

Q1. (a) Maximum amount of material that can be sent from node A to node H through the given network = $3+5+2+1+1+1+1 = 14$

		Iteration 1							
		Starting Flow Matrix							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		8.00	0.00	2.00	0.00	10.00	0.00	0.00
N2	B	8.00		5.00	2.00	3.00	0.00	0.00	0.00
N3	C	0.00	5.00		0.00	2.00	0.00	0.00	10.00
N4	D	2.00	2.00	0.00		1.00	5.00	3.00	0.00
N5	E	0.00	3.00	2.00	1.00		0.00	4.00	1.00
N6	F	10.00	0.00	0.00	5.00	0.00		2.00	0.00
N7	G	0.00	0.00	0.00	3.00	4.00	2.00		6.00
N8	H	0.00	0.00	10.00	0.00	1.00	0.00	6.00	

		Iteration 2: Breakthrough flow = 3.00							
		Labels: (1)[0,-]{6}[10.00,1]{4}[5.00,6]{7}[3.00,4]{8}[6.00,7]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		8.00	0.00	2.00	0.00	7.00	0.00	0.00
N2	B	8.00		5.00	2.00	3.00	0.00	0.00	0.00
N3	C	0.00	5.00		0.00	2.00	0.00	0.00	10.00
N4	D	2.00	2.00	0.00		1.00	8.00	0.00	0.00
N5	E	0.00	3.00	2.00	1.00		0.00	4.00	1.00
N6	F	13.00	0.00	0.00	2.00	0.00		2.00	0.00
N7	G	0.00	0.00	0.00	6.00	4.00	2.00		3.00
N8	H	0.00	0.00	10.00	0.00	1.00	0.00	9.00	

		Iteration 3: Breakthrough flow = 5.00							
		Labels: (1)[0,-]{2}[8.00,1]{3}[5.00,2]{8}[10.00,3]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		3.00	0.00	2.00	0.00	7.00	0.00	0.00
N2	B	13.00		0.00	2.00	3.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	2.00	0.00	0.00	5.00
N4	D	2.00	2.00	0.00		1.00	8.00	0.00	0.00
N5	E	0.00	3.00	2.00	1.00		0.00	4.00	1.00
N6	F	13.00	0.00	0.00	2.00	0.00		2.00	0.00
N7	G	0.00	0.00	0.00	6.00	4.00	2.00		3.00
N8	H	0.00	0.00	15.00	0.00	1.00	0.00	9.00	

		Iteration 4: Breakthrough flow = 2.00							
		Labels: (1)[0,-]{6}[7.00,1]{4}[2.00,6]{2}[2.00,4]{5}[3.00,2]{7}[4.00,5]{8}[3.00,7]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		3.00	0.00	2.00	0.00	5.00	0.00	0.00
N2	B	13.00		0.00	4.00	1.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	2.00	0.00	0.00	5.00
N4	D	2.00	0.00	0.00		1.00	10.00	0.00	0.00
N5	E	0.00	5.00	2.00	1.00		0.00	2.00	1.00
N6	F	15.00	0.00	0.00	0.00	0.00		2.00	0.00
N7	G	0.00	0.00	0.00	6.00	6.00	2.00		1.00
N8	H	0.00	0.00	15.00	0.00	1.00	0.00	11.00	

		Iteration 5: Breakthrough flow = 1.00							
	Labels:	(1)[0,-]{6}[5.00,1]{7}[2.00,6]{4}[6.00,7]{5}[1.00,4]{3}[2.00,5]{8}[5.00,3]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		3.00	0.00	2.00	0.00	4.00	0.00	0.00
N2	B	13.00		0.00	4.00	1.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	3.00	0.00	0.00	4.00
N4	D	2.00	0.00	0.00		0.00	10.00	1.00	0.00
N5	E	0.00	5.00	1.00	2.00		0.00	2.00	1.00
N6	F	16.00	0.00	0.00	0.00	0.00		1.00	0.00
N7	G	0.00	0.00	0.00	5.00	6.00	3.00		1.00
N8	H	0.00	0.00	16.00	0.00	1.00	0.00	11.00	

