VE320
Intro to Semiconductor Devices

HOMEWORK 1

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1.

$$E = h\nu$$
$$\nu = \frac{c}{\lambda}$$

Thus,

$$\lambda = \frac{hc}{E}$$

The maximum wavelength of light for the photoelectric emission of electrons for gold is:

$$\lambda_{\mathrm{max}} = 2.53 \times 10^{-7} \text{ m}$$

The maximum wavelength of light for the photoelectric emission of electrons for cesium is:

$$\lambda_{\mathrm{max}} = 6.53 \times 10^{-7} \text{ m}$$

2.
(a)

$$p = mv = \frac{h}{\lambda}$$
$$\lambda = 5.5 \times 10^{-7} \text{ m}$$

Thus, the electron momentum is

$$p = \frac{h}{\lambda} = 1.20 \times 10^{-27} \text{ kg} \cdot \text{m/s}$$

The electron velocity is

$$v = \frac{h}{m\lambda} = 1.32 \times 10^{3} \text{ m/s}$$

(b)

$$\lambda = 5.5 \times 10^{-7} \text{ m}$$

Thus, the electron momentum is

$$p = \frac{h}{\lambda} = 1.51 \times 10^{-27} \text{ kg} \cdot \text{m/s}$$

The electron velocity is

$$v = \frac{h}{m\lambda} = 1.65 \times 10^{3} \text{ m/s}$$

(c) Yes, the momentum of the photon is equal to the momentum of the electron.
3.

(a) Simple cubic
(i) (100) The surface density of atoms is

$$\frac{10^{-4}}{(4.50 \times 10^{-10})^2} = 4.94 \times 10^{14}\ \mathrm{cm}^{-2}$$

(ii) (110) The surface density of atoms is

$$\frac{10^{-4}}{\sqrt{2} \times (4.50 \times 10^{-10})^2} = 3.49 \times 10^{14}\ \mathrm{cm}^{-2}$$

(iii) (111) The surface density of atoms is

$$\frac{\frac{1}{2} \times 10^{-4}}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{6}}{2} \times (4.50 \times 10^{-10})^2} = 2.85 \times 10^{14}\ \mathrm{cm}^{-2}$$

(b) Body-centered cubic
(i) (100) The surface density of atoms is

$$\frac{10^{-4}}{(4.50 \times 10^{-10})^2} = 4.94 \times 10^{14}\ \mathrm{cm}^{-2}$$

(ii) (110) The surface density of atoms is

$$\frac{2 \times 10^{-4}}{\sqrt{2} \times (4.50 \times 10^{-10})^2} = 6.98 \times 10^{14}\ \mathrm{cm}^{-2}$$

(iii) (111) The surface density of atoms is

$$\frac{\frac{1}{2} \times 10^{-4}}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{6}}{2} \times (4.50 \times 10^{-10})^2} = 2.85 \times 10^{14}\ \mathrm{cm}^{-2}$$

(c) Face-centered cubic
(i) (100) The surface density of atoms is

$$\frac{2 \times 10^{-4}}{(4.50 \times 10^{-10})^2} = 9.88 \times 10^{14}\ \mathrm{cm}^{-2}$$

(ii) (110) The surface density of atoms is

$$\frac{2 \times 10^{-4}}{\sqrt{2} \times (4.50 \times 10^{-10})^2} = 6.98 \times 10^{14}\ \mathrm{cm}^{-2}$$

(iii) (111) The surface density of atoms is

$$\frac{2 \times 10^{-4}}{\frac{\sqrt{2}}{2} \times \frac{\sqrt{6}}{2} \times (4.50 \times 10^{-10})^2} = 1.14 \times 10^{15}\ \mathrm{cm}^{-2}$$

4. The average electron energy is

$$E = \frac{3kT}{2} = 6.21 \times 10^{-21}\ \mathrm{J} = 3.88 \times 10^{-2}\ \mathrm{eV}$$

The average electrons momentum is

$$p = \sqrt{2m_e E} = \sqrt{3m_e kT} = 1.06 \times 10^{-25}\ \mathrm{kg \cdot m/s}$$

The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{3m_e kT}} = 6.23 \times 10^{-9}\ \mathrm{m}$$

5.

(a) The energy levels are

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

The width

$$a = 10^{-9} \text{ m}$$

The first three energy levels are

$$E_1 = \frac{\pi^2\hbar^2}{2m_e a^2} = 6.02 \times 10^{-20} \text{ J} = 0.376 \text{ eV}$$

$$E_2 = \frac{4\pi^2\hbar^2}{2m_e a^2} = 2.41 \times 10^{-19} \text{ J} = 1.504 \text{ eV}$$

$$E_3 = \frac{9\pi^2\hbar^2}{2m_e a^2} = 5.42 \times 10^{-19} \text{ J} = 3.384 \text{ eV}$$

(b) The wavelength of a photon that might be emitted is

$$\lambda = \frac{hc}{E_3 - E_2} = \frac{8cm_e a^2}{5h} = 6.59 \times 10^{-7} \text{ m}$$

6.

