VE320 Homework 4 Yihua Liu 刘翊华 SID: 518021910998

1 (a) $N_a = 10^{16}\,cm^{-3}$ $N_d = 0$, $n_i = 1.5\times10^{10}\,cm^{-3}$

$$p_0 = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a-N_d}{2}\right)^2 + n_i^2} \approx \frac{N_a}{2} + \sqrt{\frac{N_a^2}{4}+n_i^2} = 10^{16}\,cm^{-3}$$

$$n_0 = \frac{n_i^2}{p_0} = \frac{(1.5\times10^{10})^2}{10^{16}} = 2.25\times10^4\,cm^{-3}$$

$$D_n\frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E\frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t} \Rightarrow$$

Uniformly doped: $\frac{\partial(\delta n)}{\partial x} = 0$, excess carrier uniformly generated: $D_n\frac{\partial^2(\delta n)}{\partial x^2} = 0$

$g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$ Homogeneous: $\delta n = Ae^{-\frac{t}{\tau_{n0}}}$ particular: $\delta n = g'\tau_{n0}$

$\delta n = g'\tau_{n0} + Ae^{-\frac{t}{\tau_{n0}}}$ $t=0$ $\delta n = 0 \Rightarrow A + g'\tau_{n0} = 0 \Rightarrow A = -g'\tau_{n0} \Rightarrow \delta n = g'\tau_{n0}(1-e^{-\frac{t}{\tau_{n0}}})$

$g' = 8\times10^{20}\,cm^{-3}\cdot s^{-1}$ $\tau_{n0} = 5\times10^{-7}\,s$

$\delta n = 4\times10^{14}(1-e^{-2\times10^6 t})$

$\sigma = e(\mu_n n + \mu_p p) = e\mu_n n_0 + e\mu_p p_0 + e(\mu_n+\mu_p)\delta n \approx e\mu_p p_0 + e(\mu_n+\mu_p)\delta n$

$\mu_n = 900\,cm^2/V\cdot s$, $\mu_p = 380\,cm^2/V\cdot s$

$\sigma = 0.691 - 0.0820e^{-2\times10^6 t}$ $(\Omega\cdot cm)^{-1}$

$= 0.609 + 0.0820(1-e^{-2\times10^6 t})$ $(\Omega\cdot cm)^{-1}$ $(t>0)$

(b) (i) $t=0$ $\sigma = 0.609\ (\Omega\cdot cm)^{-1}$

(ii) $t=\infty$ $\sigma = 0.691\ (\Omega\cdot cm)^{-1}$

2. (a) $0 \le t \le 2\times10^{-6}\,s$

$$D_n\frac{\partial^2(\delta n)}{\partial x^2} + \mu_n E\frac{\partial(\delta n)}{\partial x} + g' - \frac{\delta n}{\tau_{n0}} = \frac{\partial(\delta n)}{\partial t}$$

$D_n\frac{\partial^2(\delta n)}{\partial x^2} = 0$ $\mu_n E\frac{\partial(\delta n)}{\partial x} = 0$ $g' = 0$ $\frac{\partial(\delta n)}{\partial t} + \frac{\delta n}{\tau_{n0}} = 0 \Rightarrow \delta n = Ae^{-\frac{t}{\tau_{n0}}}$

$t=0$ $\delta n = g'\tau_{n0} = A$

$g' = 10^{21}\,cm^{-3}\cdot s^{-1}$ $\Rightarrow \delta n = g'\tau_{n0}e^{-\frac{t}{\tau_{n0}}}$

$\tau_{n0} = 5\times10^{-7}\,s$

$\delta n = 5\times10^{14}e^{-2\times10^6 t}\ cm^{-3}$

$\to 2\times10^{-6}\,s$ $\delta n = Ae^{-\frac{t-2\times10^{-6}}{\tau_{n0}}} + B$ $t=\infty$ $\delta n = g'\tau_{n0} = B = 5\times10^{14}\,cm^{-3}$

$t = 2\times10^{-6}\,s$, $\delta n = A + 5\times10^{14} = 9.158\times10^{12}\,cm^{-3} \Rightarrow A = -4.908\times10^{14}$

$\delta n = -4.908\times10^{14}e^{-2\times10^6(t-2\times10^{-6})} + 5\times10^{14}$

$= -4.908\times10^{14}e^{-2\times10^6 t + 4} + 5\times10^{14}$

$$\delta n = \begin{cases} 5\times10^{14}e^{-2\times10^6 t}, & 0 \le t \le 2\times10^{-6}\,s \\ -4.908\times10^{14}e^{-2\times10^6 t+4} + 5\times10^{14}, & t > 2\times10^{-6}\,s \end{cases} \quad (cm^{-3})$$

