UDPS1203/2103 Marking Scheme(May2016):

Q1.

(a) Therefore, $\theta(t) = 300 - 25t - 240e^{-0.1t}$.

(b) $x = 0$ is a regular singular point.

$x = 3$ is an irregular singular point.

(c) $= \frac{1}{2}a - \frac{1}{3}ae^{-t} - \frac{1}{6}ae^{-2t}$

Q2.

(a) $\therefore y = [c_1 + c_2 \ln|x+1|](x+1)^{5/2}$

(b)(i) Therefore, the general solution is $\begin{pmatrix} I \\ V \end{pmatrix} = c_1 e^{-\frac{1}{2}t}\begin{pmatrix} \cos\frac{1}{2}t \\ 4\sin\frac{1}{2}t \end{pmatrix} + c_2 e^{-\frac{1}{2}t}\begin{pmatrix} \sin\frac{1}{2}t \\ -4\cos\frac{1}{2}t \end{pmatrix}$

(ii) $\begin{pmatrix} I \\ V \end{pmatrix} = 2e^{-\frac{1}{2}t}\begin{pmatrix} \cos\frac{1}{2}t \\ 4\sin\frac{1}{2}t \end{pmatrix} - \frac{3}{4}e^{-\frac{1}{2}t}\begin{pmatrix} \sin\frac{1}{2}t \\ -4\cos\frac{1}{2}t \end{pmatrix}$.

(iii) Since the eigenvalues have negative real parts, all solutions converge to the origin.

(c) $\therefore y(t) = \frac{6}{5}e^{-t} + \frac{18}{5}\sin 2t - \frac{6}{5}\cos 2t$.

Q3.

(a) $y_p = Ae^{5x} + (Bx^3 + Cx^2 + Dx + E)\cos x + (Fx^3 + Gx^2 + Hx + I)\sin x$.

(b) $= c_1\cos x + c_2\sin x - \frac{15}{16}x\cos x + \frac{5}{8}\cos x\sin 2x - \frac{5}{64}\cos x\sin 4x + \frac{5}{8}\sin^5 x$

(c) $= a_0\left(1 - x^2 + \frac{1}{6}x^4 + \dots\right) + a_1\left(x - \frac{1}{3}x^3 - \frac{1}{12}x^4 + \dots\right) + \left(\frac{3}{2}x^2 + \frac{1}{2}x^3 + \dots\right)$

Q4.

(a) $=\frac{1}{5}\cos t+\frac{2}{5}\sin t-\frac{1}{5}e^{-t}[\cos t+3\sin t]-e^{-\left(t-\frac{\pi}{2}\right)}\cos(t)u_{\pi/2}(t)$

(b) $=c_1\begin{pmatrix}5\\1\end{pmatrix}e^{-t}+c_2\begin{pmatrix}-2\\1\end{pmatrix}e^{6t}-\frac{1}{18}\begin{pmatrix}7\\1\end{pmatrix}e^{-3t}+\begin{pmatrix}-3\\0\end{pmatrix}e^{t}$

(c) $y''+4y'+20y=0$.

Q5.

(a)(i) $\Rightarrow x=a\mp\frac{1}{\sqrt{2(kt+C)}}$

(ii) $\therefore \frac{1}{(a-c)^2}\ln|a-x|+\frac{1}{(c-a)(a-x)}-\frac{1}{(a-c)^2}\ln|c-x|=kt+C$

(b) $=a_0\left((x+1)+\frac{1}{2}(x+1)^2-\frac{1}{24}(x+1)^4-\frac{7}{480}(x+1)^5+\ldots\right)$