(a) The de Broglie wavelength of an electron is

$$\lambda = 8.5 \times 10^{-9} \text{ m}$$

The electron momentum is

$$p = \frac{h}{\lambda} = 7.80 \times 10^{-26} \text{ kg} \cdot \text{m/s}$$

The electron velocity is

$$v = \frac{p}{m_e} = \frac{h}{m_e \lambda} = 8.56 \times 10^4 \text{ m/s}$$

The electron energy is

$$E = \frac{p^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = 3.34 \times 10^{-21} \text{ J} = 2.08 \times 10^{-2} \text{ eV}$$

(b) The velocity of the electron is

$$v = 8 \times 10^3 \text{ m/s}$$

The electron energy is

$$E = \frac{1}{2}m_e v^2 = 2.92 \times 10^{-23} \text{ J} = 1.82 \times 10^{-4} \text{ eV}$$

The electron momentum is

$$p = m_e v = 7.29 \times 10^{-27} \text{ kg} \cdot \text{m/s}$$

The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v} = 9.09 \times 10^{-8} \text{ m} = 909 \text{ Å}$$

7. Since

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 \mathrm{d}x = 1,$$

we have

$$\int_{-\infty}^{-1}|\Psi(x,t)|^2\mathrm{d}x+\int_{-1}^{3}|\Psi(x,t)|^2\mathrm{d}x+\int_{3}^{\infty}|\Psi(x,t)|^2\mathrm{d}x=1$$

thus,

$$\int_{-\infty}^{-1}0\mathrm{d}x+\int_{-1}^{3}|A(\cos{(\frac{\pi x}{2})})e^{-j\omega t}|^2\mathrm{d}x+\int_{3}^{\infty}0\mathrm{d}x=1.$$

Then,

$$\int_{-1}^{3}A^2\cos^2{(\frac{\pi x}{2})}\mathrm{d}x=1$$

Since

$$\int\cos^2{(\frac{\pi x}{2})}\mathrm{d}x=\frac{x}{2}+\frac{\sin\pi x}{2\pi},$$

we have

$$2A^2=1.$$

Therefore,

$$A=\frac{\sqrt{2}}{2}.$$

8.

(a) The time-independent Schrodinger's wave equation is

$$\frac{-\hbar^2}{2m}\cdot\frac{1}{\psi(x)}\cdot\frac{\partial^2\psi(x)}{\partial x^2}+V(x)=E. \tag{1}$$

or

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi=E\psi. \tag{2}$$

Condition: for $x<0$, $V(x)=V_0$, so

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=(E-V_0)\psi.$$

Thus, the general solution of ψ is

$$\psi_1(x)=Ae^{-jk_1x}+Be^{jk_1x}.$$

Condition: for $0\le x\le a$, $V(x)=0$, so

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}=E\psi.$$

Thus, the general solution of ψ is

$$\psi_2(x)=Ce^{-jk_2x}+De^{jk_2x}.$$

Condition: for $x>a$, $V(x)=\infty$, so the equation

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi=E\psi$$

has the general solution of ψ of

$$\psi_3(x)=0.$$

Solving the equations, we have

$$k_1=\sqrt{\frac{2m(E-V_0)}{\hbar^2}}$$

$$k_2=\sqrt{\frac{2mE}{\hbar^2}}$$

Besides, later we would calculate the integral of ψ_1 and note that $\psi_1(x) < \infty$ when $x < a$, so $A = 0$, then $\psi_1(x) = Be^{jk_1x}$.

In addition, later we would calculate the integral of ψ_2 and need to use Eq. (2.29) in the textbook as an alternative particular form of solution to the Schrodinger's equation:

$$\psi_2(x) = A_1 \cos k_2x + A_2 \sin k_2x.$$

Therefore, the wave solutions that apply in each region are

$$\psi(x) = \begin{cases} Be^{j\sqrt{\frac{2m(E-V_0)}{\hbar^2}}x}, & x < 0 \\ A_1 \cos\sqrt{\frac{2mE}{\hbar^2}}x + A_2 \sin\sqrt{\frac{2mE}{\hbar^2}}x, & 0 \le x \le a \\ 0, & x > a \end{cases}$$

