UM-SJTU Joint Institute

Physics Laboratory
(Vp141)

Laboratory Report

Exercise 4
Measurement of the Speed of Sound

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1 Introduction

The Objective of the exercise is to experiment on the damped and driven oscillations both with the Pohl resonator. Besides, we will observe and quantify the phenomenon of the mechanical resonance further as for the driven oscillations.

If the amplitude of the resonance system decreases gradually due to friction or medium resistance or other energy consumption, the resonance is called damped oscillation. Denote the damping coefficient as b and the stiffness as k, we have

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \tag{1}$$

Solving the differential equation, if $b^2 < 4km$, it is an underdamped regime

$$x(t) = Ae^{-\frac{b}{2m}t}\cos\left(\sqrt{{\omega_0}^2 - \frac{b^2}{4m^2}}t + \varphi\right) \tag{2}$$

If $b^2 > 4km$, it is an overdamped regime

$$x(t) = C_1 e^{-(\frac{b}{2m}+\sqrt{\frac{b^2}{4m^2}-(\omega_0)^2})t} + C_2 e^{-(\frac{b}{2m}-\sqrt{\frac{b^2}{4m^2}-(\omega_0)^2})t} \tag{3}$$

If $b^2 = 4km$, it is a critical regime

$$x(t) = (D_1 + D_2 t)e^{-\frac{b}{2m}t} \tag{4}$$

If a periodic force exerts on a resonance system, the resonance is called forced or driven oscillation. For a system endowed with single degree of freedom, the driving force has the form

$$F = F_0 \cos\omega t \tag{5}$$

where F_0 is the amplitude and ω is the angular frequency. Then, the kinetic motion for the linear driven oscillation can be written in the form of derivatives:

$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx + F_0\cos\omega t \tag{6}$$

In this experiment, we study a rotating system instead of a translational system that the kinetic equation can be rewriten as

$$I\frac{d^2\theta}{dt^2} = -k\theta - b\frac{d\theta}{dt} + \tau_0\cos\omega t \tag{7}$$

where I is the moment of intertia, τ_0 is the amplitude, ω is the angular frequency. The second item is the damping torque and the third item is the periodic driving torque.

Drawing an analogy between the translational motions and rotating motions, we simplify the equation by

$$\omega_0^2 = \frac{k}{I}, \qquad 2\beta = \frac{b}{I}, \qquad \mu = \frac{\tau_0}{I}, \tag{8}$$

Eq. (7) can be rewritten as

$$\frac{d^2\theta}{dt^2} + 2\beta\frac{d\theta}{dt} + \omega_0^2\theta = \mu\cos\omega t \tag{9}$$

Consider a critical condition that the driving torque $\mu = 0$, the equation can express the motion for a damped harmonic oscillator. Another critical condition is that the damping in the system $\beta = 0$ so that it becomes a simple harmonic oscillator with ω_0, the natural angular frequency.

Seperating the transient term $\theta_t r$ and the steady-state term $\theta_s t$ from the time function of the angle $\theta(t)$,

$$\theta(t) = \theta_{tr}(t) + \theta_{st} \cos \omega t + \varphi \tag{10}$$

Analyze the differential equation quantitatively, the trasient solution θ_{tr} decrease to 0 exponentially as $t \to \infty$. On the other hand, the steady-sate solution θ_{st} multiplied by cosine function is endowed with the amplitude

$$\theta_{st} = \frac{\mu}{\sqrt{(\omega_0^2 - \omega^2) + 4\beta^2\omega^2}} \tag{11}$$

The lags φ from the equilibrium position to the position when time is t in the form of the tangent of phase shift is

$$\tan \varphi = \frac{2\beta\omega}{\omega^2 - \omega_0^2}, \tag{12}$$

where $-\pi \leq \varphi < 0$. Different from the transient term θ_{tr}, both the amplitude and the phase shift are not determined by the initial conditions.

Looking for the maximum value of θ_{st} as a function of ω, the resonance angular frequency is

$$\omega_{\text{res}} = \omega = \sqrt{\omega_0^2 - 2\beta^2} \tag{13}$$

Substitute ω into Eq.11, the amplitude of the resonance angle is

$$\theta_{\text{res}} = \theta_{st}(\omega_{\text{res}}) = \frac{\mu}{2\beta\sqrt{\omega_0^2 - \beta^2}} \tag{14}$$

When the damping coefficient $\beta \to 0$, the resonance angular frequency $\omega_{\text{res}} \to \omega_0$. Under this circumstance, the amplitude of the steady-state oscillations is close to its maximum value. Taking different values of β that $\beta_1 > \beta_2 > \beta_3$ for comparing the amplitude θ related to the ratio $frac\omega\omega_0$ and $\beta_1 < \beta_2$ for comparing the phase shift φ related to $frac\omega\omega_0$ in Figure 1.

