University of Tunku Abdul Rahman
Faculty of Science
Session: 01/2020
UDPS2073 Mathematical Statistics

Test

Instruction:

Read the following before you start:

1. Fill in all your particulars on the attached answer scripts.
2. Generate Appendix 1 from Appendix1_UDPS2073text_[Your ID].Rmd according to the attached instruction manual.
3. Answer **all** questions in **55 minutes**. Marks carried by each question are indicated in brackets.
4. Submit your Appendix 1 and your answer script. Name your answer script as UDPS2073_Answer_[Your ID].pdf

Student's Name:____

Student's ID: ____

Question:

Q1. Suppose Y has a probability density distribution, $f(y)$ as stated in Appendix 1.

(a) Find the distribution function of Y. (4 marks)

(b) Show that $\frac{Y}{\theta}$ is a pivotal quantity. (4 marks)

Q2. Suppose X is a random variable from a binomial distribution with the parameters n and θ, the value of n is stated in Appendix 1.

(a) Prove the equation stated in Appendix 1 Question 2 (a) is true. Hence, show that $x \cdot \left(1 - \frac{x}{n}\right)$ is a biased estimator for the variance of X. (5 marks)

(b) Find the unbiased estimator for the variance of X based on the result in part (a). (3 marks)

Q3. Suppose that X_1, X_2, X_3 be a sample of size $n = 3$ from a distribution with unknown mean, $E(X_i) = \mu$, where the variance, $Var(X_i) = \sigma^2$ is a known positive number.

(a) Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ as shown in Appendix 1 are unbiased estimators for μ. (4 marks)

(b) Given X_i are independent. Compare the variance of $\hat{\theta}_1$ and $\hat{\theta}_2$ and conclude which estimator is more consistent. (5 marks)

APPENDIX 2

Distribution	Probability Mass Function / Probability Density Function	Mean	Variance	Moment – Generating Function
Binomial	$p(x) = {}^nC_x p^x (1-p)^{n-x}$, $x = 0, 1, \ldots, n$	np	$np(1-p)$	$\left[pe^t + (1-p)\right]^n$
Poisson	$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, \ldots$	λ	λ	$\exp\left[\lambda(e^t - 1)\right]$
Geometry	$g(x; p) = p\ q^{x-1};\quad x = 1,2,3,\ldots$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pe^t}{1-qe^t}$
Uniform	$f(x) = \frac{1}{\theta_2 - \theta_1}$, $\theta_1 \le x \le \theta_2$	$\frac{\theta_1 + \theta_2}{2}$	$\frac{(\theta_2 - \theta_1)^2}{12}$	$\frac{e^{t\theta_2} - e^{t\theta_1}}{t(\theta_2 - \theta_1)}$
Normal	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$, $-\infty < x < \infty$	μ	σ^2	$\exp\left(\mu t + \frac{t^2\sigma^2}{2}\right)$
Exponential	$f(x) = \frac{1}{\beta} e^{-x/\beta}$, $x > 0$	β	β^2	$(1-\beta t)^{-1}$
Gamma	$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}$, $x > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Chi – square	$f(x) = \frac{1}{\Gamma(\nu/2) 2^{\nu/2}} x^{(\nu/2)-1} e^{-x/2}$, $-\infty < x < \infty$	ν	2ν	$(1-2t)^{-\nu/2}$
Beta	$f(x) = \begin{cases} \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)}, & 0 < x < 1 \\ 0, & \textit{elsewhere} \end{cases}$ $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	