(b) Applying the boundary conditions:
① at $x = 0$, $\psi_1 = \psi_2$, $B = A_2$;
② at $x = 0$, $\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x}$, $jk_1B = k_2A_1$;
③ at $x = a$, $\psi_2 = \psi_3 = 0$, $A_1 \cos k_2x + A_2 \sin k_2x = 0 \Rightarrow A_2 = -A_1 \tan k_2a$;
④ $\int_{-\infty}^{\infty} |\psi|^2 \mathrm{d}x = \int_{-\infty}^{0} |\psi_1|^2 \mathrm{d}x + \int_0^a |\psi_2|^2 \mathrm{d}x + \int_a^{\infty} |\psi_3|^2 \mathrm{d}x = \int_{-\infty}^{0} \psi_1\psi_1^* \mathrm{d}x + \int_0^a \psi_2\psi_2^* \mathrm{d}x = 1.$

We would assume that the wave function solution $\psi(x)$ is a real function, then $\psi(x) = \psi^*(x)$. To prove $\psi(x)$ can be chosen to be real, consider Eq. (1) and (2), we have

$$\frac{\hbar^2}{2m}\psi'' + (E - V(x))\psi = 0. \tag{3}$$

Since $(\psi'')^* = (\psi^*)''$, the complex conjugation of Eq. (3) is

$$\frac{\hbar^2}{2m}(\psi^*)'' + (E - V(x))\psi^* = 0.$$

By superposition, we can derive the real and the imaginary parts of ψ that are real solutions to the equation:

$$\psi_{re} = \frac{1}{2}(\psi + \psi^*), \qquad \psi_{im} = \frac{1}{2j}(\psi - \psi^*).$$

Hence, we have proved that solution $\psi(x)$ of the time-independent Schrodinger equation can always be chosen to be real.

Using the assumption above, the integral becomes

$$\begin{aligned}\int_{-\infty}^{\infty} |\psi|^2 \mathrm{d}x &= \int_{-\infty}^{0} B^2 e^{2jk_1x} + \int_0^a (A_1 \cos k_2x + A_2 \sin k_2x)^2 \mathrm{d}x \\ &= \frac{B^2}{2j} e^{2jk_1x}\Big|_{-\infty}^{0} + \frac{(2k_2x + \sin 2k_2x)A_1^2 - 2\cos 2k_2xA_1A_2 + (2k_2x - \sin 2k_2x)A_2^2}{4k}\Big|_0^a \\ &= \frac{B^2}{2j} + \frac{(2k_2a + \sin 2k_2a)A_1^2 + 2(1 - \cos 2k_2a)A_1A_2 + (2k_2a - \sin 2k_2a)A_2^2}{4k} \\ &= 1.\end{aligned}$$

Since only three constants remain unknown, we only need three equations. Therefore, the set of equations that result from applying the boundary conditions are

$$B = A_2 \tag{4}$$

$$jk_1B = k_2A_1 \tag{5}$$

$$A_2 = -A_1 \tan k_2a \tag{6}$$

(c) Using the set of equations above in (b), combining Eq. (4) and (5),

$$A_1 = \frac{jk_1 A_2}{k_2}. \tag{7}$$

Combining Eq. (6) and (7),

$$A_2 = -\frac{jk_1 A_2}{k_2} \tan k_2 a,$$

then,

$$-\tan k_2 a = \frac{jk_2}{k_1}.$$

Thus,

$$\sqrt{\frac{E}{V_0 - E}} = -\tan\left(\sqrt{\frac{2mE}{\hbar^2}}a\right)$$

The solutions of E as the intersections comprising a discrete set. For example, suppose $V_0 = 10$ eV and $a = 10^{-9}$ m,

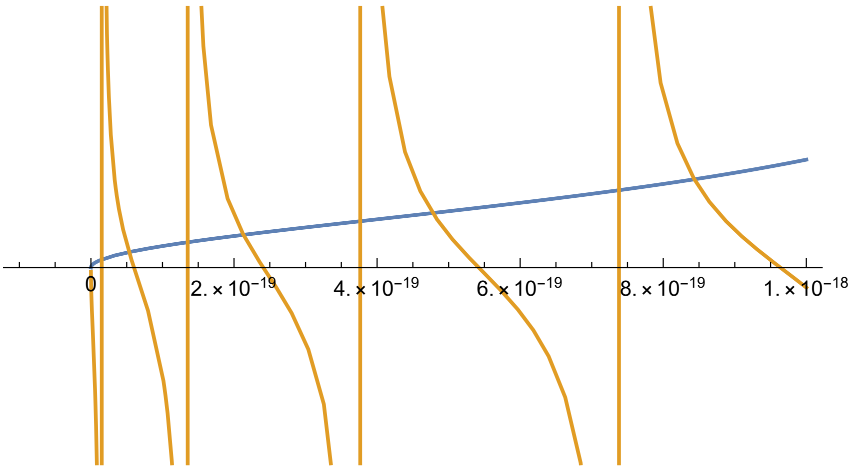

Figure 1. Graph of $\sqrt{\frac{E}{V_0 - E}}$ and $-\tan\left(\sqrt{\frac{2mE}{\hbar^2}}a\right)$.

Therefore, the energy levels of the electron are quantized.