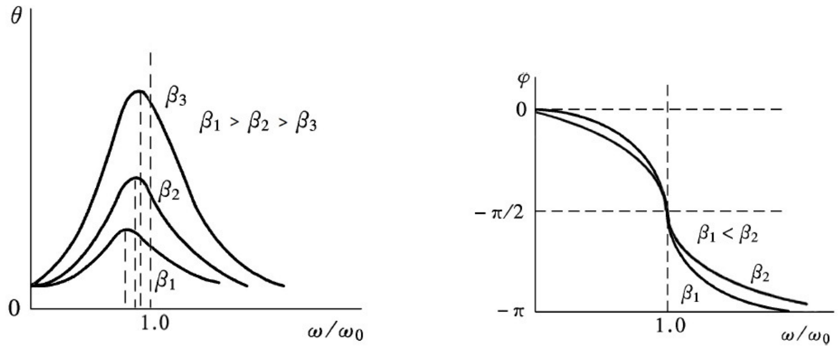

Figure 1. The dependence of the amplitude (left) and phase shift (right) of steady-state driven oscillations [1].

Looking for the homogeneous equation of Eq. (9) conditioned initally by $\theta(t) = \theta_0 e^{-\beta t} \cos(\omega_{\mathrm{f}} t + \alpha)$, we have $\theta_1 = \theta_0 e^{-\beta T}, \theta_2 = \theta_0 e^{-\beta(2T)}, ..., \theta_n = \theta_0 e^{-\beta(nT)}$. Hence, the damping coefficient β is

$$\ln \frac{\theta_j}{\theta_i} = \ln \frac{\theta_0 e^{-\beta(iT)}}{\theta_0 e^{-\beta(jT)}} = (j-i)\beta T \tag{15}$$

where we can calculate the damping coefficient β by the period T.

2 Experimental setup

The BG-2 Pohl resonator is mainly composed by a vibrometer and a control box. The vibrometer is shown in Figure 2.

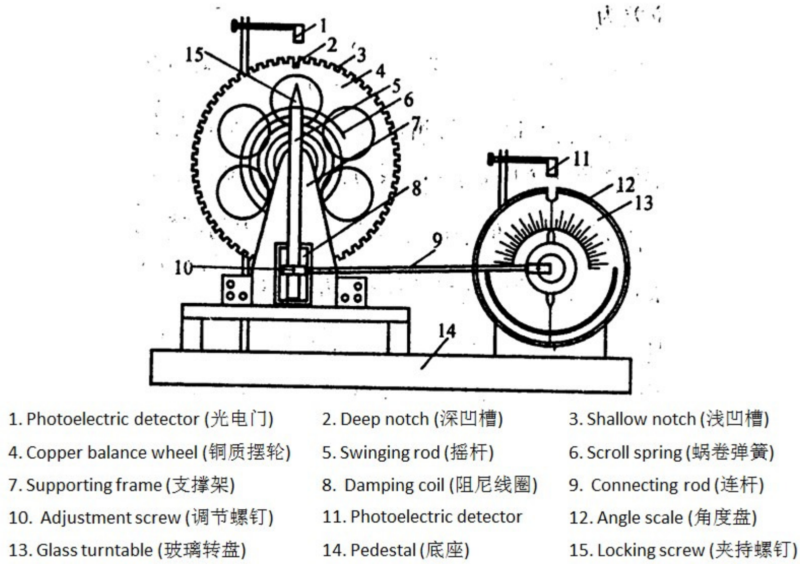

Figure 2. The vibrometer [2].

The equipment is supported by the balance wheel. The spring mounted on the supporting frame is to provide an elastic restoring torque to the wheel that is attached to it. At the equilibrium position, the balance whell will rotate. The notches on the edge of the balance whell. Some are shallow while one of them is deep that is used to set a photoelectric detector. The detector connected to the electronic control box can measure the amplitude and the period of an oscillation.

At the bottom of the supporting frame on the pedestal, a pair of coils are configured and the balance wheel fit well into the gap between them, connected by adjustment screws. The mechanism of the electromagnetic damping force is the electromagnetic induction. Changing the current carried by the pair of coils, the magnitude of the damping force changes. On the right side, a motor is used to drive the eccentric whell with a rod.

The electric control box that can control the speed of the motor precisely is shown in Figure 3.

