VE320
Intro to Semiconductor Devices

HOMEWORK 9

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Yihua Liu 518021910998

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1. (a) Since $V_{SG} > -V_T$ and $V_{SD} < V_{SG} + V_T$, the drain current I_D for $V_{SG} = 0.8$ V, $V_{SD} = 0.25$ V is

$$\begin{aligned} I_D &= \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left[2\left(V_{SG} + V_T\right) V_{SD} - V_{SD}^2\right] \\ &= \frac{1.0 \times 10^{-4}}{2} \times 15 \times \left[2 \times (0.8 - 0.4) \times 0.25 - 0.25^2\right] \\ &= 1.031 \times 10^{-4}\,\text{A}. \end{aligned}$$

(b) Since $V_{SD} > V_{SG} + V_T$, the drain current I_D for $V_{SG} = 0.8$ V, $V_{SD} = 1.0$ V is

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left(V_{SG} + V_T\right)^2 = \frac{1.0 \times 10^{-4}}{2} \times 15 \times (0.8 - 0.4)^2 = 1.2 \times 10^{-4}\,\text{A}.$$

(c) Since $V_{SD} > V_{SG} + V_T$, the drain current I_D for $V_{SG} = 1.2$ V, $V_{SD} = 1.0$ V is

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left(V_{SG} + V_T\right)^2 = \frac{1.0 \times 10^{-4}}{2} \times 15 \times (1.2 - 0.4)^2 = 4.8 \times 10^{-4}\,\text{A}.$$

(d) Since $V_{SD} > V_{SG} + V_T$, the drain current I_D for $V_{SG} = 1.2$ V, $V_{SD} = 2.0$ V is

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left(V_{SG} + V_T\right)^2 = \frac{1.0 \times 10^{-4}}{2} \times 15 \times (1.2 - 0.4)^2 = 4.8 \times 10^{-4}\,\text{A}.$$

2. (a) Assume the transistor operates in the saturation region, then the drain current is

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left(V_{SG} + V_T\right)^2$$

$$10^{-4} = \frac{1.2 \times 10^{-4}}{2} \times 20 \times \left(0 + V_T\right)^2$$

Solving the equation, the V_T value is

$$V_T = 0.2887\,\text{V}.$$

Then we can check that

$$V_{SD} = 1\,\text{V} > V_{SG} + V_T = 0.2887\,\text{V}$$

so the transistor does operate in the saturation region.

(b) Since $V_{SD} > V_{SG} + V_T$, the drain current I_D for $V_{SG} = 0.4$ V, $V_{SB} = 0$ V, and $V_{SD} = 1.5$ V is

$$I_D = \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left(V_{SG} + V_T\right)^2 = \frac{1.2 \times 10^{-4}}{2} \times 20 \times (0.4 + 0.2887)^2 = 5.69 \times 10^{-4}\,\text{A}.$$

(c) Since $V_{SG} > -V_T$ and $V_{SD} < V_{SG} + V_T$, the value of I_D for $V_{SG} = 0.6$ V, $V_{SB} = 0$ V, and $V_{SD} = 0.15$ V is

$$\begin{aligned} I_D &= \frac{k_p'}{2} \cdot \frac{W}{L} \cdot \left[2\left(V_{SG} + V_T\right) V_{SD} - V_{SD}^2\right] \\ &= \frac{1.2 \times 10^{-4}}{2} \times 20 \times \left[2 \times (0.6 + 0.2887) \times 0.15 - 0.15^2\right] \\ &= 2.929 \times 10^{-4}\,\text{A}. \end{aligned}$$

3. (a) The difference between E_{Fi} and E_F is

$$\phi_{fp} = \frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right) = \frac{300k}{e} \ln\left(\frac{10^{15}}{1.5 \times 10^{10}}\right) = 0.287\,\text{V}.$$