		Iteration 6: Breakthrough flow = 1.00							
	Labels:	(1)[0,-]{6}[4.00,1]{7}[1.00,6]{5}[6.00,7]{3}[1.00,5]{8}[4.00,3]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		3.00	0.00	2.00	0.00	3.00	0.00	0.00
N2	B	13.00		0.00	4.00	1.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	4.00	0.00	0.00	3.00
N4	D	2.00	0.00	0.00		0.00	10.00	1.00	0.00
N5	E	0.00	5.00	0.00	2.00		0.00	3.00	1.00
N6	F	17.00	0.00	0.00	0.00	0.00		0.00	0.00
N7	G	0.00	0.00	0.00	5.00	5.00	4.00		1.00
N8	H	0.00	0.00	17.00	0.00	1.00	0.00	11.00	

		Iteration 7: Breakthrough flow = 1.00							
	Labels:	(1)[0,-]{2}[3.00,1]{4}[4.00,2]{7}[1.00,4]{5}[5.00,7]{8}[1.00,5]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		2.00	0.00	2.00	0.00	3.00	0.00	0.00
N2	B	14.00		0.00	3.00	1.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	4.00	0.00	0.00	3.00
N4	D	2.00	1.00	0.00		0.00	10.00	0.00	0.00
N5	E	0.00	5.00	0.00	2.00		0.00	4.00	0.00
N6	F	17.00	0.00	0.00	0.00	0.00		0.00	0.00
N7	G	0.00	0.00	0.00	6.00	4.00	4.00		1.00
N8	H	0.00	0.00	17.00	0.00	2.00	0.00	11.00	

		Iteration 8: Breakthrough flow = 1.00							
	Labels:	(1)[0,-]{2}[2.00,1]{5}[1.00,2]{7}[4.00,5]{8}[1.00,7]							
		N1	N2	N3	N4	N5	N6	N7	N8
		A	B	C	D	E	F	G	H
N1	A		1.00	0.00	2.00	0.00	3.00	0.00	0.00
N2	B	15.00		0.00	3.00	0.00	0.00	0.00	0.00
N3	C	0.00	10.00		0.00	4.00	0.00	0.00	3.00
N4	D	2.00	1.00	0.00		0.00	10.00	0.00	0.00
N5	E	0.00	6.00	0.00	2.00		0.00	3.00	0.00
N6	F	17.00	0.00	0.00	0.00	0.00		0.00	0.00
N7	G	0.00	0.00	0.00	6.00	5.00	4.00		0.00
N8	H	0.00	0.00	17.00	0.00	2.00	0.00	12.00	

Q1. (b) Company at node A should purchase the materials from supplier at node H from economic perspective.

Maximum amount of material can be sent from node H to node A = $5+3+2+1+2+1 = 14$

Iteration 1									
Starting Flow Matrix									
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	10.00	0.00	1.00	0.00	6.00	0.00
N2	B	0.00		5.00	2.00	3.00	0.00	0.00	8.00
N3	C	10.00	5.00		0.00	2.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		1.00	5.00	3.00	2.00
N5	E	1.00	3.00	2.00	1.00		0.00	4.00	0.00
N6	F	0.00	0.00	0.00	5.00	0.00		2.00	10.00
N7	G	6.00	0.00	0.00	3.00	4.00	2.00		0.00
N8	A	0.00	8.00	0.00	2.00	0.00	10.00	0.00	

Iteration 2: Breakthrough flow = 5.00									
Labels: (1)[0,-](3)[10.00,1](2)[5.00,3](8)[8.00,2]									
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	5.00	0.00	1.00	0.00	6.00	0.00
N2	B	0.00		10.00	2.00	3.00	0.00	0.00	3.00
N3	C	15.00	0.00		0.00	2.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		1.00	5.00	3.00	2.00
N5	E	1.00	3.00	2.00	1.00		0.00	4.00	0.00
N6	F	0.00	0.00	0.00	5.00	0.00		2.00	10.00
N7	G	6.00	0.00	0.00	3.00	4.00	2.00		0.00
N8	A	0.00	13.00	0.00	2.00	0.00	10.00	0.00	

Iteration 3: Breakthrough flow = 3.00									
Labels: (1)[0,-](7)[6.00,1](5)[4.00,7](2)[3.00,5](8)[3.00,2]									
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	5.00	0.00	1.00	0.00	3.00	0.00
N2	B	0.00		10.00	2.00	6.00	0.00	0.00	0.00
N3	C	15.00	0.00		0.00	2.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		1.00	5.00	3.00	2.00
N5	E	1.00	0.00	2.00	1.00		0.00	7.00	0.00
N6	F	0.00	0.00	0.00	5.00	0.00		2.00	10.00
N7	G	9.00	0.00	0.00	3.00	1.00	2.00		0.00
N8	A	0.00	16.00	0.00	2.00	0.00	10.00	0.00	