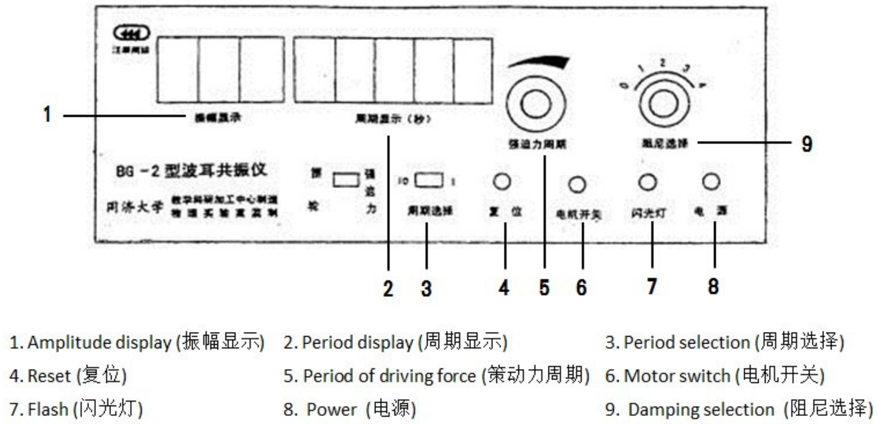

Figure 3. The front panel of the control box [3].

The electric control box provides 8 options on the front panel. The function Amplitude Display can display the amplitude of the balance wheel. Period Display can display the period of the oscillation in two modes. Period Selection can switch two periods of the oscillation, "1" meaning single and "10" meaning the time of 10, the latter one used in this experiment.

The knob Period of the driving force can precisely change the period of the driving force. Motor Switch switches the status of motor, in "off" when measuring the damping coefficient and the natural angular frequency. The knob Damping Selection can switch the level of the damping force by adjusting the current in the coils at the bottom of the balance wheel, ranging from "0" (no current) to "5" (current of around 0.6 A), 6 options in total, of which "2", "3", and "4" used in this experiment. The strobe can generate a flash according to the phase difference in angle scale to read the time period.

3 Measurement

3.1 Natural Angular Frequency

Before doing experiments, we check the position of the photoelectric gate above the balanced wheel that there is enough space between them.

We first turn the Damping Selection knob to "0". Then, we carefully rotate the balance wheel to the initial angular position $\theta_0 \approx 150°$. Then, we release it and record the time of 10 periods. We repeat for four times and calculate the natural angular frequency ω_0.

At first, we mistook the initial angle as approximately 120° and we found that the readings of the time of 10 periods slightly changes. Hence, it is important to keep the initial angles of each time of measurement almost the same to minimize errors.

3.2 Damping Coefficient

We first turn the Damping Selection knob to "2", and keep it unchanged during this part. Then, we rotate the balance wheel to the initial angle of about 150° carefully. Then,we release it and record the angle of each period that is the amplitude and time of 10 periods. Since when the vibrometer may release the first reading before or after the copper balance wheel reaches the endpoint, we dismiss the first reading and start reading from the second reading.

Having recorded all required data, we start to solve the homogeneous equation of Eq. (15). Since we read the time of 10 periods, we have $j - i = 5$. Then, Eq. (15) can be rewritten as

$$\beta = \frac{1}{5T} \ln \frac{\theta_i}{\theta_{i+5}}. \tag{16}$$

We then obtain the value of the average period T by Eq. (16).

During our experiment, we did not miss any readings with our camera. However, if we miss some readings, we should repeat the measurement recording them. Besides, by repeating the measurement, our results would be more precise.

3.3 θ_{st} vs. ω and φ vs. ω Characteristics of Forced Oscillations

At first, we keep the Damping Selection at "2" and set the speed of the motor. We record the position of the motor knob in case we need to repeat the measurement. After the motor begin working and the oscillation reaches a steady state, we record the amplitude θ_{st}, the period T, and the phase shift φ.

Then, we repeat operating the motor and recording data but change the speed of motor to change a small phase shift $\Delta\varphi$. We generally adjust the amplitude to $90°$ where there is a maximum value for it is a resonance point and to $0°$ or $180°$. In total we have more than 15 data collected for plotting.

As we observe, when the amplitude is going to reach the maximum value, it changes rapidly as φ and θ_{st} change. Besides, when φ and θ_{st} reach their limit, they cannot reach the endpoint $0°$ or $180°$.

Next, we can choose Damping Selection "1" or "3". We choose "3" and repeat steps above. We can no longer change the Damping Selection to ensure the accuracy of the measurement until it is entirely completed. Having collected demanding data, we plot the $\theta_{\mathrm{st}}(\omega)$ characteristics with ω/ω_0 on the horizontal axis and θ_{st} on the vertical axis and two sets of data plotted together. We choose "3" partly because we find the values in previous measurement choosing "2" is large enough.

