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1(a)  $n_a = 10^{16} \text{ cm}^{-3}$   $n_d = 0$ ,  $p_i = 1.5 \times 10^{10} \text{ cm}^{-3}$

$$p_0 = \frac{n_a - n_d}{2} + \sqrt{\left(\frac{n_a - n_d}{2}\right)^2 + p_i^2} = \frac{n_a}{2} + \sqrt{\frac{n_a^2}{4} + p_i^2} = 10^{16} \text{ cm}^{-3}$$

$$k_0 = \frac{n_i^2}{p_0} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

$$D_n \frac{\partial^2 (dn)}{\partial x^2} + \mu_n E \frac{\partial (dn)}{\partial x} + g' - \frac{dn}{\tau_{n0}} = \frac{\partial (dn)}{\partial t} \Rightarrow$$

Uniformly doped:  $\frac{\partial (dn)}{\partial x} = 0$ , excess carrier uniformly generated:  $p_n \frac{\partial^2 (dn)}{\partial x^2} = 0$

$$g' - \frac{dn}{\tau_{n0}} = \frac{\partial (dn)}{\partial t} \quad \text{Homogeneous: } dn = A e^{-\frac{x}{L_{n0}}} \quad \text{particular: } dn = g' \tau_{n0}$$

$$dn = g' \tau_{n0} + A e^{-\frac{x}{L_{n0}}} \quad \text{at } x=0: dn=0 \Rightarrow A = -g' \tau_{n0} \Rightarrow dn = g' \tau_{n0} (1 - e^{-\frac{x}{L_{n0}}})$$

$$g' = 8 \times 10^{20} \text{ cm}^{-3} \cdot \text{s}^{-1} \quad L_{n0} = 5 \times 10^{-7} \text{ s}$$

$$dn = 4 \times 10^{14} (1 - e^{-2 \times 10^6 x})$$

$$\delta = e(\mu_n n + \mu_p p) = e\mu_n n_0 + e\mu_p p_0 + e(\mu_n + \mu_p) dn \approx e\mu_n p_0 + e(\mu_n + \mu_p) dn$$

$$\mu_n = 900 \text{ cm}^2/\text{V}\cdot\text{s}, \mu_p = 380 \text{ cm}^2/\text{V}\cdot\text{s}$$

$$\delta = 0.691 - 0.9820 e^{-2 \times 10^6 x} \quad (\mu\text{cm})$$

$$= 0.691 + 0.9820 (1 - e^{-2 \times 10^6 x}) \quad (\mu\text{cm}) \quad (x \geq 0)$$

(b) (i)  $t=0 \quad \delta = 0.691 \quad (\mu\text{cm})$   
 (ii)  $t=\infty \quad \delta = 0.691 \quad (\mu\text{cm})$

2 (a)  $0 \leq x \leq 2 \times 10^{-6} \text{ s}$

$$p_n \frac{\partial^2 (dn)}{\partial x^2} + \mu_n E \frac{\partial (dn)}{\partial x} + g' - \frac{dn}{\tau_{n0}} = \frac{\partial (dn)}{\partial t}$$

$$D_n \frac{\partial^2 (dn)}{\partial x^2} = 0 \quad \mu_n E \frac{\partial (dn)}{\partial x} = 0 \quad g' = 0 \quad \frac{\partial (dn)}{\partial t} + \frac{dn}{\tau_{n0}} = 0 \Rightarrow dn = A e^{-\frac{t}{\tau_{n0}}}$$

$$g' = 10^{21} \text{ cm}^{-3} \cdot \text{s}^{-1} \Rightarrow dn = g' \tau_{n0} e^{-\frac{t}{\tau_{n0}}}$$

$$\tau_{n0} = 5 \times 10^{-7} \text{ s}$$

$$dn = 5 \times 10^{14} e^{-2 \times 10^6 t} \quad \text{cm}^{-3}$$

$$\delta = A e^{-\frac{t}{\tau_{n0} + \beta}} \quad t=0 \quad dn = g' \tau_{n0} = \beta = 5 \times 10^{14} \text{ cm}^{-3}$$

$$t = 2 \times 10^{-6} \text{ s}, dn = A + 5 \times 10^{14} = 9.58 \times 10^{12} \text{ cm}^{-3} \Rightarrow A = 4.908 \times 10^{14}$$

$$dn = \begin{cases} 5 \times 10^{14} e^{-2 \times 10^6 t} + 5 \times 10^{14} & (0 \leq t \leq 2 \times 10^{-6} \text{ s}) \\ -4.908 \times 10^{14} e^{-2 \times 10^6 t} + 4 & t \geq 2 \times 10^{-6} \text{ s} \end{cases}$$



