UNIVERSITY TUNKU ABDUL RAHMAN

FACULTY OF SCIENCE

UDPS2293 QUEUING MODELS

ASSIGNMENT

TRIMESTER JANUARY 2020

LOCATION: THE ALLEY

LECTURER: MR LOOI SING YAN

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1.0 Introduction

In Malaysia, bubble tea shops are now being popular and trendy. To attract more customers, many factors are taken into consideration. One of the main factors is the customers' queuing time as it will affect customers' overall satisfaction. The study of queue or waiting lines can be done to predict the length of the queues in order for bubble tea shops to plan efficiently and execute their business decisions appropriately.

In our assignment, we use queuing theory to study the waiting line in The Alley, a bubble tea shop in Kampar. We choose to collect data on 28th February 2020 (Friday) during 12PM to 3PM and 6PM to 9PM, which represent the non-peak hour and the peak hour respectively. Since we targeted our queuing system to be a single server model, we selected the order counter to perform our study. There is only one counter prepared by The Alley for customers to order their drinks, and the length of the queue is not limited. A customer is considered as entering the queuing system when he starts queuing. After he paid the money and left the counter, he is considered as leaving the queuing system. The counter for collecting the drinks is not taken into consideration in this assignment.

The objectives of this study is to investigate the queuing system at order counter in The Alley in term of its arrival rate, service rate, the characteristics and performance of the queuing system and the idleness of the server. Besides that, we compare the queuing system between peak hour and non-peak hour. Simulation is also done for further investigation. This paper seeks to provide some useful information for applying queuing theory in a real-case situation.

2.0 Data and Methodology

In this study, we are going to determine the characteristics and steady-state performances of the queuing system at order counter in The Alley Kampar. The Kendall Lee Notation for this queuing system is $M|M|1:GD|\infty|\infty$. This notation implies that the arrival time and service time are exponentially distributed and there is only one counter operating for queuing system. The queue discipline for this queuing system is First Come

First Serve (FCFS) basis. Moreover, the maximum allowable number of customer and the customer population size are infinite. So, in this model, the λ_{lost}=0 as there is no limit for the customer so no customer will be lost.

There are some assumptions for this study. Firstly, the arrival times and service times are random which means the occurrence of an event is not influenced by the length of the time that has elapsed since the occurrence of last event. Next, the arrivals queue is in single-waiting line and single service line. The arrivals are independent. For our study which is M/M/1 queue, arrivals have an exponential distribution with the rate of λ, where $1/\lambda$ is the mean arrival time. Service times have an exponential distribution with rate of μ, where $1/\mu$ is the mean service time.

For data collection, we have choose one day for observations which is on 28th February 2020 (Friday). It is conducted at two periods of time which is non-peak hour, 12PM to 3PM and peak hour, 6PM to 9PM. We record the time for the person enter into the queue as their arrival time. The service time will be recorded once customers start to order the drink. Service time ended once the customers leave the counter. After the observation and all the data have been collected, we calculated the inter-arrival time by calculating the differences of the arrival time of the current customer and the previous customer. Moreover, we also calculated the service time by calculating the differences of the service time of the current customer and the previous customer. In order to calculate the arrival rate (λ) and service rate (μ), we need the formula below:

$$\lambda = \frac{1}{inter - arrival\ time}$$

$$\mu = \frac{1}{service\ time}$$

*** λ and μ are calculated in minute in this study, and we have converted into hour by multiply them with 60.*

After that, we used the arrival rate (λ) and service rate (μ) to perform the simulation technique to compute steady-state measures of performances by using the Microsoft Excel. We stimulate 500 customers in order to achieve the steady-state measures. In order to perform the simulation of this research, we have to calculate the random inter-arrival time and service time by using the formula below:

$$Inter - arrival\ time = \frac{ln(1 - rand())}{-\lambda}$$

$$Service\ time = \frac{ln(1 - rand())}{-\mu}$$

*** $rand()$ is the function to generate the random number in Microsoft Excel.*

After the inter-arrival time and the service has been generated by using Microsoft Excel, we can obtain the arrival time, service time, leaving time, waiting time in this system. After that, we calculated the simulated average time spent waiting in line (W_q), average time spent waiting in system (W_s), average number of the customers in system (L_s) and average number of the customers in queue (L_q).