4 Results

4.1 Natural Angular Frequency

The period of oscillations was measured in the procedure in section 3.1. We calculate the average value of the period T according to the results in Table 1 as

$$\bar{T} = \frac{1}{40} \sum_{i=1}^{4} T_i = 1.5782 \pm 0.0003 \text{ s}$$

with the relative uncertainty 0.02%.

Measurement	$10T$ [s]±0.001[s]
1	15.781
2	15.783
3	15.783
4	15.779

Table 1: Measurement of the natural frequency.

Hence, the natural angular frequency is

$$\omega_0 = \frac{2\pi}{\bar{T}} = 3.9814 \pm 0.0008 \text{ rad/s}$$

with the relative uncertainty 0.02%.

4.2 Damping Coefficient

The amplitude of angle was measured in the procedure in section 3.2.

Amplitude [°]±1[°]		Amplitude [°]±1[°]		$\ln \frac{\theta_i}{\theta_{i+5}}$
θ_0	130	θ_5	81	0.4731
θ_1	118	θ_6	73	0.4802
θ_2	107	θ_7	66	0.4832
θ_3	98	θ_8	60	0.4906
θ_4	89	θ_9	54	0.4997
The average value of $\ln \frac{\theta_i}{\theta_{i+5}}$				0.4854

Table 2: Measurement of the damping coefficient.

The period is $T = 15.832$ [s] $\pm$ 0.001 [s]. $u_\theta = 1°$. We denote $q_i = \ln(\theta_i/\theta_{i+5})$, then by the uncertainty propagation formula

$$\Delta_{q_i,B} = (\sqrt{(\frac{u_\theta}{\theta_{i+5}})^2 + (\frac{u_\theta}{\theta_i})^2}$$

we have

i	$Delta_{q_i,B}$
1	0.0145
2	0.0161
3	0.0178
4	0.0195
5	0.0217

Table 3. Type-B uncertainties for q_i.

$$u_q = \sqrt{\Delta^2_{q_i,B} + \Delta^2_{q_i,A}} = 0.026$$

$$u_\beta = \sqrt{(-\frac{q}{5T^2})^2 {u_T}^2 + (\frac{1}{5T^2})^2 {u_q}^2} = 0.0032 \text{ s}^{-1}$$

The damping coefficient is calculated from the amplitude by using Eq. (16) in Table 2.

$$\beta = \frac{1}{5T} \ln \frac{\theta_i}{\theta_{i+5}} = (6.13 \pm 0.32) \times 10^{-2}\ \mathrm{s}^{-1}$$

with the relative uncertainty 5.2%.

4.3 θ_{st} vs. ω and φ vs. ω Characteristics of Forced Oscillations

We use different damping selection mode 2 and 3 and the value of times 10 of the period, φ, and θ is shown in Table 3 and Table 4 respectively. The method is introduced in section 3.3.

For damping selection is 2,

Measurement	$10T$ [s]$\pm$0.001[s]	φ[°]$\pm$1[°]	θ[°]$\pm$1[°]
1	14.879	165	32
2	14.930	164	34
3	15.019	163	38
4	15.155	160	44
5	15.232	158	50
6	15.374	153	62
7	15.472	147	76
8	15.543	140	90
9	15.667	120	126
10	15.711	109	136
11	15.752	98	143
12	15.787	90	144
13	15.822	82	143
14	15.883	69	136
15	15.954	58	123
16	16.010	51	112
17	16.119	40	91
18	16.211	33	78
19	16.303	28	67
20	16.411	24	59

Table 4. θ vs. ω and φ vs. ω characteristics (Damping Selection: 2).

For damping selection is 3,

Measurement	$10T$ [s]$\pm$0.001[s]	φ[°]$\pm$1[°]	θ[°]$\pm$1[°]
1	16.408	26	58
2	16.334	30	63
3	16.222	36	74
4	16.133	42	85
5	16.065	48	95
6	16.012	54	104
7	15.953	62	114
8	15.883	73	124
9	15.838	82	128
10	15.794	91	128
11	15.765	98	128
12	15.733	105	124
13	15.672	118	114
14	15.615	128	99
15	15.557	135	87
16	15.464	144	71
17	15.371	150	60
18	15.200	157	46
19	14.856	163	31

Table 5. θ vs. ω and φ vs. ω characteristics (Damping Selection: 3).

