Physics Laboratory I
Vp141

Exercise 1

Collisions and Projectile Motion

This lab manual is based on materials provided by the Department of Physics, Shanghai Jiaotong University. rev 1.8

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1 Pre-lab Reading

Sections 3.3, 8.1–8.4, as well as a basic understanding of the kinetic energy and the gravitational potential energy and the energy conservation law from Chapter 7 (Young and Freedman).

2 Objectives

The objective of this exercise is to study various concepts related to collisions and the projectile motion.

In the experiment, an object (a ball) will collide with a target (another ball), which in turn will start moving in projectile motion. The fundamental laws of the momentum and the mechanical energy conservation will be applied to investigate the relationship between the initial parameters of the experiment and the position of the target after it hits the ground. The role of the material the target is made of, will also be studied.

Your ultimate goal in this exercise is to be able to effectively predict the trajectory of the target ball, so that it hits a specific point.

3 Theoretical Background

3.1 Mechanical Energy in Pendulum Motion

Figure 1 shows a pendulum. Assuming that the string is massless, and neglecting the air resistance, the (conservative) gravitational force is the only force that does work on the ball. Therefore, we can apply the energy conservation law.

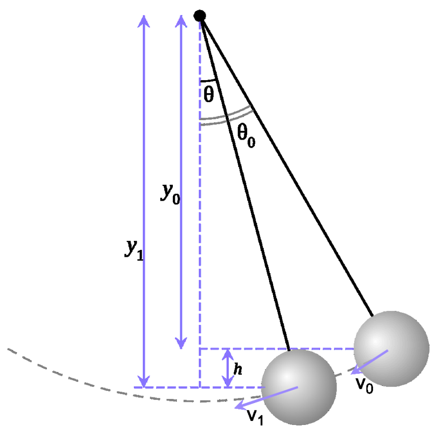


Figure 1. Pendulum system.

The gravitational potential energy of the ball is $U_{\text{grav}} = mg(l - y)$, where l is the length of the string and the reference level for the potential energy has been chosen to go through the center of the ball when it is at the lowest position. The kinetic energy of the ball with mass m is $K = mv^2/2$. Since the total mechanical energy is conserved, for any two positions of the ball (see Figure 1) we have

$$mgh = \frac{1}{2}m\left(v_1^2 - v_0^2\right),$$

where $h = y_1 - y_2$.

3.2 Collisions

Collision is a short-time interaction between two (or more than two) bodies, resulting in a change in motion of bodies. In general, in any collision in which external forces are not present, the momentum is conserved and the total momentum of the system has the same value before and after the collision (the conservation of momentum).

If, over the same period of time time, the external forces acting on a body are much smaller than the internal forces (that is the impulse of the external forces is negligible), we can treat the system as if there were no external forces and use the conservation of momentum principle.

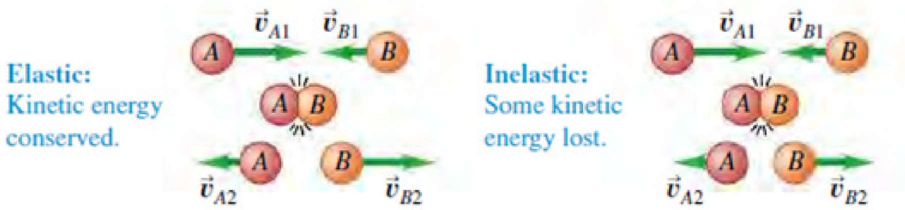


Figure 2. Classification of collisions.

We can classify collisions with respect to changes of various forms of the mechanical energy. If the potential energy just before and just after a collision is the same, then it is enough to consider changes of the kinetic energy. The kinetic energy is conserved in *elastic collisions*, whereas in *inelastic collisions* it decreases (see Figure 2).

3.2.1 Elastic Collisions

According to the above classification, an elastic collision is the one in which the kinetic energy is conserved. Let us consider a one-dimensional elastic collision between two bodies A and B (Figure 2, left panel), where all the velocity vectors lie along the same line. From the fact that the kinetic energy remains constant, we have

$$\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2, \tag{1}$$

and the conservation of the momentum yields

$$m_A v_{A1} + m_B v_{B1} = m_A v_{A2} + m_B v_{B2}, \tag{2}$$

where the signs of the velocities are chosen correspondingly, to indicate whether a body moves to the left or to the right.