(b) From (a), we have

(i) $t=0$ $\quad \delta n = 5\times 10^{10}\ cm^{-3}$

(ii) $t = 2\times 10^{-6}$ $\quad \delta n = 9.158\times 10^{12}\ cm^{-3}$

(iii) $t=\infty$ $\quad \delta n = 5\times 10^{10}\ cm^{-3}$

(c) See attached pages

3. Steady-state $\frac{\partial \delta p}{\partial t} = 0$ $\quad E=0$ $\quad$ minority carrier lifetime is infinite: $\frac{\delta p}{\tau_{p0}} = 0$

Thus, $D_p \frac{\partial^2(\delta p)}{\partial x^2} + g' = 0$ $\quad g' = G_0'$. $\quad -L<x<+L$

$$D_p \frac{\partial^2(\delta p)}{\partial x^2} + G_0' = 0 \Rightarrow \frac{\partial^2(\delta p)}{\partial x^2} = -\frac{G_0'}{D_p}$$

Solving the ODE, general solution, $\delta p = -\frac{G_0'}{2D_p}x^2 + C_1 x + C_2$

when $-3L\le x\le -L$, $L\le x\le 3L$, $g'=0$ $\quad D_p \frac{\partial^2(\delta p)}{\partial x^2} = 0$ $\Rightarrow \delta p = C_3 x + C_4$ $\ (-3L\le x\le -L)$

$\delta p(-3L) = \delta p(3L) = 0$ $\qquad \delta p = C_5 x + C_6$ $\ (L\le x\le 3L)$

$\Rightarrow -3LC_3 + C_4 = 3LC_5 + C_6 = 0$

BC: $\delta p(L) = -\frac{G_0'}{2D_p}L^2 + C_1 L + C_2 = C_5 L + C_6$

$\delta p(-L) = -\frac{G_0'}{2D_p}L^2 - C_1 L + C_2 = -C_3 L + C_4$

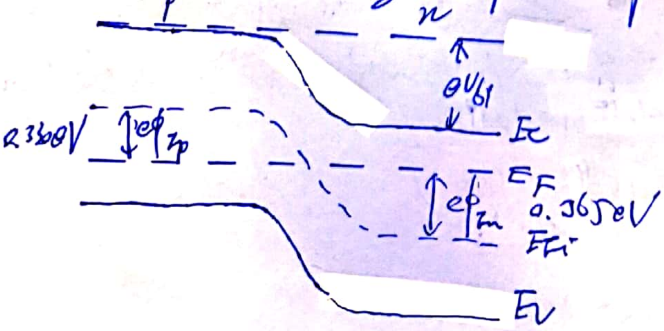
continuous: $\frac{\partial(\delta p(L))}{\partial x} = -\frac{G_0'}{D_p}L + C_1 = C_5$, $\quad \frac{\partial(\delta p(-L))}{\partial x} = \frac{G_0'}{D_p}L + C_1 = C_3$

Solving the equations, $C_1 = 0$, $C_4 = C_6 = \frac{3G_0'}{D_p}L^2$, $C_3 = \frac{G_0'}{D_p}L$, $C_5 = -\frac{G_0'}{D_p}L$

$C_2 = \frac{5G_0'}{2D_p}L^2$

Therefore, $$\delta p(x) = \begin{cases} \frac{G_0'}{D_p}x + \frac{3G_0'}{D_p}L^2, & -3L\le x<-L \\ -\frac{G_0'}{2D_p}x^2 + \frac{5G_0'}{2D_p}L^2, & -L\le x\le L \\ -\frac{G_0'}{D_p}x + 3\frac{G_0'}{D_p}L^2, & L<x\le 3L \end{cases}$$

4. (a) The energy-band diagram for the pn junction:



(b) p-region:

$N_a = n_i \exp\left(\frac{E_{Fi}-E_F}{kT}\right)$

$= 1.5\times10^{10}\exp\left(\frac{0.330\, e}{300k}\right)$

$= 5.25\times10^{15}\ cm^{-3}$

n-region: $N_d = n_i \exp\left(\frac{E_F-E_{Fi}}{kT}\right)$

$= 1.5\times10^{10}\exp\left(\frac{0.365\, e}{300k}\right)$

$= 2.03\times10^{16}\ cm^{-3}$

(c) $V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$

$= 0.330 + 0.365$

$= 0.695\ V$

5.(a) (i) $x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]}$ $\quad x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)}$

$W = x_n + x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e}\frac{N_a+N_d}{N_a N_d}}$ $\quad x_n = 25\%\, W \Rightarrow N_d = 3N_a$