The x coordinates ω/ω_0 can be calculated by

$$\frac{\omega}{\omega_0} = \frac{2\pi/T}{3.98 \pm 0.001 \text{ rad/s}}$$

Substituting ω for T, the direct relations between ω/ω_0 and θ and φ is shown in Table 5 and Table 6 respectively

For Damping Selection is 2,

Measurement	ω/ω_0	$\varphi[°]\pm1[°]$	$\theta[°]\pm1[°]$
1	1.061	165	32
2	1.057	164	34
3	1.051	163	38
4	1.042	160	44
5	1.036	158	50
6	1.027	153	62
7	1.020	147	76
8	1.016	140	90
9	1.008	120	126
10	1.005	109	136
11	1.002	98	143
12	1.000	90	144
13	0.998	82	143
14	0.994	69	136
15	0.990	58	123
16	0.986	51	112
17	0.979	40	91
18	0.974	33	78
19	0.968	28	67
20	0.962	24	59

Table 7: θ vs. ω/ω_0 and φ vs. ω/ω_0 characteristics (Damping Selection: 2).

For Damping Selection is 3,

Measurement	ω/ω_0	$\varphi[°]\pm1[°]$	$\theta[°]\pm1[°]$
1	0.962	26	58
2	0.967	30	63
3	0.973	36	74
4	0.979	42	85
5	0.983	48	95
6	0.986	54	104
7	0.990	62	114
8	0.994	73	124
9	0.997	82	128
10	1.000	91	128
11	1.001	98	128
12	1.003	105	124
13	1.007	118	114
14	1.011	128	99
15	1.015	135	87
16	1.021	144	71
17	1.027	150	60
18	1.039	157	46
19	1.063	163	31

Table 8: θ vs. ω/ω_0 and φ vs. ω/ω_0 characteristics (Damping Selection: 3).

Hence, using Tale 5 and Table 6, we can plot the $\theta_{\mathrm{st}}(\omega)$ characteristics, with

ω/ω_0 on the horizontal axis θ_{st} on the vertical axis with damping selection 2 and 3 on the same graph. Similarly, $\varphi(\omega)$ can be plotted with the same quantities of the horizontal and the vertical axis as the plot of $\theta_{\mathrm{st}}(\omega)$ characteristics.

The uncertainties u_Q that we plot error bars are list in Table 9.

Table 9: The uncertainties u_Q.

$frac\omega\omega_0$ DS-2	u_Q DS-2	frac$\omega\omega_0$ DS-2	u_Q DS-2
0.962	0.017337	0.962	0.017331
0.968	0.017114	0.967	0.017178
0.974	0.016924	0.973	0.016947
0.979	0.016736	0.979	0.016764
0.986	0.016514	0.983	0.016626
0.990	0.016401	0.986	0.016518
0.994	0.016257	0.990	0.016399
0.998	0.016135	0.994	0.016257
1.000	0.016065	0.997	0.016167
1.002	0.015995	1.000	0.016079
1.005	0.015913	1.001	0.016021
1.008	0.015826	1.003	0.015957
1.016	0.015581	1.007	0.015836
1.020	0.015441	1.011	0.015723
1.027	0.01525	1.015	0.015608
1.036	0.014974	1.021	0.015426
1.042	0.014826	1.027	0.015244
1.051	0.014566	1.039	0.014913
1.057	0.014398	1.063	0.014258
1.061	0.014301		

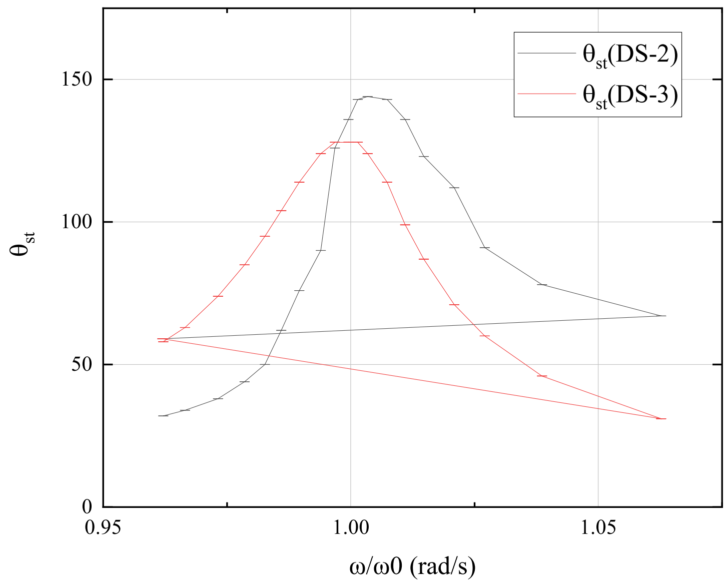

Figure 4. θ vs. ω/ω_0 characteristics.

