

Chapter 3

3.1 (a) No, because $f(4)$ is negative; (b) Yes; (c) No, because $f(1) + f(2) + f(3) + f(4) = \frac{18}{19}$ is less than 1.

3.2 (a) No, because $f(1)$ is negative; (b) Yes; (c) No, because $f(0) + f(1) + f(2) + f(3) + f(4) + f(5)$ is greater than 1.

3.3 $f(x) > 0$ for each value of x and

$$\sum_{x=1}^k f(x) = \frac{2}{k(k+1)}(1+2+\dots+k) = \frac{2}{k(k+1)} \cdot \frac{k(k+1)}{2} = 1$$

3.4 (a) $c(1+2+3+\dots+5) = 1$; thus $C = \frac{1}{15}$

(b) $c\left(5 + \frac{5}{2} + \frac{5}{3} + \frac{5}{4} + 1\right) = 1$; thus, $c = \frac{12}{137}$

(c) $\sum_{x=1}^k f(x) = c \sum_{x=1}^k x^2 = cS(k, 2)$

From Theorem A.1 we obtain $S(k, 2) = \frac{1}{6}k(k+1)(2k+1)$

Thus, for $f(x)$ to be a distribution function, $c = \frac{6}{k(k+1)(2k+1)}$, $k \neq 0$.

(d) $\sum_{x=1}^{\infty} f(x) = c \sum_{x=1}^{\infty} \left(\frac{1}{4}\right)^x$

The right-hand sum is a geometric progression with $a = 1$ and $r = 1/4$.

For $x = 1$ to n , this sum equals

$$S_n = \frac{1 - \left(\frac{1}{4}\right)^n}{1 - \frac{1}{4}} \rightarrow \frac{1/4}{3/4} = \frac{1}{3} \text{ as } n \rightarrow \infty. \text{ Therefore, } c = 3.$$

3.5 For $f(x) = (1-k)k^x$ to converge to 1, $0 < k < 1$.

3.6 For $c > 0$, $f(x)$ diverges. For $c = 0$, $f(x) = 0$ for all x , and it cannot be a density function

3.9 (a) No, because $F(4) > 1$; (b) No, because $F(2) < F(1)$; (c) Yes.

$$3.10 \quad f(0) = \frac{4}{20} = \frac{1}{5}; \quad f(1) = \frac{2 \cdot 6}{20} = \frac{12}{20} = \frac{3}{5}, \quad F(2) = \frac{4}{20} = \frac{1}{5}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ 1/5 & 0 \leq x < 1 \\ 4/5 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

$$3.11 \quad (\mathbf{a}) \quad \frac{5}{6} - \frac{1}{3} = \frac{1}{2}; \quad (\mathbf{b}) \quad \frac{1}{2} - \frac{1}{3} = \frac{1}{6}; \quad (\mathbf{c}) \quad f(1) = \frac{1}{3}, \quad f(4) = \frac{1}{6}, \quad f(6) = \frac{1}{3} \text{ and } f(10) = \frac{1}{6},$$

0 elsewhere.

$$3.12 \quad F(x) = \begin{cases} 0 & x < 1 \\ 1/15 & 1 \leq x < 2 \\ 3/15 & 2 \leq x < 3 \\ 6/15 & 3 \leq x < 4 \\ 10/15 & 4 \leq x < 5 \\ 1 & 5 \leq x \end{cases}$$

$$3.13 \quad (\mathbf{a}) \quad \frac{3}{4} \quad (\mathbf{b}) \quad \frac{3}{4} - \frac{1}{2} = \frac{1}{4} \quad (\mathbf{c}) \quad \frac{1}{2} \quad (\mathbf{d}) \quad 1 - \frac{1}{4} = \frac{3}{4}$$

$$(\mathbf{e}) \quad \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \quad (\mathbf{f}) \quad 1 - \frac{3}{4} = \frac{1}{4}$$

$$3.14 \quad f(1) = \frac{3}{25}, \quad f(2) = \frac{4}{25}, \quad f(3) = \frac{5}{25}, \quad f(4) = \frac{6}{25}, \quad f(5) = \frac{7}{25}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ 3/25 & 1 \leq x < 2 \\ 7/25 & 2 \leq x < 3 \\ 12/25 & 3 \leq x < 4 \\ 18/25 & 4 \leq x < 5 \\ 1 & 5 \leq x \end{cases}$$

$$F(1) = \frac{6}{50} = \frac{3}{25}, \quad F(2) = \frac{14}{50} = \frac{7}{25}, \quad F(3) = \frac{24}{50} = \frac{12}{25}, \quad F(4) = \frac{36}{50} = \frac{18}{25}, \quad F(5) = \frac{50}{50} = 1, \text{ checks}$$

$$3.15 \quad (\mathbf{a}) \quad P(x > x_1) = 1 - P(x \leq x_1) = 1 - F(x_1) \text{ for } i = 1, 2, \dots, n$$

$$(\mathbf{b}) \quad P(x > x_i) = 1 - P(x < x_i) = 1 - F(x_{i-1}) \text{ for } i = 2, \dots, n \text{ and}$$

$$P(x \geq x_1) = 1$$

$$3.16 \quad F(x) = \begin{cases} 0 & x \leq 2 \\ \frac{1}{5}(x-2) & 2 < x < 7 \\ 1 & 7 \leq x \end{cases}$$

$$3.17 \quad (a) \quad \int_{-\infty}^{\infty} f(x) dx = \int_2^7 \frac{1}{5} dx \cdot \frac{1}{5} \cdot x \Big|_2^7 = \frac{1}{5}(7-2) = 1$$

$$(b) \quad \int_3^5 \frac{1}{5} dx = \frac{1}{5}(5-3) = \frac{2}{5}$$

$$3.18 \quad (a) \quad f(x) \geq 0, \quad 0 < x < \infty, \quad \text{and} \quad \int_0^{\infty} f(x) dx = \int_0^{\infty} e^{-x} dx = e^0 = 1$$

$$(c) \quad P(x > 1) = \int_1^{\infty} e^{-x} dx = e^{-1}$$

$$3.19 \quad (a) \quad f(x) \geq 0, \quad 0 < x < 1 \quad \text{and} \quad \int_0^1 f(x) dx = 1$$

$$(c) \quad P(0.1 < x < 0.5) = \int_{0.1}^{0.5} 3x^2 dx = 0.124$$

$$3.20 \quad (a) \quad \int_2^{3.2} \frac{1}{8}(y+1) dy = \frac{1}{8} \left(\frac{y^2}{2} + y \right) \Big|_2^{3.2} = \frac{1}{8}(8.32 - 4) = 0.54$$

$$(b) \quad \int_{2.9}^{3.2} \frac{1}{8}(y+1) dy = \frac{1}{8} \left(\frac{y^2}{2} + y \right) \Big|_{2.9}^{3.2} = \frac{1}{8}(8.32 - 7.105) = 0.1519$$

$$3.21 \quad \int_2^y \frac{1}{8}(t+1) dt = \frac{1}{8} \left(\frac{t^2}{2} + t \right) \Big|_2^y = \frac{1}{8} \left(\frac{y^2}{2} + y \right) - \frac{1}{8} \cdot 4 = \frac{1}{8} \left(\frac{y^2}{2} + y - 4 \right)$$