		Iteration 4: Breakthrough flow = 2.00							
	Labels:	(1)[0,-1]{3}[5.00,1]{5}[2.00,3]{7}[7.00,5]{4}[3.00,7]{6}[5.00,4]{8}[10.00,6]							
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	3.00	0.00	1.00	0.00	3.00	0.00
N2	B	0.00		10.00	2.00	6.00	0.00	0.00	0.00
N3	C	17.00	0.00		0.00	0.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		1.00	3.00	5.00	2.00
N5	E	1.00	0.00	4.00	1.00		0.00	5.00	0.00
N6	F	0.00	0.00	0.00	7.00	0.00		2.00	8.00
N7	G	9.00	0.00	0.00	1.00	3.00	2.00		0.00
N8	A	0.00	16.00	0.00	2.00	0.00	12.00	0.00	

		Iteration 5: Breakthrough flow = 1.00							
	Labels:	(1)[0,-1]{7}[3.00,1]{5}[3.00,7]{4}[1.00,5]{6}[3.00,4]{8}[8.00,6]							
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	3.00	0.00	1.00	0.00	2.00	0.00
N2	B	0.00		10.00	2.00	6.00	0.00	0.00	0.00
N3	C	17.00	0.00		0.00	0.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		2.00	2.00	5.00	2.00
N5	E	1.00	0.00	4.00	0.00		0.00	6.00	0.00
N6	F	0.00	0.00	0.00	8.00	0.00		2.00	7.00
N7	G	10.00	0.00	0.00	1.00	2.00	2.00		0.00
N8	A	0.00	16.00	0.00	2.00	0.00	13.00	0.00	

		Iteration 6: Breakthrough flow = 2.00							
	Labels:	(1)[0,-1]{7}[2.00,1]{6}[2.00,7]{4}[8.00,6]{8}[2.00,4]							
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	3.00	0.00	1.00	0.00	0.00	0.00
N2	B	0.00		10.00	2.00	6.00	0.00	0.00	0.00
N3	C	17.00	0.00		0.00	0.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		2.00	4.00	5.00	0.00
N5	E	1.00	0.00	4.00	0.00		0.00	6.00	0.00
N6	F	0.00	0.00	0.00	6.00	0.00		4.00	7.00
N7	G	12.00	0.00	0.00	1.00	2.00	0.00		0.00
N8	A	0.00	16.00	0.00	4.00	0.00	13.00	0.00	

		Iteration 7: Breakthrough flow = 1.00							
	Labels:	(1)[0,-1]{5}[1.00,1]{7}[6.00,5]{4}[1.00,7]{6}[4.00,4]{8}[7.00,6]							
		N1	N2	N3	N4	N5	N6	N7	N8
		H	B	C	D	E	F	G	A
N1	H		0.00	3.00	0.00	0.00	0.00	0.00	0.00
N2	B	0.00		10.00	2.00	6.00	0.00	0.00	0.00
N3	C	17.00	0.00		0.00	0.00	0.00	0.00	0.00
N4	D	0.00	2.00	0.00		2.00	3.00	6.00	0.00
N5	E	2.00	0.00	4.00	0.00		0.00	5.00	0.00
N6	F	0.00	0.00	0.00	7.00	0.00		4.00	6.00
N7	G	12.00	0.00	0.00	0.00	3.00	0.00		0.00
N8	A	0.00	16.00	0.00	4.00	0.00	14.00	0.00	

Maximum amount of material can be sent from node D to node A = $5+3+2+2+1 = 13$

Iteration 1									
Starting Flow Matrix									
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		2.00	0.00	1.00	5.00	3.00	0.00	2.00
N2	B	2.00		5.00	3.00	0.00	0.00	0.00	8.00
N3	C	0.00	5.00		2.00	0.00	0.00	10.00	0.00
N4	E	1.00	3.00	2.00		0.00	4.00	1.00	0.00
N5	F	5.00	0.00	0.00	0.00		2.00	0.00	10.00
N6	G	3.00	0.00	0.00	4.00	2.00		6.00	0.00
N7	H	0.00	0.00	10.00	1.00	0.00	6.00		0.00
N8	A	2.00	8.00	0.00	0.00	10.00	0.00	0.00	

