

No: _____

(d) Distribution function of \hat{N}_2 , Distribution of ratio of X_1, X_2, \dots, X_n to N is

$$P(Y_{(n)} \leq y) = P(Y_1 \leq y) P(Y_2 \leq y) \dots P(Y_n \leq y), \quad \frac{Y_i}{N} = (X_i - \bar{X}) \frac{1}{\sigma}.$$
$$= \left(\frac{y}{N}\right)^n$$

$$P(Y_{(n)} \geq y) = 1 - \left(\frac{y}{N}\right)^n \approx 1 - \frac{y^n}{N^n} = (1 - \frac{y}{N})^n = (1 - \frac{\bar{X}}{N})^n = (1 - \frac{\bar{X}}{N})^{\frac{1}{N}} = (1 - \frac{\bar{X}}{N})^{\frac{1}{N}} = \frac{1}{N}$$

$$E(\hat{N}_2) = \sum_{y=1}^N P(Y_{(n)} \geq y)$$

$$= \sum_{y=1}^N [1 - \left(\frac{y}{N}\right)^n]$$

$$= N - \frac{1}{N^n} \sum_{y=1}^N y^n$$

$$\sum_{y=1}^N y^n \approx \int_0^N x^n dx = \left[\frac{x^{n+1}}{n+1} \right]_0^N$$

$$= \frac{N^{n+1}}{n+1}$$

$$\sum_{y=1}^N y^n \approx \frac{N^{n+1}}{n+1}$$

$$E(\hat{N}_2) = N - \frac{1}{N^n} \sum_{y=1}^N y^n$$

$$\approx N - \frac{1}{N^n} \left(\frac{N^{n+1}}{n+1} \right)$$

$$\approx N - \frac{N}{n+1}$$

$$\approx \frac{nN + N - N}{n+1}$$

$$\approx \left(\frac{n}{n+1} \right) N$$

$$E\left(\frac{n+1}{n} \cdot \hat{N}_2\right) \approx N$$

$$\text{Let } \hat{N}_3 = \frac{n+1}{n} \cdot \hat{N}_2 = \frac{n+1}{n} \cdot Y_{(n)}^{\frac{1}{n}} (1+N)$$

$$E(\hat{N}_3) = E\left(\frac{n+1}{n} \cdot \hat{N}_2\right) \approx N$$

$\therefore \hat{N}_3$ is approximately unbiased (for) N .