac{1}{4} + 1 = 2$

$\frac{1}{T}\int_T |x(t)|^2 dt = \frac{1}{2\pi}\int_0^{2\pi} (1+\sin t + \sin 4t)^2\, dt = 2$

So $P = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$

we've verified the Parseval's relation.

(h) $H(j\omega) = \frac{1}{-\omega^2+j\omega+1}$

$\omega=4 \quad H(4j) = \frac{1}{4j-15} \quad \omega=-4 \quad H(-4j) = \frac{-1}{4j+15}$

$\omega=1 \quad H(j) = -j \quad \omega=-1 \quad H(-j) = j$

$\omega=0 \quad H(0) = 1$

$$y(t) = 1 + \frac{e^{jt}}{2j}(-j) - \frac{e^{-jt}}{2j}j + \frac{e^{j4t}}{2j}\frac{1}{4j-15} + \frac{e^{-j4t}}{2j}\frac{1}{4j+15}$$

$$\boxed{y(t) = 1 - \frac{e^{jt}}{2} - \frac{e^{-jt}}{2} - \frac{e^{j4t}}{8+30j} + \frac{e^{-j4t}}{-8+30j}}$$

power density spectrum see attached page

(i) $\omega = \pm 4\omega_0$ component of $x(t)$ is attenuated

This RLC circuit is a $\boxed{\text{bandpass}}$ filter

11. $\omega_0 = \frac{2\pi}{T} = 12$

$$y(t) = \sum_{k=-\infty}^{\infty} H(j12k)e^{j12kt}$$

$|12k| < 100 \Rightarrow |k| \leq 8 \quad H(j12k) = 1 = a_k$

$|12k| > 100 \Rightarrow |k| \geq 9 \quad H(j12k) = 0 = a_k$

$$= x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j12kt}$$

hence for $\boxed{k \leq -9 \text{ or } k \geq 9}$ it's guaranteed that a_k must be zero

12. see attached page

13. If the FS coefficients of $x(t)$ is periodic with period N,

$a_k = a_{k-N}, \quad x(t) = x(t)e^{j(\frac{2\pi}{T})Nt}$

It is possible only when $x(t) = 0\ \forall t$ or $\frac{2\pi}{T}N = 2\pi k$

Therefore $x(t)$ can be expressed by $x(t) = \sum_{k=-\infty}^{\infty} g[k]\,\delta(t-\frac{kT}{N})$

14. Since $\sin^5 x = \frac{\sin 5x - 5\sin x + 15\sin x - 5\sin 3x}{16}$

$$\sin^5(3t) = \frac{\sin 15t + 10\sin 3t - 5\sin 9t}{16}$$

$$y(t) = 7\sin(3t) + a\,7\,\frac{\sin 15t + 10\sin 3t - 5\sin 9t}{16}$$

$$= \frac{49}{16}\sin(3t) + \frac{7}{160}\sin(15t) - \frac{7}{32}\sin(9t)$$ (continued)

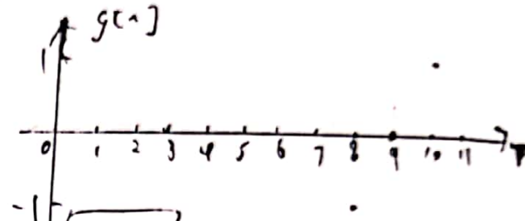
$w_0 = 3 \quad w_2 = 9,\ w_3 = 15$

Total power $P = \frac{(\frac{119}{16})^2}{2} + \frac{(\frac{7}{160})^2}{2} + \frac{(\frac{7}{32})^2}{2}$

power in fundamental $P_f = \frac{(\frac{119}{16})^2}{2}$

$THD = (1 - \frac{P_f}{P}) \times 100\% = 8.988 \times 10^{-4} \times 100\% = 0.09\%$

THD for this amplifier $\boxed{THD = 0.09\%}$

Graph: 

15. (a) The graph see the attached page

The fundamental period of $g[n]$ is $\boxed{10}$.