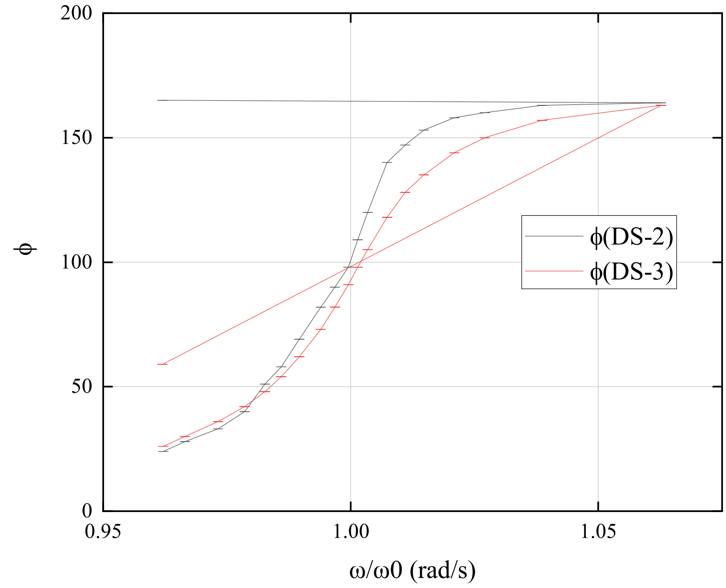

Figure 5. φ vs. ω/ω_0 characteristics .

5 Conclusions and discussion

In the experiment, we first calculate the natural angular frequency.

$$\omega_0 = 3.9814 \pm 0.0008 \text{ rad/s}$$

Then we calculate the factor β which is the damping coefficient. Both are damping oscillations.

$$\beta = (6.13 \pm 0.32) \times 10^{-2} \text{ s}^{-1}$$

Finally, we study the driven oscillations based on the natural angular frequency we derived before and get the relationshp between the ratio $\frac{\omega}{\omega_0}$ and θ and φ. We note that the more damping coefficient, the more the ratio of the resonance and the natural frequency. Besdies, the more the phase shift, the more ratio $\frac{\omega}{\omega_0}$ and θ and φ. Taking damping into consideration, comparing the damping selection 2 and 3, we find that the higher the damping coefficient, the more the slope of the curve.

When it comes to the control of errors, we try to accurately read and record the readings carefully. Besides, since the reading of the phase shift is not steady, the reading of the phase shift may not be very accurate.

A more obvious thing is that when the angle is far from 90 degrees and near to 0 or 180 degrees, the two flashes varies more than when the angle is around 90 degrees. Sometimes, the difference between two readings can be more than 3 degrees. We just take the average of the two readings approximately and repeatedly to possibly narrow the width.

A Measurement uncertainty analysis

UM-SJTU Joint Institute, Physics Laboratory I
Measurement Uncertainty Analysis Worksheet*
Exercise 5

WS-1 Natural Angular Frequency

The uncertainty for ten periods is found first. Then the result for the natural frequency is given along with its uncertainty.

The type-B uncertainty for T_{10} is $\Delta_{T_{10},B} = 0.001$ s. To find the type-A uncertainty, we first find the standard deviation

$$s_{T_{10}} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(T_{10,i} - \overline{T}_{10})^2} = \underline{0.001915}\ [\underline{\ s\ }].$$

We have $n = \underline{\ 4\ }$, so the type-A uncertainty $\Delta_{T_{10},A}$ is calculated as

$$\Delta_{T_{10},A} = \frac{t_{0.95}}{\sqrt{n}} s_{T_{10}} = \underline{1.6001} \times \underline{0.001915} = 0.003064\ [\underline{\ s\ }].$$

Hence the uncertainty for T_{10} is given by

$$u_{T_{10}} = \sqrt{\Delta^2_{T_{10},A} + \Delta^2_{T_{10},B}} = \underline{0.003}\ [\underline{\ s\ }].$$

The period is found indirectly by measuring the ten periods. Therefore, its uncertainty u_T of a single period is found by applying the uncertainty propagation formula

$$\boxed{u_T} = \sqrt{\left(\frac{\partial T}{\partial T_{10}} u_{T_{10}}\right)^2} = \frac{u_{T_{10}}}{10} = \boxed{3 \times 10^{-4}\ [\underline{\ s\ }]},$$

Hence the period is given by

$$\boxed{T = 1.5782 \pm 0.0003\ [\underline{\ s\ }]},$$

*Created by Peng Wenhao, edited by Fan Yixing, Ye Haojie, Mateusz Krzyzosiak [rev. 1.3]

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with relative uncertainty

$$\boxed{u_{rT}} = \frac{u_T}{T} \times 100\% = \boxed{0.02}\,\%$$

The natural angular frequency ω_0 is found from the formula $\omega_0 = 2\pi/T$, so by the uncertainty propagation formula and the fact that

$$\frac{\partial \omega_0}{\partial T} = -\frac{2\pi}{T^2},$$

we obtain

$$\boxed{u_{\omega_0}} = \left|\frac{\partial \omega_0}{\partial T} u_T\right| = \boxed{0.0008 \quad [\mathrm{s}^{-1}]}$$

with the relative uncertainty

$$\boxed{u_{\mathrm{r},\omega_0}} = \frac{u_{\omega_0}}{\omega_0} \times 100\% = \boxed{0.02\,\%}$$

WS-2 Damping Coefficient

The damping coefficient is found indirectly form measurements of the period T and the amplitude θ as $\beta = \frac{1}{5T}\ln(\theta_i/\theta_{i+5})$.