3.0 Result and Analysis

We have collected 21 observation during non-peak hour and 60 observation during peak hour in The Alley. *Table 9* and *Table 10* in Appendix show the collected data.

	n	λ (per hour)	μ (per hour)	ρ or $\bar{c}$	p_0
Non-Peak Hour	21	7.98	20.57	0.3880	0.6120
Peak Hour	60	21.18	28.13	0.7529	0.2471

Table 1. Measurements of performance during non-peak hour and peak hour

	Ls	Lq	Ws (min)	Wq (min)
Non-Peak Hour	0.6341	0.2460	4.7660	1.8493
Peak Hour	3.0476	2.2947	8.6349	6.5016

Table 2. Measurements of performance during non-peak hour and peak hour

From the *Table 1* above, we observe that The Alley is having more customers during the peak hour. This causes us to obtain a lower arrival rate for non-peak hour (about 8 customers per hour) compared to peak hour (about 21 customers per hour). Similarly, the service rate of non-peak hour (about 21 customers per hour) is also lower than the service rate of peak hour (about 28 customers per hour). As a result, the system is always in a stable condition since the service rate is always higher than the arrival rate. Furthermore, it is shown that the server is having a higher utilization rate during peak hour (75%) compared to non-peak hour (39%). The probability of having no customer in the shop (p_0) is much higher during non-peak hour (61%) compared to peak hour (25%).

Based on *Table 2*, we found out the average number of customers in the service system, L_s, the average number of customers waiting in line, L_q, the average time spent waiting in the system, W_s, the average time spent waiting in line, W_q, during peak hour are all higher than non-peak hour. This indicated that there are more customers in the system and queue during the peak hour and they have to spend more time in the system and queue compared to non-peak hour.

Average Service Time per Cup (min)		
Number of Cup	**Non-Peak Hour**	**Peak Hour**
1	2.3833	2.1000
2	1.8000	1.0000
3	1.3167	0.9333
4	N/A	1.1667
5	1.0833	0.2500

Table 3. Average service time per cup

According to *Table 3*, the average service time per cup for peak hour is always shorter than non-peak hour. It is probably because when there is more customers waiting in the line, they will try to take shorter time to order their drinks. Moreover, we observe that the average service time is decreasing for non-peak hour.

Simulation

For further investigation, 500 customers are being simulated with the peak hour information (λ=21.28 and μ=28.13) by using Microsoft Excel. The simulation model can be considered a long run system. It is usually closer to the actual system and will increase the accuracy. As shown in *Figure 1* and *Figure 2* in Appendix, the simulated inter-arrival time and service time are both following the exponential distribution. Thus, the simulation model is appropriate for this study.

Firstly, we would like to determine whether the two servers system is suitable for The Alley. Besides that, we would like to investigate the number of seats that should be prepared by The Alley so that most of the customers can wait on the seats. Lastly, we would like to determine the performance when The Alley is having constant service time.

A. Two Servers (M|M|2:GD|∞|∞)

$$\lambda = 21.83 \text{ customers per hour}$$

$$\mu = \begin{cases} 28.05 \text{ customers per hour}, n = 1 \\ 56.10 \text{ customers per hour}, n = 2,3,\ldots \end{cases}$$

ρ	p_0	p_1	$P(n \geq 2)$	Ls	Lq	Ws (min)	Wq (min)
0.3891	0.4398	0.3422	0.2180	0.9171	0.1389	2.5208	0.3817

Table 4. Measurements of performance of two servers system

The measurements of performance of two servers system are listed in *Table 4* above. The average number of customers in the system is lesser than 1 and the average time spent waiting for the service is only 0.38 minutes. Furthermore, customers will have 78.2% opportunity to enjoy the service directly once they enter The Alley. This is a good news for the customers.