$$F(y) = \begin{cases} 0 & y \leq 2 \\ \frac{1}{8} \left(\frac{y^2}{2} + y - 4 \right) & 2 < y < 4 \\ 1 & 4 \leq y \end{cases}$$

$$(a) \quad F(3.2) = \frac{1}{8} \left(\frac{3.2^2}{2} + 3.2 - 4 \right) = 0.54$$

$$(b) \quad F(3.2) = F(2.9) = 0.54 - \frac{1}{8} \left(\frac{2.9^2}{2} + 2.9 - 4 \right) = 0.54 - 0.3881 = 0.1519$$

$$3.22 \quad (a) \quad 1 = \int_0^4 \frac{c}{\sqrt{x}} dx = c \int_0^4 x^{-1/2} dx = c \frac{x^{1/2}}{1/2} \Big|_0^4 = 4c \quad c = \frac{1}{4}$$

$$(b) \quad P\left(x < \frac{1}{4}\right) = \int_0^{1/4} \frac{1}{4\sqrt{x}} dx = \frac{1}{4} \int_0^{1/4} x^{-1/2} dx = \frac{1}{4} \frac{\sqrt{x}}{1/2} \Big|_0^{1/4} = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(x > 1) = 1 - \int_0^1 \frac{1}{4\sqrt{x}} dx = 1 - \frac{1}{2} \sqrt{x} \Big|_0^1 = \frac{1}{2}$$

$$3.23 \quad F(x) = \frac{1}{2} \sqrt{x}$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{2} \sqrt{x} & 0 < x < 4 \\ 1 & 4 \leq x \end{cases}$$

$$F\left(\frac{1}{4}\right) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \quad \text{and} \quad 1 = F(1) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$3.24 \quad F(z) = k \int_0^z z e^{-z^z} dz = k \int_0^z \frac{1}{2} e^{-u} du = \frac{k}{2} [1 - e^{-u}] = \frac{k}{2} (1 - e^{-z^z}) \quad k = 2$$

$$3.25 \quad F(z) = \begin{cases} 0 & z \leq 0 \\ 1 - e^{-z^z} & z > 0 \end{cases}$$

$$3.26 \quad P\left(x < \frac{1}{4}\right) = (3x^2 - 2x^3) \Big|_0^{1/4} = \frac{3}{16} - \frac{1}{32} = \frac{5}{32}$$

$$P\left(x > \frac{1}{2}\right) = \int_{1/2}^1 6x(1-x) dx = (3x^2 - 2x^3) \Big|_{1/2}^1 = 1 - \left(\frac{3}{4} - \frac{1}{4}\right) = \frac{1}{2}$$

$$3.27 \quad F(x) = \int_0^x 6x(1-x) dx = 3x^2 - 2x^3 \quad F(x) = \begin{cases} 0 & x \leq 0 \\ 3x^2 - 2x^3 & 0 < x < 1 \\ 1 & 1 \leq x \end{cases}$$

$$P\left(x < \frac{1}{4}\right) = \frac{3}{16} - \frac{2}{64} = \frac{5}{32} \quad \text{and} \quad P\left(x > \frac{1}{2}\right) = 1 - \left(\frac{3}{4} - \frac{2}{8}\right) = \frac{1}{2}$$

$$3.28 \quad F(x) = \int_0^x x \, dx = \frac{x^2}{2} \quad 0 \text{ to } 1$$

$$F(x) = \frac{1}{2} + \int_1^x (2-x) \, dx = \frac{1}{2} + \left(2x - \frac{x^2}{2} \right) \Big|_1^x = \frac{1}{2} + 2x - \frac{x^2}{2} - \frac{3}{2}$$

$$= 2x - \frac{x^2}{2} - 1$$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{x^2}{2} & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

$$3.29 \quad F(x) = \int_0^x \frac{1}{3} \, dx = \frac{1}{3}x \quad 0 \text{ to } 1 \quad F(x) = \frac{1}{3} \quad 1 \text{ to } 2$$

$$F(x) = \frac{1}{3}(x-2) \quad 2 \text{ to } 4 \quad F(x) = \begin{cases} 0 & x \leq 0 \\ \frac{1}{3}x & 0 < x < 1 \\ \frac{1}{3} & 1 \leq x \leq 2 \\ \frac{1}{3}(x-1) & 2 < x < 4 \\ 1 & 4 \leq x \end{cases}$$

$$= \frac{1}{3}(x-1)$$

$$3.30 \quad (a) \quad \int_{0.8}^1 x \, dx + \int_1^{1.2} (2-x) \, dx = \frac{x^2}{2} \Big|_{0.8}^1 + \left(2x - \frac{x^2}{2} \right) \Big|_1^{1.2} = \left(\frac{1}{2} - 0.32 \right) + \left(2.4 - 0.72 - 2 + \frac{1}{2} \right) = 0.36$$

$$(b) \quad F(1.2) - F(0.8) = 2(1.2) - \frac{(1.2)^2}{2} - 1 - \left(\frac{(0.8)^2}{2} \right)$$

$$= 2.4 - 0.72 - 1 - 0.32 = 0.36$$

$$3.31 \quad x \leq 0$$

$$F(x) = 0$$

$$0 < x \leq 1$$

$$F(x) = \frac{x^2}{4}$$

$$F(1) = \frac{1}{4}$$

$$1 < x \leq 2$$

$$F(x) = \frac{1}{2}x - \frac{1}{4}$$

$$F(2) = \frac{3}{4}$$

$$2 < x < 3$$

$$F(x) = \frac{3}{2}x - \frac{x^2}{4} - \frac{5}{4}$$

$$F(3) = 1$$

$$3 \leq x$$

$$F(x) = 1$$

$$3.32 \quad (a) \quad F\left(\frac{1}{2}\right) - F\left(-\frac{1}{2}\right) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}; \quad F(3) - F(2) = 1 - 1 = 0$$

$$3.33 \quad \frac{dF}{dx} = \frac{1}{2}, \quad f(x) = \frac{1}{2} \text{ for } -1 < x < 1; \quad 0 \text{ elsewhere}$$

$$P\left(-\frac{1}{2} < x < \frac{1}{2}\right) = \frac{1}{2} \cdot 1 = \frac{1}{2}; \quad P(2 < x < 3) = 0$$

$$3.34 \quad (a) \quad F(5) = 1 - \frac{9}{25} = \frac{16}{25}$$

$$(b) \quad 1 - F(8) = 1 - 1 + \frac{9}{64} = \frac{9}{64}$$

$$3.35 \quad \frac{dF}{dy} = \frac{18}{y^2} \text{ for } y > 0; \text{ elsewhere}$$

$$(a) \quad \int_3^5 \frac{18}{y^2} dy = -\frac{9}{y^2} \Big|_3^5 = -\frac{9}{25} + 1 = \frac{16}{25}; \quad (b) \quad \int_8^\infty \frac{18}{y^2} dy = -\frac{9}{y^2} \Big|_8^\infty = 0 + \frac{9}{64} = \frac{9}{64}$$

$$3.37 \quad P(x \leq 2) = F(2) = 1 - 3e^{-2} = 1 - 3(0.1353) = 1 - 0.4074 = 0.5926$$

