UNIVERSITI TUNKU ABDUL RAHMAN

ACADEMIC YEAR 2015/2016

SEPTEMBER EXAMINATION

UDPS1203 DIFFERENTIAL EQUATIONS

WEDNESDAY, 30 SEPTEMBER 2015 TIME : 2.00 PM – 4.00 PM (2 HOURS)

BACHELOR OF SCIENCE (HONS)
STATISTICAL COMPUTING AND OPERATIONS RESEARCH

Instructions to Candidates:

SECTION A: Answer **ALL** questions.
SECTION B: Answer **any ONE (1) out of TWO (2)** questions.

For Section B, if more than one question is answered, then only the first question (or the first question appear in the answer booklet) will be marked.

Marking
Scheme

Solution:

Q1.

(a) (i)

$$T = M + Ae^{-kt}$$

(ii)

$$70.77^\circ$$

(b)

$$y(t) = -e^{-t} + 3$$

(c)

$$\begin{cases} v_1' \cos t + v_2' \sin t = 0 & (1) \\ -v_1' \sin t + v_2' \cos t = f(t) & (2) \end{cases}$$

$(1)\sin t + (2)\cos t$

$$v_2' = f(t)\cos t,\ v_1' = -v_2' \frac{\sin t}{\cos t} = -f(t)\sin t$$

$$v_1(t) = -\int_0^t f(s)\sin s ds,\ v_2(t) = \int_0^t f(s)\cos s ds$$

$$y_p(t) = y_2(t)v_2(t) + y_1(t)v_1(t)$$

$$= \sin t \int_0^t f(s)\cos s ds - \cos t \int_0^t f(s)\sin s ds$$

$$= \int_0^t f(s)\sin t \cos s ds - \int_0^t f(s)\cos t \sin s ds$$

$$= \int_0^t f(s)(\sin t \cos s - \cos t \sin t) ds$$

$$= \int_0^t f(s)\sin(t - s) ds$$

Q2.

(a) (i)

$$y^2\frac{dy}{dx}+2y^3=x$$
$$v'=3y^2y'$$
$$\frac{1}{3}\frac{dv}{dx}+2v=x$$
$$\frac{dv}{dx}+6v=3x$$

(ii)

$$y=\sqrt[3]{\frac{x}{2}-\frac{1}{12}+ce^{-6x}}$$

(b)

$$\cos 3t+\frac{1}{3}\int_0^t \sin\left[3(t-v)g(v)\right]dv$$

(c)

$$y_p=x\cos x+x^2\sin x$$

Q3.

(a) Therefore, the general solution is $\mathbf{x}=c_1\begin{pmatrix}\cos 3t\\ \frac{1}{5}\cos 3t+\frac{3}{5}\sin 3t\end{pmatrix}+c_2\begin{pmatrix}\sin 3t\\ \frac{1}{5}\sin 3t-\frac{3}{5}\cos 3t\end{pmatrix}$

(b) Thus, $x=0$ is a regular singular point.

Thus, $x=5$ is an irregular singular point.

(c) Therefore the general solution is a power series of

$$=a_0\left(1+\frac{1}{6}x^3+\ldots\right)+a_1\left(x+\frac{1}{12}x^4+\ldots\right)+\left(\frac{3}{2}x^2+\frac{1}{2}x^3+\frac{1}{4}x^4+\frac{9}{40}x^5+\ldots\right)$$

Q4.

(a) Therefore, the general solution is

$$\therefore\ y=\left[c_1+c_2\ln|x+3|\right](x+3)^5$$

(b) (i) $\therefore\ \rho_{\min}=3$.

(ii) $\therefore\ \rho_{\min}=\sqrt{13}$.

(c) $$=c_1\begin{pmatrix}1\\2\end{pmatrix}+c_2\begin{pmatrix}1\\-\frac{1}{2}\end{pmatrix}e^{-5t}+\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix}te^{2t}-\begin{pmatrix}\frac{1}{4}\\\frac{1}{2}\end{pmatrix}e^{2t}+\begin{pmatrix}\frac{1}{25}\\\frac{9}{50}\end{pmatrix}\sin 5t+\begin{pmatrix}-\frac{1}{25}\\\frac{1}{50}\end{pmatrix}\cos 5t$$

Q5.

(a) $$2e^{-t}+te^{-t}+\left[\frac{1}{2}-e^{-3}\left(t-4e^3\right)e^{-t}+\frac{e^6-2e^3}{2}e^{-2t}\right]u(t-3)$$

(b) $$=a_0\left((x-1)+\frac{1}{2}(x-1)^2+\frac{1}{6}(x-1)^3+\frac{1}{72}(x-1)^4+\ldots\right)$$