On the other hand, to investigate whether this system is suitable for The Alley, we determine the probability of no customer in the system and the proportion of time that one server is idle. The probability of no customer in the system is 0.4398, while the proportion of time that one server is idle is 0.6109. Since the idleness for a particular server is more than 0.5, it is not recommended for The Alley to install another order counter.

B. One Server (M|M|1:GD|∞|∞)

Since the two servers system is being rejected, we recommend The Alley to remain its one server system. Same simulated data is used in this system.

λ (per hour)	μ (per hour)	ρ or $\bar{c}$	p_0	L_s	L_q	W_s (min)	W_q (min)
21.83	28.05	0.7782	0.2218	3.5096	2.7313	9.6464	7.5073

Table 5. Measurements of performance for one server system

The measurements of performance of one servers system are listed in *Table 5* above. We can observe that the utilization rate in long run is higher (77.8%) as compared with the collected data. However, all L_s, L_q, W_s, and W_q for simulated data are having a higher value compared with collected data. This indicates that longer queue and longer service time might happen in the long run.

n	Probability, pn	Cumulative, Pn
0	0.2218	0.2218
1	0.1726	0.3943
2	0.1343	0.5286
3	0.1045	0.6332
4	0.0813	0.7145
5	0.0633	0.7778
6	0.0493	0.8271
7	0.0383	0.8654
8	0.0298	0.8953
9	0.0232	0.9185

Table 6. TORA output for one server system

In order to investigate the number of seats that should be prepared by The Alley, we used TORA to find the cumulative probability for the numbers of customer in system. From *Table 6*, we observe that 83% of the customers can be seated if The Alley prepare 5 seats in the shop. If The Alley hopes 92% of its customers to be seated while waiting the service, it is recommended to prepare 8 seats in the shop.

Performance measure	Peak Hour		
	Collected data	Simulated data	Error percentage (%)
W_s	8.6349	9.6464	11.71
W_q	6.5016	7.5073	15.47

Table 7. Comparison for W_s and W_q for the collected data and simulated data

From the simulated data, we can obtained the average waiting time in system, W_s is equal to 9.6464 minutes. However, from the observation, we obtained the value of W_s is equal to 8.6349 minutes. The error percentage we obtained from W_s is equal to 11.71% which means that this stimulation is acceptable.

In addition, we also obtained the average waiting time in queue, W_q is equal to 7.5073 minutes for simulated data. However, the W_q that we have obtained from the observation is equal to 6.5016 minutes. The error percentage we obtained from W_q is equal to 15.47%, which is also acceptable.

C. Constant Service Time (M|G|1:GD|∞|∞)

In this model, we consider The Alley is having a constant service time which is 0.04 hour. Same simulation data for inter-arrival time is used in this model. Meanwhile, we calculate the average service time of the previous simulated data as the constant service time in the model.

$\boldsymbol{\lambda}$ **(per hour)**	$\boldsymbol{E(t)}$ **(hour)**	$\boldsymbol{Var(t)}$ **(hour)**	$\boldsymbol{p_0}$	$\boldsymbol{L_s}$	$\boldsymbol{L_q}$	$\boldsymbol{W_s}$ **(min)**	$\boldsymbol{W_q}$ **(min)**
21.83	0.04	0.00	0.2218	2.1439	1.3657	5.8927	3.7536

Table 8. Measurements of performance for constant service time

The measurements of performance are listed in *Table 8* above. We can observe that the idleness of the server remains the same as previous model. While the L_s, L_q, W_s, and W_q for constant service time are having a lower value compared with exponential service time. This shows that constant service time allows the customers to have a shorter queue and also shorter waiting time. However, it is difficult to achieve as the most of the service time depend on the time of customers ordering their drinks. If they are hesitated, the service time will be longer.