3) From (a), we have

(i)  $t=0$   $n = 5 \times 10^{10} \text{ cm}^{-3}$

(ii)  $t=10^{-6}$   $n = 9.158 \times 10^{12} \text{ cm}^{-3}$

(iii)  $t=\infty$   $n = 5 \times 10^{10} \text{ cm}^{-3}$

(d) See attached pages

3. Steady-state  $\frac{\partial sp}{\partial x} = 0$   $E=0$  minority carrier lifetime is infinite.  $\frac{\partial p}{\partial x} = 0$

Thus,  $D_p \frac{\partial^2 sp}{\partial x^2} + g' = 0$   $g' = g_0' \quad -L \leq x \leq L$

$D_p \frac{\partial^2 sp}{\partial x^2} + g_0' = 0 \Rightarrow \frac{\partial^2 sp}{\partial x^2} = -\frac{g_0'}{D_p}$

Solving the ODE, general solution,  $sp = -\frac{g_0'}{2D_p} x^2 + C_1 x + C_2$

When  $-3L \leq x \leq -L$ ,  $g' = 0$   $D_p \frac{\partial^2 sp}{\partial x^2} = 0 \Rightarrow sp = C_3 x + C_4$   
 $sp = C_5 x + C_6 \quad L \leq x \leq 3L$

$sp(-3L) = sp(3L) = 0$

$\Rightarrow -3LC_3 + C_4 = 3LC_5 + C_6 = 0$

BC:  $sp(L) = -\frac{g_0'}{2D_p} L^2 + C_1 L + C_2 = C_5 L + C_6$

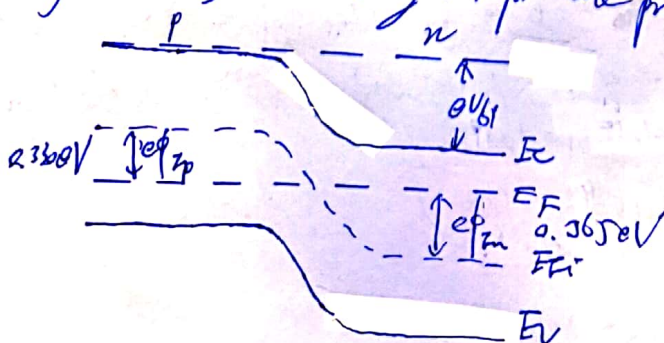
$sp(-L) = -\frac{g_0'}{2D_p} L^2 - C_1 L + C_2 = -C_3 L + C_4$

continuity:  $\frac{\partial sp(L)}{\partial x} = -\frac{g_0'}{D_p} L + C_1 = C_5$ ,  $\frac{\partial sp(-L)}{\partial x} = \frac{g_0'}{D_p} L + C_1 = C_3$

Solving the equations,  $C_1 = 0$ ,  $C_4 = C_6 = \frac{3g_0' L^2}{D_p}$ ,  $C_3 = \frac{g_0'}{D_p} L$ ,  $C_5 = -\frac{g_0'}{D_p} L$   
 $C_2 = \frac{5g_0' L^2}{2D_p}$

Therefore,  $sp(x) = \begin{cases} \frac{g_0'}{D_p} x + \frac{3g_0' L^2}{D_p} & -3L \leq x < -L \\ -\frac{g_0'}{2D_p} x + \frac{5g_0' L^2}{2D_p} & -L \leq x \leq L \\ -\frac{g_0'}{D_p} x + \frac{3g_0' L^2}{D_p} & L < x \leq 3L \end{cases}$

4. (a) The energy-band diagram for the pn junction:





(b) p-region:

$$n_a = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \\ = 1.5 \times 10^{10} \exp\left(\frac{0.330 \text{ eV}}{300 \text{ K}}\right) \\ = 5.25 \times 10^{15} \text{ cm}^{-3}$$

n-region:

$$n_d = n_i \exp\left(\frac{E_F - E_i}{kT}\right) \\ = 1.5 \times 10^{10} \exp\left(\frac{0.365 \text{ eV}}{300 \text{ K}}\right) \\ = 2.03 \times 10^{16} \text{ cm}^{-3}$$

$$(c) V_{bi} = |\phi_{Fn}| + |\phi_{Fp}| \\ = 0.330 + 0.365 \\ = 0.695 \text{ V}$$

$$5. (a) (i) x_n = \sqrt{\frac{2\epsilon_s \epsilon_0 V_{bi}}{e} \left[ \frac{N_a}{N_d} \right] \left[ \frac{1}{N_a + N_d} \right]} \quad x_p = \sqrt{\frac{2\epsilon_s \epsilon_0 V_{bi}}{e} \left[ \frac{N_d}{N_a} \right] \left[ \frac{1}{N_a + N_d} \right]}$$

$$W = x_n + x_p = \sqrt{\frac{2\epsilon_s \epsilon_0 V_{bi}}{e} \frac{N_a + N_d}{N_a N_d}}$$

$$V_{bi} = V_t \ln \frac{N_a N_d}{n_i^2} = \frac{kT}{e} \ln \frac{N_a N_d}{n_i^2}$$

$$N_a = \frac{n_i}{\sqrt{3}} \exp\left(\frac{eV_{bi}}{kT}\right) = \frac{n_i}{\sqrt{3}} \exp\left(\frac{eV_{bi}}{2kT}\right) = 7.97 \times 10^{15} \text{ cm}^{-3}$$

$$(ii) N_d = \sqrt{3} n_i \exp\left(\frac{eV_{bi}}{kT}\right) = 2.39 \times 10^{16} \text{ cm}^{-3}$$

$$(iii) x_n = \sqrt{\frac{2\epsilon_s \epsilon_0 V_{bi}}{3e} \frac{\sqrt{3}}{n_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = \sqrt{\frac{\sqrt{3}\epsilon_s \epsilon_0 V_{bi}}{6en_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = 9.800 \times 10^{-6} \text{ cm}$$

$$(iv) x_p = \sqrt{\frac{6\epsilon_s \epsilon_0 V_{bi}}{e} \frac{\sqrt{3}}{n_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = \sqrt{\frac{3\sqrt{3}\epsilon_s \epsilon_0 V_{bi}}{2en_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = 2.940 \times 10^{-5} \text{ cm}$$

$$(v) |E_{max}| = \frac{eN_a x_n}{\epsilon_s} = \frac{\sqrt{3} en_i \exp\left(\frac{eV_{bi}}{kT}\right)}{\epsilon_s} \sqrt{\frac{\sqrt{3}\epsilon_s \epsilon_0 V_{bi}}{6en_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = \sqrt{\frac{\sqrt{3} V_{bi} en_i}{2\epsilon_s} \exp\left(\frac{eV_{bi}}{kT}\right)} \\ = 3.62 \times 10^4 \text{ V/cm}$$

$$(b) (i) n_i = 1.8 \times 10^6 \text{ cm}^{-3}, V_{bi} = 1.180 \text{ V}$$

$$N_a = \frac{n_i}{\sqrt{3}} \exp\left(\frac{eV_{bi}}{kT}\right) = 8.478 \times 10^{15} \text{ cm}^{-3}$$

$$(ii) N_d = \sqrt{3} n_i \exp\left(\frac{eV_{bi}}{kT}\right) = 2.543 \times 10^{16} \text{ cm}^{-3}$$

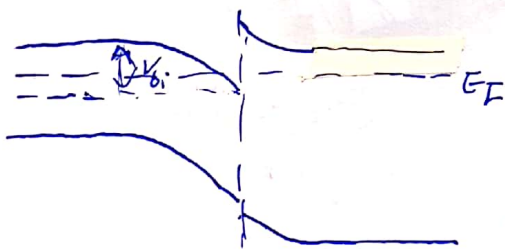
$$(iii) x_n = \sqrt{\frac{\sqrt{3}\epsilon_s \epsilon_0 V_{bi}}{6en_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = 1.286 \times 10^{-5} \text{ cm}$$

$$(iv) x_p = \sqrt{\frac{3\sqrt{3}\epsilon_s \epsilon_0 V_{bi}}{2en_i} \exp\left(-\frac{eV_{bi}}{kT}\right)} = 3.888 \times 10^{-5} \text{ cm}$$

$$(v) |E_{max}| = \sqrt{\frac{\sqrt{3} V_{bi} en_i}{2\epsilon_s} \exp\left(\frac{eV_{bi}}{kT}\right)} = 4.552 \times 10^4 \text{ V/cm}$$