$$P(1 < x < 3) = F(3) - F(1) = 1 - 4e^{-2} - 1 + 2e^{-1} - 4e^{-2}$$

$$= 2(0.3679) - 4(0.0498) = 0.7358 - 0.1992 = 0.5366$$

$$P(x > 4) = 1 - F(4) = 5e^{-4} = 5(0.0183) = 0.0915$$

$$3.38 \quad \frac{dF}{dx} = xe^{-x} \text{ for } x > 0; \quad 0 \text{ elsewhere}$$

$$3.39 \quad (a) \quad \text{for } x \leq 0 \quad F(x) = 0$$

$$(b) \quad \text{for } 0 < x < 0.5 \quad F(x) = \frac{1}{2}x$$

$$(c) \quad \text{for } 0.5 \leq x < 1 \quad F(x) = \frac{1}{2}\left(x - \frac{1}{2}\right) + \frac{3}{4} = \frac{1}{2}(x+1)$$

$$(d) \quad \text{for } x \geq 1 \quad f(x) = 0$$

$$3.40 \quad (a) \quad f(x) = 0; \quad (b) \quad f(x) = \frac{1}{2}; \quad (c) \quad f(x) = \frac{1}{2}; \quad (d) \quad f(x) = 0$$

$$3.41 \quad P(Z = -2) = \frac{-2+4}{8} = \frac{1}{4}, \quad P(Z = 2) = \frac{1}{4}, \quad P(-2 < Z < 1) = \frac{5}{8} - \frac{1}{4} = \frac{3}{8}$$

$$\text{and } P(0 \leq z \leq 2) = 1 - \frac{1}{2} = \frac{1}{2}$$

3.42 (a) $\frac{1}{20}$; (b) $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$; (c) $\frac{1}{6} + \frac{1}{4} + \frac{1}{12} = \frac{1}{2}$; (d) $\frac{1}{6} + \frac{1}{24} + \frac{1}{40} = \frac{28}{120} = \frac{7}{30}$

3.43 (a) $\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$; (b) 0; (c) $\frac{1}{12} + \frac{1}{6} + \frac{1}{24} = \frac{7}{24}$; (d) $1 - \frac{1}{120} = \frac{119}{120}$

3.44 $c(2+5+10+1+4+9+2+5+10+10+13+18) = 1$
 $c = \frac{1}{89}$

3.45 (a) $\frac{1}{89}(10+9+10) = \frac{29}{89}$; (b) $\frac{1}{89}(1+4) = \frac{5}{89}$

(c) $\frac{1}{89}(9+5+10+13+18) = \frac{55}{89}$

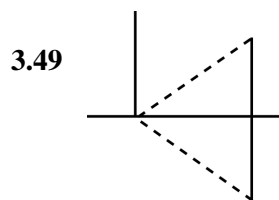
3.46 (a) $k(0+2+8+0-1+2) = 1$
 $f(3, 1)$ differs in sign from all other terms

3.47

		x			
		0	1	2	3
y	0	0	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$
	1	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{2}{15}$
	2	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{2}{15}$	$\frac{1}{6}$
		density			

		y			
		0	1	2	3
0		0	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{5}$
1		$\frac{1}{30}$	$\frac{2}{15}$	$\frac{3}{10}$	$\frac{8}{15}$
2		$\frac{1}{10}$	$\frac{3}{10}$	$\frac{3}{5}$	1
		joint distribution function			

3.48 (a) $P(x \leq -\infty, y \leq -\infty) = 0$
 (b) $P(x \leq \infty, y \leq \infty) = 1$
 (c) $F(b, c) = F(a, c) +$ three probabilities
 $F(b, c) \geq F(a, c)$

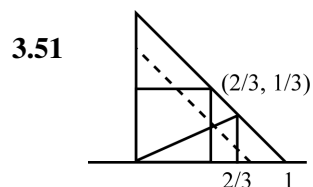


$$k \int_0^1 \int_{-x}^x x(x-y) dy dx = k \int_0^1 \left(x^2 y - \frac{xy^2}{2} \right) \Big|_{-x}^x dx$$

$$k \int_0^1 \left(x^3 - \frac{x^3}{2} + x^3 + \frac{x^3}{2} \right) dx = k \int_0^1 2x^3 dx = \frac{k}{2} = 1$$

$$k = 2$$

$$\begin{aligned}
 3.50 \quad & 24 \int_0^{1/2} \int_0^{1/2-x} xy \, dy \, dx = 24 \int_0^{1/2} \frac{xy^2}{2} \Big|_0^{1/2-x} dx = 12 \int_0^{1/2} x \left(\frac{1}{2} - x \right)^2 dx \\
 & = 12 \int_0^{1/2} \left(\frac{x}{4} - x^2 + x^3 \right) dx = 12 \left[\frac{x^2}{8} - \frac{x^3}{3} + \frac{x^4}{4} \right] \Big|_0^{1/2} = 12 \left[\frac{1}{32} - \frac{1}{24} + \frac{1}{64} \right] \\
 & = \frac{12}{64 \cdot 3} (6 - 8 + 3) = \frac{12}{3 \cdot 64} = \frac{1}{16}
 \end{aligned}$$

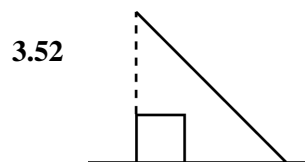


(a) $\frac{1}{2}$

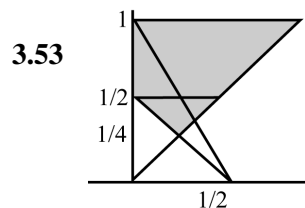
(b) $1 - 2 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} = 1 - \frac{4}{9} = \frac{5}{9}$

(c) $2 \left(\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} \right) = \frac{3}{9} = \frac{1}{3}$

$$F(x, y) = 2xy \text{ for } x > 0, y > 0, x + y < 1$$



(a) $2 \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{2}$



$$\begin{aligned}
 & \int_{1/4}^{1/2} \frac{1}{y} \int_{1/2-y}^y dx \, dy + \int_{1/2}^1 \frac{1}{y} \int_0^y dx \, dy \\
 & = 1 - \frac{1}{2} \ln 2 = 1 - 0.3466 = 0.6534
 \end{aligned}$$

3.54 $\frac{\partial F}{\partial y} \frac{\partial F}{\partial x} = 2xe^{x^2} \cdot 2ye^{-y^2} = 4xy^{-x^2} e^{-y^2} = 4xye^{-(x^2+y^2)} \quad x > 0, y > 0$
and $f(x, y) = 0$ elsewhere

3.55 $\int_1^2 2xe^{-x^2} dx \int_1^2 2ye^{-y^2} dy = \left[\int_1^4 e^{-u} du \right]^2 = \left(-e^{-u} \Big|_1^4 \right)^2 = (e^{-1} - e^{-4})^2$

3.56 $\frac{\partial F}{\partial x} = e^{-x} - e^{-x-y} \frac{\partial^2 F}{\partial x \partial y} = e^{-x-y} \quad x > 0, y > 0$
 $= 0$ elsewhere

3.57 $\int_2^3 e^{-x} dx \int_2^3 e^{-y} dy = \left[-e^{-x} \Big|_2^3 \right]^2 = (e^{-2} - e^{-3})^2$