4.0 Conclusion and Recommendation

This study has discussed the application of single channel M|M|1 queuing model in The Alley Kampar. Applying this model we have obtained that, during the non-peak hour period, customers are expected to spend 4.8 minutes in the ordering system in The Alley Kampar and they have to wait around 1.8 minutes in the queue before entering to the system. Moreover, there are average 0.63 customer in the system and 0.25 customer queuing in the line. The system is idle for 61.20% of the time.

In contrast, during the peak hour period, the observations show that customers have to spend 8.6 minutes in the ordering system and 6.5 minutes to queue in the system on average. Furthermore, 3.05 customers are expected to be in the ordering system and 2.29 customers in the queue. The percentage of time that the system is idle is 24.71%.

For further investigation, a simulation model is developed based on the information of peak hour period. Firstly, we introduce a two servers system for The Alley. It successfully cut down the waiting time and the length of the queue. However, due to the high idleness of the servers (61.1%), we do not recommended The Alley to install a second server.

Since two servers system is not ideal, we recommend The Alley to remain its one server system. We observe that longer queue and service time might be occurred in the long run. On the other hand, the utilization rate is higher compared with the collected data. In addition, we also recommend The Alley to prepare 5 seats in the shop so that 83% of the customers can be seated while waiting for their turn to be served.

Lastly, we consider The Alley to have a constant service time of 0.04 hour. We find that the idleness of the server remains the same as previous model. While the constant service time are having a shorter queue and also shorter waiting time compared with exponential service time. However, constant service time is somehow difficult to be achieved as the most of the service time depend on the time of customers ordering their drinks.