6. (a)



$$\begin{aligned}
 (b) \quad n_0 &= n_i \exp\left(\frac{E_F - E_{Fi}}{kT}\right) & n_{01} &= n_{d1} = 10^{16} \text{ cm}^{-3} \\
 & & n_{02} &= n_{d2} = 10^{15} \text{ cm}^{-3} \\
 E_{F1} - E_{Fi} &= kT \ln\left(\frac{n_{01}}{n_i}\right) = 5.55 \times 10^{-20} \text{ J} = 0.347 \text{ eV} \\
 E_{F2} - E_{Fi} &= kT \ln\left(\frac{n_{02}}{n_i}\right) = 4.60 \times 10^{-20} \text{ J} = 0.287 \text{ eV} \\
 V_{bi} &= \frac{\Delta(E_F - E_{Fi})}{e} = 0.0595 \text{ V}
 \end{aligned}$$

(c) The charge on the left of the junction is equal to the charge on the right of the junction.  
 with concentration  $10^{16} \text{ cm}^{-3}$  and concentration  $10^{15} \text{ cm}^{-3}$   
 There are carriers in the depletion region decreasing from left to right from  $10^{16} \text{ cm}^{-3}$  to  $10^{15} \text{ cm}^{-3}$

$$\begin{aligned}
 7. (a) \quad V_{bi} &= \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right) \Rightarrow N_A N_D = n_i^2 \exp\left(\frac{eV_{bi}}{kT}\right) \\
 &= 2.740 \text{ V} = 80 N_A V
 \end{aligned}$$

$$N_A = \frac{n_i}{2.75} \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.76 \times 10^{15} \text{ cm}^{-3}$$

$$N_D = 4.5 N_A \exp\left(\frac{eV_{bi}}{2kT}\right) = 2.20 \times 10^{17} \text{ cm}^{-3}$$

$$(b) \quad x_p = \sqrt{\frac{2\epsilon_s \epsilon_0 (N_D + N_A)}{q n_i^2} \exp\left(\frac{-eV_{bi}}{kT}\right)} = \sqrt{\frac{5.5 \epsilon_s \epsilon_0 (N_D + N_A)}{810 n_i^2} \exp\left(\frac{-eV_{bi}}{2kT}\right)} = 2.79 \times 10^{-6} \text{ m}$$

$$x_n = \sqrt{\frac{640.8 \epsilon_s \epsilon_0 (N_D + N_A)}{18 n_i^2} \exp\left(\frac{-eV_{bi}}{kT}\right)} = 2.23 \times 10^{-4} \text{ m}$$

$$\begin{aligned}
 (c) \quad |\bar{E}_{max}| &= \frac{20^{16} \text{ cm}^{-3} \cdot eV_{bi}}{W} = 2 \times 10^{16} \times \sqrt{\frac{e}{2.5 \epsilon_s \epsilon_0} \frac{N_A N_D}{N_A + N_D}} = 1.48 \sqrt{\frac{20^{16} n_i^2}{2.5 \epsilon_s \epsilon_0}} \exp\left(\frac{eV_{bi}}{2kT}\right) \\
 &= 9.51 \times 10^4 \text{ V/cm}
 \end{aligned}$$

$$(d) \quad C_i = \sqrt{\frac{e \epsilon_s N_A N_D}{2W \epsilon_s \epsilon_0 (N_A + N_D)}} = 4.59 \times 10^9 \text{ F/cm}^2$$

$$8. (a) \quad V_{bi} = \frac{kT}{e} \ln\left(\frac{n_0 \cdot p_0}{n_i^2}\right) = 0.58 \text{ V}, \quad n_{02} = \frac{N_D - N_A}{2} + \sqrt{\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2} = 10^{14} \text{ cm}^{-3}$$

$$(b) \quad x_n = \sqrt{\frac{2\epsilon_s \epsilon_0 N_A}{e N_D (N_A + N_D)}} = 2.66 \times 10^{-4} \text{ cm}$$

$$x_p = \sqrt{\frac{2\epsilon_s \epsilon_0 N_D}{e N_A (N_A + N_D)}} = 5.32 \times 10^{-6} \text{ cm}$$

$$(c) \quad x_n = \sqrt{\frac{2\epsilon_s \epsilon_0 (N_D - N_A)}{e N_A (N_A + N_D)}} = 3 \times 10^{-3} \text{ cm}$$

$$V_k = \frac{x_n^2 e}{2\epsilon_s} \frac{N_A}{N_D} (N_A + N_D) - V_{bi} = 70.4 \text{ V}$$

