MODELLING VEHICLE TRAFFIC FLOW WITH PARTIAL DIFFERENTIAL EQUATIONS

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DISCIPLINE IN LEADERSHIP

MODELLING VEHICLE TRAFFIC FLOW WITH PARTIAL DIFFERENTIAL EQUATIONS

BY

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DECLARATION

I hereby declare that this submission is my own work towards the award of the BSc. Mathematics degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

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DEDICATION

This project is dedicated to the Almighty God for granting me guidance, strength, intelligence and grace to accomplish this project.

I also dedicate this work to my family. I owe you a debt of gratitude for your prayers, support and understanding during my period of studies at Presbyterian University College.

My final dedication goes to my supervisor, Mr. Gabriel O. Fosu for his encouragement and support from beginning of this project to the end.

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ABSTRACT

Traffic congestion is a condition on road networks that occurs as use increases, and is characterized by slower speeds, longer trip times, and increased vehicular queuing. The most common example is the physical use of roads by vehicles. The challenge of traffic flow has motivated many researchers to model traffic flow at both the macroscopic and microscopic levels. This study seeks to model the continuous and the discontinuous behaviour of vehicles by using the traffic flow parameters; flow, density and velocity with the use of partial differential Equations (PDEs) based on Lighthill, Whitham and Richard (LWR) model. Using regression analysis the Flow-Density curve was found to be quadratic of the form

$$q(k) = 1.145k - 0.0107k^2$$

The method of characteristics was used to solve both the non-linear homogeneous and the inhomogeneous PDE and the model yielded characteristics curve of the form

$x = (1.145 - 2.01x_0)t + x_0$

and

$x = (1.145 - 0.0214A(x_0)\ e^t)t + x_0$

also, the homogeneous solution of the PDE is obtained as

$$k(x,t) = \frac{x - 1.145t}{1 - 2.01t}$$

the inhomogeneous solution of the PDE is obtained as

$x_0 = x + (0.0214\beta - 1.145)t$

$k = \beta e^t$

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

In this section, we take a look at transportation, urban transportation, traffic congestion,the difficulties of urban traffic congestion, causes and effects of urban traffic congestion and traffic flow network in Accra.

1.1.1 TRANSPORTATION

Transportation is the movement of people, goods, animals and data from one location to another through a medium. There are various modes of transport or travel, in Ghana it is mostly done by road, even though we do have rail, air and water based travel as well. Arasan (2012) states that, transportation is an activity of life processes and seeks to provide access to various activities that satisfy mobility needs of humankind . According to Eddington. (2006), an effective transportation system is significantly important in sustaining economic growth in contemporary economies since it provides linkages between different parts of the country and the global world. It links to work, deliver products to market, underpins logistics and supply chain, and support local and international trade. A good-established transportation system is not only a key to national growth but also serves as catalyst for economic development of a country.

1.1.2 URBAN TRANSPORTATION

Municipalities or urban areas are the engine of economic growth in most developing countries, and transport provides the arteries for movement. In

Ghana, an urban area is defined as any settlement inhabited by 5,000 or more individuals. Urban transportation refers to the system of transportation that provides access and mobility for people and goods within cities. Elements of urban transportation include public transit (collective transport) and non-motorized transport (pedestrians, cyclists). Effective urban transport systems are essential to economic activity and quality of life. Commercial activities depend on urban transportation systems to ensure the mobility of its customers, employees and suppliers. The urban transport services cover a range of important social and economic services such as leisure trips; business journeys; commuting; shopping; trips to places of education and merchandise distribution. Rodrigue (2009), report that transportation infrastructure is one of the key factors that directly affect urban transportation effectiveness and capacity within the metropolis. Transportation infrastructure mainly includes roads, parking lots, vehicles and transportation terminals. Urban traffic management system is also an important component which can properly control and guide the distribution of traffic flows on roads.

1.1.3 TRAFFIC CONGESTION

Traffic congestion is a condition on road networks that occurs as use increases, and is characterized by slower speeds, longer trip times, and increased vehicular queuing. The most common example is the physical use of roads by vehicles. When traffic demand is great enough that the interaction between vehicles slows the speed of the traffic stream, this results in some congestion. Rodrigue (2009) states that congestion can be perceived as unavoidable consequences of scarce transport facilities such as road space, parking area, road signals and effective traffic management. It is argued that urban congestion mainly concerns two domains of circulation, passengers and freight which share the same infrastructure.

1.1.4 THE DIFFICULTIES OF URBAN TRAFFIC CONGESTION

Because of the ever-growing population rate in our cities, there are high levels of traffic congestions on our streets. That is, due to increasing demand for urban trips, urban traffic congestion is a problem for both developed and developing countries. Downie (2008) opines that traffic congestion occurs when the volume of vehicular traffic is greater than the available road capacity, a point commonly referred to as saturation. He describes a number of specific circumstances which cause or aggravate congestion. Most of such circumstances are concerned with reduction in the capacity of road at a given point or over a certain length, or increase in the number of vehicles required for the movement of people and goods. He further argues that economic surge in various economies has resulted in a massive increase in the number of vehicles that overwhelms transport infrastructure, thus causing congestion on roads in cities. He adds that high urban mobility rate also contributes to the congestion menace. The massive use of cars does not only have an impact on traffic congestion but also leads to decline in public transit efficiency, thereby creating commuting difficulties in cities. Indeed the over-dependence on cars has tremendously increased the demand for transport infrastructure. Unfortunately the supply of transport infrastructure has never been commensurate with the growth of mobility needs. Consequently, several vehicles spend most of the time in traffic as a result of traffic space limitation (Yan and Crookes, 2010).

1.1.5 CAUSES AND EFFECTS OF URBAN TRAFFIC CONGESTION

One of the main reasons for urban traffic congestion is increased number of vehicles on the road. Because of the increased rate in adult population, there is more demand for vehicles. As the number of vehicles increase the level of congestion also increases. This is why in smaller towns and villages congestion is almost unheard of.

Again lack of proper infrastructure is a cause of congestion. Councils and national governments fail to act on the looming threat of heavy congestion until it happens. The city does not expand along with an increasingly vehicle reliant population. A single street with a lane on each side might be ok for now but will not suffice in ten years after the population has increased. Authorities often fail to convert this into a dual roadway.

Alternate routes are also a problem. Cities have limited capacity to expand due to poor funding and planning restrictions preventing building on certain spaces. Cities are forced to work with the routes they already have. If the authorities cannot increase the number of lanes it leads to congestion.

One of the effects of urban traffic congestion is delays. The first thing many people think of when it comes to congested roadways is the delay. During the morning commute there is additional stress because delays caused by traffic can make people late for work. And at the end of the day, the afternoon rush hour is again a frustrating time because the workday is done and people want to get home to relax, and traffic is preventing it. These delays are universal to everyone who has to maneuver through congested roads.

Again disruption in an individual's time management is an effect of road congestion. It is a secondary effect of traffic congestion related to delays because of people's inability to estimate travel times. Those who regularly travel congested areas know approximately how long it usually takes to get through a particular area depending on the time of day or the day of the week. These experienced city drivers have to build in time "just in case" the traffic is bad. This takes away from leisure time and time to do other tasks throughout the day. Also, on a day when the traffic is unusually light the built in extra time may be of no use and the person arrives too early.

Moreover, fuel consumption and pollution is also an effect of traffic congestion.

The stopping and starting in traffic jams burns fuel at a higher rate than the smooth rate of travel on the open highway. This increase in fuel consumption costs commuters additional money for fuel and it also contributes to the amount of emissions released by the vehicles. These emissions create air pollution and are related to global warming.

Road Rage is an effect of traffic congestion; it is a senseless reaction to traffic that is common in congested traffic areas. If someone is not driving as fast as the person behind him thinks he should, or someone cuts in front of someone else it can lead to an incident that is dangerous to the offender and those around him on the road. Road rage often manifests itself as shouting matches on the road, intentional tailgating, retaliatory traffic maneuvers and mostly a lack of attention being paid to the traffic around the people involved. It is basically a temper tantrum by frustrated drivers in traffic.

Emergency vehicles can also suffer as an effect of traffic congestion. When you dial 911 and request a police officer, an ambulance or a fire truck and the emergency vehicle is unable to respond in an appropriate amount of time because of traffic congestion it can be a danger to you and your property. Systems are available that help alleviate the problem by allowing the emergency crews to automatically change the traffic lights to keep the line moving.

1.1.6 TRAFFIC FLOW NETWORK IN ACCRA.

In mathematics and engineering traffic flow is the study of interactions between vehicles, drivers and infrastructure (including highways and traffic control devices), with the aim of understanding and developing an optimal road network with efficient movement of traffic and minimal traffic congestion problems. Nevertheless, even with the advent of significant computer processing power, to date there has been no satisfactory general theory that can be consistently applied to real flow conditions. Current traffic models use a mixture of empirical and theoretical techniques .These models are then developed into traffic forecasts, to take account of proposed local or major changes, such as increased vehicle use, change land use or change in mode of transport (with people moving from cars for example), and to identify areas of congestion where the network needs to be adjusted. With the knowledge of traffic flow we then proceed to a case study for this study in Accra – Ghana. Accra is the largest city in Ghana and its road network is linked up by major and minor road network. The inner and the outer road network are marked by modern traffic light and roundabout to govern the flow of vehicle. A case study would be done on a suburb of Accra, which stretch of network is marked by influx of minor network. At the various intersection of network are set of traffic light which manages the traffic flow. Heavy Congestion on the inner and outer stretch of road in Accra often occurs from 6:30am-10:00am and 4:30pm-10:00pm.