(b) $a_k = \frac{1}{10}\sum_{n=0}^{9} x[n] e^{-j\frac{\pi}{5}nk}$

$= \frac{1}{10}(1 + e^{-j\frac{\pi}{5}k} + e^{-j\frac{2\pi}{5}k} + e^{-j\frac{3\pi}{5}k} + e^{-j\frac{4\pi}{5}k} + e^{-j\pi k} + e^{-j\frac{6\pi}{5}k} + e^{-j\frac{7\pi}{5}k})$

the Fourier coefficient of $g[n]$ denoted as b_k is

$b_k = \frac{1}{10}(e^{-j\frac{\pi}{5}0k} - e^{-jk\frac{\pi}{5}8}) = \frac{1}{10}(1 - e^{-j\frac{8\pi}{5}k})$

Thus the Fourier series coefficients of $g[n]$ is $\boxed{b_k = \frac{1}{10}(1 - e^{-j\frac{8\pi}{5}k})}$

(c) $g[n] = x[n] - x[n-1] \overset{FS}{\longleftrightarrow} b_k = (1 - e^{-jk\frac{2\pi}{N}})a_k$

$N = 10$

So $\boxed{a_k = \frac{1 - e^{-j\frac{8}{5}\pi k}}{10(1 - e^{-j\frac{\pi}{5}k})}}$.

16. (a) $w_0 = 4\pi \quad x(t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2} = \sum_{k=-\infty}^{\infty} a_k e^{j4\pi k t}$

So $a_1 = \frac{1}{2}\ a_{-1} = \frac{1}{2}$

The FS coefficients of $x(t)$ is $\boxed{\begin{matrix} a_1 = \frac{1}{2} \\ a_{-1} = \frac{1}{2} \\ a_k = 0, k \neq \pm 1 \end{matrix}}$

(b) $w_0 = 4\pi \quad y(t) = \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} = \sum_{k=-\infty}^{\infty} b_k e^{j4\pi k t}$

So $b_1 = \frac{1}{2j}\quad b_{-1} = -\frac{1}{2j}$

The FS coefficients of $y(t)$ is $\boxed{\begin{matrix} b_1 = \frac{1}{2j} \\ b_{-1} = -\frac{1}{2j} \\ b_k = 0, k \neq \pm 1 \end{matrix}}$

(c) The FS coefficients of $z(t)$ is c_k, then

$$c_k = \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

$$c_0 = a_{-1} b_1 + a_1 b_{-1} = \frac{1}{4j} - \frac{1}{4j} = 0$$

$$c_1 = a_{-1} b_2 + a_1 b_0 = 0, \quad c_{-1} = a_{-1} b_0 + a_1 b_{-2} = 0$$

$$c_2 = a_{-1} b_3 + a_1 b_1 = \frac{1}{4j} \quad c_{-2} = a_{-1} b_{-1} + a_1 b_{-3} = -\frac{1}{4j}$$

Thus, the FS coefficients of $z(t) = x(t) y(t)$ is $\boxed{c_2 = \frac{1}{4j},\ c_{-2} = -\frac{1}{4j},\ c_k = 0,\ k \neq \pm 2}$

(d) $z(t) = x(t) y(t) = \cos(4\pi t) \sin(4\pi t) = \frac{1}{2} \sin(8\pi t)$

$$z(t) = \frac{e^{j2(4\pi)t} - e^{-j2(4\pi)t}}{4j} \qquad \omega_0 = 4\pi$$

$$= \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}, \text{ so } c_2 = \frac{1}{4j}, \ c_{-2} = -\frac{1}{4j}$$

So the FS coefficient of $z(t)$ is $\boxed{c_2 = \frac{1}{4j},\ c_{-2} = -\frac{1}{4j},\ c_k = 0,\ k \neq \pm 2}$

It is the $\boxed{\text{same}}$ as the result with that of part (c).