3.58 $F(b, d) - F(a, d) - F(b, c) + F(a, c)$

3.59 $a = 1, b = 3, c = 1, d = 2$

$$\begin{aligned} & F(3,2) - F(1,2) - F(3,1) + F(1,1) \\ &= (1 - e^{-3})(1 - e^{-2}) - (1 - e^{-1})(1 - e^{-2}) - (1 - e^{-3})(1 - e^{-1}) + (1 - e^{-1})(1 - e^{-1}) \\ &= (1 - e^{-2})[(1 - e^{-3}) - (1 - e^{-1})] - (1 - e^{-1})[(1 - e^{-2}) - (1 - e^{-1})] \\ &= [(1 - e^{-2})(1 - e^{-1})][(1 - e^{-3}) - (1 - e^{-1})] \\ &= (e^{-1} - e^{-2})(e^{-1} - e^{-3}) = 0.074 \end{aligned}$$

3.60 $F(2,2) - F(1,2) - F(2,1) + F(1,1)$

$$\begin{aligned} &= (1 - e^{-4})(1 - e^{-4}) - (1 - e^{-1})(1 - e^{-4}) - (1 - e^{-1})(1 - e^{-4}) + (1 - e^{-1})(1 - e^{-1}) \\ &= (1 - e^{-4})[(1 - e^{-4}) - (1 - e^{-1})] - (1 - e^{-1})[(1 - e^{-4}) - (1 - e^{-1})] \\ &= (1 - e^{-4})(e^{-1} - e^{-4}) - (1 - e^{-1})(e^{-1} - e^{-4}) \\ &= (e^{-1} - e^{-4})(e^{-1} - e^{-4}) = (e^{-1} - e^{-4})^2 \end{aligned}$$

3.61 $F(3,3) - F(2,3) - F(3,2) + F(2,2)$

$$\begin{aligned} &= (1 - e^{-3} - e^{-3} + e^{-6}) \\ &\quad - (1 - e^{-2} - e^{-3} + e^{-5}) - (1 - e^{-2} - e^{-2} + e^{-5}) + (1 - e^{-2} - e^{-2} + e^{-4}) \\ &= e^{-4} - 2e^{-5} + e^{-6} = (e^{-2} - e^{-3})^2 \quad \text{QED} \end{aligned}$$

3.62 $x = 1, 2$

$y = 1, 2, 3$

$z = 1, 2$

$(1 + 2 + 2 + 4 + 3 + 6 + 2 + 4 + 4 + 8 + 6 + 12)k = 1$

$$k = \frac{1}{54}$$

3.63 (a) $\frac{1}{54}(1 + 2) = \frac{1}{18}$

(b) $\frac{1}{54}(8 + 6) = \frac{14}{54} = \frac{7}{27}$

3.64 (a) $\frac{1}{54}(1 + 2 + 2 + 4) = \frac{9}{54} = \frac{1}{6}$; (b) 0; (c) 1

3.65 $\int_0^1 \int_0^{1-z} \int_0^{1-y-z} xy(1-z) dx dy dz$

$$\int_0^1 \int_0^{1-z} \frac{1}{2}(1-y-z)^2 y(1-z) dy dz$$

$$k \int_0^1 \int_0^{1-z} \int_0^{1-y-z} xy(1-z) dx dy dz = 1 \quad k = 144$$

$$3.66 \quad \int_0^{1/2} \int_0^{1/2-x} \int_0^{1-x-y} 144 \, xy(1-z) \, dz \, dy \, dx = 0.15625$$

$$\begin{aligned}
 3.68 \quad (a) \quad & \frac{1}{3} \int_0^{1/2} \int_0^{1/2} \int_0^{1/2} (2x+3y+z) \, dz \, dy \, dx \\
 &= \frac{1}{3} \int_0^{1/2} \int_0^{1/2} \left[(2x+3y)z + \frac{z^2}{2} \right]_0^{1/2} dy \, dx \\
 &= \frac{1}{3} \int_0^{1/2} \int_0^{1/2} \left(x + \frac{3}{2}y + \frac{1}{8} \right) dy \, dx \\
 &= \frac{1}{3} \int_0^{1/2} \left(xy + \frac{3}{4}y^2 + \frac{1}{8}y \right) \Big|_0^{1/2} dx = \frac{1}{3} \int_0^{1/2} \left(\frac{1}{2}x + \frac{3}{16} + \frac{1}{16} \right) dx \\
 &= \frac{1}{3} \left(\frac{1}{16} + \frac{3}{32} + \frac{1}{32} \right) = \frac{1}{3} \cdot \frac{6}{32} = \frac{1}{16}
 \end{aligned}$$

$$3.69 \quad (a) \quad g(-1) = \frac{1}{4}, \quad g(1) = \frac{3}{4}$$

$$(b) \quad h(-1) = \frac{5}{8}, \quad h(0) = \frac{1}{4}, \quad h(1) = \frac{1}{8}$$

$$(c) \quad f(-1|-1) = \frac{1/8}{1/8+1/2} = \frac{1}{5}; \quad f(1|-1) = \frac{1/2}{1/8+1/2} = \frac{4}{5}$$

$$3.70 \quad (a) \quad g(0) = \frac{1}{12} + \frac{1}{4} + \frac{1}{8} = \frac{1}{120} = \frac{7}{15}; \quad g(1) = \frac{1}{6} + \frac{1}{4} + \frac{1}{20} = \frac{7}{15}$$

$$g(2) = \frac{1}{24} + \frac{1}{40} = \frac{1}{15}$$

$$(b) \quad h(0) = \frac{1}{12} + \frac{1}{6} + \frac{1}{24} = \frac{7}{24}; \quad h(1) = \frac{1}{4} + \frac{1}{4} + \frac{1}{40} = \frac{21}{40}$$

$$h(2) = \frac{1}{8} + \frac{1}{20} = \frac{7}{40}; \quad h(3) = \frac{1}{120}$$

$$(c) \quad f(0|1) = \frac{1/4}{21/40} = \frac{10}{21}; \quad f(1|1) = \frac{10}{21}; \quad f(2|1) = \frac{1/40}{21/20} = \frac{1}{21}$$

$$\begin{aligned}
 (d) \quad & w(0|0) = \frac{1/12}{56/120} = \frac{5}{28}; \quad w(1|0) = \frac{1/4}{56/120} = \frac{15}{28}; \quad w(2|0) = \frac{1/8}{56/120} = \frac{15}{56} \\
 & w(3|0) = \frac{1/120}{56/120} = \frac{1}{56}
 \end{aligned}$$

$$3.71 \quad (a) \quad m(x, y) = \frac{xy}{108}(1+2) = \frac{xy}{36} \text{ for } x = 1, 2, 3; \quad y = 1, 2, 3$$

$$(b) \quad n(x, z) = \frac{xz}{108}(1+2+3) = \frac{xz}{18} \text{ for } x = 1, 2, 3; \quad z = 1, 2$$

$$(c) \quad g(x) = \frac{x}{36}(1+2+3) = \frac{x}{6} \text{ for } x = 1, 2, 3$$

$$(d) \quad \phi(z|1, 2) = \frac{z/64}{2/36} = \frac{z}{3} \text{ for } z = 1, 2$$

$$(e) \quad \psi(y, z|3) = \frac{yz/36}{1/2} = \frac{yz}{18} \text{ for } y = 1, 2, 3; \quad z = 1, 2$$