1.2 STATEMENT OF THE PROBLEM

With the ever-increasing population growth and their demand for vehicles, traffic congestion has become a genuine problem, which is why the subject has substantial research interest in developed and developing countries, and has recently drawn the consideration of both road engineers and statistical researchers. Because of productive time lost in long traffic jams, traffic congestion continues to be the leading source of problems in the economic growth of the country and as such needs to be dealt with. In Ghana, traffic congestion in recent years has become severe leading to major negative impact in diverse areas which include economic development. In recent years, in cities like Accra, most of its productive hours that maximize economic growth are spent in long traffic jam that amount to delay at work place and those who manage to arrive early wake up at dawn in order to avoid the traffic and spent most of the working hours feeling sleepy which eventually affect economic growth negatively. The passion for this study came from the observation that most cities in Ghana are enduring some levels of congestion on their freeway networks. Some major cities experience traffic congestion for three to eight hours daily. Moreover, when traffic congestion occurs in our cities, it causes major environmental hazards due to excessive emission of poisonous gases in the cities. This further imposes health threat and stress to all individual that are exposed to it. Traffic congestion has become a matter of concern in Ghana because although about 95% of our transportation is done by road, existing road infrastructure has not been improved significantly and in most cases, it is not viable to extend traffic infrastructures due to costs, limited available space, and environmental impact. Further, irrespective of these challenges, as the population grows, more vehicles are imported into the system to compete the existing vehicles. Hence it is important to look at effective management of vehicle traffic flow between the existing traffic lights in Accra from Kwame Nkrumah Circle to Adabraka as a case study based on the continuous and the discontinuous behavior of the traffic light nodes that imposes much traffic jam on our roads in the cities in Ghana.

1.3 OBJECTIVE OF THE STUDY

The purpose of this study is to model the traffic flow of vehicles in a genuine urban dynamic traffic situation of long traffic congestion on the limited number of road on the peak hours. Hence the main objectives of the study are as follows:

1. To use partial differential Equations (PDEs) based on Lighthill and Whitham (1955) and Richard (1956) (LWR) model to model traffic flow between two traffic light nodes in Accra from Kwame Nkrumah Circle to Adabraka in Ghana as the study zone.

2. Use regression analysis to obtain the quadratic relationship between the flux (flow) and the density of vehicles within traffic light nodes.

3. To use method of characteristics to solve the both nonlinear homogeneous and nonlinear inhomogeneous PDE.

1.4 METHODOLOGY

This study is based on Lighthill, Whitham and Richard (LWR) macroscopic model. This would be employed to generate first order nonlinear partial differential equations. The method of characteristics would be used to solve the systems of PDEs. A primary data would be collected between the Kwame Nkrumah Circle traffic light node and the Adabraka traffic light node about 150 metres interval in Accra through out the day. Data count such as density and time would be obtained from tagged cars that travel from one traffic light node to the other by stand-by observers. Microsoft excel software would be needed for running the data to establish some fundamental relationship among the traffic parameter such as density, flow (flux) and speed.

1.5 SIGNIFICANT OF THE STUDY

Effective traffic controlling is very important for the lasting success of any country's development; because without traffic congestions, individuals have more productive hours at their disposal and that promotes economic development and prevents most countries from going bankrupt. If traffic congestion is not properly coped with, it may lead to the crumpling of the economy. It is therefore justifiable because it helps traffic engineers to verify whether traffic properties and characteristics such as speed(velocity), density and flow among others determines the effectiveness of traffic flow. Hence the LWR model can be used by engineers of road network to plan ahead to prevent excessive traffic jam.

1.6 LIMITATION

Some of the problems relating to the success of the study are as follows;

1. Unavailability of up-to-date road equipment to record accurate road information in the cities.

2. Restricted access to extensive set of data variables.

1.7 THESIS ORGANIZATION

• The first chapter deals with the background of study,its significance and limitations.

• Chapter 2 contains the literature review and the contributions of other researchers regarding vehicle traffic flow and the applications on real world networks.

• In Chapter 3 the methodology is presented as mathematical treatment and logical presentation of formulation and models of solution.

• Chapter 4 deals with the data collection, analysis and result.

• Chapter 5 synthesis the whole study and presents the conclusion, summary and recommendation

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

This chapter contains the general overview of research done on transportation system, traffic flow theory and macroscopic traffic flow models.

2.2 TRANSPORTATION SYSTEM

Effective transportation systems lead to the efficient movement of goods and people, which significantly contribute to the quality of life in every society. In the heart of every economic and social development, there is always a transportation system. Meanwhile, traffic congestion has been increasing worldwide because of increased motorization, urbanization, population growth, and changes in population density. This threatens the social and economic prosperity of communities all over the world. Congestion reduces utilization of the transportation infrastructure and increases travel time, air pollution, and fuel consumption. Therefore, managing and controlling transportation systems becomes a high priority task for every community, as it constitutes a matter of survival and prosperity for humanity. In the search for meeting the demand for more traffic capacity, it has been realized repeatedly that building more roads is no longer a feasible solution due to the high cost and scarcity of land especially in metropolitan areas. In addition, the length of time that it takes to build additional roads and the disruption that this introduces to the rest of the traffic network makes the option of building new roads as the worst case scenario. The current highway transportation system runs almost open loop whereas traffic lights at streets are still lacking the intelligence that is necessary to reduce delays and speed up traffic flows. The recent advances in electronics, communications, controls, computers, and sensors provide an opportunity to develop appropriate transportation management policies and strategies in order to effectively utilize the existing infrastructure rather than building new road systems. The use of technologies will help provide accurate traffic data, implement control actions, and in general reduce the level of uncertainty and randomness that exists in today's transportation networks. The successful implementation of intelligent transportation systems will require a good understanding of the dynamics of traffic on a local as well as global system level and the effect of associated phenomena and disturbances such as shock wave generation and propagation, congestion initiation and so on. In addition, the understanding of human interaction within the transportation system is also crucial. Transportation systems and traffic phenomena constitute highly complex dynamical problems where simplified mathematical models are not adequate for their analysis. There is a need for more advanced methods and models in order to analyse the causality, coupling, feedback loops, and chaotic behavior involved in transportation problem situations. Traffic modelling can facilitate the effective design and control of today's complex transportation systems.

2.3 TRAFFIC CONGESTION

Levinson (2003) states that traffic congestion management has the goal to optimize transportation flow of people and goods particularly in the metropolitan area .To define what is meant by traffic congestion, the executives report of Organization of Economic co-operation and Development (OCED) and European Conference of Ministers of Transport (ECMT), January, 2004 stated that there is no single definition for congestion since congestion takes on many faces, occurs in many different contexts and caused by many different process. However, below are some of their definitions of congestion:

I. Congestion is a situation in which demand for road spacing exceeds supply

II. Congestion is the impedance vehicles impose on each other due to the speed flow relationship, in condition where the use of a transport system approaches capacity.

III. Congestion is essentially a relative phenomenon that is linked to difference between the road system performance that users expect and how the system actually performs.

Susan Grant-Muller and Laird (2007), describes Traffic congestion as a widely recognized transport cost and is a significant factor in transport system performance evaluation which affects transport planning decisions. To individual motorists, congestion is a cost they bear, but each motorist also imposes congestion on other road users. The existence of congestion can be explained by the fact that each additional vehicle imposes more total delay on others than they bear, resulting in economically excessive traffic volumes. Congestion can be recurrent (regular, occurring on a daily, weekly or annual cycle) or non-recurrent (traffic incidents, such as accidents and disabled vehicles) as revealed by ECMT(January,2000). Some congestion costs analyst only consider recurrent, others include both. Economist, William and Vickrey (1969), identified six types of congestion as follows:

2.3.1 TYPES OF TRAFFIC CONGESTION

1. Simple interaction on homogeneous roads: where two vehicles travelling close together delay one another.

2. Multiple interactions on homogeneous roads: where several vehicles interact.

3. Bottlenecks: where several vehicles are trying to pass through narrowed lanes.

4. “Trigger neck” congestion: when an initial narrowing generates a line of vehicles interfering with a flow of vehicles not seeking to follow the jammed itinerary.

5. Network control congestion: where traffic controls programmed for peak-hour traffic inevitably delay off-peak hour traffic.

6. Congestion due to network morphology: where traffic congestion reflects the state of traffic on all itineraries and for all modes. The cost of intervention for a given segment of roadway increases through possible interventions on other segments of the road, due to the effect of triggered congestion.

Most congestion cost analysis concentrates on the second and third types of congestion: congestion arising from interactions between multiple vehicles on homogeneous road section and bottleneck congestion. Others types are overlooked or assumed to be included in the types that are measured. Another often overlooked factor that complicates economic analysis is that congestion reduces some costs. Moderate highway congestion reduces traffic speeds to levels that maximize vehicle throughput and vehicle fuel efficiency, and although congestion tends to increase crash rates per vehicle-mile, the crashes that occur tend to be less severe, reducing injuries and deaths (Vickrey, 1969).

