iii). According to the conservation of energy, i.e.,

Total energy = potential energy + kinetic energy:

We get $U(r(t)) + \frac{m}{2} r'(t)^2 = const.$ ①

At the initial point, $t=0$ $r(t)=x_1$.

the initial displacement is x_1 ~~and~~

Also, the initial velocity of the object is 0. So $r'(0)=0$

Substitute them into the energy equation

Then ~~we~~ we get:

$$const = U(r(0)) = U(x_1)$$

Since velocity $= r'(t)$, we deduce from ① that

$$r'(t) = \sqrt{\frac{2}{m}(U(x_1)-U(x))}$$

For a small interval of time, $dt = \frac{ds}{v}$.

Integrate it, we get, $T(x_1) = \int_0^T dt = \int_0^{x_1} \frac{1}{v} ds$

$$= \sqrt{\frac{m}{2}} \int_0^{x_1} \frac{1}{\sqrt{U(x_1)-U(x)}} dx \quad ②$$

Then ~~of~~ substitute ② as follows. $y^2 = \frac{U(x)}{U(x_1)}$ ③

~~From~~ Before substitution, we deduce from ~~that~~ ③:

$$y = \sqrt{\frac{U(x)}{U(x_1)}} \qquad \frac{dy}{dx} = \frac{U'(x)}{2\sqrt{U(x)U(x_1)}}.$$

So ② can be written as $\int_0^{x_1} \frac{1}{\sqrt{U(x_1)-U(x)}} dx = \int_0^{x_1} \frac{dx}{\sqrt{U(x_1)\left(1-\frac{U(x)}{U(x_1)}\right)}}$

$$= \int_0^1 \frac{2\sqrt{U(x)U(x_1)}\, dy}{\sqrt{U(x_1)}\sqrt{1-y^2}\; U'(x)} = \int_0^1 \frac{2\sqrt{U(x)}}{U'(x)\sqrt{1-y^2}} dy$$

(where $x = U^{-1}[U(x_1)y^2]$, derived from ③

So define $T(x_1) = \sqrt{\frac{m}{2}} \int_0^1 \frac{2\sqrt{U(x)}}{U'(x)\sqrt{1-y^2}} dy$.

For fixed y, when x_1 increase, x will increase because $x = U^{-1}[U(x_1)y^2]$, where U and U^{-1} are strictly increasing functions.

If T doesn't change when x_1 changes, it will obviously not change when x changes. So $T'(x) = 0$.

Denote $H(x,y) := \int_0^1 \frac{2\sqrt{U(x)}}{U'(x)\sqrt{1-y^2}} dy$.

Differentiate $T(x)$, we get:

$$\cancel{T'(x)}\ DT(x) = \sqrt{\frac{m}{2}} \int_0^1 DH(\cdot, y)\, dy$$

$$= \sqrt{\frac{m}{2}} \int_0^1 \left[\frac{\sqrt{U(x)}}{U'(x)}\right]' \frac{2}{\sqrt{1-y^2}} dy.$$

~~Since $DT(x) = 0$ for all x,~~

(Since x can be kept independent of y by changing x_1, we can continue to write the following equations)

$$= \sqrt{\frac{m}{2}} \cdot \left[\frac{\sqrt{U(x)}}{U'(x)}\right]' \int_0^1 \frac{2}{\sqrt{1-y^2}} dy$$

(Calculate the integral) $= \pi \sqrt{\frac{m}{2}} \left[\frac{\sqrt{U(x)}}{U'(x)}\right]' = 0.$

Therefore, $\left[\frac{\sqrt{U(x)}}{U'(x)}\right]' = 0$, ~~i.e. $U(x) = C\sqrt{U(x)}$~~

Since $U(x) > 0$ and $U'(x) > 0$ for all $x \in \mathbb{R}$, we can say $U'(x) = C\sqrt{U(x)}$ $(C > 0)$

iv). Now we get $U'(s) = C\sqrt{U(s)}$, which can be written as
$$\frac{dU}{ds} = C\sqrt{U}.$$

To solve the equation:
$$\frac{dU}{\sqrt{U}} = c \cdot ds,$$
$$\Rightarrow \int \frac{dU}{\sqrt{U}} = \int c \cdot ds$$
$$\Rightarrow 2\sqrt{U} + C_0 = c \cdot s \quad (C_0 \in R). \quad ④$$

Since $U(0) = 0$, we can get $C_0 = 0$. ⑤

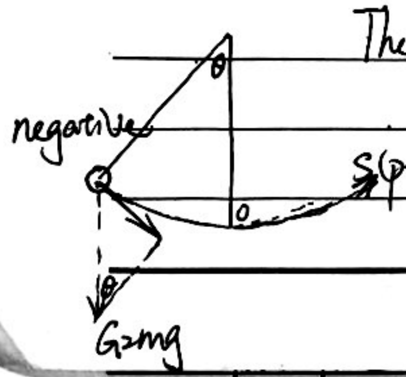
Therefore, ~~$U = \frac{c^2 s^2}{4}$~~ we can get $U = \frac{c^2 s^2}{4}$ from ④

According to Newton's Second Law of Motion,
$$F = m \cdot a = m \frac{d^2 s}{dt^2} \quad ⑥$$
where m is the mass.

From the property of potential energy, ~~an object in the potential field is exerted~~ a force of $F(s) = -U'(s)$ is exerted on an object in the potential field.

From ⑤, we deduce that $F(s) = -U'(s) = \frac{-c^2 s}{2}$.
$$\text{So } \frac{d^2 s}{dt^2} = a = \frac{F(s)}{m} = -\frac{c^2 s}{2m} \quad k = \frac{c^2}{2m}.$$

In the model of simple pendulum:



The force tangential to the trajectory:
$$F = -mg\sin\theta, \text{ so } \frac{d^2 s}{dt^2} = a = \frac{F}{m} = -g\sin\theta.$$
$$S = \theta \cdot l$$
$$\text{So } k = \frac{\frac{d^2 s}{dt^2}}{-S} = \frac{g\sin\theta}{\theta l}$$ ~~is a constant~~.

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$$k(\theta) = \frac{g\sin\theta}{\theta l} \quad ⑦$$
$$k'(\theta) = \frac{\theta\cos\theta - \cancel{\tan\theta}\sin\theta}{\theta^2} = \frac{\theta - \tan\theta}{\frac{\theta^2}{\cos\theta}} \quad ⑧$$

In ⑧, we know that $\tan\theta > \theta$ on ~~Eo~~ $(0, \frac{\pi}{2})$

since $(\tan\theta)' = \sec^2 x > 1$, $(\theta)' = 1$ and $\tan\theta = \theta$ when $\theta = 0$.

So $k'(\theta) < 0$. k isn't a constant.

In conclusion, simple pendulum doesn't satisfy the relation of $\frac{d^2 s}{dt^2} = -ks$.