Given the masses m_A and m_B and the initial velocities v_{A1} and v_{B1}, we can solve the system of two equations to find the two final velocities v_{A2} and v_{B2}.

In a particular case, when the body B is at rest before the collision ($v_{B1} = 0$), and the two bodies have the same mass ($m_A = m_B = m$), the equations (1) and (2) can be simplified to

$$\frac{1}{2}mv_{A1}^2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2 \qquad \text{and} \qquad mv_{A1} = mv_{A2} + mv_{B2},$$

respectively. Solving these two equations, we can derive $v_{A2} = 0$ and $v_{B2} = v_{A1}$. That is, the body A stops and transfers all its momentum and kinetic energy to the body B, that was initially at rest.

3.2.2 Inelastic Collisions

According to the classification of collisions outline before, the total kinetic energy decreases in inelastic collisions. In a one-dimensional inelastic collision between two bodies A and B (Figure 2, left panel), that loss of the kinetic energy can be calculated as

$$\Delta K = (\frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2) - (\frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2). \tag{3}$$

A special class of inelastic collisions are completely inelastic collisions. In such collisions, the two colliding objects stick to each other after the collision, moving as a single body.

3.3 Projectile Motion

A projectile is an object that is given an initial velocity, at an angle to the earth surface, and then follows a path determined entirely by the effects of gravitational force and air resistance. In the present experiment, the air resistance is neglected. Consequently, the projectile motion is effectively a superposition of horizontal motion with constant velocity and vertical motion with constant acceleration. Let us consider a particular case, where the initial velocity of magnitude v_0 is horizontal (and the air resistance is ignored). With the choice of the coordinate system as in Figure 3, the x and y components of the particle's position vector can be found as

$$x = v_0 t \qquad \text{and} \qquad y = \frac{1}{2}gt^2.$$

4 Apparatus and Measurement Procedure

4.1 Apparatus

The measurement equipment consists of the following elements: electronic balance, steel ball, copper ball, aluminium ball, target paper, and the collision apparatus shown in Figure 4.

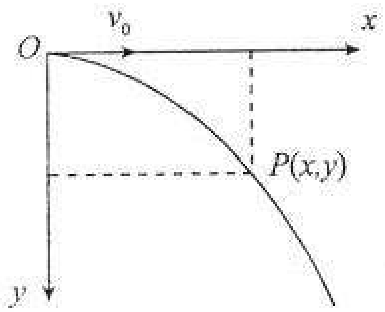


Figure 3. Projectile motion.

The upper end of the ball support is conical and flat at the top so that the contact area between the ball and the support is minimal. In that way, the effect of friction can be reduced when the ball starts moving after the collision. The ball support is equipped with a weak magnet to allow to adjust the center of mass of the ball, so that is in line with the center of the support.

There is an electromagnet at the vertical column. The electromagnet can be moved vertically up and down along the column, and the height can be read off from a marked grading. The centers of mass of the two balls should be adjusted to the same height before the experiment, so that the ball on the string hits the ball on the support face-on, once the electromagnetic is switched off. After the impact, the target (impacted) ball will start moving in projectile motion and will eventually fall on the target box.

4.2 Measurement Procedure

1. Adjust the slideway so that it is horizontal with a bubble level.

2. On a sheet of paper draw several concentric circles with diameters increasing sequentially by 2 cm. This will be your target paper.

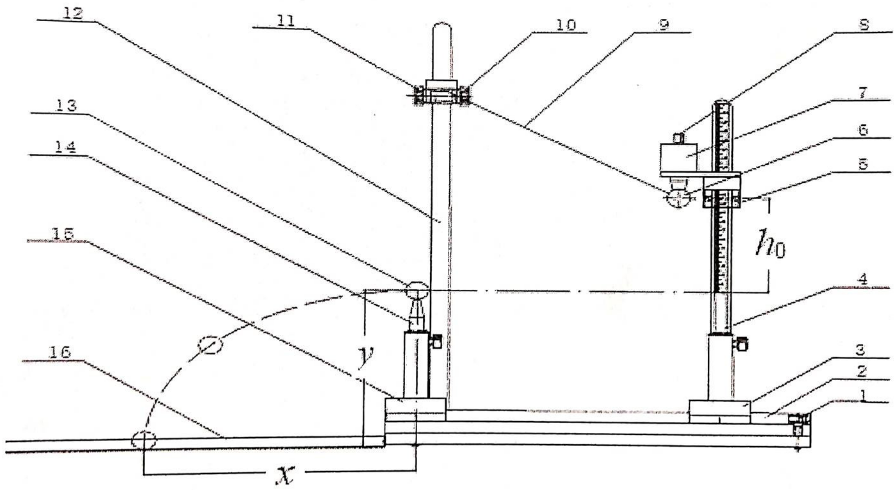
3. Place the target paper in the target box and measure the horizontal distance x between the bull's-eye and the ball supporter.