2.4 BASIC FREEWAY TRAFFIC FLOW THEORY

According to Austroads (2008), a freeway is generally known as an uninterrupted facility because traffic on the mainline is not interrupted by a control device such as a traffic signal. Traffic flow on a freeway segment is therefore relatively easier to model analytically, especially in free-flow or uncongested conditions. This section provides the basic analytical framework for freeway traffic models and also briefly reviews the tools currently used for automatic freeway control.

2.4.1 TRAFFIC FLOW THEORY

Traffic flow theory comprises the study of the movement of individual drivers and vehicles between two points and the interactions they make with one another and plays a vital role in the progress of overall social productivity. In the 1950s, James Lighthill and Gerard Whitham, two experts in fluid dynamics, (and, independently P. Richards), modelled the flow of car traffic along a single road using the same equations describing the flow of water Lighthill and Whitham (1955),Whitham (1955) and Richard (1956). The basic idea is to consider traffic on a large scale so that cars are taken as small particles and to assume the conservation of the number of cars. The LWR model is described by a single conservation law, a special partial differential equation where the dependent variable, the car density, is a conserved quantity, i.e. a quantity which can neither be created nor destroyed. Traffic flow when likened to fluid flow has several parameters associated with it. These parameters could then provide information regarding the nature of traffic flow, which would help a traffic analyst in detecting any variation in flow characteristics. Thus, understanding traffic behavior would require a through knowledge of these traffic stream parameters and their mutual relationships. The LWR model is presented in some detail including a description of an analytical solution using the method of characteristics. In formulating a mathematical model for a continuum traffic flow, there are basic steps that are often used as giving by Haight. (1963);

Haberman (1977); Banks (1992); Michalopoulos and Lyrintzis. (1993); Bellomo and Delitala. (2002) and Bellemans and Moor. (2002) below:

1. Identify appropriate conservation laws (e.g. mass, momentum, energy, etc) and their corresponding densities and fluxes.

2. Write the corresponding equations using conservation law and close the system of equations by proposing appropriate relationships between the fluxes and the densities.

2.5 HISTORY OF TRAFFIC MODEL

Attempts to produce a mathematical theory of traffic flow dates back to the 1920s, when Frank Knight first produced an analysis of traffic equilibrium, which was refined into first and second principles of equilibrium by Wardrop (1952).

2.5.1 TRAFFIC FLOW MODELLING

The study of traffic flow and in particular vehicular traffic flow, is carried out with the aim of understanding and assisting in the prevention and remedy of traffic congestion problems. The first attempts to develop a mathematical theory for traffic flow date back to the 1930s Adams (1937); Greenshields (1935a); Greenshields (1935b), but despite the continuous research activity in the area we do not have yet a satisfactory mathematical theory to describe real traffic flow conditions. This is because traffic phenomena are complex and nonlinear, depending on the interactions of a large number of vehicles.

Moreover, vehicles do not interact simply by following the laws of physics, but are also influenced by the psychological reactions of human drivers. As a result we observe chaotic phenomena such as cluster formation and backward propagating shockwaves of vehicle speed/density that are difficult if at all possible to be accurately described with mathematical models (Bose and Ioannou, 2000). According to a state of the art report of the Transportation Research Board Gartner and Rathi. (2001), mathematical models for traffic flow may be classified as: Traffic Stream Characteristics Models, Human Factor Models, Car Following Models, Continuum Flow Models, Macroscopic Flow Models, Traffic Impact Models, Unsignalized Intersection Models, Signalized Intersection Models and Traffic Simulation Models. Below we describe briefly each of the above categories.

• Traffic stream characteristics: Hall (1996) theory involves various mathematical models, which have been developed to characterize the relationships among the traffic stream variables of speed, flow, and concentration or density.

• Human factor modeling: Koppa (1999), deals with salient performance aspects of the human element in the context of the human-machine interactive system. These include perception reaction time, control movement time, responses to: traffic control devices, movement of other vehicles, hazards in the roadway, and how different segments of the population differ in performance. Further, human factors theory deals with the kind of control performance that underlies steering, braking, and speed control. Human factors theory provides the basis for the development of car following models.

• Car following models: Rothery (1992), examine the manner in which individual vehicles (and their drivers) follow one another. In general, they are developed from a stimulus-response relationship, where the response of successive drivers in the traffic stream is to accelerate or decelerate in proportion to the magnitude of the stimulus. Car following models recognize that traffic is made up of discrete particles or driver-vehicle units and it is the interactions between these units that determine driver behavior, which affects speed-flow-density patterns.

• Continuum models: Kuhne (1997) are concerned more with the overall statistical behavior of the traffic stream rather than with the interactions between the particles.

• Following the continuum model paradigm, macroscopic flow models Williams (1997), discard the microscopic view of traffic in terms of individual vehicles or individual system components (such as links or intersections) and adopt instead a macroscopic view of traffic in a network. Macroscopic flow models consider variables such as flow rate, speed of flow, density and ignore individual responses of vehicles.

• Ardekani and Jamei. (1992) deal with traffic safety, fuel consumption and air quality models. Traffic safety models describe the relationship between traffic flow and accident frequency.

• Unsignalized intersection theory: Troutbeck and Brilon (1997) deals with gap acceptance theory and the headway distributions used in gap acceptance calculations.

• Traffic flow at signalized intersections: Rouphail and Li (1996) deals with the statistical theory of traffic flow, in order to provide estimates of delays and queues at isolated intersections, including the effect of upstream traffic signals.

• Traffic simulation modelling: Lieberman and Rathi. (1996) deals with the traffic models that are embedded in simulation packages and the procedures that are being used for conducting simulation experiments. Mathematically the problem of modelling vehicle traffic flow can be solved at two main observation scales: the microscopic and the macroscopic levels.

2.6 MICROSCOPIC TRAFFIC FLOW MODEL.

In the microscopic level, every vehicle is considered individually, and therefore for every vehicle we have an equation that is usually an ordinary differential equation (ODE). The microscopic model involves separate units with characteristics such as speed, acceleration, and individual driver-vehicle interaction. It may be classified in different types based on the so-called car-following model approach. The car-following modelling approach implies that the driver adjusts his or her acceleration according to the conditions of leading vehicles.

In these models, the vehicle position is treated as a continuous function and each vehicle is governed by an ODE that depends on speed and distance of the car in the front.

Another type of microscopic model involve the use of Cellular Automata or vehicle hopping models which differs from the car-following approach in that they are fully discrete time models. They consider the road as a string of cells that are either empty or occupied by one vehicle. One such model is the Stochastic Traffic Cellular Automata Nagel and Schreckenberg (1992); Nagel (1996) model. Further, a more recent approach is currently under heavy research with the use of agent based modelling (Naiem A. and khodary I., 2010). Microscopic approaches are generally computationally intense, as each car has an ODE to be solved at each time step, and as the number of cars increases, so does the size of the system to be solved.

Analytical mathematical microscopic models are difficult to evaluate but a remedy for this is the use of microscopic computer simulation. In such microscopic traffic models, vehicles are treated as discrete driver-vehicle units moving in a computer-simulated environment.

2.7 MACROSCOPIC TRAFFIC FLOW MODELS

At a macroscopic level, we use the analogy of fluid dynamics models, where we have a system of partial differential equations, which involves variables such density, speed, and flow rate of traffic stream with respect to time and space.

Macroscopic flow models Williams (1996), discard the real view of traffic in terms of individual vehicles or individual system components such as links or intersections and adopt instead a macroscopic fluid view of traffic in a network. The relationship between density, velocity, and flow is presented to derive the equation of conservation of vehicles, which is the main governing equation for scalar macroscopic traffic flow models.

The macroscopic models for traffic flow, whether they are one-equation or a system of equations, are based on the physical principle of conservation. When physical quantities remain the same during some process, these quantities are said to be conserved. Putting this principle into a mathematical representation, it becomes possible to predict the density and velocity patterns at a future time. The model's characteristics are shifted toward the parameters such as flow rate, concentration, or traffic density, and average speed, all being functions of one-dimensional space and time. This class of models is represented by partial differential equations. Modelling vehicular traffic via macroscopic models is achieved using fluid flow theory in a continuum responding to local or non-local influences. The mathematical details of such models are less than those of the microscopic ones.

The drawback of macroscopic modelling is the assumption that traffic flow behaves like fluid flow, which is a rather harsh approximation of reality. Vehicles tend to interact among themselves and are sensitive to local traffic disturbances, phenomena that are not captured by macroscopic models. On the other hand, macroscopic models are suitable for studying large-scale problems and are computationally less intense especially after approximating the partial differential equation with a discrete time finite order equation.

2.8 FUNDAMENTAL RELATION

There is unique relation among the three macroscopic traffic flow parameters density, flow and speed. Under uninterrupted flow conditions, these traffic variables are all related by $q = v.k$. Where q is flow, v is speed and k is density. (This relationship represents the fundamental equation of traffic flow).

2.9 MEASUREMENT OF TRAFFIC STREAM PROPERTIES

Traffic flow is generally constrained along a one-dimensional pathway (e.g. a travel lane). A time-space diagram provides a graphical depiction of the flow of vehicles along a pathway over time. Time is measured along the horizontal axis, and distance is measured along the vertical axis. Traffic flow in a time-space diagram is represented by the individual trajectory lines of individual vehicles. Vehicles following each other along a given travel lane will have parallel trajectories, and trajectories will cross when one vehicle passes another. Time-space diagrams are useful tools for displaying and analyzing the traffic flow characteristics of a given roadway segment over time (e.g. analyzing traffic flow congestion). There are three main variables to visualize a traffic stream: speed (v), density (k), and flow(q)

2.9.1 SPEED

Speed in traffic flow is defined as the distance covered per unit time. Speed is one of the basic parameters of traffic flow and time mean speed and space mean speed are the two representations of speed. The speed of every individual vehicle is almost impossible to track on a roadway; therefore, in practice, average speed is based on the sampling of vehicles over a period of time or area and is calculated and used in formulas. If speed is measured by keeping time as reference it is called time mean speed, and if it is measured by space reference it is called space mean speed.

