UM-SJTU Joint Institute

Physics Laboratory
(Vp141)

Laboratory Report

Exercise 2
Measurement of Fluid Viscosity

Name: Yihua Liu ID:518021910998 Group 9

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1 Introduction

The Objective of the exercise is to experiment on the fluid viscosity that determines the fluid's flow. We can describe and measure the properties of some fluids with high viscosity by Stoke's method. For objects moving in a fluid, a drag force exerted on it in the opposite direction to its velocity is related to the shape, the speed, and the internal friction in the fluid quantified by the viscosity coefficient denoted as η.

Considering the simplest situation – a spherical object with the radius R and its speed v in an infinite volume of a liquid, the expression for the magnitude of the drag force is

$$F_1 = 6\pi\eta v R \tag{1}$$

If it falls vertically downwards in a fluid, the viscous force $\mathbf{F_1}$ and the buoyancy force $\mathbf{F_2}$ upwards and the gravity $\mathbf{F_3}$ downwards, of which $\mathbf{F_2}$ is

$$F_2 = \frac{4}{3}\pi R^3 \rho_1 g \tag{2}$$

and the weight of the object

$$F_3 = mg = \frac{4}{3}\pi R^3 \rho_2 g \tag{3}$$

where rho_2 is the density of the object. By balance equation, we have

$$\eta = \frac{2}{9} g R^2 \frac{\rho_2 - \rho_1}{v_t} = \frac{2}{9} g R^2 \frac{(\rho_2 - \rho_1)t}{s} = (3mg - 4\rho_1 \pi g R^3)\frac{s}{18\pi t} \tag{4}$$

where the speed v_t is a constant, s is the distance that it traveled and t the time from the origin until the object reaches terminal speed. Taking into consideration that finally the velocity of the motion is a constant, Eq. (4) has the following experimental form

$$\eta = \frac{mg - \frac{4}{3}\pi R^3 \rho g}{6\pi v_t R(1 + 2.4\frac{R}{R_c})} = \frac{(mg - \frac{4}{3}\pi R^3 \rho g)t}{6\pi s R(1 + 2.4\frac{R}{R_c})} \tag{5}$$

where R_c is the radius of an infinite long cylindral container.

2 Experimental setup

In this experiment, we use a set of Stoke's viscosity measurement device (see Figure 1) including conducting pipe, semiconductor laser generator, 3-D ajusting bracket, kerosene oil, and graduated flask. For measurement devices including micrometer, calliper, densimeter, electronic scales, stopwatch, and thermometer, we can measure various physical quantities of the motion of the small metal ball.

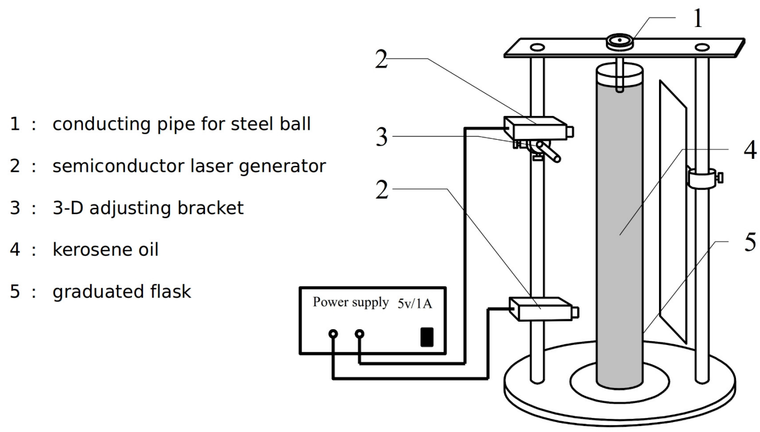

Figure 1. Stokes' viscosity measurement apparatus.

To measure the positions of upper and lower laser beam, we use rule with maximum uncertainty $u = \pm 0.5$ mm. As the diameter of metal balls is very small, we use micrometer with maximum uncertainty $u = \pm 0.005$ mm. For stop watches, their undertainty $u = \pm 0.01$ s. The uncertainty of electronic scale used for measuring mass is $u = \pm 0.001$ g. Besides, we use the calliper with the uncertainty $u = \pm 0.02$ mm and the densimeter with $u = \pm 0.0005 \mathrm{g/cm}^3$.

3 Measurement

3.1 Adjustment of the Stokes' viscosity measurement device

We first adjust the knobs beneath the base to ensure that the plumb aims just at center of the the base. Then we adjust the beams in order to make them parallel and aim at the plumb line by turning on the two lasers. After finishing adjustment, we remove the plumb and put the graduated flask together with castor oil at the center of the base. Finally, on the top of the Stokes' device we place the guilding pipe.