$$3.72 \quad (a) \quad g(0) = \frac{5}{12}, \quad g(1) = \frac{1}{2}; \quad g(2) = \frac{1}{12} \quad G(x) = \begin{cases} 0 & x < 0 \\ 5/12 & 0 \leq x < 1 \\ 11/12 & 1 \leq x < 2 \\ 1 & 2 \leq x \end{cases}$$

$$(b) \quad \begin{aligned} f(0|1) &= \frac{2/9}{7/18} = \frac{4}{7} \\ f(1|1) &= \frac{1/6}{7/18} = \frac{3}{7} \end{aligned} \quad F(x|1) = \begin{cases} 0 & x < 0 \\ 4/7 & 0 \leq x < 1 \\ 1 & 1 \leq x \end{cases}$$

$$3.73 \quad (a) \quad f(x) = \frac{1}{2} \text{ for } x = -1, 1; \quad g(y) = \frac{1}{2} \text{ for } y = -1, 1; \quad \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}, \text{ independent}$$

$$(b) \quad f(0) = \frac{2}{3}, \quad f(1) = \frac{1}{3}, \quad g(0) = \frac{1}{3}, \quad g(1) = \frac{2}{3}$$

$$f(0, 0) = \frac{1}{3} \neq \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9} \quad \text{not independent}$$

$$3.74 \quad (a) \quad \frac{1}{4} \int_0^2 (2x + y) dy = \frac{1}{4} \left[2xy + \frac{y^2}{2} \right]_0^2 = \frac{1}{4} (4x + 2) = \frac{1}{2} (2x + 1) \text{ for } 0 < x < 1$$

$$= 0 \text{ elsewhere}$$

$$(b) \quad f\left(y \middle| \frac{1}{4}\right) = \frac{\frac{1}{4} \left(\frac{1}{2} + y \right)}{\frac{1}{2} \cdot \frac{3}{2}} = \frac{1}{6} (2y + 1) \text{ for } 0 < y < 2$$

$$= 0 \text{ elsewhere}$$

$$3.75 \quad (a) \quad \frac{1}{4} \int_0^1 (2x + y) dx = \frac{1}{4} (x^2 + xy) \Big|_0^1 = \frac{1}{4} (1 + y) \quad \text{for } 0 < y < 2$$

$$= 0 \text{ elsewhere}$$

$$(b) \quad f(x|1) = \frac{\frac{1}{4}(2x+1)}{\frac{1}{4}(2)} = \frac{1}{2}(2x+1) \quad \text{for } 0 < x < 1$$

$$= 0 \text{ elsewhere}$$

$$3.76 \quad (a) \quad f(x) = 24 \int_0^{1-x} (y - xy - y^2) dy = 24 \left[\frac{y}{2} - \frac{xy^2}{2} - \frac{y^3}{3} \right] \Big|_0^{1-x}$$

$$= 12(1-x)^2 - 12x(1-x)^2 - 8(1-x)^3$$

$$= 12(1-x)^3 - 8(1-x)^3 = 4(1-x)^3$$

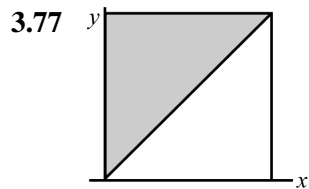
$$f(x) = \begin{cases} 4(1-x)^3 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad g(y) = 24 \int_0^{1-y} (y - xy - y^2) dy = 24 \left[y(1-y) - \frac{1}{2} y(1-y)^2 - y^2(1-y) \right]$$

$$= 24(1-y) \left[1 - \frac{1}{2}(1-y) - y \right] = 24y \left(\frac{1}{2} - \frac{y}{2} \right) (1-y)$$

$$= \begin{cases} 12y(1-y)^2 & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$f(x, y) \neq f(x) \cdot g(y)$ not independent



$$(a) \quad g(x) = \int_x^1 \frac{1}{y} dy = \ln y \Big|_x^1 = \ln 1 - \ln x = \begin{cases} -\ln x & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad h(y) = \int_0^y \frac{1}{y} dx = \frac{1}{y} (y - 0) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\frac{1}{y} \neq 1 \cdot (-\ln x) \text{ not independent}$$

$$3.78 \quad (a) \quad f(x_2|x_1, x_3) = \frac{(x_1 + x_2)e^{-x_3}}{\left(x_2 + \frac{1}{2}\right)e^{x_2}} = \frac{x_1 + x_2}{x_1 + \frac{1}{2}}$$

$$f\left(x_2\left|\frac{1}{3}, 2\right.\right) = \frac{\frac{1}{3} + x_2}{\frac{1}{3} + \frac{1}{2}} = \begin{cases} \frac{2+6x^2}{5} & 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad g(x_2, x_3|x_1) = \frac{(x_1 + x_2)e^{-x_3}}{x_2 + \frac{1}{2}} = \begin{cases} \left(\frac{1}{2} + x_2\right)e^{-x_3} & 0 < x_2 < 1, x_3 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$3.79 \quad g(x) = \int_{-\infty}^{\infty} f(x, y) dy \quad G(x) = \int_0^x \int_{-\infty}^{\infty} f(x, y) dy = F(x, \infty)$$

$$G(x) = F(x, \infty) = \begin{cases} 1 - e^{-x^2} & x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$3.80 \quad M(x_1, x_3) = \int_{-\infty}^{\infty} f(x_1, x_2, x_3) dx_2 = F(x_1, \infty, x_3)$$

$$G(x_1) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x_1, x_2, x_3) dx_2 dx_3 = F(x_1, \infty, \infty)$$

$$(a) \quad M(x_2, x_3) = \begin{cases} 0 & x_1 \leq 0 \text{ or } x_3 \leq 0 \\ \frac{1}{2}x_1(x_1 + 1)(-1 - e^{-x_3}) & 0 < x_1 < 1, x_3 > 0 \\ 1 - e^{-x_3} & x_1 \geq 1, x_3 > 0 \end{cases}$$

$$(b) \quad G(x_1) = \begin{cases} 0 & x_1 \leq 0 \\ \frac{1}{2}x_1(x_1 + 1) & 0 < x_1 < 1 \\ 1 & 1 \leq x_1 \end{cases}$$

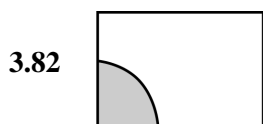
$$3.81 \quad g(x_1) = \begin{cases} x_1 + \frac{1}{2} & 0 < x_1 < 1 \\ 0 & \text{elsewhere} \end{cases} \quad h(x_2) = \begin{cases} x_2 + \frac{1}{2} & 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$\phi(x_3) = \begin{cases} e^{-x_3} & x_3 > 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$f(x_1, x_2, x_3) \neq g(x_1) \cdot h(x_2) \cdot \phi(x_3) \quad \text{not independent}$$

$$m(x_1, x_3) = g(x_1)\phi(x_3) \quad \text{independent}$$

$$n(x_2, x_3) = h(x_2)\phi(x_3) \quad \text{independent}$$



$$(a) \quad g(x, y) = \begin{cases} \frac{1}{6} & 0 < x < 2, 0 < y < 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad 1 - \frac{\pi/4}{6} = 1 - \frac{\pi}{24}$$

$$(a) \quad g(0) = \frac{5}{14}, \quad g(1) = \frac{5}{28}, \quad g(2) = \frac{3}{28}$$

$$(b) \quad \phi(0|0) = \frac{3.28}{10/28} = \frac{3}{10}, \quad \phi(1|0) = \frac{6/28}{10/28} = \frac{6}{10}, \quad \phi(2|0) = \frac{1/28}{10/28} = \frac{1}{10}$$

