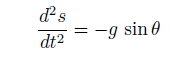
Given the equation , now we want to find a parametrization of this cycloid.

The second law gives us:



Combining these two equations

s=g/K sinθ

Differentiate the equation

ds = g/K cosθ dθ (1)

On the other hand, ds=. And Since the normal component of the speed is 0:

(2)

Combining the two equation, we get ds = dx / cos θ,

substitute (1) into it

From equation(2) we get

Multiply both sides with dt and integrate both sides, we get

Applying the initial conditions x = 0 and y = −g/2k , both when θ = 0, we finally get

Now we can show such curve is a cycloid

(1)

(2)

Since the conservation of mechanical energy gives us ,

,plugging into

From equation (1) and (2), we can get

Integrate both sides

Now considering an intermediate point

Since the conservation of mechanical energy gives us ,

,plugging into

Then integrate both sides, after computing it with computer software

That shows T is the same for any point of the curve. Therefore, the curve is a cycloid.

Reference

Weisstein, Eric W. "Tautochrone Problem." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/TautochroneProblem.html