The threshold voltage is

$$\begin{aligned} V_{TN} &= \frac{Q_{SD}'(\max)}{C_{\text{ox}}} + V_{FB} + 2\phi_{fp} \\ &= ((Q_{SD}'(\max) - Q_{ss}')\left(\frac{t_{\text{ox}}}{\epsilon_{\text{ox}}}\right) + \phi_{ms} + 2\phi_{fp} \\ &= ((eN_a x_{dT} - Q_{ss}')\left(\frac{t_{\text{ox}}}{\epsilon_{\text{ox}}}\right) + \phi_{ms} + 2\phi_{fp} \\ &= \left((\sqrt{4eN_a\epsilon_s\phi_{fp}} - Q_{ss}'\right)\left(\frac{t_{\text{ox}}}{\epsilon_{\text{ox}}}\right) + \phi_{ms} + 2\phi_{fp} \\ &= \left(\sqrt{4e \times 10^{15} \times 0.01 \times 11.7\epsilon_0\phi_{fp}} - 5 \times 10^{10}e\right)\left(\frac{4 \times 10^{-6}}{0.01 \times 3.9\epsilon_0}\right) - 1 + 2\phi_{fp} \\ &= -0.359\,\text{V}. \end{aligned}$$

(b) It is possible to apply a V_{SB} voltage such that $V_T = 0$. The body effect coefficient is

$$\gamma = \frac{2\epsilon_s eN_a}{C_{\text{ox}}} = \sqrt{2e \times 11.7 \times 0.01\epsilon_0 \times 10^{15}}\frac{4 \times 10^{-6}}{3.9 \times 0.01\epsilon_0} = 0.211\,\text{V}^{1/2}.$$

The value of ΔV_T is

$$\Delta V_T = -V_{TN} = \gamma\left(\sqrt{2\phi_{fp} + V_{SB}} - \sqrt{2\phi_{fp}}\right)$$

Solving the equation, the value of V_{SB} is

$$V_{SB} = \left(\frac{-V_{TN}}{\gamma} + \sqrt{2\phi_{fp}}\right)^2 - 2\phi_{fp} = 5.46\,\text{V}.$$

4. (a) The transconductance is

$$\begin{aligned} g_{ms} &= \frac{k_n' W(V_{GS} - V_T)}{L} \\ &= \mu_n \frac{\epsilon}{t_{\text{ox}}}\frac{W}{L}(V_{GS} - V_T) \\ &= 400 \times \frac{3.9 \times 0.01\epsilon_0}{4.75 \times 10^{-6}} \times 10 \times (5 - 0.65) \\ &= 1.265 \times 10^{-3}\,\text{A/V}. \end{aligned}$$

We have the relation

$$g_m' = 0.8g_m = \frac{g_m}{1 + g_m r_s}.$$

Solving the equation, the value of source resistance is

$$r_s = 197.6\,\Omega.$$

(b) The ratio is

$$\begin{aligned}\frac{g'_m}{g_m} &= \frac{1}{1+\mu_n \frac{\epsilon}{t_{\text{ox}}}\frac{W}{L}(V_{GS}-V_T)r_s} \\ &= \frac{1}{1+400\times\frac{3.9\times 0.01\epsilon_0}{4.75\times 10^{-6}}\times 10\times(3-0.65)r_s} \\ &= 0.881.\end{aligned}$$

Therefore, g_{ms} is reduced from its ideal value when $V_G = 3$ V by 11.9 %.

5. (a) The total current is

$$I = 10^6\times 10^{-15}\exp\left(\frac{V_{GS}}{2.1V_t}\right) = 10^{-9}\exp\left(\frac{eV_{GS}}{630k}\right).$$

At $V_{GS} = 0.5$ V, the total current is

$$I = 10^{-9}\exp\left(\frac{0.5e}{630k}\right) = 9.996\times 10^{-6}\,\text{A}.$$

At $V_{GS} = 0.7$ V, the total current is

$$I = 10^{-9}\exp\left(\frac{0.7e}{630k}\right) = 3.979\times 10^{-4}\,\text{A}.$$

At $V_{GS} = 0.9$ V, the total current is

$$I = 10^{-9}\exp\left(\frac{0.9e}{630k}\right) = 1.584\times 10^{-2}\,\text{A}.$$