$V_{bi} = V_t \ln\frac{N_a N_d}{n_i^2} = \frac{kT}{e}\ln\frac{N_a N_d}{n_i^2}$ $\quad \Rightarrow N_a N_d = n_i^2 \exp\left(\frac{eV_{bi}}{kT}\right)$, $\quad n_i = 1.5\times10^{10}\ cm^{-3}$, $V_{bi} = 0.710\ V$

$N_a = \frac{n_i}{\sqrt3}\sqrt{\exp\left(\frac{eV_{bi}}{kT}\right)} = \frac{n_i}{\sqrt3}\exp\left(\frac{eV_{bi}}{2kT}\right) = 7.97\times10^{15}\ cm^{-3}$

(ii) $N_d = \sqrt3 n_i \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.39\times10^{16}\ cm^{-3}$

(iii) $x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{3e}\frac{\sqrt3}{4n_i}\exp\left(\frac{-eV_{bi}}{2kT}\right)} = \sqrt{\frac{\sqrt3\varepsilon_s V_{bi}}{6en_i}\exp\left(\frac{-eV_{bi}}{2kT}\right)} = 9.800\times10^{-6}\ cm$ $\quad \boxed{\varepsilon_s = 1.00\times11.7\,\varepsilon_0}$

(iv) $x_p = \sqrt{\frac{6\varepsilon_s V_{bi}}{e}\frac{\sqrt3}{4n_i}\exp\left(-\frac{eV_{bi}}{2kT}\right)} = \sqrt{\frac{3\sqrt3\varepsilon_s V_{bi}}{2en_i}\exp\left(-\frac{eV_{bi}}{2kT}\right)} = 2.940\times10^{-5}\ cm$

(v) $|E_{max}| = \frac{eN_d x_n}{\varepsilon_s} = \sqrt3 en_i \exp\left(\frac{eV_{bi}}{2kT}\right)\sqrt{\frac{\sqrt3 V_{bi}}{6n_i e\varepsilon_s}\exp\left(\frac{-eV_{bi}}{2kT}\right)} = \sqrt{\frac{\sqrt3 V_{bi} en_i}{2\varepsilon_s}\exp\left(\frac{eV_{bi}}{2kT}\right)}$

$= 3.62\times10^{4}\ V/cm$

(b) (i) $n_i = 1.8\times10^{6}\ cm^{-3}$, $V_{bi} = 1.180\ V$

$N_a = \frac{n_i}{\sqrt3}\exp\left(\frac{eV_{bi}}{2kT}\right) = 8.478\times10^{15}\ cm^{-3}$

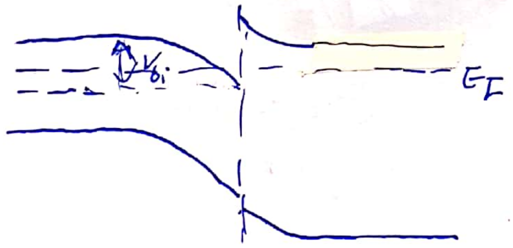
(ii) $N_d = \sqrt3 n_i \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.543\times10^{16}\ cm^{-3}$

(iii) $x_n = \sqrt{\frac{\sqrt3\varepsilon_s V_{bi}}{6en_i}\exp\left(\frac{-eV_{bi}}{2kT}\right)} = 1.296\times10^{-5}\ cm$ $\quad \boxed{\varepsilon_s = 1.00\times13.1\,\varepsilon_0}$

(iv) $x_p = \sqrt{\frac{3\sqrt3\varepsilon_s V_{bi}}{6en_i}\exp\left(-\frac{eV_{bi}}{2kT}\right)} = 3.888\times10^{-5}\ cm$

(v) $|E_{max}| = \sqrt{\frac{\sqrt3 V_{bi} en_i}{2\varepsilon_s}\exp\left(\frac{eV_{bi}}{2kT}\right)} = 4.552\times10^{4}\ V/cm$

6. (a)



(b) $n_0 = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right]$ $n_{01} = N_{d1} = 10^{16}\ cm^{-3}$

$n_{02} = N_{d2} = 10^{15}\ cm^{-3}$

$E_{F_1} - E_{Fi1} = kT \ln\left(\frac{n_{01}}{n_i}\right) = 5.55\times10^{-20} J = 0.347 eV$

$E_{F_2} - E_{Fi2} = kT \ln\left(\frac{n_{02}}{n_i}\right) = 4.60\times10^{-20} J = 0.287 eV$