The uncertainty each single measurement of the amplitude is $u_\theta =$ 1 °, so the uncertainty of the logarithm of the quotient of them $q_i = \ln(\theta_i/\theta_{i+5})$ is found from the uncertainty propagation formula

$$\Delta_{q_i,B} = \sqrt{\left(\frac{\partial(\ln(\theta_i/\theta_{i+5}))}{\partial\theta_i}\right)^2 u_\theta^2 + \left(\frac{\partial(\ln(\theta_i/\theta_{i+5}))}{\partial\theta_{i+5}}\right)^2 u_\theta^2} = \sqrt{\left(\frac{u_\theta}{\theta_{i+5}}\right)^2 + \left(\frac{u_\theta}{\theta_i}\right)^2}$$

For example, for $i = 1$,

$$\Delta_{q_1,B} = \sqrt{\left(\frac{1}{81}\right)^2 + \left(\frac{1}{130}\right)^2} = 0.0145.$$

The results for all five sequences of measurements are given in Table WS-1.

i	$\Delta_{q_i,B}$
1	0.0145
2	0.0161
3	0.0178
4	0.0195
5	0.02[illegible]7

Table WS-1: Type-B uncertainties for q_i.

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The overall type-B uncertainty for the quotient can be estimated as the maximum of uncertainties listed in in Table WS-1

$$\Delta_{q,B} = \underline{0.047}.$$

To estimate the type-**A** uncertainty of q, the standard deviation of q is calculated as

$$s_q = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(q_i - \bar{q})^2} = \underline{0.0102}$$

Hence the type-A uncertainty for $n = 5$ is calculated as

$$\Delta_{q,A} = \frac{t_{0.95}}{\sqrt{n}} s_q = \underline{1.43} \times \underline{0.0102} = \underline{\cancel{0.0143}}\ 0.0146$$

and the combined uncertainty

$$u_q = \sqrt{\Delta^2_{q,B} + \Delta^2_{q,A}} = \sqrt{\underline{0.0217^2 + 0.0146^2}} = \underline{0.026}.$$

A single measurement for ten periods is recorded as $T_{10} = \underline{15.832} \pm \underline{0.01}$ [$\underline{s}$]. Hence $T = \underline{1.5832} \pm \underline{0.001}$ [$\underline{s}$].

Then the uncertainty propagation equation is used to calculate the uncertainty for the damping coefficient $\beta = \frac{1}{5T}q$ as

$$\boxed{u_\beta} = \sqrt{\left(\frac{\partial\beta}{\partial T}\right)^2 u_T^2 + \left(\frac{\partial\beta}{\partial q}\right)^2 u_q^2} = \sqrt{\left(-\frac{q}{5T^2}\right)^2 u_T^2 + \left(\frac{1}{5T}\right)^2 u_q^2}$$

$$= \sqrt{\left(-\frac{0.456}{5\times1.5832^2}\right)^2 \times 0.001^2 + \left(\frac{1}{5\times1.5832}\right)^2 \times 0.026^2} = \boxed{\underline{0.0032}\ [\underline{s^{-1}}]}$$

with relative uncertainty

$$\boxed{u_{r,\beta}} = \frac{u_\beta}{\beta} \times 100\% = \boxed{\underline{5.2}\ \%}$$

WS-3 The θ_{st}-ω and φ-ω Characteristics of Forced Oscillations

On the graphs included in the report, the uncertainty is shown in the form of error bars.[1] In both the φ vs. (ω/ω_0) graph and the θ_{st} vs. (ω/ω_0) graph, the

[1] Please follow this part to find the uncertainties and mark them on the graphs of the phase shift φ vs. (ω/ω_0) graph and the amplitude of steady-state oscillations θ_{st} vs. (ω/ω_0).