3.2 Measurement of the (constant) velocity of a falling ball

After the Stokes' device is settled, we measure the vertical distance s between the two laser beams for three. Then we repeat doing this for three times. After the measurement, we prepare a metal ball in the guiding pipe and measure the time period t with astopwatch when the ball passes through the first and the second beam for six times.

3.3 Measurement of constant quantities

In this experiment, to minimize errors, we measure the mass of the ball m instead of the ball density ρ_2 just by electronic scales. To make the result more precise, we weight 40 metal balls and calculate the average mass of a single ball. Although we do not need to calculate the ball density, we are supposed to examine the diameter of the metal balls using micrometers for ten times. Then we calculate the average value as the diameter of the metal balls.

3.4 Calculation of the viscosity coefficient η

Using the formula Eq. (5), we calculate the value of viscosity coefficient η.

4 Results

4.1 Distance

The distance between the two laser beams was measured in the procedure in section 3.2.

$$\bar{x}_{A,1} = \frac{1}{3}\sum_{i=1}^{3} x_{A,i} = 194.8 \pm 0.7 \text{ mm}$$

and

$$\bar{x}_{B,1} = \frac{1}{3}\sum_{i=1}^{3} x_{B,i} = 60.2 \pm 0.7 \text{ mm}$$

The distance traveled of the small metal ball is

$$\bar{S} = \bar{x}_{A,1} - \bar{x}_{B,1} = 134.7 \pm 0.7 \text{ mm}$$

with the relative uncertainty 0.5%.

distance x [mm] $\pm$ 0.5 [mm]			
$x_{A,1}$	195.0	$x_{B,1}$	60.0
$x_{A,2}$	194.5	$x_{B,2}$	60.0
$x_{A,3}$	195.0	$x_{B,3}$	60.5

Table 1. Distance measurement data.

4.2 Time

The time that the small metal ball traveled from the first laser beam to the second was also measured in section 3.2.

$$\bar{t} = \frac{1}{6}\sum_{i=1}^{6} t_i = 6.60 \pm 0.03 \text{ s}$$

with the relative uncertainty 0.5%.

time t [s] $\pm$ 0.01 [s]			
t_1	6.60	t_4	6.62
t_2	6.59	t_5	6.62
t_3	6.59	t_6	6.57

Table 2. Time measurement data.

4.3 Diameters of the Balls

The diameters of the balls was examined in section 3.3 with a micrometer.

$$\bar{d} = \frac{1}{10}\sum_{i=1}^{10} t_i = 1.992 \pm 0.006 \text{ mm}$$

with the relative uncertainty 0.3%.

diameter d [mm] $\pm$ 0.005 [mm]			
d_1	1.995	d_6	1.990
d_2	1.990	d_7	1.990
d_3	1.990	d_8	2.000
d_4	1.990	d_9	1.990
d_5	1.995	d_{10}	1.990

Table 3. Measurement data for the diameters of the balls.

4.4 Inner Diameter of the Flask

The inner diameter of the flask was measured in section 3.3 with a calliper.

$$\bar{D} = \frac{1}{6}\sum_{i=1}^{6} t_i = 62.33 \pm 0.03 \text{ mm}$$

with the relative uncertainty 0.05%.

diameter D [mm] $\pm$ 0.02 [mm]			
D_1	62.30	D_4	62.34
D_2	62.34	D_5	62.32
D_3	62.32	D_6	62.36

Table 4. Measurement data for the inner diameter of the flask.

4.5 Other Physcial Quantities

density of the castor oil ρ_1	0.9570 [g/cm^3] $\pm$ 0.0005 [g/cm^3]
mass of 40 metal balls m	1.312 [g] $\pm$ 0.001 [g]
temperature in the lab T	25.4 [°C] $\pm$ 0.2 [°C]
acceleration due to gravity in the lab g	9.81 [m/s^2]

Table 5. Values of other physical quantities.

4.6 Viscosity Coefficient

We first convert the quantities in the measurement to quantities in the formula Eq. (5). The mass of a single ball $m = \frac{1.312 \text{ g}}{40} = 0.0328 \text{ g} = 3.28 \times 10^{-5}$ kg. The radius of a metal ball is $R = \frac{d}{2} = 0.996 \text{ mm} = 9.96 \times 10^{-4}$ m. The radius of the long cylindrical container is $R_c = \frac{D}{2} = 31.165 \text{ mm} = 3.1165 \times 10^{-2}$ m. The density of the castor oil is $\rho = 0.9570 \text{ g/cm}^3 = 9.570 \times 10^2 \text{ kg/m}^3$.