Iteration 2: Breakthrough flow = 5.00									
Labels: (1)[0,-]-{5}[5.00,1]-{8}[10.00,5]									
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		2.00	0.00	1.00	0.00	3.00	0.00	2.00
N2	B	2.00		5.00	3.00	0.00	0.00	0.00	8.00
N3	C	0.00	5.00		2.00	0.00	0.00	10.00	0.00
N4	E	1.00	3.00	2.00		0.00	4.00	1.00	0.00
N5	F	10.00	0.00	0.00	0.00		2.00	0.00	5.00
N6	G	3.00	0.00	0.00	4.00	2.00		6.00	0.00
N7	H	0.00	0.00	10.00	1.00	0.00	6.00		0.00
N8	A	2.00	8.00	0.00	0.00	15.00	0.00	0.00	

Iteration 3: Breakthrough flow = 3.00									
Labels: (1)[0,-]-{6}[3.00,1]-{7}[6.00,6]-{3}[10.00,7]-{2}[5.00,3]-{8}[8.00,2]									
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		2.00	0.00	1.00	0.00	0.00	0.00	2.00
N2	B	2.00		8.00	3.00	0.00	0.00	0.00	5.00
N3	C	0.00	2.00		2.00	0.00	0.00	13.00	0.00
N4	E	1.00	3.00	2.00		0.00	4.00	1.00	0.00
N5	F	10.00	0.00	0.00	0.00		2.00	0.00	5.00
N6	G	6.00	0.00	0.00	4.00	2.00		3.00	0.00
N7	H	0.00	0.00	7.00	1.00	0.00	9.00		0.00
N8	A	2.00	11.00	0.00	0.00	15.00	0.00	0.00	

Iteration 4: Breakthrough flow = 2.00									
Labels: (1)[0,-]-{2}[2.00,1]-{3}[8.00,2]-{7}[13.00,3]-{6}[9.00,7]-{5}[2.00,6]-{8}[5.00,5]									
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		0.00	0.00	1.00	0.00	0.00	0.00	2.00
N2	B	4.00		6.00	3.00	0.00	0.00	0.00	5.00
N3	C	0.00	4.00		2.00	0.00	0.00	11.00	0.00
N4	E	1.00	3.00	2.00		0.00	4.00	1.00	0.00
N5	F	10.00	0.00	0.00	0.00		4.00	0.00	3.00
N6	G	6.00	0.00	0.00	4.00	0.00		5.00	0.00
N7	H	0.00	0.00	9.00	1.00	0.00	7.00		0.00
N8	A	2.00	11.00	0.00	0.00	17.00	0.00	0.00	

		Iteration 5: Breakthrough flow = 2.00							
	Labels:	(1)[0,-]{8}[2.00,1]							
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		0.00	0.00	1.00	0.00	0.00	0.00	0.00
N2	B	4.00		6.00	3.00	0.00	0.00	0.00	5.00
N3	C	0.00	4.00		2.00	0.00	0.00	11.00	0.00
N4	E	1.00	3.00	2.00		0.00	4.00	1.00	0.00
N5	F	10.00	0.00	0.00	0.00		4.00	0.00	3.00
N6	G	6.00	0.00	0.00	4.00	0.00		5.00	0.00
N7	H	0.00	0.00	9.00	1.00	0.00	7.00		0.00
N8	A	4.00	11.00	0.00	0.00	17.00	0.00	0.00	

		Iteration 6: Breakthrough flow = 1.00							
	Labels:	(1)[0,-]{4}[1.00,1]{6}[4.00,4]{7}[5.00,6]{3}[9.00,7]{2}[4.00,3]{8}[5.00,2]							
		N1	N2	N3	N4	N5	N6	N7	N8
		D	B	C	E	F	G	H	A
N1	D		0.00	0.00	0.00	0.00	0.00	0.00	0.00
N2	B	4.00		7.00	3.00	0.00	0.00	0.00	4.00
N3	C	0.00	3.00		2.00	0.00	0.00	12.00	0.00
N4	E	2.00	3.00	2.00		0.00	3.00	1.00	0.00
N5	F	10.00	0.00	0.00	0.00		4.00	0.00	3.00
N6	G	6.00	0.00	0.00	5.00	0.00		4.00	0.00
N7	H	0.00	0.00	8.00	1.00	0.00	8.00		0.00
N8	A	4.00	12.00	0.00	0.00	17.00	0.00	0.00	

Since total cost are same, the company should purchase the materials from supplier at node H to obtain more material.