5.0 Appendix

Figure 1. Inter-arrival time distribution for the simulated data

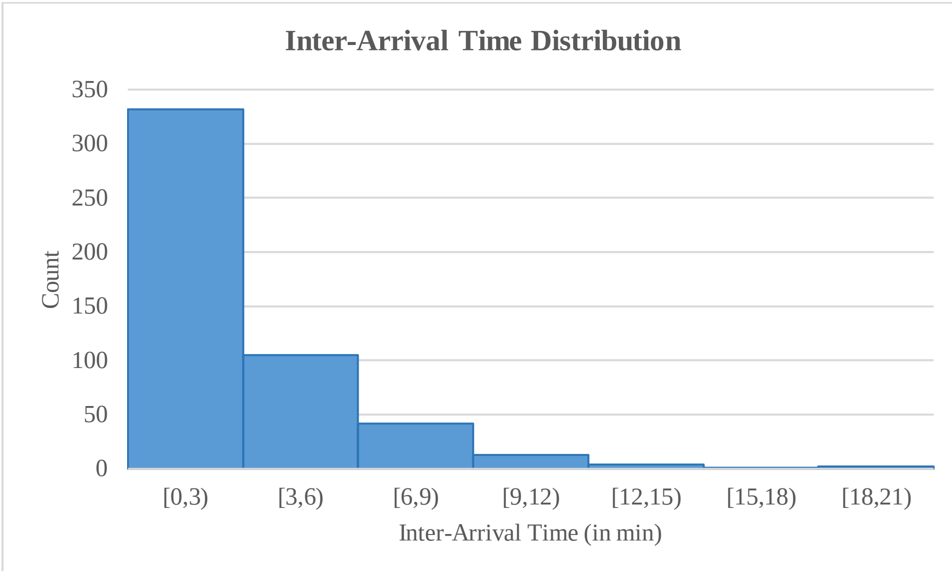

Figure 2. Service time distribution for the simulated data

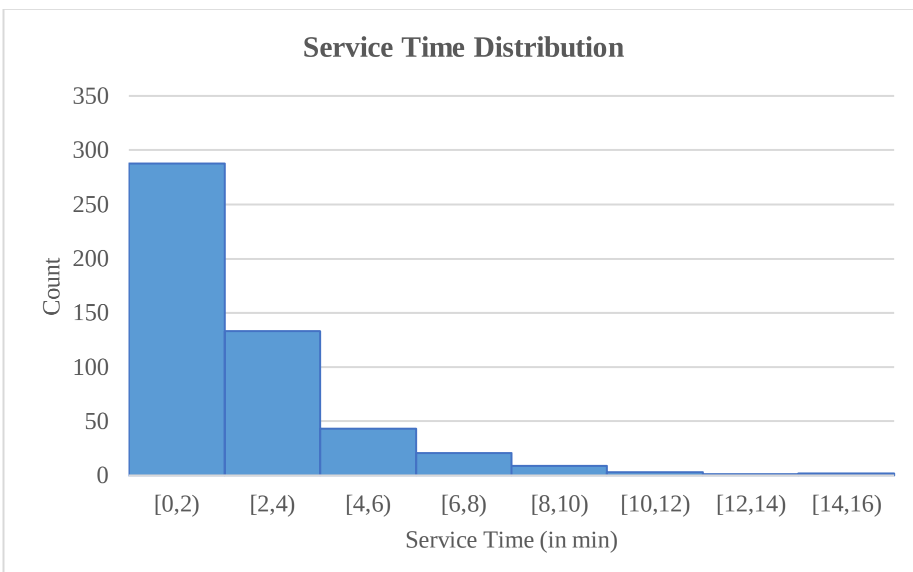

Table 9. Data collected during non-peak hour, 12.00 p.m. to 3.00 p.m.

n	Arrival Time	Service Time	Leaving Time	Cups
1	12:25:13 PM	12:25:13 PM	12:30:37 PM	5
2	12:33:27 PM	12:33:27 PM	12:35:52 PM	1

3	12:45:41 PM	12:45:41 PM	12:48:22 PM	3
4	12:52:49 PM	12:52:49 PM	12:55:01 PM	2
5	1:22:39 PM	1:22:39 PM	1:24:03 PM	1
6	1:47:15 PM	1:47:15 PM	1:50:47 PM	1
7	2:02:25 PM	2:02:25 PM	2:05:19 PM	1
8	2:04:20 PM	2:05:19 PM	2:08:04 PM	1
9	2:05:32 PM	2:08:04 PM	2:10:13 PM	1
10	2:10:12 PM	2:10:13 PM	2:15:29 PM	3
11	2:12:28 PM	2:15:29 PM	2:17:37 PM	1
12	2:14:23 PM	2:17:37 PM	2:21:44 PM	2
13	2:15:51 PM	2:21:44 PM	2:23:21 PM	1
14	2:20:22 PM	2:23:21 PM	2:25:30 PM	1
15	2:24:36 PM	2:25:30 PM	2:29:33 PM	2
16	2:35:15 PM	2:35:15 PM	2:37:26 PM	1
17	2:39:51 PM	2:39:51 PM	2:42:43 PM	1
18	2:45:55 PM	2:45:55 PM	2:49:59 PM	2
19	2:48:46 PM	2:49:59 PM	2:52:12 PM	1
20	2:50:16 PM	2:52:12 PM	2:54:59 PM	1
21	2:55:40 PM	2:55:40 PM	2:58:00 PM	1

Table 10. Data collected during peak hour, 6.00 p.m. to 9.00 p.m.

n	Arrival Time	Service Time	Leaving Time	Cups
1	6:12:32 PM	6:12:32 PM	6:14:45 PM	1
2	6:15:36 PM	6:15:36 PM	6:17:17 PM	1
3	6:22:03 PM	6:22:03 PM	6:25:04 PM	3
4	6:25:16 PM	6:25:16 PM	6:26:28 PM	1
5	6:29:15 PM	6:29:15 PM	6:31:12 PM	2
6	6:30:00 PM	6:31:12 PM	6:34:35 PM	2
7	6:33:01 PM	6:34:35 PM	6:36:14 PM	1
8	6:33:06 PM	6:36:14 PM	6:37:54 PM	1
9	6:36:11 PM	6:37:54 PM	6:38:47 PM	1
10	6:40:44 PM	6:40:44 PM	6:42:01 PM	5
11	6:44:28 PM	6:44:28 PM	6:47:50 PM	1

