No:

Q4. (a) Let $Y_1, Y_2, \cdots, Y_n$ be values of independent n draws.

$$P(Y_i = k) = \frac{1}{N}, \quad k = 1, 2, \cdots, N.$$

$$\mu_1' = E(Y) = \sum_{k=1}^{N} kP(Y_i = k) = \sum_{k=1}^{N} k\frac{1}{N} = \frac{N}{2}\left[2\left(\frac{1}{N}\right) + (N-1)\left(\frac{1}{N}\right)\right]$$

$$\bar{Y} = \frac{1}{2}(2+N-1) = \frac{1}{2}(N+1)$$

$\therefore$ The estimator of N is

$$\hat{N}_1 = 2\bar{Y} - 1$$

(b) $$E(\hat{N}_1) = E(2\bar{Y}-1) = 2E(\bar{Y}) - 1 = 2\left[\frac{1}{2}(N+1)\right] - 1 = N$$

$$\begin{aligned}
Var(\hat{N}_1) &= Var(2\bar{Y}) \\
&= 4\,Var(\bar{Y}) \\
&= \frac{4}{n} Var(Y) \\
&= \frac{4}{n}\left[E(Y^2) - E(Y)^2\right] \\
&= \frac{4}{n}\left[\sum_{i=1}^{N}\frac{i^2}{N} - \frac{(N+1)^2}{4}\right] \\
&= \frac{4}{n}\left[\frac{1}{6N}(N)(N+1)(2N+1) - \frac{(N+1)^2}{4}\right] \\
&= \frac{4}{n}\left[\frac{2(N+1)(2N+1) - 3(N+1)^2}{12}\right] \\
&= \frac{4}{n}\left(\frac{N+1}{12}\right)(4N+2-3N-3) \\
&= \frac{1}{3n}(N+1)(N-1) \\
&= \frac{1}{3n}(N^2-1)
\end{aligned}$$

(c) Let $I(x) = 1 / 0$ depending on whether the condition x is true.

$$\begin{aligned}
L(Y_1, Y_2, \cdots, Y_n \mid \theta) &= \prod_{i=1}^{n} \frac{1}{N} \cdot I(Y_i \leq N) \\
&= \frac{1}{N^n} \prod_{i=1}^{n} I(Y_i \leq N) \\
&= \frac{1}{N^n} \cdot I(Y_{(n)} \leq N)
\end{aligned}$$

For L to be maximized, N should be chosen as small as possible subject to the constraint that $Y_{(n)} \leq N$.

So, we must choose N as small such that $I(Y_{(n)} \leq N) = 1$.

This gives that $Y_{(n)} = \hat{N}_2$.

$\therefore$ The maximum likelihood estimator of N is

$$\hat{N}_2 = Y_{(n)}$$