3.83

Heads	Tails	Probability	H-T
0	4	1/16	-4
1	3	4/16	-2
2	2	6/16	0
3	1	4/16	2
4	0	1/16	4

3.84

1	2	3
1	3	4
1	4	5
2	3	5
2	4	6
3	4	7

(a)

x	3	4	5	6	7
$f(x)$	1/6	1/6	2/6	1/6	1/6

$$F(x) = \begin{cases} 0 & x < 3 \\ 1/6 & 3 \leq x < 4 \\ 2/6 & 4 \leq x < 5 \\ 4/6 & 5 \leq x < 6 \\ 5/6 & 6 \leq x < 7 \\ 1 & 7 \leq x \end{cases}$$

$$3.85 \quad P(H) = \frac{2}{3}$$

$$(a) \quad P(0) = \frac{1}{27}, \quad P(1) = \frac{6}{27}, \quad P(2) = 3 \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{12}{27}, \quad P(3) = \frac{8}{27}$$

$$(b) \quad \frac{1}{27} + \frac{6}{27} + \frac{12}{27} = \frac{19}{27}$$

$$3.86 \quad F(x) = \begin{cases} 0 & x < 0 \\ 1/27 & 0 \leq x < 1 \\ 7/27 & 1 \leq x < 2 \\ 19/27 & 2 \leq x < 3 \\ 1 & 3 \leq x \end{cases} \quad \begin{array}{ll} (a) & 1 - \frac{7}{27} = \frac{20}{27} \\ (b) & 1 - \frac{19}{27} = \frac{8}{27} \end{array}$$

$$3.87 \quad F(V) = \begin{cases} 0 & V < 0 \\ 0.40 & 0 \leq V < 1 \\ 0.70 & 1 \leq V < 2 \\ 0.90 & 2 \leq V < 3 \\ 1 & 3 \leq V \end{cases}$$

$$3.88 \quad (a) \quad 0.20 + 0.10 = 0.30$$

$$(b) \quad 1 - 0.70 = 0.30$$

$$3.89 \quad \text{Yes; } f(x) \geq 0 \text{ for } x = 2, 3, \dots, 12 \text{ and } \sum_{x=2}^{12} f(x) = 1$$

$$3.90 \quad \begin{array}{cccccccccccccc} S & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ & 1/36 & 3/36 & 6/36 & 10/36 & 15/36 & 21/36 & 26/36 & 30/36 & 33/36 & 35/36 & 1 \end{array}$$

$$3.91 \quad (a) \quad \frac{1}{5}(228.65 - 227.5) = 0.23; \quad (b) \quad \frac{1}{5}(231.66 - 229.34) = 0.464;$$

$$(c) \quad \frac{1}{5}(232.5 - 229.85) = 0.53$$

$$3.92 \quad F(x) = \frac{1}{288} \int (36 - x^2) dx + c = \frac{1}{288} \left(36x - \frac{x^3}{3} \right) + \frac{1}{2} \text{ so that } F(-6) = 0 \text{ and } F(6) = 1.$$

$$(a) \quad F(-2) = \frac{1}{288} \left(-72 + \frac{8}{3} \right) + \frac{1}{2} = \frac{1}{288} \cdot \frac{-208}{3} + \frac{1}{2} = \frac{7}{27}$$

$$(b) \quad F(6) - F(1) = 1 - \frac{1}{288} \left(36 - \frac{1}{3} \right) - \frac{1}{2} = 1 - \frac{1}{288} \cdot \frac{107}{3} - \frac{1}{2} = \frac{757}{864} - \frac{1}{2} = \frac{325}{854}$$

$$(c) \quad F(3) - F(1) = \frac{1}{288} (108 - 9) - \frac{1}{288} \left(36 - \frac{1}{3} \right) = \frac{99}{288} - \frac{1}{288} \cdot \frac{107}{3} = \frac{190}{288 \cdot 3} = \frac{95}{432}$$

$$(d) \quad 0$$

$$3.93 \quad F(x) = \int \frac{1}{30} e^{-x/30} dx + c = \frac{1}{30} \frac{e^{-x/30}}{-1/30} + c = c - e^{-x/30} = 1 - e^{-x/30}$$

$$(a) \quad F(18) = 1 - e^{-18/30} = 1 - e^{-0.6} = 1 - 0.5488 = 0.4512$$

$$(b) \quad F(36) - F(27) = e^{-27/30} - e^{-36/30} = e^{-0.9} - e^{-1.2} = 0.4066 - 0.3012 = 0.1054$$

$$(c) \quad 1 - F(48) = e^{-48/30} = e^{-1.6} = 0.2019$$

$$3.94 \quad F(x) = \int \frac{20,000}{(x+100)^3} dx + c = \frac{20,000}{-2(x+100)^2} + 1 = -\frac{10,000}{(x+100)^2} + 1$$

$$(a) \quad 1 - F(200) = \frac{10,000}{300^2} = \frac{1}{9}$$

$$(b) \quad f(100) = 1 - \frac{10,000}{40,000} = \frac{3}{4}$$

$$3.95 \quad (a) \quad 1 - F(10) = \frac{25}{10^2} = 0.25 = \frac{1}{4}$$

$$(b) \quad F(8) = 1 - \frac{25}{8^2} = \frac{39}{64}$$

$$(c) \quad F(15) - F(12) = \frac{25}{12^2} - \frac{25}{15^2} = \frac{25(25-16)}{15^2-16} = \frac{1}{16}$$

$$3.96 \quad F(x) = \frac{1}{9} \int_0^x e^{-x/3} dx + c = \frac{1}{9} \frac{e^{-x/3}}{-1/3} \left(-\frac{1}{3}x - 1 \right) + c = c - e^{-x/3} \left(\frac{1}{3}x + 1 \right)$$

$$c = 1$$

$$(a) \quad F(6) = 1 - 3e^{-2} = 1 - 3e^{-2} = 1 - 3(0.1353) = 0.5491$$

$$(b) \quad 1 - F(9) = 4e^{-3} = 4(0.0498) = 0.1992$$

$$3.97 \quad (0,0,2) = \binom{3}{0} \binom{2}{0} \binom{3}{2} = 3 \quad f(0,0) = \frac{3}{28}, \quad f(0,1) = \frac{6}{28}, \quad f(0,2) = \frac{1}{28}$$

$$(1,0,1) = \binom{3}{1} \binom{2}{0} \binom{3}{1} = 9 \quad f(1,0) = \frac{9}{28}, \quad f(2,0) = \frac{3}{28}, \quad f(1,1) = \frac{6}{28}$$

$$(0,1,1) = \binom{3}{0} \binom{2}{1} \binom{3}{1} = 6$$

$$(2,0,0) = \binom{3}{2} \binom{2}{0} \binom{3}{0} = 3$$

$$(1,1,0) = \binom{3}{1} \binom{2}{1} \binom{3}{0} = 6$$

$$(0,2,0) = \binom{3}{0} \binom{2}{2} \binom{3}{0} = 1$$

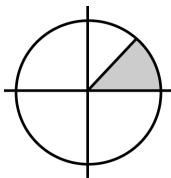
3.98 (b)

		x		
		0	1	2
y	0	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{36}$
	1	$\frac{1}{3}$	$\frac{1}{6}$	
	2	$\frac{1}{4}$		

$$3.99 \quad f(0,3) = \frac{1}{8}, \quad f(1,2) = \frac{3}{8}, \quad f(2,1) = \frac{3}{8}, \quad f(3,0) = \frac{1}{8}$$

$$g(0,-3) = \frac{1}{8}, \quad g(1,1) = \frac{3}{8}, \quad g(2,1) = \frac{3}{8}, \quad g(3,3) = \frac{1}{8}$$

