Name : Ngu Yi Hui
ID : 18ADB01438
Course : SCOR.

No: UDPS 2073

Assignment.

Q1. Let Y_1 = arrival time for the first person. It follows a uniform distribution over 1-hour period.

$f(y_1) = 1$, for $0 \le y_1 \le 1$, and 0 elsewhere.

Let Y_2 = arrival time for the second person. It follows a uniform distribution over 1-hour period.

$f(y_2) = 1$, for $0 \le y_2 \le 1$, and 0 elsewhere.

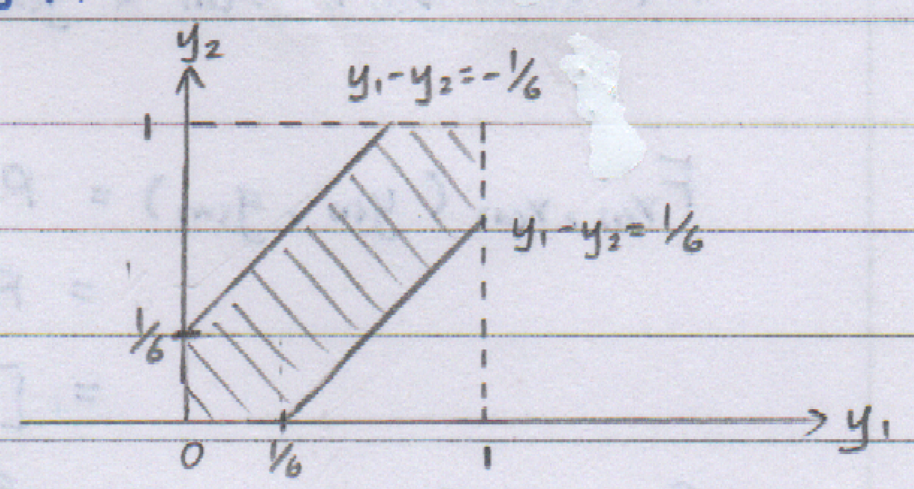
Since Y_1 and Y_2 are independent,

$f(y_1, y_2) = f(y_1) \times f(y_2) = 1$, for $0 \le y_1 \le 1$, $0 \le y_2 \le 1$, and 0 elsewhere.

Each agrees to wait a maximum of 10 minutes for the other.

So, they will meet if $|Y_1 - Y_2| \le 10/60$.

Probability that they will meet $= P(|Y_1 - Y_2| \le 1/6)$



$$P(|Y_1 - Y_2| \le 1/6) = 1 - 2 \times (1/2 \times 5/6 \times 5/6)$$
$$= 1 - 25/36$$
$$= 11/36$$

$\therefore$ Probability that they will meet $= 11/36$

Q2. Given $U = \frac{Y_1}{Y_1 + Y_2}$,

Let $Z = Y_1 + Y_2$.

$\rightsquigarrow$ $Y_1 = UZ$, $Y_2 = Z - UZ$

$\frac{dy_1}{du} = z$, $\frac{dy_1}{dz} = u$, $\frac{dy_2}{du} = -z$, $\frac{dy_2}{dz} = 1-u$

$$J = \begin{vmatrix} z & u \\ -z & 1-u \end{vmatrix} = z - zu + zu = z$$

Since Y_1 and Y_2 are independent,

$f(y_1, y_2) = f(y_1) \cdot f(y_2) = e^{-(y_1 + y_2)}$, for $y_1 > 0$, $y_2 > 0$, and 0 elsewhere.

$g(u, z) = e^{-z} \cdot |z| = ze^{-z}$, for $0 < u < 1$, $z > 0$,

and $h(u, z) = 0$ elsewhere.

Probability density of U,

$$h(u) = \int_0^\infty ze^{-z}\, dz$$
$$= -ze^{-z} - e^{-z} \Big|_0^\infty$$
$$= 0 - (-1)$$
$$= 1 \quad , \text{for } 0 < u < 1 ,$$

and $h(u) = 0$, elsewhere.

di		int
z	+	e^{-z}
1	−	$-e^{-z}$
0		e^{-z}