Q2. The maximum flow for the given network is obtained as 19 by using LINGO, which obtain the same answer with TORA.

Lingo 18.0 - Solution Report - Lingo1

File Edit Solver Window Help

Lingo Model - Lingo1

```

MAX = X0;
X12 <= 3;
X13 <= 6;
X14 <= 10;
X32 <= 5;
X34 <= 8;
X35 <= 9;
X24 <= 8;
X25 <= 16;
X45 <= 12;
X0 - X12 - X14 - X13 = 0;
X12 + X32 - X24 - X23 = 0;
X13 - X32 - X34 - X35 = 0;
X24 + X14 + X34 - X45 = 0;
X25 + X45 + X35 - X0 = 0;
X0 >= 0;
X12 >= 0;
X13 >= 0;
X14 >= 0;
X24 >= 0;
X25 >= 0;
X32 >= 0;
X34 >= 0;
X35 >= 0;
X45 >= 0;

```

Solution Report - Lingo1

Global optimal solution found.

Objective value: 19.00000

Infeasibilities: 0.000000

Total solver iterations: 3

Elapsed runtime seconds: 0.33

Model Class: LP

Total variables: 11

Nonlinear variables: 0

Integer variables: 0

Total constraints: 25

Nonlinear constraints: 0

Total nonzeros: 40

Nonlinear nonzeros: 0

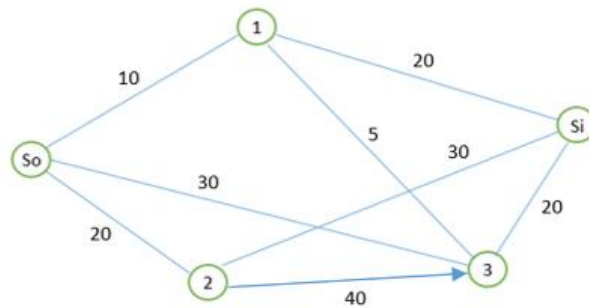
Variable	Value	Reduced Cost
X0	19.00000	0.000000
X12	3.000000	0.000000
X13	6.000000	0.000000
X14	10.00000	0.000000
X32	0.000000	0.000000
X34	0.000000	0.000000
X35	6.000000	0.000000
X24	0.000000	0.000000
X25	3.000000	0.000000
X45	10.00000	0.000000
X23	3.000000	0.000000

Row	Slack or Surplus	Dual Price
1	19.00000	1.000000
2	0.000000	1.000000
3	0.000000	1.000000
4	0.000000	1.000000
5	5.000000	0.000000
6	8.000000	0.000000

For Help, press F1

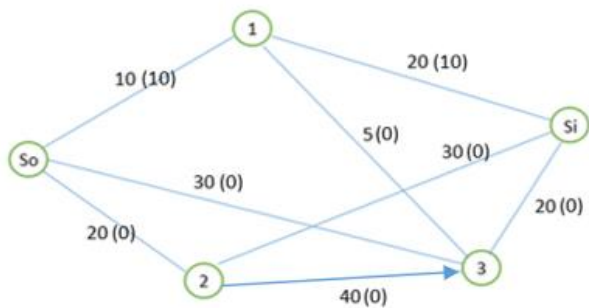
Q3. The amount of maximum flow from the source to sink is not always similar to the amount of maximum flow from sink to source for all network flow problems. When one of the arcs in the network is unidirectional, the amount of maximum flow from the source to sink will be different with the amount of maximum flow from sink to source.

Example shown as below:

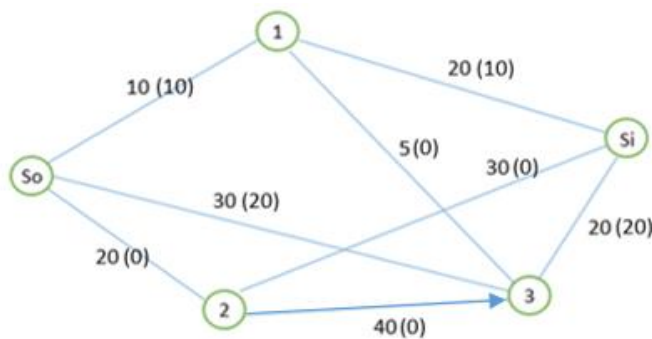


From Source to Sink

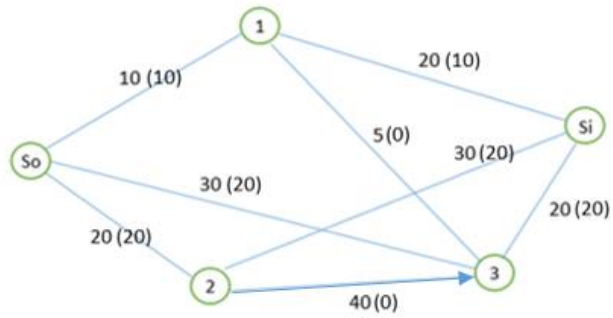
Iteration 1: So – 1 – Si (10)



Iteration 2: So – 3 – Si (20)



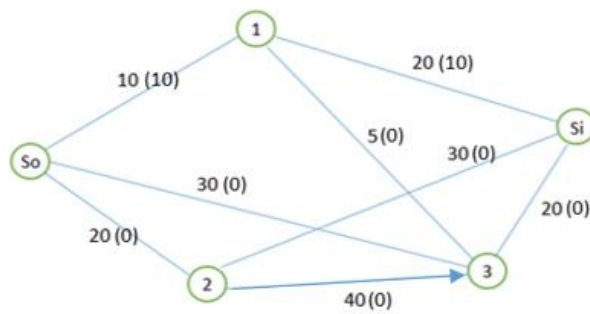
Iteration 3: So – 2 – Si (20)



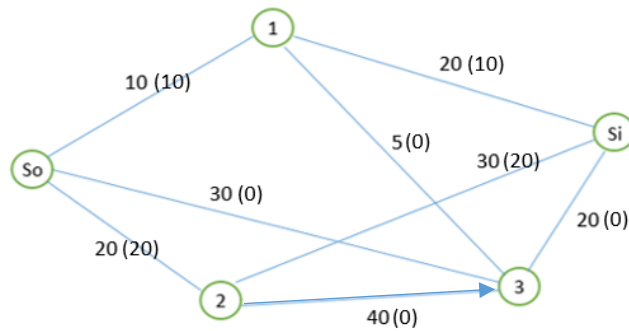
\therefore Amount of maximum flow from source to sink = $10+20+20 = 50$

From Sink to Source

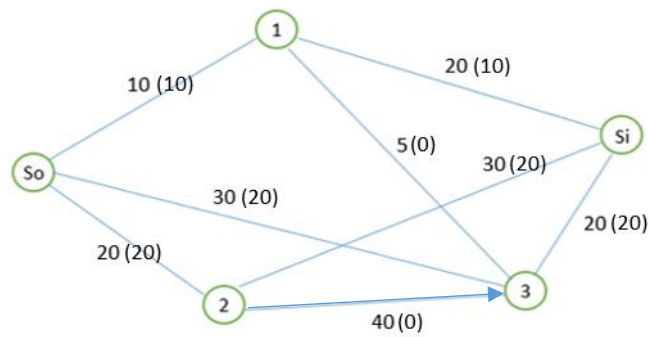
Iteration 1: Si – 1 – So (10)



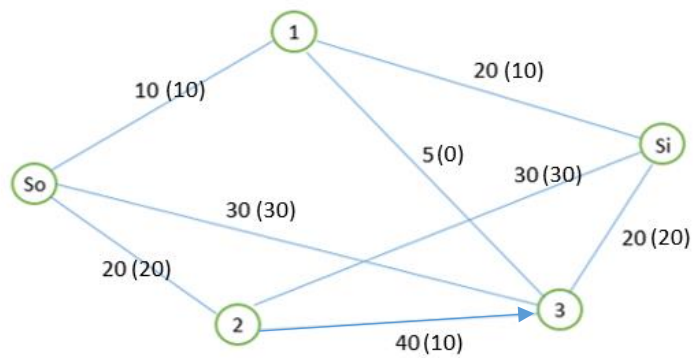
Iteration 2: Si – 2 – So (20)



Iteration 3: Si – 3 – So (20)



Iteration 3: Si – 2 – 3 – So (10)



\therefore Amount of maximum flow from sink to source = $10+20+20+10 = 60$

The amount of maximum flow from the source to sink is not always similar to the amount of maximum flow from sink to source for all network flow problems.