The distance is $s = 134.7$ mm $= 0.1347$ m. The time is $t = 6.60$ s. By Eq. (5), we can calculate the viscosity coefficient η of the castor oil

$$\eta = \frac{(m - \frac{4}{3}\pi R^3 \rho)gt}{6\pi s R(1 + 2.4\frac{R}{R_c})} = 0.686 \text{ Pa} \cdot \text{s}$$

5 Conclusions and discussion

In this experiment, we use Stokes' method to measure the viscosity coefficient of castor oil. However, there are some corrections. For example, to minimize type-B uncertainties, we use the mass of the ball instead of the ball density to calculate the viscosity coefficient. Besides, the final uncertainty turns out to be great because of some possible erros when measuring the diamter of one single metal ball. When using a micrometer, it is important not to compress the ball but also keep it contact compactly.

On the other hand, there are probably some remaining castor oil on the surface of the metal balls so that when measuring the mass of 40 metal balls the result might be greater than expected.

Besides, when applying the formula Eq. (5), it is actually a simulation based on experiments rather than theories, since the cylindrical container cannot have infinite length.

To improve this experiment, I think carefully cleaning the ball can reduce errors when measuring the mass of 40 metal balls.

A Measurement uncertainty analysis

A.1 Uncertainty of distance measurements

$$\sigma_{x_A} = \sqrt{\frac{\sum_{i=1}^{3}(\bar{x}_A - x_{A,i})^2}{3-1}} = 0.289 \text{ mm}$$

$$\Delta_A = \frac{t_{0.95}}{\sqrt{3}}\sigma_{x_A} = 0.534 \text{ mm}$$

$$\Delta_B = 0.5 \text{ mm}$$

$$u_{x_A} = \sqrt{\Delta_A^2 + \Delta_B^2} = 0.7 \text{ mm}$$

$$\sigma_{x_B} = \sqrt{\frac{\sum_{i=1}^{3}(\bar{x}_B - x_{B,i})^2}{3-1}} = 0.289 \text{ mm}$$

$$\sigma_{x_A} = \sigma_{x_B}$$

$$u_{x_B} = u_{x_A} = \sqrt{\Delta_A^2 + \Delta_B^2} = 0.7 \text{ mm}$$

$$u_s = 0.7 \text{ mm}$$

$$u_{rs} = \frac{u_s}{s} \times 100\% = 0.5\%$$

$$s = 134.7 \pm 0.7 \text{ mm}$$

A.2 Uncertainty of time measurements

$$\sigma_t = \sqrt{\frac{\sum_{i=1}^{6}(\bar{t} - t_i)^2}{6-1}} = 0.0194 \text{ s}$$

$$\Delta_A = \frac{t_{0.95}}{\sqrt{6}}\sigma_t = 0.0254 \text{ s}$$

$$\Delta_B = 0.01 \text{ mm}$$

$$u_t = \sqrt{\Delta_A^2 + \Delta_B^2} = 0.03 \text{ s}$$

$$u_{rt} = \frac{u_t}{t} \times 100\% = 0.5\%$$

$$t = 6.60 \pm 0.03 \text{ s}$$

A.3 Uncertainty of measurements for the diameters of the balls

$$\sigma_d = \sqrt{\frac{\sum_{i=1}^{10}(\bar{t} - t_i)^2}{10-1}} = 0.00350 \text{ mm}$$

$$\Delta_A = \frac{t_{0.95}}{\sqrt{10}}\sigma_d = 0.00354 \text{ mm}$$

$$\Delta_B = 0.005 \text{ mm}$$

$$u_d = \sqrt{\Delta_A^2 + \Delta_B^2} = 0.006 \text{ mm}$$

$$u_{rd} = \frac{u_d}{d} \times 100\% = 0.3\%$$

$$d = 1.992 \pm 0.006 \text{ mm}$$

A.4 Uncertainty of measurements for the inner diameter of the flask

$$\sigma_D = \sqrt{\frac{\sum_{i=1}^{6}(\bar{t}-t_i)^2}{6-1}} = 0.0210 \text{ mm}$$

$$\Delta_A = \frac{t_{0.95}}{\sqrt{6}}\sigma_D = 0.0274 \text{ mm}$$

$$\Delta_B = 0.02 \text{ mm}$$

$$u_D = \sqrt{\Delta_A^2 + \Delta_B^2} = 0.03 \text{ mm}$$

$$u_{rD} = \frac{u_D}{D} \times 100\% = 0.05\%$$

$$D = 62.33 \pm 0.03 \text{ mm}$$

A.5 Uncertainty of measurements for the mass of one ball

$$u_m = \frac{0.001 \text{ g}}{40} = 2.5 \times 10^{-5} \text{ g}$$

$$m = (3.28 \pm 0.0025) \times 10^{-5} \text{ kg}$$

A.6 Uncertainty of measurements for the density of the castor oil

Since the measurements of the density of the castor oil were single measurements with type-B uncertainty of 0.0005 g/cm^3, the uncertainty is $u_\rho = 0.0005$ g/cm^3.