3.100 (a) Probability = 1/8



$$(b) \quad \frac{1}{\pi} \cdot \pi \cdot \frac{1}{4} = \frac{1}{4}$$

$$3.101 (a) \quad \int_{0.2}^{0.3} \int_2^{\infty} 5pe^{-ps} ds dp = \int_{0.2}^{0.3} -5e^{-ps} \Big|_2^{\infty} dp$$

$$= \int_{0.2}^{0.3} 5e^{-2p} dp = \frac{5 \cdot e^{-2p}}{-2} \Big|_{0.2}^{0.3} = \frac{5}{2} (e^{-0.4} - e^{-0.6}) = 0.3038$$

$$(b) \quad \int_{0.25}^{0.30} \int_0^1 5pe^{-ps} ds dp = \int_{0.25}^{0.30} -5e^{-ps} \Big|_0^1 dp = \int_{0.25}^{0.30} 5(1 - e^{-p}) dp$$

$$= 5[p + e^{-p}]^{0.30} = 5(0.30 + e^{-0.30} - 0.25 - e^{-0.25}) = 0.01202$$

$$3.102 (a) \quad \frac{2}{5} \int_0^{0.4} \int_0^{0.4} (2x + 3y) dx dy = \frac{2}{5} \int_0^{0.4} (x^2 + 3xy) \Big|_0^{0.4} dy$$

$$= \frac{2}{5} \int_0^{0.4} ((0.16) + 1.2y) dy$$

$$= \frac{2}{5} \left[(0.16)(0.4) + \frac{1.2(0.16)}{2} \right] = 0.064$$

$$\begin{aligned}
 \text{(b)} \quad \frac{2}{5} \int_0^{0.5} \int_{0.8}^1 (2x+3y) \, dx \, dy &= \frac{2}{5} \int_0^{0.5} (x^2 + 3xy) \Big|_{0.8}^1 \, dy \\
 &= \frac{2}{5} \int_0^{0.5} [(1+3y) - (0.64+2.4y)] \, dy = \frac{2}{5} \int_0^{0.5} (0.6y + 0.36) \, dy \\
 &= \frac{2}{5} (0.3y^2 + 0.36y) \Big|_0^{0.5} = \frac{2}{5} (0.075 + 0.18) = 0.102
 \end{aligned}$$

$$\text{3.103 (a)} \quad g(0) = \frac{5}{14}, \quad g(1) = \frac{15}{28} \text{ and } g(2) = \frac{3}{28}$$

$$\text{(b)} \quad \phi(0|0) = \frac{3}{10}, \quad \phi(1|0) = \frac{6}{10}, \text{ and } \phi(2|0) = \frac{1}{10}$$

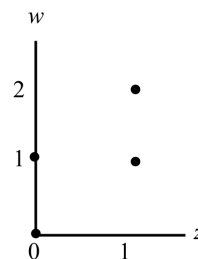
$$\begin{aligned}
 \text{3.104 (a)} \quad \int_{0.3}^1 \int_0^1 \frac{2}{5} (x+4y) \, dy \, dx &= \frac{2}{5} \int_{0.3}^1 (xy + 2y^2) \Big|_0^1 \, dx = \frac{2}{5} \int_{0.3}^1 (x+2) \, dx \\
 &= \frac{2}{5} \left[\frac{x^2}{2} + 2x \right]_{0.3}^1 \\
 &= \frac{2}{5} \left(\frac{1}{2} + 2 - \frac{0.09}{2} - 0.6 \right) = \frac{2}{5} (1.855) = 0.742
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad g(x) &= \frac{2}{5} \int_0^1 (x+4y) \, dy = \frac{2}{5} (x+2) \\
 g(y|x) &= \frac{(2/5)(x+4y)}{(2/5)(x+2)}, \quad g(y|0.2) = \frac{4y+0.2}{2.2} \\
 \frac{1}{2.2} \int_0^{0.5} (4y+0.2) \, dy &= \frac{1}{2.2} (0.5+0.1) = \frac{0.6}{2.2} = 0.273
 \end{aligned}$$

$$\begin{aligned}
 \text{3.105 (a)} \quad f(0,0) &= \frac{48}{52} \cdot \frac{47}{51} = \frac{188}{221}, \quad f(0,1) = \frac{48}{52} \cdot \frac{4}{51} = \frac{16}{221} \\
 f(1,0) &= \frac{4}{52} \cdot \frac{48}{51} = \frac{16}{221}, \quad f(1,1) = \frac{48}{52} \cdot \frac{4}{51} = \frac{16}{221}, \quad f(1,2) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}
 \end{aligned}$$

$$\text{(b)} \quad g(0) = \frac{188+16}{221} = \frac{204}{221}, \quad g(1) = \frac{16+1}{221} = \frac{17}{221}$$

$$\text{(c)} \quad \phi(0|1) = \frac{16/221}{17/221} = \frac{16}{17}, \quad \phi(1|1) = \frac{1/221}{17/221} = \frac{1}{17}$$



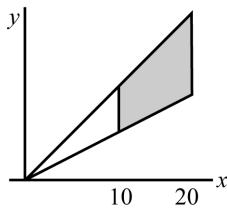
$$3.106 \quad f(p, s) = 5pe^{-ps} \quad 0.2 < p < 0.4 \quad s > 0$$

$$(a) \quad 5p \int_0^{\infty} e^{-ps} ds = 5p \frac{e^{-ps}}{-p} = -5e^{-ps} \Big|_0^{\infty} = \begin{cases} 5 & 0.2 < p < 0.4 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad \frac{f(p, s)}{g(s)} = \frac{5pe^{-ps}}{5} = \begin{cases} pe^{-ps} & \text{for } s > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$(c) \quad \int_0^3 \frac{1}{4} e^{-(1/4)s} ds = \left[e^{-s/4} \right]_0^3 = 1 - e^{-0.75}$$