(b) The total power dissipated in the chip is

$$P = I_D V_D D = 5I_D.$$

The total power dissipated in the chip at $V_{GS} = 0.5$ V is

$$P = 5I_D = 4.998\times 10^{-5}\,\text{W}.$$

The total power dissipated in the chip at $V_{GS} = 0.7$ V is

$$P = 5I_D = 1.989\times 10^{-3}\,\text{W}.$$

The total power dissipated in the chip at $V_{GS} = 0.9$ V is

$$P = 5I_D = 7.919\times 10^{-2}\,\text{W}.$$

6. (a) (i) The difference between E_{Fi} and E_F is

$$\phi_{fp} = \frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right) = \frac{300k}{e}\ln\left(\frac{4\times 10^{16}}{1.5\times 10^{10}}\right) = 0.3825\,\text{V}.$$

The threshold voltage is

$$\begin{aligned}V_{TN} &= \left((\sqrt{4eN_a\epsilon_s\phi_{fp}} - Q'_{ss}\right)\left(\frac{t_{\text{ox}}}{\epsilon_{\text{ox}}}\right) + \phi_{ms} + 2\phi_{fp} \\ &= \left(\sqrt{4e\times 4\times 10^{16}\times 0.01\times 11.7\epsilon_0\phi_{fp}} - 4\times 10^{10}e\right)\left(\frac{1.2\times 10^{-6}}{0.01\times 3.9\epsilon_0}\right) - 0.5 + 2\phi_{fp} \\ &= 0.593\,\text{V}.\end{aligned}$$

The drain-source saturation voltage is

$$V_{DS} = V_{GS} - V_{TN} = 1.25 - 0.593 = 0.657\,\text{V}.$$

The value of ΔL is

$$\Delta L = \sqrt{\frac{2\epsilon}{eN_a}} \left(\sqrt{\phi_{fp} + V_{DS} + \Delta V_{DS}} - \sqrt{\phi_{fp} + V_{DS}} \right).$$

ΔL for $\Delta V_{DS} = 1$ V is

$$\Delta L = \sqrt{\frac{2 \times 0.117\epsilon_0}{4 \times 10^{16} e}} \left(\sqrt{\phi_{fp} + V_{DS} + 1} - \sqrt{\phi_{fp} + V_{DS}} \right) = 7.346 \times 10^{-8}\,\text{m}.$$

(ii) ΔL for $\Delta V_{DS} = 2$ V is

$$\Delta L = \sqrt{\frac{2 \times 0.117\epsilon_0}{4 \times 10^{16} e}} \left(\sqrt{\phi_{fp} + V_{DS} + 1} - \sqrt{\phi_{fp} + V_{DS}} \right) = 1.302 \times 10^{-7}\,\text{m}.$$

(iii) ΔL for $\Delta V_{DS} = 4$ V is

$$\Delta L = \sqrt{\frac{2 \times 0.117\epsilon_0}{4 \times 10^{16} e}} \left(\sqrt{\phi_{fp} + V_{DS} + 1} - \sqrt{\phi_{fp} + V_{DS}} \right) = 2.203 \times 10^{-7}\,\text{m}.$$

(b) We want

$$\frac{\Delta L}{L} = 0.12,$$

using result from (a) (iii), the minimum channel length is

$$L = 1.836 \times 10^{-6}\,\text{m}.$$

7. (a) (i) The ideal drain current for $V_{GS} = 0.8$ V is

$$I_D = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 = \frac{7.5 \times 10^{-5}}{2} \times 10 \times (0.8 - 0.35)^2 = 7.594 \times 10^{-5}\,\text{A}.$$

(ii) The drain current if $\lambda = 0.02\,\text{V}^{-1}$ is

$$I_D' = I_D(1 + \lambda V_{DS}) = I_D(1 + 0.02 \times 1.5) = 7.822 \times 10^{-5}\,\text{A}.$$

(iii) The output resistance for $\lambda = 0.02\,\text{V}^{-1}$ is

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02 I_D} = 6.584 \times 10^5\,\Omega.$$

(b) (i) The ideal drain current for $V_{GS} = 1.25$ V is

$$I_D = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 = \frac{7.5 \times 10^{-5}}{2} \times 10 \times (1.25 - 0.35)^2 = 3.0375 \times 10^{-4}\,\text{A}.$$