2.9.2 TIME MEAN SPEED v_t

Time mean speed is the average of all vehicles passing a point over a duration of time. It is the simple average of spot speed. Time mean speed is measured by taking a reference area on the roadway over a fixed period of time. In practice, it is measured by the use of loop detectors. Loop detectors, when spread over a reference area, can record the signature of vehicles and can track the speed of each individual vehicle. However, average speed measurements obtained from this method are not accurate because instantaneous speeds averaged among several vehicles cannot account for the difference in travel time for the vehicles that are traveling at different speeds over the same distance. Time mean speed v_t is given by,

$$v_t = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{2.1}$$

where v_i is the spot speed of i^{th} vehicle, and n is the number of observations. In many speed studies, speeds are represented in the form of frequency table. Then the time mean speed is given by,

$$v_t = \frac{\sum_{i=1}^{n} q_i v_i}{\sum_{i=1}^{n} q_i} \tag{2.2}$$

where q_i is the number of vehicles having speed v_i , and n is the number of such speed categories.

2.9.3 SPACE MEAN SPEED v_s

Space mean speed is the speed measured by taking the whole roadway segment into account. Consecutive pictures or video of a roadway segment track the speed of individual vehicles, and then the average speed is calculated. It is considered more accurate than the time mean speed. The data for space calculating space mean speed may be taken from satellite pictures, a camera, or both. The space mean speed also averages the spot speed. This is derived as below. Consider unit length of a road, and let v_i be the spot speed of i^th vehicle. Let t_i be the time the vehicle takes to complete unit distance and is given by $\frac{1}{v_i}$. If there are n such vehicles, then the average travel time t_s is given by,

$$t_s = \frac{\sum t_i}{n} = \frac{1}{n}\sum \frac{1}{v_i} \tag{2.3}$$

If t_{av} is the average travel time, then average speed $v_s = \frac{1}{t_s}$. Therefore, from the above equation,

$$v_s = \frac{n}{\sum_n^{i=1} \frac{1}{v_i}}. \tag{2.4}$$

This is simply the harmonic mean of the spot speed. If the spot speeds are expressed as a frequency table, then,

$$v_s = \frac{\sum_{i=1}^{n} q_i}{\sum_{i=1}^{n} \frac{q_i}{v_i}} \tag{2.5}$$

where q_i vehicles will have v_i speed and n_i is the number of such observations.

Numerical Example;

If the spot speeds are 50, 40, 60, 54 and 45, then find the time mean speed and space mean speed.

Solution.

Time mean speed v_t is the average of spot speed. Therefore,

$$v_t = \frac{\sum v_i}{n} = \frac{50 + 40 + 60 + 54 + 45}{5} = \frac{249}{5} = 49.8. \tag{2.6}$$

Space mean speed is the harmonic mean of spot speed. Therefore,

$$v_s = \frac{n}{\sum v_i} = \frac{5}{\frac{1}{50} + \frac{1}{40} + \frac{1}{60} + \frac{1}{54} + \frac{1}{45} = \frac{5}{0.12}} = 48.82. \tag{2.7}$$

2.9.4 DENSITY

Density (k) is defined as the number of vehicles per unit area of the roadway. In traffic flow, the two most important densities are the critical density (kc) and jam density (kj). The maximum density achievable under free flow is kc, while kj is minimum density achieved under congestion. In general, jam density is seven times the critical density. Inverse of density is spacing (s), which is the distance between two vehicles.
The density (k) within a length of roadway (L) at a given time (t_1) is equal to the inverse of the average spacing of the n vehicles.

2.9.5 FLOW

Flow (q) is the number of vehicles passing a reference point per unit of time, and is measured in vehicles per hour. The inverse of flow is headway (h), which is the time that elapses between the i^{th} vehicle passing a reference point in space and the $i + 1$ vehicle. In congestion, h remains constant. As a traffic jam forms, h approaches infinity. $q = kv$
$q = 1/h$
The flow (q) passing a fixed point (x_1) during an interval (T) is equal to the inverse of the average headway of the m vehicles.

2.10 FUNDAMENTAL DIAGRAMS OF TRAFFIC FLOW

The relation between flow and density, density and speed, speed and flow, can be represented with the help of some curves. They are referred to as the fundamental diagrams of traffic flow.

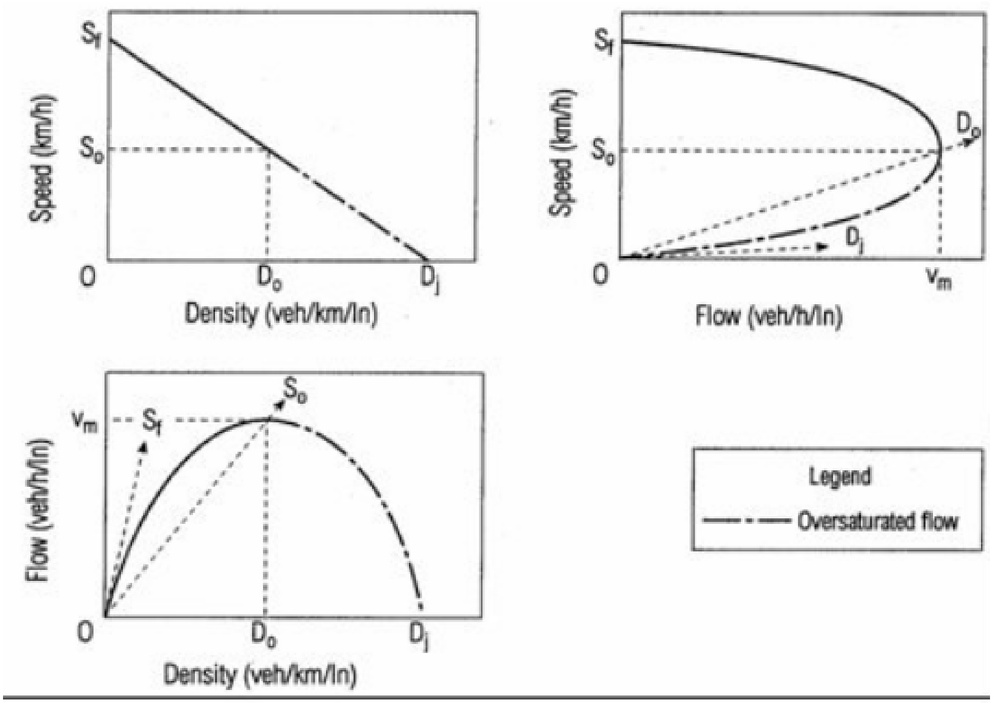

Figure 2.1: Fundamental diagrams of traffic flow

2.10.1 FLOW DENSITY CURVE

The flow and density varies with time and location. The relation between the density and the corresponding flow on a given stretch of road is referred to as one of the fundamental diagram of traffic flow. Some characteristics of an ideal flow-density relationship is listed below:

1. When the density is zero, flow will also be zero,since there is no vehicles on the road.

2. When the number of vehicles gradually increases the density as well as flow increases.

3. When more and more vehicles are added, it reaches a situation where vehicles can not move. This is referred to as the jam density or the maximum density. At jam density, flow will be zero because the vehicles are not moving.

4. There will be some density between zero density and jam density, when the flow is maximum. The relationship is normally represented by a parabolic curve.

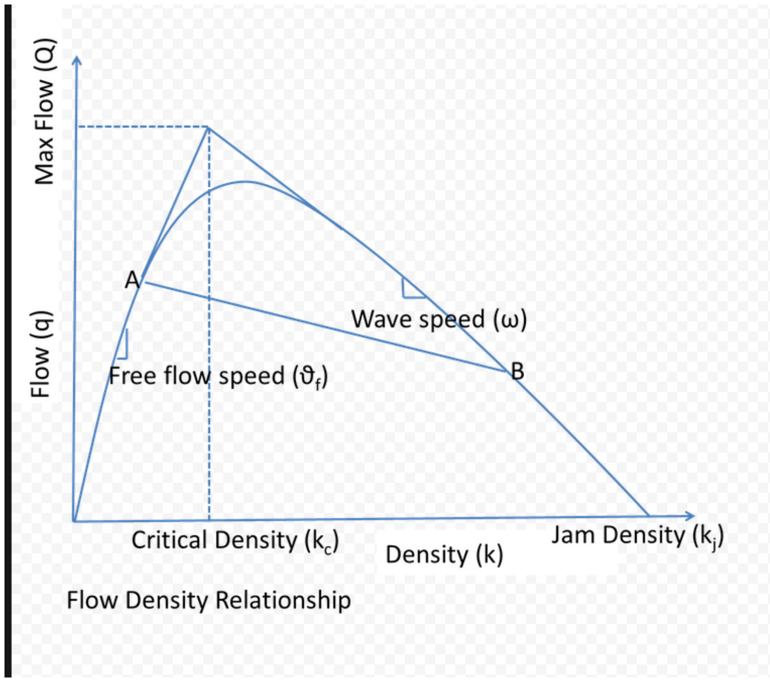

Figure 2.2: Flow density curve

2.10.2 SPEED DENSITY DIAGRAM

Similar to the flow-density relationship, speed will be maximum, referred to as the free flow speed, and when the density is maximum, the speed will be zero. The most simple assumption is that this variation of speed with density is linear. Corresponding to the zero density, vehicles will be flowing with their desire speed, or free flow speed. When the density is jam density, the speed of the vehicles becomes zero. It is also possible to have non-linear relationships.