3.107



$$(a) \quad \frac{1}{25} \frac{20-x}{x} \int_{x/2}^x dy = \begin{cases} \frac{20-x}{50} & 10 < x < 20 \\ 0 & \text{elsewhere} \end{cases}$$

$$(b) \quad \phi(y|x) = \frac{\frac{1}{25} \left(\frac{20-x}{x} \right)}{\frac{20-x}{50}} = \frac{2}{x}, \quad \phi(y|12) = \begin{cases} 1/6 & 6 < y < 12 \\ 0 & \text{elsewhere} \end{cases}$$

$$(c) \quad \frac{1}{6} (12 - 8) = \frac{1}{6} \cdot 4 = \frac{2}{3}$$

$$3.108 \quad f(x, y) = \frac{2}{5} (2x + 3y)$$

$$g(x) = \frac{2}{5} \left[2xy + \frac{3y^2}{2} \right] \Big|_0^1 = \frac{2}{5} \left(2x + \frac{3}{2} \right) \\ = \begin{cases} \frac{4}{5}x + \frac{3}{5} & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(y) = \frac{2}{5} (x^2 + 3xy) \Big|_0^1 \\ = \begin{cases} (2/5)(1 + 3y) & 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

$$f(x, y) \neq g(x)h(y)$$

$$3.109 \text{ (a)} \quad f(x_1, x_2, x_3) = \begin{cases} \frac{(20,000)^3}{(x_2+100)^3(x_2+100)^3(x_3+100)^3} & x_1 > 0, \ x_2 > 0, \ x_3 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\begin{aligned} \text{(b)} \quad & \int_0^{100} \frac{20,000}{(x_1+100)^3} dx_1 \int_0^{100} \frac{20,000}{(x_2+100)^3} dx_2 \int_{200}^{\infty} \frac{20,000}{(x_3+100)^3} dx_3 \\ &= \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{1}{9} = \frac{1}{16} \end{aligned}$$

$$3.110 \text{ (a)} \quad \begin{array}{l} 5 | 9 \ 4 \ 5 \ 7 \ 9 \ 9 \ 8 \\ 6 | 1 \ 3 \ 5 \ 0 \ 2 \ 1 \ 7 \ 0 \ 8 \ 4 \ 5 \ 2 \ 0 \ 2 \ 1 \ 3 \ 1 \end{array}$$

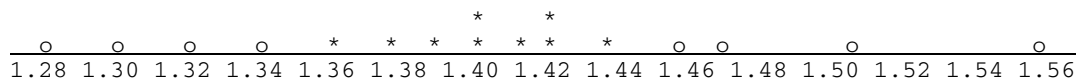
$$\begin{array}{l} \text{(b)} \quad 5f | 4 \\ 5s | 9 \ 5 \ 7 \ 9 \ 9 \ 8 \\ 6f | 1 \ 3 \ 0 \ 2 \ 0 \ 1 \ 0 \ 4 \ 2 \ 0 \ 2 \ 1 \ 3 \ 1 \\ 6s | 5 \ 7 \ 8 \ 5 \end{array}$$

(c) The double-stem display is more informative.

3.111 * = Station 105 ○ = Station 107



3.112 * = Lathe A ○ = Lathe B



3.115	Class Limits	Frequency
	40.0 – 44.9	5
	45.0 – 49.9	7
	50.0 – 54.9	15
	55.0 – 59.9	23
	60.0 – 64.9	29
	65.0 – 69.9	12
	70.0 – 74.9	8
	75.0 – 79.9	<u>1</u>
		100

3.116

Class Limits	Frequency
3.0 – 4.9	15
5.0 – 6.9	25
7.0 – 8.9	17
9.0 – 10.9	11
11.0 – 12.9	8
13.0 – 14.9	<u>4</u>
	80

3.117 The class boundaries are: 39.95, 44.95, 49.95, 54.95, 59.95, 64.95, 69.95, 79.95;
the class interval is 5;
the class marks are: 42.45, 47.45, 52.45, 57.45, 62.45, 67.45, 72.45, 77.45.

3.118 The class boundaries are: 2.95, 4.95, 6.95, 8.95, 10.95, 12.95, 14.95;
the class interval is 2;
the class marks are: 3.95, 5.95, 7.95, 9.95, 11.95, 13.95.

3.119

Class Limits	Frequency	Class Boundary	Class Mark
0 – 1	12	–0.5 – 1.5	0.5
2 – 3	7	1.5 – 3.5	2.5
4 – 5	4	3.5 – 5.5	4.5
6 – 7	5	5.5 – 7.5	6.5
8 – 9	1	7.5 – 9.5	8.5
10 – 11	0	9.5 – 11.5	10.5
12 – 13	<u>1</u>	11.5 – 13.5	12.5
	30		

3.120

Class Limits	Frequency	Percentage
3.0 – 4.9	15	18.75%
5.0 – 6.9	25	31.25
7.0 – 8.9	17	21.25
9.0 – 10.9	11	13.75
11.0 – 12.9	8	10.00
13.0 – 14.9	<u>4</u>	<u>5.00</u>
	80	100.00

3.121

Class Limits	Frequency	Percentage
40.0 – 44.9	5	5.0%
45.0 – 49.9	7	7.0
50.0 – 54.9	15	15.0
55.0 – 59.9	23	23.0
60.0 – 64.9	29	29.0
65.0 – 69.9	12	12.0
70.0 – 74.9	8	8.0
75.0 – 79.9	<u>1</u>	<u>1.0</u>
	100	100.0

3.122

Class Limits	Percentage	
	Shipping Department	Security Department
0 – 1	43.3%	45.0%
2 – 3	30.0	27.5
4 – 5	16.7	17.5
6 – 7	6.7	7.5
8 – 9	<u>3.3</u>	<u>2.5</u>
	100.0	100.0

The patterns seem comparable for the two departments.

3.123

Upper Class Boundary	Frequency	Cumulative Frequency
44.95	5	5
49.95	7	12
54.95	15	27
59.95	23	50
64.95	29	79
69.95	12	91
74.95	8	99
79.95	<u>1</u>	<u>100</u>
	100	

3.124

Upper Class Boundary	Frequency	Cumulative Frequency
4.95	15	15
6.95	25	40
8.95	17	57
10.95	11	68
12.95	8	76
14.95	<u>4</u>	<u>80</u>
	100	

3.125

Class Limits	Cumulative Percentage	
	Shipping Department	Security Department
1.5	43.3%	45.0%
3.5	73.3	72.5
5.5	90.0	90.0
7.5	96.7	97.5
9.5	100.0	100.0

3.126	(a)	Class Limits	Frequency	(b) No. The class interval of the last class is greater than that of the others.
		0 – 1	12	
		2 – 3	7	
		4 – 5	4	
		6 – 7	5	
		8 – 13	<u>2</u>	
			30	

3.127	(a)	Class Limits	Frequency	Class Marks	(b) Yes, [see part (a)].
		0 – 99	4	49.5	
		100 – 199	3	149.5	
		200 – 299	4	249.5	
		300 – 324	7	312.0	
		325 – 349	14	337.0	
		350 – 399	<u>6</u>	374.5	
			38		

3.130 The class marks are found from the class boundaries by averaging them; thus, the first class mark is $(2.95 + 4.95)/2 = 3.95$, and so forth.

3.135 The MINITAB output is:

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS
6.0	2 **
6.5	5 *****
7.0	4 *****
7.5	5 *****
8.0	5 *****
8.5	3 ***
9.0	2 **
9.5	2 **
10.02	2 **

3.136 The MINITAB output is:

MIDDLE OF INTERVAL	NUMBER OF OBSERVATIONS
40	1 *
45	7 *****
50	11 *****
55	21 *****
60	21 *****
65	23 *****
70	10 *****
75	6 *****