12	6:45:31 PM	6:47:50 PM	6:49:24 PM	3
13	6:45:47 PM	6:49:24 PM	6:51:40 PM	2
14	6:49:27 PM	6:51:40 PM	6:52:19 PM	1
15	6:50:33 PM	6:52:19 PM	6:55:20 PM	1
16	6:55:06 PM	6:55:20 PM	6:58:45 PM	1
17	6:56:49 PM	6:58:45 PM	7:00:38 PM	1
18	7:01:30 PM	7:01:30 PM	7:02:26 PM	1
19	7:05:52 PM	7:05:52 PM	7:09:40 PM	3
20	7:10:04 PM	7:10:04 PM	7:14:30 PM	1
21	7:11:38 PM	7:14:30 PM	7:15:24 PM	2
22	7:21:36 PM	7:21:36 PM	7:23:35 PM	1
23	7:25:02 PM	7:25:02 PM	7:30:39 PM	1
24	7:30:10 PM	7:30:39 PM	7:33:14 PM	2
25	7:36:47 PM	7:36:47 PM	7:40:00 PM	1
26	7:39:02 PM	7:40:00 PM	7:42:31 PM	1
27	7:45:09 PM	7:45:09 PM	7:49:02 PM	2
28	7:45:50 PM	7:49:02 PM	7:51:39 PM	1
29	7:51:19 PM	7:51:39 PM	7:53:56 PM	1
30	7:55:31 PM	7:55:31 PM	7:56:29 PM	1
31	7:56:11 PM	7:56:29 PM	7:58:52 PM	2
32	8:05:55 PM	8:05:55 PM	8:10:36 PM	4
33	8:11:40 PM	8:11:40 PM	8:16:40 PM	1
34	8:15:25 PM	8:16:40 PM	8:18:43 PM	2
35	8:15:26 PM	8:18:43 PM	8:21:00 PM	1
36	8:16:34 PM	8:21:00 PM	8:23:00 PM	1
37	8:17:58 PM	8:23:00 PM	8:24:00 PM	2
38	8:20:21 PM	8:24:00 PM	8:26:32 PM	1
39	8:21:21 PM	8:26:32 PM	8:28:14 PM	1
40	8:21:28 PM	8:28:14 PM	8:29:39 PM	2
41	8:21:32 PM	8:29:39 PM	8:30:07 PM	1
42	8:25:01 PM	8:30:07 PM	8:32:13 PM	1
43	8:25:58 PM	8:32:13 PM	8:35:08 PM	1
44	8:27:59 PM	8:35:08 PM	8:36:04 PM	1

45	8:29:19 PM	8:36:04 PM	8:38:45 PM	1
46	8:30:12 PM	8:38:45 PM	8:40:43 PM	1
47	8:30:52 PM	8:40:43 PM	8:42:57 PM	2
48	8:33:08 PM	8:42:57 PM	8:43:58 PM	1
49	8:35:48 PM	8:43:58 PM	8:45:11 PM	1
50	8:36:36 PM	8:45:11 PM	8:46:13 PM	1
51	8:37:10 PM	8:46:13 PM	8:47:51 PM	2
52	8:37:22 PM	8:47:51 PM	8:49:16 PM	1
53	8:38:47 PM	8:49:16 PM	8:50:49 PM	1
54	8:39:32 PM	8:50:49 PM	8:52:07 PM	2
55	8:42:42 PM	8:52:07 PM	8:53:47 PM	1
56	8:44:48 PM	8:53:47 PM	8:54:02 PM	1
57	8:52:14 PM	8:54:02 PM	8:55:10 PM	2
58	8:55:04 PM	8:55:10 PM	8:57:38 PM	1
59	8:58:33 PM	8:58:33 PM	8:59:58 PM	1
60	8:59:49 PM	8:59:58 PM	9:03:07 PM	1