A.7 Uncertainty of the viscosity coefficient

Substitute D for R_c and d for R and denote the density of the castor oil as ρ_1, we have the expression of the viscosity

$$\eta = \frac{g}{18}(\frac{6m}{\pi d} - \rho_1 d^2)\frac{t}{s}\frac{D}{D+2.4d}$$

The uncertainty of the viscosity coefficient η can be calculated as

$$u_\eta = \sqrt{(\frac{\partial\eta}{\partial d}u_d)^2 + (\frac{\partial\eta}{\partial D}u_D)^2 + (\frac{\partial\eta}{\partial s}u_s)^2 + (\frac{\partial\eta}{\partial t}u_t)^2 + (\frac{\partial\eta}{\partial\rho_1}u_{\rho_1})^2 + (\frac{\partial\eta}{\partial m}u_m)^2}$$

Calculating the six partical derivatives respectively,

$$\frac{\partial\eta}{\partial d} = -\frac{gD}{18}(\frac{6m}{\pi}\frac{D+4.8d}{d^2(D+2.4d)^2} + \rho_1\frac{2Dd+2.4d^2}{(D+2.4d)^2})\frac{t}{s}$$

$$\frac{\partial\eta}{\partial D} = \frac{g}{18}(\frac{6m}{\pi d} - \rho_1 d^2)\frac{t}{s} \cdot \frac{2.4d}{(D+2.4d)^2}$$

$$\frac{\partial\eta}{\partial s} = -\frac{g}{18}(\frac{6m}{\pi d} - \rho_1 d^2)\frac{t}{s^2} \cdot \frac{D}{D+2.4d}$$

$$\frac{\partial\eta}{\partial t} = \frac{g}{18}(\frac{6m}{\pi d} - \rho_1 d^2)\frac{1}{s} \cdot \frac{D}{D+2.4d}$$

$$\frac{\partial\eta}{\partial\rho_1} = -\frac{g}{18}\frac{td^2}{s} \cdot \frac{D}{D+2.4d}$$

$$\frac{\partial\eta}{\partial m} = \frac{g}{18}(\frac{6}{\pi d})\frac{t}{s} \cdot \frac{D}{D+2.4d}$$

We have the uncertainty of the viscosity coefficient

$$u_\eta = \sqrt{(\frac{\partial\eta}{\partial d}u_d)^2 + (\frac{\partial\eta}{\partial D}u_D)^2 + (\frac{\partial\eta}{\partial s}u_s)^2 + (\frac{\partial\eta}{\partial t}u_t)^2 + (\frac{\partial\eta}{\partial\rho_1}u_{\rho_1})^2 + (\frac{\partial\eta}{\partial m}u_m)^2} = 0.6\,\mathrm{Pa}\cdot\mathrm{s}$$

with the relative uncertainty

$$u_{r\eta} = \frac{u_\eta}{\eta} \times 100\% = 87\%.$$

B Data Sheet

UM-SJTU JOINT INSTITUTE
PHYSICS LABORATORY
DATA SHEET (EXERCISE 2)

Name: [illegible] Lin Yihua   Student ID: 518021910998

Group: 9   Date: July 12, 2019

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with a pencil or modified with a correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used. You are required to hand in the original data with your lab report, so please keep the data sheet properly.

distance x [mm] $\pm$ 0.1 [mm]					
$x_{A,1}$	195.0	$x_{B,1}$	60.0	S_1	135.0
$x_{A,2}$	194.5	$x_{B,2}$	60.9	S_2	134.5
$x_{A,3}$	195.0	$x_{B,3}$	60.5	S_3	134.5

Table 1. Distance measurement data.

time t [s] $\pm$ 0.01 [s]	
t_1	6.60
t_2	6.59
t_3	6.59
t_4	6.62
t_5	6.62
t_6	6.57

Table 2. Time measurement data.

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diameter d [mm] ± 0.005 [mm]			
d_1	1.998	d_6	1.990
d_2	1.998	d_7	1.990
d_3	1.999	d_8	2.000
d_4	1.990	d_9	1.990
d_5	1.995	d_{10}	1.990

Table 3. Measurement data for the diameters of the balls.

diameter D [mm] ± 0.02 [mm]	
D_1	62.30
D_2	62.34
D_3	62.32
D_4	62.34
D_5	62.32
D_6	62.36

Table 4. Measurement data for the inner diameter of the flask.

density of the castor oil ρ_1 [g/cm³] 0.9570 ± 0.0005 [g/cm³]
mass of 40 metal balls m [1.312] ± 0.001 [g]
temperature in the lab T [27.4] ± 0.2 [°C]
acceleration due to gravity in the lab g [m/s²] 9.81

Table 5. Values of other physical quantities.

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