$V_{bi} = \frac{\Delta(E_F - E_{Fi})}{e} = 0.0595\ V$

(c) The charge on the left of the junction is equal to the charge on the right of the junction
with concentration $10^{-16}\ cm^{-3}$ ... with concentration $10^{-15}\ cm^{-3}$

There are carriers in the depletion region decreasing from left to right from $10^{-16}\ cm^{-3}$ to $10^{-15}\ cm^{-3}$

7. (a) $V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) \Rightarrow N_a N_d = n_i^2 \exp\left(\frac{eV_{bi}}{kT}\right)$
$= 0.7460 V$ $= 80 N_a^2$

$N_a = \frac{n_i}{4\sqrt{5}} \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.76\times10^{15}\ cm^{-3}$

$N_d = 4\sqrt{5} n_i \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.20\times10^{17}\ cm^{-3}$

(b) $x_p = \sqrt{\frac{2\varepsilon_s(V_{bi}+V_R)\cdot 4\sqrt{5}}{80\, e n_i\, 81} \exp\left(\frac{-eV_{bi}}{2kT}\right)} = \sqrt{\frac{\sqrt{5}\,\varepsilon_s(V_{bi}+V_R)}{810\, e n_i} \exp\left(-\frac{eV_{bi}}{2kT}\right)} = 2.79\times10^{-6}\ m$

$x_n = \sqrt{\frac{640\sqrt{5}\,\varepsilon_s(V_{bi}+V_R)}{81\, e n_i} \exp\left(\frac{-eV_{bi}}{2kT}\right)} = 2.23\times10^{-4}\ m$

(c) $|E_{max}| = \frac{2(V_{bi}+V_R)}{W} = 2\times10.74 \times \sqrt{\frac{e}{2\varepsilon_s V_{bi}} \frac{N_a N_d}{N_a + N_d}} = 21.48 \sqrt{\frac{2\sqrt{5}\, e n_i}{81\, \varepsilon_s V_{bi}}} \exp\left(\frac{eV_{bi}}{4kT}\right)$

$= 9.51\times10^{4}\ V/cm$

(d) $C' = \sqrt{\frac{e\varepsilon_s N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}} = 4.59\times10^{-9}\ F/cm^2$

8. (a) $V_{bi} = \frac{kT}{e}\ln\left(\frac{n_{n0}\cdot p_{p0}}{n_i^2}\right) = 0.58V$, $n_{n0} = \frac{N_d - N_a}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} = 10^{14}\ cm^{-3}$

$p_{p0} = \frac{N_a - N_d}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} = 5\times10^{15}\ cm^{-3}$

(b) $x_n = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \frac{N_a}{N_d} \frac{1}{N_a + N_d}} = 2.66\times10^{-4}\ cm$

$x_p = \sqrt{\frac{2\varepsilon_s V_{bi}}{e} \frac{N_d}{N_a} \frac{1}{N_a + N_d}} = 5.32\times10^{-6}\ cm$

(c) $x_n = \sqrt{\frac{2\varepsilon_s(V_{bi}+V_R)}{e} \frac{N_a}{N_d} \frac{1}{N_a + N_d}} = 3\times10^{-3}\ cm$

$V_R = \frac{x_n^2 e}{2\varepsilon_s} \frac{N_d}{N_a}(N_a + N_d) - V_{bi} = 70.4\ V$

VE320
Intro to Semiconductor Devices

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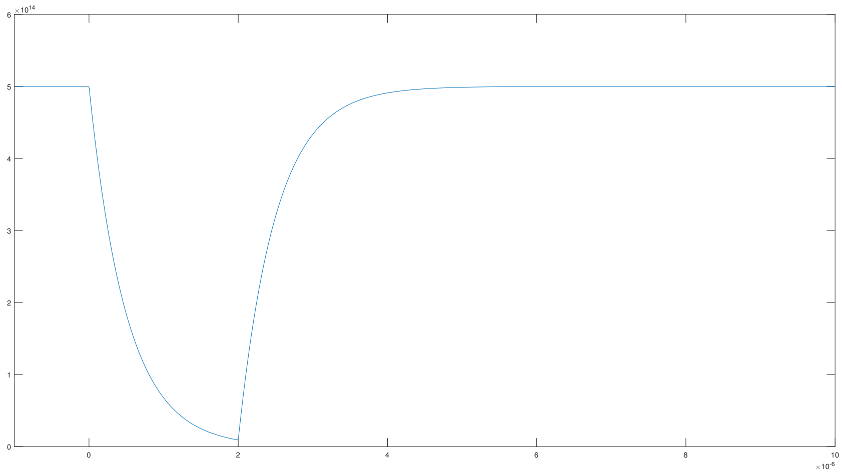

Figure 1: 2(c).