Answer: (UDPS1203 - May2014 Final Solution)

Q1.

(a) Therefore, the general solution is $\mathbf{x} = c_1\begin{pmatrix} 2\cos t - \sin t \\ \cos t \end{pmatrix} + c_2\begin{pmatrix} \cos t + 2\sin t \\ \sin t \end{pmatrix}$

(b) Therefore, the final solution is $y = |x-1|\left[3\cos\left(\sqrt{3}\ln|x-1|\right) + \frac{2}{\sqrt{3}}\sin\left(\sqrt{3}\ln|x-1|\right)\right]$

(c) $x = 0$ is an irregular singular point.

$x = \pm n\pi$ are regular singular points.

Q2.

(a) $y = y_h + y_p = e^x\left(c_1\cos x + c_2\sin x\right) + \left(\frac{\cos 2x}{4}\right)e^x\cos x + \frac{1}{2}\left(\frac{\sin 2x}{2} + x\right)e^x\sin x$

(b)Hence, $= \left(1 - e^{-(t-1)}\right)u_1(t)$

(c)

(i)$\Rightarrow$ $y = \dfrac{y_0}{\left(1 - cy_0^c kt\right)^{1/c}}$

(ii) $y(t) \to \infty$ as $\Rightarrow 1 - cy_0^c kt \to 0$. That is, as $t \to \dfrac{1}{cy_0^c k}$.

Define $T = \dfrac{1}{cy_0^c k}$, then $\lim\limits_{t\to T^-} y(t) = \infty$.

Q3.

(a) $\therefore\ y = \frac{1}{x}\left(\ln|x| + 2\right)$

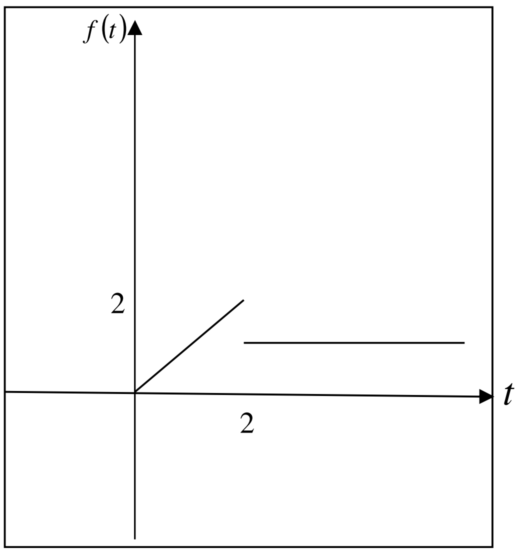
(b) $$= \begin{cases} 0, & 0 < t < 1 \\ \frac{1}{2} - e^{-(t-1)} + \frac{1}{2}e^{-2(t-1)}, & 1 < t < 2 \\ -e^{-(t-1)} + e^{-(t-2)} + \frac{1}{2}e^{-2(t-1)} - \frac{1}{2}e^{-2(t-2)}, & t > 2 \end{cases}$$

(c) Thus, $T(r) = -\frac{20}{r} + 35$.

Q4.

(a)

(i) $f(t) = t - u_2(t)(t-1) = \begin{cases} t, & 0 \le t < 2 \\ 1, & t \ge 2 \end{cases}$ (1)



(ii) $= \dfrac{1 - e^{-2s} - se^{-2s}}{s^2}$

(b) $= c_1 e^{2x} + c_2 e^{-2x} + \left(-\frac{1}{3}x - \frac{2}{9}\right)e^x - \frac{1}{8}\cos 2x$

(c) $y(t) = L^{-1}\left\{\frac{1}{s} + \frac{1}{s^2} + \frac{1}{s^3}\right\} = 1 + t + \frac{1}{2}t^2$

Q5.

(a)

(i) Thus, $y = c_1 J_5(x) + c_2 Y_5(x)$

(ii)

$$\left(\left((n+r)^2-25\right)a_n+a_{n-2}\right)x^{n+r}=0$$

$$\left((n+r)^2-25\right)a_n+a_{n-2}=0$$

$$a_n=\frac{-a_{n-2}}{(n+r)^2-25},n\ge 2$$

When $r=5$,

$$a_n=\frac{-a_{n-2}}{(n+5)^2-25},n\ge 2$$

$$a_n=\frac{-a_{n-2}}{n^2+10n},n\ge 2$$

$$a_n=\frac{-a_{n-2}}{n(n+10)},n\ge 2$$

When $r=-5$,

$$a_n=\frac{-a_{n-2}}{(n+5)^2-25},n\ge 2$$

$$a_n=\frac{-a_{n-2}}{n^2+10n},n\ge 2$$

$$a_n=\frac{-a_{n-2}}{n(n+10)},n\ge 2$$

(b) $$=c_1\begin{pmatrix}1\\2\end{pmatrix}+c_2\begin{pmatrix}1\\-1/2\end{pmatrix}e^{-5t}+\begin{pmatrix}1\\2\end{pmatrix}\ln|t|+\frac{8}{5}\begin{pmatrix}1\\2\end{pmatrix}t+\frac{4}{25}\begin{pmatrix}-2\\1\end{pmatrix}$$