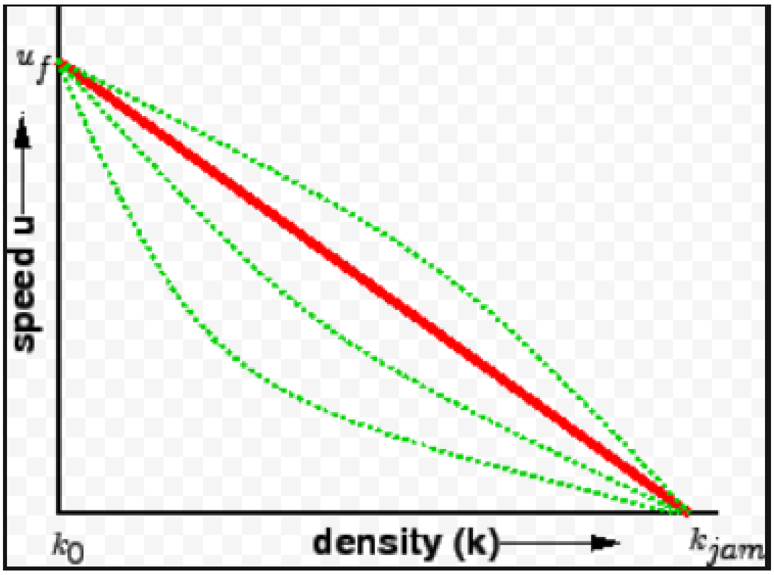

Figure 2.3: Speed density diagram

2.10.3 SPEED FLOW RELATION

The relationship between the speed and flow can be postulated as follows. The flow is zero either because there is no vehicles or there are too many vehicles so that they cannot move. At maximum flow, the speed will be in between zero and free flow speed.

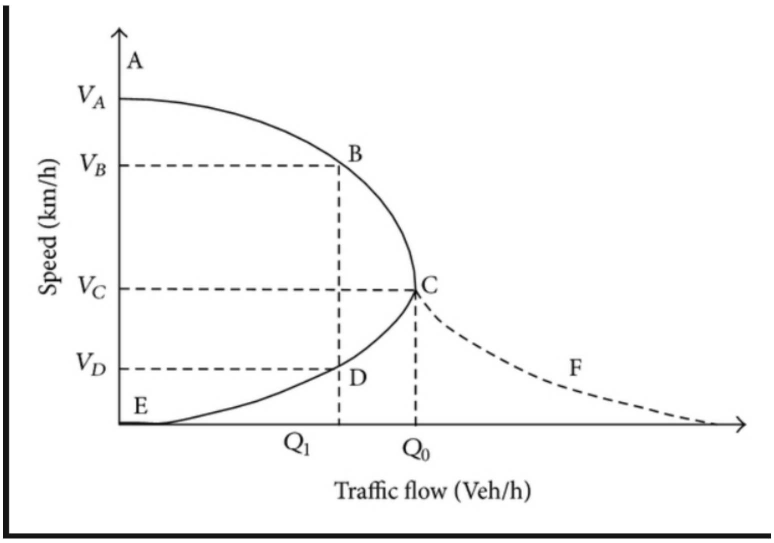

Figure 2.4: Speed flow curve

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter deals with modelling traffic flow from the macroscopic approach using partial differential equation (PDE).

3.2 LIGHTHILL-WHITHAM-RICHARDS TRAFFIC MODEL (LWR)

The mathematical model, referred to as Lighthill-Whitham-Richards (LWR) model is the simplest macroscopic traffic model, and as such, is only concerned with average behavior on a large scale. It is based on the one-dimensional continuity equation,

$$k_t + (kv)_x = 0 \tag{3.1}$$

where $k(x,t)$ is the density (in number of vehicles per unit length of a road at time t), and v is the traffic stream velocity. The subscripts denote partial differentiation. Since the model consists of one equation with two dependent variables, an additional relationship between traffic speed or flow and density needs to be specified for the system to be closed. For a deterministic model, it is assumed that there is a one-to-one relationship between speed or flow and density, which only exists at the traffic equilibrium state. For example,

$$v = V_{eq}(k) = v_{max}(1 - \frac{k}{k_{max}}) \tag{3.2}$$

where v_{max} is some maximal velocity as a property of the road, and k_{max} is the maximal bumper to bumper density. One big drawback in this type of models is that there is no agreement on the functional form of the speed-density (or flow-density) relationship. Different functional relationships, such as equation (3.2), result in different traffic flow models and are referred to as the fundamental diagram because they are the integral part of these models. The system consisting of (3.1) and (3.2) is a non-linear first order partial differential equation describing a hyperbolic conservation law where mass, or in this case density of traffic, is conserved. In this sense, although the distribution of vehicles may vary with time, the overall number of vehicles in the system will only depend on the flux into and out of the domain. Because the LWR model is a closed model consisting of a single equation and one algebraic relationship, it is classified as a first-order model.

3.3 PARTIAL DIFFERENTIAL EQUATION (PDE)

Partial differential equation is an equation involving one or more partial derivatives of an unknown function of several variables. The order of a PDE is the order of the highest-order derivative that appears in the equation. Linear PDE: The PDE $F(x, y, z, t; u, u_x, u_y, u_z, u_t, u_{x,x}, u_{x,y}, ...) = 0$ is said to be linear if the function F algebraically linear in each of the variable u, u_x, u_y..... and if the coefficient of u and its derivatives are functions only of the independent variables; otherwise it is non linear. A non linear PDE is said to be quasilinear if it is linear in the highest – order derivatives.

3.3.1 CLASSIFICATION OF FIRST ORDER PDE

The general quasilinear system of n first-order PDEs in n functions of two independent variables is of the form:

$$\sum_{j=1}^{n} a_{ij}\frac{\partial u_j}{\partial x} + \sum_{j=1}^{n} b_{ij}\frac{\partial u_j}{\partial y} = c_i, (i = 1, 2, 3, ..., n) \tag{3.3}$$

Where a_{ij}, b_{ij}, c_i may depend on $x, y, u_1, u_2, ..., u_n$ .If each a_{ij} and b_{ij} is independent of $u_1, u_2, u_3, ..., u_n$

Example, the system

$$\begin{aligned} (pu)_x + p_t &= 0 \\ uu_x + u_t &= -\frac{1}{p}p_x \\ up_x + p_t &= -\gamma p u_x \end{aligned} \tag{3.4}$$

(Duchateau, 1986) is called almost linear. If in addition each c_i depends linearly on $u_1, u_2, u_3, ..., u_n$, the system is said to be linear .The example (3.4) of systems of equation above is a quasilnear. In term of the $n * n$ matrix $A =| a_{ij} |$ and $B =| b_{ij} |$,and the column vectors $u = (u_1, u_2, ..., u_n)$ and $c = (c_1, c_2, ..., c_n)$ the system of equations can be expressed as :

$$Au_x + Bu_y = c \tag{3.5}$$

If A or B is non singular, it is usually possible to classify the systems of first -order according to types as follows: suppose $det(B) \neq 0$ and define a polynomial of degree n in λ by

$$P_n(\lambda) \equiv det(A^T - \lambda B^T) = det(A - \lambda B) \tag{3.6}$$

Hence systems of first- order PDEs is classified as follows: The PDE is said to be:

I. Elliptic, if $P_n(\lambda)$ has no real zeros.

II. Hyperbolic, if $P_n(\lambda)$ has n real, distinct zero, or if $P_n(\lambda)$ has n real roots, at least one of which is repeated and the generalized eigenvalue problem

$(A^T - \lambda B^T)t = 0$ yields n linearly independent eigenvectors t.

III. Parabolic, if $P_n(\lambda)$ has n real zeros, at least one of which is repeated, and the above generalized eigenvalue problem yields fewer than n linearly independent eigenvectors. However an exhaustive classification cannot be carried out when $P_n(\lambda)$ has both real and complex zeros. (Duchateau, 1986)

The spatial variables in PDE are usually restricted to some open Ω with boundary S; the union of Ω and S is a closure of Ω and denoted by Φ. If present the time variable is considered to run over an interval $t_1 < t < t_2$. A function

$$u = u(x, y, z, t) \tag{3.7}$$

is the solution for a given $m^{th}-$ order PDE if ,for (x, y, z) in Ω and $t_1 < t < t_2.u$ is c^m and satisfy the PDE.