measurements of φ and θ_{st} are single measurements with uncertainty __1__ °, determined by the resolution of our equipment. However, to find the uncertainty of (ω/ω_0) we need to derive it from the uncertainty propagation formula. Let us introduce symbols $Q = \frac{\omega}{\omega_0}$, $T_{10,\text{natural}} = N$ and $T_{10,\text{driven}} = D$, where the uncertainty of D is again the minimum scale (resolution) of the equipment used. Since these are single measurements, we have

$$Q = \frac{\omega}{\omega_0} = \frac{T_{10,\text{natural}}}{T_{10,\text{driven}}} = \frac{N}{D}$$

and the uncertainty of the ratio Q, found from the uncertainty propagation formula, is

$$u_Q = \sqrt{\left(\frac{\partial Q}{\partial N}u_N\right)^2 + \left(\frac{\partial Q}{\partial D}u_D\right)^2} = \sqrt{\left(\frac{u_N}{D}\right)^2 + \left(\frac{N u_D}{D^2}\right)^2}$$

In particular, with $N =$ 15.782 [s], $u_N =$ 0.003 [s], and $u_D =$ 0.001 [s], so with every set of N and D a unique uncertainty is generated. For instance,[2] for $D =$ 14.879 [s], we can calculate Q as

$$Q = \frac{N}{D} = \frac{15.782}{14.879} = 1.0607$$

with uncertainty u_Q calculated as

$$\boxed{u_Q} = \sqrt{\left(\frac{0.003}{14.879}\right)^2 + \left(\frac{15.782 \times 0.001}{14.879^2}\right)^2} = \boxed{2.14 \times 10^{-4}},$$

and

$$\boxed{u_\varphi = 1° = 0.017\ \text{rad}} \qquad \boxed{u_{\theta_{st}} = 1° = 0.017\ \text{rad}}.$$

[2]Here, based on your measurement data, give one sample calculation for a chosen value of ω/ω_0. All values of the calculated uncertainties u_Q that you have used to plot error bars, should be given in the *Results* section, where tables with the data for the plots φ vs. (ω/ω_0) and θ_{st} vs. (ω/ω_0) is included.

B Data Sheet

UM-SJTU JOINT INSTITUTE

PHYSICS LABORATORY

DATA SHEET (EXERCISE 5)

Name: [illegible] Liu Yihang Student ID: [illegible]

Name: [illegible] Xi Guanghan Student ID: 518021910778

Group: 7 Date: June 21, 2019

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with pencil or modified by correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used.

You are required to hand in the original data with your lab report, so please keep the data sheet properly.

	$10T$ [s] ± 0.001 [s]
1	15.781
2	15.783
3	15.783
4	15.779

Table 1. Measurement of the natural frequency.

Damping Selection: 2

Amplitude [°] ± 1 [°]		Amplitude [°] ± 1 [°]		$\ln(\theta_i/\theta_{i+5})$
θ_0	130	θ_5	81	0.4731
θ_1	118	θ_6	73	0.4802
θ_2	107	θ_7	66	0.4832
θ_3	98	θ_8	60	0.4906
θ_4	89	θ_9	54	0.4997
The average value of $\ln(\theta_i/\theta_{i+5})$				.4854

$10T$ = 15.832 [s] ± 0.001 [s]

Table 2. Measurement of the damping coefficient.

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1

Damping Selection: 2

	10T [s] ± 0.001 [s]	φ [°] ± 1 [°]	θ [°] ± 1 [°]
1	~~14.956~~ 14.879	~~184~~ 165	~~35~~ ~~34~~ 32
2	~~14.907~~ 14.980	~~164~~ ~~165~~ 164	~~35~~ 34
3	15.019	163	38
4	15.155	160	44
5	15.272	158	50
6	15.374	153	62
7	15.472	147	~~77~~ 76
8	15.543	140	90
9	15.667	120	126
10	15.711	109	136
11	15.752	98	143
12	15.787	90	144
13	15.822	82	142
14	15.883	67	136
15	15.954	58	~~[illegible]~~ 123
16	16.010	51	112
17	16.119	40	91
18	16.211	33	78
19	16.303	28	67
20	16.411	24	~~58~~ 59
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22			
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Table 3. θ vs. ω and φ vs. ω characteristics.

2

Damping Selection: #3

	10T [s] ± 0.001 [s]	φ [°] ± 1 [°]	θ [°] ± 1 [°]
1	16.408	26	58
2	16.334	30	63
3	16.222	36	74
4	16.133	42	85
5	16.065	48	95
6	16.012	54	104
7	15.953	62	114
8	15.863	73	124
9	15.838	82	128
10	15.794	91	128
11	15.765	98	128
12	15.733	105	124
13	15.672	118	114
14	15.615	128	99
15	15.557	135	87
16	15.464	144	71
17	15.371	150	60
18	15.200	157	48
19	14.856	163	31
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Table 4. θ vs. ω and φ vs. ω characteristics.

Instructor's signature:

3