(ii) The drain current if $\lambda = 0.02\,\text{V}^{-1}$ is

$$I_D' = I_D(1 + \lambda V_{DS}) = I_D(1 + 0.02 \times 1.5) = 3.129 \times 10^{-4}\,\text{A}.$$

(iii) The output resistance for $\lambda = 0.02\,\text{V}^{-1}$ is

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.02 I_D} = 1.646 \times 10^5\,\Omega.$$

8. (a) (i) The maximum drain current in the original device is

$$I_D = \frac{k_n'}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2 = \frac{1.5 \times 10^{-4}}{2} \times \frac{6.0}{1.2} \times (3 - 0.45)^2 = 2.438 \times 10^{-3}\,\text{A}.$$

(ii) The maximum drain current in the scaled device is

$$\begin{aligned}
I_D &= \frac{k_n'}{2k} \cdot \frac{kW}{kL} \cdot (kV_{GS} - V_T)^2 \\
&= \frac{1.5 \times 10^{-4}}{2 \times 0.65} \times \frac{6.0}{1.2} \times (3 \times 0.65 - 0.45)^2 \\
&= 1.298 \times 10^{-3}\,\mathrm{A}.
\end{aligned}$$

(b) (i) The maximum power dissipation in the original device is

$$P = I_D V_D = 3I_D = 7.315 \times 10^{-3}\,\mathrm{W}.$$

(ii) The maximum power dissipation in the scaled device is

$$P = I_D k V_D = 1.95 I_D = 2.531 \times 10^{-3}\,\mathrm{W}.$$

9. The difference between E_{Fi} and E_F is

$$\phi_{fp} = \frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right) = \frac{300k}{e} \ln\left(\frac{2 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3646\,\mathrm{V}.$$

The maximum charge width is

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fp}}{eN_a}} = \sqrt{\frac{4 \times 11.7 \times 0.01\epsilon_0 \phi_{fp}}{2 \times 10^{16} e}} = 2.171 \times 10^{-5}\,\mathrm{cm}.$$

The threshold voltage shift is

$$\begin{aligned}
\Delta V_T &= -\frac{eN_a x_{dT}}{C_{\mathrm{ox}}} \left[\frac{r_j}{L}\left(\sqrt{1 + \frac{2x_{dT}}{r_j}} - 1\right)\right] \\
&= -\frac{2 \times 10^{16} e x_{dT} \times 8 \times 10^{-7}}{3.9 \times 0.01\epsilon_0} \left[\frac{0.30}{0.70}\left(\sqrt{1 + \frac{2x_{dT}}{3 \times 10^{-5}}} - 1\right)\right] \\
&= -0.0390\,\mathrm{V}.
\end{aligned}$$

The equivalent long-channel threshold voltage is

$$V_{TO} = V_T - \Delta V_T = 0.3890\,\mathrm{V}.$$

10. The difference between E_{Fi} and E_F is

$$\phi_{fp} = \frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right) = \frac{300k}{e} \ln\left(\frac{3 \times 10^{16}}{1.5 \times 10^{10}}\right) = 0.3751\,\mathrm{V}.$$

The maximum charge width is

$$x_{dT} = \sqrt{\frac{4\epsilon_s \phi_{fp}}{eN_a}} = \sqrt{\frac{4 \times 11.7 \times 0.01\epsilon_0 \phi_{fp}}{3 \times 10^{16} e}} = 1.798 \times 10^{-5}\,\mathrm{cm}.$$

The shift in threshold voltage due to narrow-channel effects is

$$\Delta V_T = \frac{eN_a x_{dT}}{C_{\mathrm{ox}}}\left(\frac{\xi x_{dT}}{W}\right) = \frac{3 \times 10^{16} e x_{dT} \times 8 \times 10^{-7}}{3.9 \times 0.01\epsilon_0}\left(\frac{\frac{\pi}{2} x_{dT}}{2.2 \times 10^{-4}}\right) = 0.02571\,\mathrm{V}.$$