3.3.2 INITIAL AND BOUNDARY CONDITION.

Initial conditions: These are conditions that must be satisfied throughout Ω at the instant when consideration of the physical system begins. A typical initial condition prescribes some combination of u and its time derivatives. The prescribed initial and boundary-condition functions and any inhomogeneous term in PDE are said to comprise the data in the problem modelled by the PDE. The solution is said to depend continuously on the data if small change in the data produce correspondingly small changes in the solution. Hence a problem is said to be well posed if:

I. A solution of the problem exists.

II. The solution is unique.

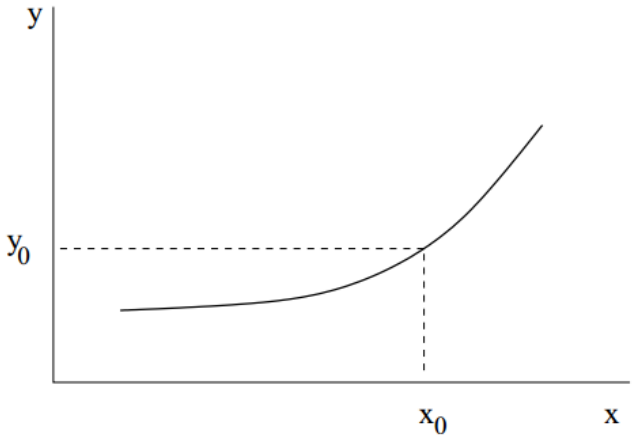

Figure 3.1: Initial condition

Boundary Conditions: These are conditions that must be satisfied at points on the boundary S of the spatial region Ω in which PDE holds.

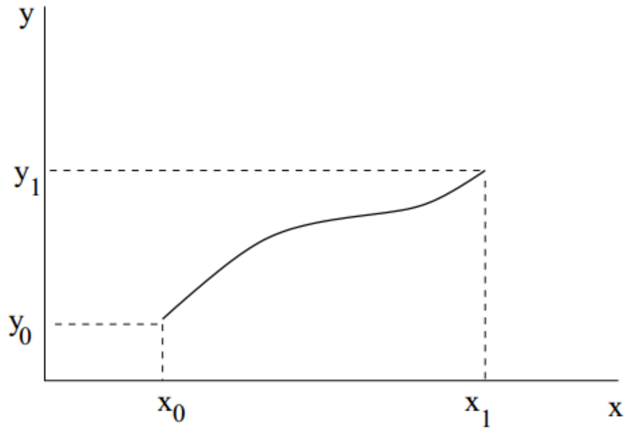

Figure 3.2: Boundary condition

3.4 THE PDE MODEL BASED ON LWR MODEL

The macroscopic traffic model developed first by Lighthill and Whitham (1955), Whitham (1955) and Richard (1956) shortly called LWR model is most suitable for correct description of traffic flow. In this model, vehicles in traffic flow are considered as particles in fluid: further the behaviour of traffic flow is modelled by the method of fluid dynamics and formulated by hyperbolic partial differential equation (PDE) .The macroscopic traffic flow model is used to study traffic flow by collective variables such as traffic flow rate (flux) $q(x,t)$, traffic speed $v(x,t)$ and traffic density $k(x,t)$ all of which are functions of space, $x \in \ R$ and time $t \in \Re^{+}$.

There is unique relation among the three macroscopic traffic flow parameters density, flow and speed. Under uninterrupted flow conditions, these traffic variables are all related by the equation: $q = v.k$.

This relationship is called the fundamental equation of traffic flow.

3.5 METHOD OF CHARACTERISTICS.

The method of characteristics is a method which can be used to solve the initial value problem for general first order partial differential equation. Consider the first order linear PDE

$a(x,t)u_x + b(x,t)u_t + c(x,t)u = 0$

With initial condition $u(x,0) = f(x)$

The goal of method of characteristics when applied is to change coordinates from (x,t) to a new system (x_0, s) in which the partial differential equation becomes ordinary differential equation along certain curves in the x - t plane. Such curves are called the characteristics curves. The new variable s will vary, and new variable x_0 will be constant along the characteristics curves. The variable x_0 will change along the initial curve in the x - t plane (along the line t =0).

When $\frac{dx}{ds} = a(x,t)$ and $\frac{dt}{ds} = b(x,t)$, we have

$\frac{du}{ds} = \frac{dx}{ds}u_x + \frac{dt}{ds}u_t = a(x,t)u_x + b(x,t)u_t$

and along the characteristics curves, the PDE becomes an ODE

$\frac{du}{ds} + c(x,t)u = 0.$

Suppose that the function $u(x,t)$ satisfies the following quasilinear PDE:

$$\frac{\partial u}{\partial t} + c(u,x,t)\frac{\partial u}{\partial x} = Q(u,x,t) \tag{3.8}$$

The rate of change of $u(x,t)$ along the trajectory $x(t)$ will be given by

$$\frac{du}{dt}(x(t),t) = \frac{\partial u}{\partial t}(x(t),t) + \frac{dx(t)}{dt}\frac{\partial u}{\partial x}(x(t),t) \tag{3.9}$$

Now comparing (3.8) with (3.9), we see that if we let

$$\frac{dx}{dt} = c(u,x,t) \tag{3.10}$$

then the LHS of (3.8) coincides with the RHS of (3.9) so that we have

$$\frac{du}{dt} = Q(u,x,t) \tag{3.11}$$

Equation (3.8) and (3.9) are the ordinary differential equations that result from the observation of $u(x,t)$ along the trajectory x(t). The characteristic curve x(t) is the solution of equation (3.8). Note that it may not always be possible to solve these two equations explicitly. However, they may provide useful qualitative behaviour of the solutions. This analytical method is used to solve the macroscopic LWR traffic model in conjunction with Greenshield's flow density relation. Let consider the function $k(x,t)$ at each point of the (x,t) plane. Along any curve in the (x,t) plane, x and k can be considered as functions of t, therefore the total derivative of $k(x(t),t)$ is:

$$\frac{dk(x(t),t)}{dt} = \frac{\partial k}{\partial t} + \frac{dx}{dt}\frac{\partial k}{\partial x} \tag{3.12}$$

The conservation equation where the flux changes with the density is:

$$\frac{\partial k}{\partial t} + \frac{dq}{dk}\frac{dk}{dx} = 0 \tag{3.13}$$

From Equations (3.12) and (3.13) we obtain

$$\frac{dk}{dt} = 0 \qquad \text{and} \qquad \frac{dx}{dt} = \frac{dq}{dk} = c(k) \tag{3.14}$$

Thus, there exists a certain curve x along which the solution k is constant. Such curves are called characteristics of the non- linear hyperbolic equation. (Greenberg, 1959). Therefore, the governing equation for the system can be expressed as:

$$\frac{\partial k(x,t)}{\partial t} + c(k)\frac{\partial k(x,t)}{\partial x} = 0 \tag{3.15}$$

CHAPTER 4

ANALYSIS AND RESULTS

4.1 INTRODUCTION

This chapter deals with data collection and their analysis. The data to be used in this study is collected in Accra from Kwame Nkrumah Circle to Adabraka in the Greater Accra region.

4.1.1 COLLECTION OF DATA

The data was collected on January 19, 2016. The field data were collected at various times within the day, where the civil servants and traders from their home are going to work, at work, on break and have close from work. The traffic count was done manually using the stand-by observer method along 150m way of Accra road (Ghana) between (Kwame Nkrumah Circle traffic light and Adabraka traffic light). The site is a level grade straight segment between two traffic nodes. In all a one hour thirty three minutes and fifty seconds data were collected for the study using the stand-by Observer method. Basically in the stand-by observer method, two observers stand by the roadside where vehicle travelling along a known section of road in one direction and records the number of vehicles entering and leaving the road segment.Samples of vehicle are tagged and the time they enter and leave the traffic nodes are recorded. The speed and flow obtained are based on the relation:
$q = k.v$

4.1.2 ANALYSIS

The essential part of any data collection process is to be analyzed and presented in a format that is easily comprehensible. Below is an illustration of a simplified manual counts data analysis, as transformed from field data collection. In all, sixty (60) runs by the observers was done to collect the data and the results are presented in table below.

Table 4.1: Data collected

Time	Density	Velocity	Flow
195	78	0.769230769	60
178	64	0.842696629	53.93258427
186	83	0.806451613	66.93548387
157	97	0.955414013	92.67515924
191	86	0.785340314	67.53926702
189	103	0.793650794	81.74603175
135	33	1.111111111	36.66666667
131	26	1.145038168	29.77099237
142	31	1.056338028	32.74647887
139	35	1.079136691	37.76978417
131	36	1.145038168	41.22137405
144	29	1.041666667	30.20833333
209	77	0.717703349	55.26315789
214	92	0.700934579	64.48598131
207	89	0.724637681	64.49275362
198	78	0.757575758	59.09090909
219	72	0.684931507	49.31506849
204	69	0.735294118	50.73529412
234	94	0.641025641	60.25641026
221	85	0.678733032	57.69230769
228	93	0.657894737	61.18421053
238	107	0.630252101	67.43697479
215	87	0.697674419	60.69767442
223	92	0.67264574	61.88340807
179	58	0.837988827	48.60335196
186	72	0.806451613	58.06451613

Table 4.2: Data collected

Time	Density	Velocity	Flow
191	74	0.785340314	58.11518325
173	69	0.867052023	59.8265896
171	67	0.877192982	58.77192982
182	71	0.824175824	58.51648352
195	67	0.769230769	51.53846154
178	71	0.842696629	59.83146067
186	79	0.806451613	63.70967742
157	81	0.955414013	77.38853503
191	69	0.785340314	54.18848168
189	93	0.793650794	73.80952381
135	38	1.111111111	42.22222222
131	32	1.145038168	36.64122137
142	29	1.056338028	30.63380282
139	32	1.079136691	34.5323741
131	31	1.145038168	35.49618321
144	37	1.041666667	38.54166667
209	59	0.717703349	42.34449761
214	71	0.700934579	49.76635514
207	54	0.724637681	39.13043478
198	63	0.757575758	47.72727273
219	81	0.684931507	55.47945205
204	83	0.735294118	61.02941176
234	96	0.641025641	61.53846154
221	91	0.678733032	61.76470588
228	86	0.657894737	56.57894737
238	91	0.630252101	57.35294118
215	98	0.697674419	68.37209302
223	79	0.67264574	53.13901345
179	63	0.837988827	52.79329609
186	51	0.806451613	41.12903226
191	68	0.785340314	53.40314136
173	54	0.867052023	46.82080925
171	74	0.877192982	64.9122807
182	84	0.824175824	69.23076923

INTERPRETATION OF RESULTS

The aim of this study is to build up the regression model of the correlation of traffic speed, rate of flow, and density, and predict the trend of traffic flow characteristics.