4. Measure the initial height y of the target ball with the rulers provided.

5. Use the electronic scales to measure the mass m_1 of the target (impacted) ball. The mass m_2 of the impacting ball is the same as m_1.

6. Assuming that the collision is elastic, calculate the theoretical initial height h_0 of the impacting ball needed to make the target ball hit the bull's-eye.

7. Adjust the height of the impacting ball to be h_0 and press the button to release the impacting ball so that it hits the target ball placed on the support. Record the distance x_1 between the landing point of the target ball and the ball support.



1. Adjustment Screw (调节螺钉)	2. Slideway (导轨)	3. Slider (滑块)	4. Column (立柱)
5. Marking board (刻线板)	6. Impacting ball (撞击球)	7. Electromagnet (电磁铁)	8. Armature Screw (衔铁螺钉)
9. Cycloid (摆线)	10. Locking screw (锁紧螺钉)	11. Adjustment Knob (调节旋钮)	12. Column (立柱)
13. Impacted ball (被撞球)	14. Ball supporter (载球支柱)	15. Slider (滑块)	16. Target box (靶盒)

Figure 4. Experimental setup.

Repeat for six times and calculate the average value. Does the ball hit the bull's-eye successfully? If not, calculate the total loss of the mechanical energy ΔE.

8. Knowing the loss of the mechanical energy ΔE during the collision, calculate how much the height needs to be increased Δh. Subsequently, calculate the initial height $h = h_0 + \Delta h$ of the impacting ball needed to make the target ball hit the bull's-eye in the real case.

9. Fix the value of y and find the height h for different x. Choose more than six different values of x, repeat the steps 7-9, and find the corresponding values of h.

10. **Game Time.** Please follow instructions given in the lab session.

11. **Optional.** Change the steel target ball to ones made of copper and aluminium. In each case, measure and compare the loss of the mechanical energy ΔE.

5 Caution

- When the impacting ball is kept in place by the electromagnet, make sure that the string is not slack.

- The lock screw should be put in place before starting the experiment.

- Make sure that the impacting ball is attached closely with the armature screw. Before the experiment, adjust the centers of mass of the two balls so that they are at the same level when the two balls are about to collide.
- Prevent balls from falling to the ground during the experiment.

6 Preview Questions

- What is a collision? What is the difference between elastic collisions and inelastic collisions?
- What is a completely inelastic collision?
- Give examples of elastic and inelastic collisions in our daily life.
- In an inelastic collision, such as the collision of two balls in this lab experiment, part of the kinetic energy just before and just after the collision is not the same. Where does the ,,missing" kinetic energy go?
- What is projectile motion? Write down equations for the vertical and the horizontal components of the projectile's velocity, neglecting air resistance.
- Assuming that the collision of balls in this experiment is elastic, find the theoretical value of the initial height of the impacting ball h_0. Express the result in terms of x, y, m_1, m_2, and g, where x is the horizontal distance between the bull's-eye and the ball support, y is the initial height of the impacted ball, and m_1, m_2 are masses of the impacting ball and the impacted ball respectively, and g is the acceleration due to gravity.

  In the special case, when $m_1 = m_2 = m$, give the simplified equation.
- Assuming that both balls have mass m, and the collision between them is elastic, we can calculate the theoretical value of the initial height of the impacting ball for which the range of the impacted (target ball) is x. However, the experimentally measured distance x_1 is different from that value. Derive an equation for the total loss of the mechanical energy ΔE and calculate the value Δh, by which the initial height should be increased in order to compensate the range for that loss.
- What are possible factors that lead to the loss of the mechanical energy in the experiment?