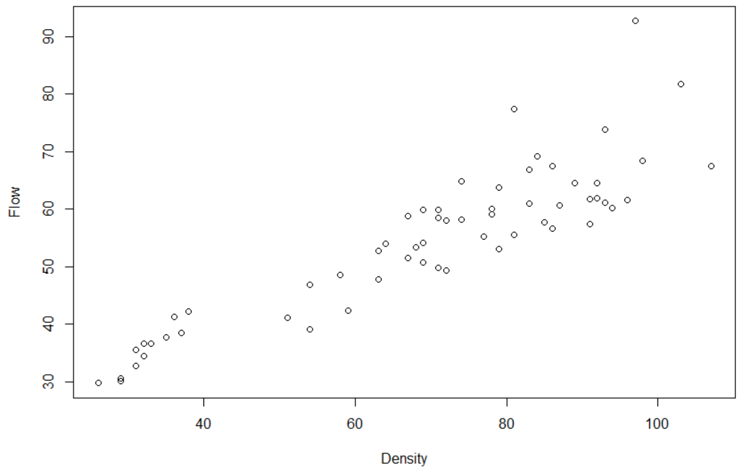

Figure 4.1: Flow density curve

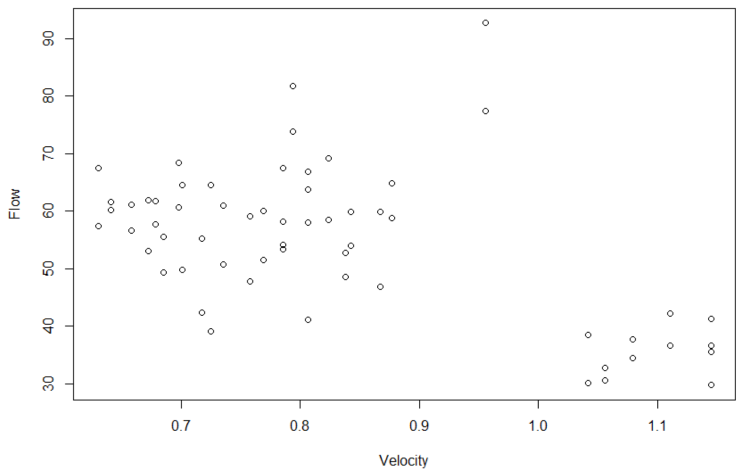

Figure 4.2: Flow velocity curve

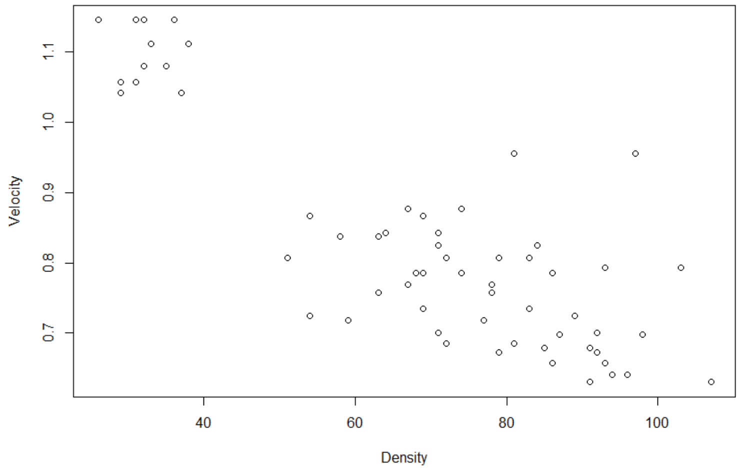

Figure 4.3: Density velocity line

DENSITY FLOW CURVE.

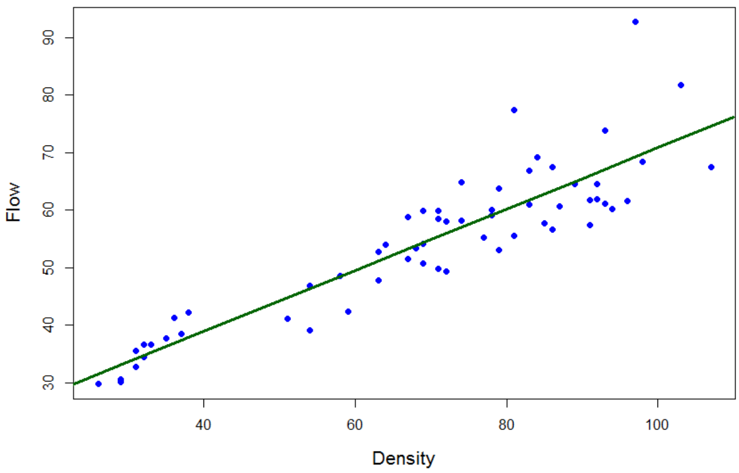

Figure 4.4: Flow density curve

The density flow curve shown above represents the free flow part of the curve. Here, flow is yet to attain its maximum point in which density will start to decrease. Therefore, from the graph, it can be seen that as density increases, flow also increases and vice versa. That is, as the number of cars on the road increases, the number of cars passing a reference point per unit of time also increases.

FLOW VELOCITY CURVE.

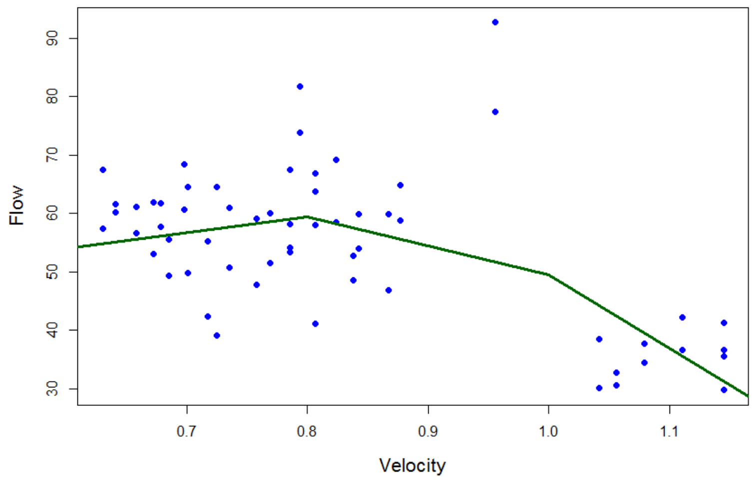

Figure 4.5: Flow velocity curve

The flow velocity curve shows clearly that as velocity increases, flow decreases. This represents the congested flow part of the curve meaning the maximum flow had been achieved. That is, as the speed of cars on the road increases, the number of cars passing a reference point per unit of time decreases.

LINEAR DENSITY VELOCITY GRAPH

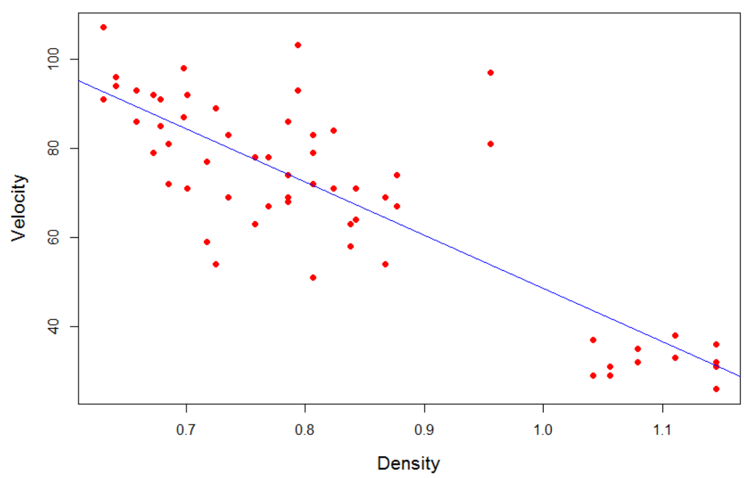

Figure 4.6: Linear density velocity graph

The linear density velocity graph shows that as density increases, velocity decreases and vice versa. That is, as the number of cars on the road increases, the speed of cars on the road decreases.

4.2 ANALYSIS OF RESULTS

This part of study discusses the flow – density relation from the fundamental diagrams.

4.2.1 FLOW DENSITY CURVE

For the LWR model, it was determined that a model for predicting densities on the basis of flow would be the most effective procedure for predicting traffic operations in the basic free way section of the roadways. The model of the result of regression for the flow rate versus density is indicated as follows: Using the fundamental equation:

$$q(k) = v_{max}(k - \frac{k^2}{k_{max}}) \tag{4.1}$$

$$q(k) = 1.145038(k - \frac{k^2}{107}) \tag{4.2}$$

$$q(k) = 1.145k - 0.0107k^2 \tag{4.3}$$

$$q(k) = k(1.145 - 0.0107k) \tag{4.4}$$

Comparing equation (4.4) with the LWR flow-density relation,

$q = k.v$

we have

$v_{max} = 0.0191 m/s$, $k_{max} = 107 veh/m$ and $q_{max} = 1.545 veh/s$

4.2.2 DETERMINATION OF MAXIMUM FLOW, OPTIMUM DENSITY AND OPTIMUM VELOCITY.

As shown above, $q = 1.145k - 0.0107k^2$

at the maximum point,

$\frac{dq}{dk} = 1.145 - 0.0107(2)k = 0$

$1.145 - 0.0107(2)k = 0$

$0.0214k = 1.145$

$k = 53.505veh/m = k_0 =$ density at maximum flow

Substituting k = 53.505 into $q = 1.145k - 0.0107k^2$ gives

$q = -0.0107(53.505)^2 + 1.145(53.505)$

$q = -30.632 + 61.263$

$q = 30.631veh/min$

$q = 0.511veh/s$

Velocity at maximum flow is $v_0 = \frac{q_{max}}{k_0} = \frac{0.511}{53.505} = 0.0096m/s$

4.2.3 NONLINEAR HOMOGENEOUS PARTIAL DIFFERENTIAL EQUATION

Now differentiating equation (4.3) and substituting into the governing equation $\frac{\partial k(x,t)}{\partial t} + c(k)\frac{\partial k(x,t)}{\partial x} = 0$, from the LWR model, we have

$$\frac{\partial k}{\partial t} + (1.145 - 0.0214k)\frac{\partial k}{\partial x} = 0 \tag{4.5}$$

This is a first order partial differential equation. Since the solution is $k = k(x, t)$ taken the derivative with respect to t, we get

$$\frac{\partial k(x(t), t)}{\partial t} = \frac{\partial k}{\partial x} + \frac{dx}{dt}\frac{\partial k}{\partial x} \tag{4.6}$$

Comparing the equation (4.5) and (4.6)

$\frac{dk}{dt} = 0$ and $\frac{dx}{dt} = \frac{dq}{dk} = 1.145 - 0.0214k$

Thus on a certain curve x(t) the solution k(x,t) of Equation 4.3 is a constant.

4.2.4 METHOD OF CHARACTERISTICS

The graph of x(t) to the ordinary equation

$\frac{dx}{dt} = \frac{dq}{dk} = 1.145 - 0.0214k$

is called a characteristic curve.

The solution k(x,t) is thus constant along the characteristic curves.

The method of characteristics can be used to find a solution for the initial boundary problem. An initial boundary problem assumes (besides the differential equation) two extra equations:

- Initial values: the density values at time t = 0.

$$k(x,0) = f(x) = \alpha + \beta x \tag{4.7}$$

- Boundary values: the density values at distance x = 0.

$$k(0,t) = g(t) \tag{4.8}$$

And $x = x_c$ where $k(x_c, t) = h(t)$

It is assumed that the initial density distribution f(x) is given by a linear function

$$k(x,0) = \lambda x \tag{4.9}$$

With the boundary conditions, $k(0,0) = 0$ and $k(x_c, 0) = \frac{k_{max}}{2} = 53.505$

This implies that

$\lambda = \frac{k(x_c,0)}{x_c} = \frac{53.505}{0.57} = 93.87$

Therefore, the initial condition

$k(x,0) = 93.87x$

Applying the method of characteristics to

$$\frac{\partial k}{\partial t} + (1.145 - 0.0214k)\frac{\partial k}{\partial x} = 0$$

along with the initial condition $k(x, 0) = 93.87x$ at $t = 0$

where $k = k(x, t)$ is the unknown to be determined.

The traffic density measured by the stand-by observer method depends on time and the position, k(x(t),t).

The characteristic ordinary differential equations is given as

$\frac{dx}{dt} = \frac{dq}{dk} = 1.145 - 0.0214k$

Therefore, $x = (1.145 - 0.0214k)t + \lambda$

Thus

$x = (1.145 - 0.0214k)t + x_0$

is the characteristic curve which starts at $x = x_0$ when t = 0.

Substituting $k(x_0, 0) = 93.87x_0$ into the above equation, we have

$x = (1.145 - 2.01x_0)t + x_0$

For $x_0 = 0$, k =0 and x = 1.145t.

for $x_0 = 0.57$, k = 53.506 and x = -0.0007t + 0.57.

The solution of the non linear homogeneous is obtained by making x_0 the subject of the characteristic equation. Thus:

$$x_0 = \frac{x - 1.145t}{1 - 2.01t} \tag{4.10}$$

Since $k(x, t) = F(x_0)$, the solution of the homogeneous non linear PDE is given as

$$k(x, t) = \frac{x - 1.145t}{1 - 2.01t} \tag{4.11}$$

where $k(x, t)$ is the density (in number of vehicles per unit length of a road at time t), x is the position and t is time. This represents the complementary function of the general solution.

4.2.5 NON LINEAR INHOMOGENEOUS PARTIAL DIFFERENTIAL EQUATION

Let $\frac{dk}{dt} = \frac{\partial k}{\partial t} + \frac{\partial k}{\partial x}\frac{dx}{dt} = k$

$\frac{dk}{dt} = k$

When integrated, it becomes

$lnk = t + c$

$\Longrightarrow k = k_0\ e^t$

$\Longrightarrow k = A(x_0)\ e^t$

When

$\frac{dx}{dt} = (1.145 - 0.0214k)$

$\Longrightarrow \frac{dx}{dt} = (1.145 - 0.0214A(x_0)\ e^t)$

Hence

$x = 1.145t - 0.0214A(x_0)\ e^t t + x_0$

Therefore

$x_0 = x - 1.145t + 0.0214A(x_0)\ e^t t$

$\Longrightarrow x_0 = x + (0.0214A(x_0) - 1.145)t$

When $A(x_0) = \beta$ then:

$x_0 = x + (0.0214\beta - 1.145)t$

$k = \beta e^t$

Where k is density, β is a constant and t is time. This represents the particular solution of the general solution.

CHAPTER 5

CONCLUSION, SUMMARY AND RECOMMENDATIONS

5.1 SUMMARY AND CONCLUSION

5.1.1 SUMMARY.

Traffic flow theory comprises the study of the movement of individual drivers and vehicles between two points and the interactions they make with one another that plays a vital role in the progress of overall social productivity. Traffic congestion is a condition on road networks that occurs as use increases, and is characterized by slower speeds, longer trip times, and increased vehicular queuing. The most common example is the physical use of roads by vehicles. With the ever-increasing population growth and their demand for vehicles, traffic congestion has become a genuine problem, the challenge of traffic flow has motivated many researchers to model traffic flow at both the macroscopic and microscopic levels. This study seeks to model the continuous and the discontinuous behaviour of vehicles by using the traffic flow parameters; flow, density and velocity with the use of partial differential Equations (PDEs) based on Lighthill, Whitham and Richard (LWR) model in order to help traffic engineers to verify whether traffic properties and characteristics such as speed(velocity), density and flow among others determines the effectiveness of traffic flow.

5.1.2 CONCLUSION.

The results of the study agree with those of LWR model, which gives a quadratic relationship between flow and density. The study of a section of the roadway between two traffic light nodes, using regression analysis, gave the flow density curve a quadratic equation of the form

$$q(k) = 1.145k - 0.0107k^2$$

where: q = average rate of flow, vehicle per second, and

k = average traffic density, vehicle per metre.

The density variable is significant to the rate of flow. In this case, it is explained by the variability of the dependent variable, the average rate of flow.

The model indicated that the critical density is 53.505 vehicles per metre, which coincides with the rate of flow at 0.511 vehicles per second (capacity).

The method of characteristics was used to solve both the non-linear homogeneous and the inhomogeneous PDE and the model yielded characteristics curve of the form

$x = (1.145 - 2.01x_0)t + x_0$

and

$x = (1.145 - 0.0214A(x_0)\ e^t)t + x_0$

which starts at $x = x_0$ when t = 0.

also, the homogeneous solution of the PDE is obtained as

$$k(x,t) = \frac{x - 1.145t}{1 - 2.01t}$$

the inhomogeneous solution of the PDE is obtained as

$x_0 = x + (0.0214A(x_0) - 1.145)t$

$k = \beta e^t$

5.2 RECOMMENDATIONS

To prevent traffic congestion and to improve efficiency of our road networks, traffic flow should be designed to move at a density corresponding to maximum traffic flow. Traffic light signal which literally stops traffic flow and then permits it to go (in intervals yielding the density corresponding maximum flow) would result in an increased flow of cars on the road. Thus momentarily stopping traffic would actually result in an increase flow. It is recommended that to obtain accurate and more reliable information on the count rate on the road segment, loop detectors should be installed on the various road in the cities of Ghana.

It is recommended to use other methods such as the finite difference method to solve the kinematic wave model by LWR instead of the method of characteristics.

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