1. Differences among Cubic Spline, Lagrange, and Hermite Polynomial.

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| Cubic Spline Polynomial | Lagrange Polynomial | Hermite Polynomial |
| It forces the function f(x) to include all the data points over a close interval for x. | | |
| All the data points need to be sorted according to x-coordinate. | It is flexible, the data points do not need to be sorted according to x-coordinate. | It is flexible, the data points do not need to be sorted according to x-coordinate as it involves Lagrange coefficient for a data set given. |
| It divides the whole interval into sub-divisions to perform the piecewise polynomial approximation where the cubic polynomial is used for each successive pair of nodes. | It is the least accurate method compared with Cubic Spline Polynomial and Hermite Polynomial. | It is more accurate than Lagrange Polynomial, because its first derivative is equal to the first derivative of f(x) at all data points given. |
| It is more accurate when it is well-fitted by the polynomial over each sub-intervals because it can retain the shape of the curve. | It will be more accurate when higher degree of Lagrange Polynomial is performed as it includes more data points about the function. | It is complex to be performed even for small value of n. |

1. Compare Bisection Method, Newton Method, and False Position Method.

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| Bisection Method | Newton Method | False Position Method |
| It is a close method which narrows down an interval that contains a root of the continuous function f(x). | It is an open method as the iteration begins at an initial estimation (p0) which is close to the root for the continuous f(x). | It is a close method which requires the interval [a,b] to include the root of the continuous function f(x). |
| An initial interval [a,b] that contains a root is given. | It needs only one initial approximation which is close to the root (p). | It needs two initial approximation (p0,p1) which are close to the root (p), but the two initial approximation (p0,p1) cannot be too close to each other. |
| It cuts the interval into two halves, then make sure the next iteration is performed by the half interval that contains the root of the f(x). | It uses the help of tangent line to approximate the solution, thus the assumption of f’(xn)≠0 (n=0,1,2,…,n) must be met. | It make use of the secant line to find the solution, thus the condition f(p0)≠f(p1) must be met in the first iteration. |
| It is simple, but it converges relatively slow. | It usually converges faster than Bisection Method, but requires that model information providing the derivative exists. | Its convergence rate usually faster than Bisection Method, |
| It is not efficient because a good intermediate approximation might be overlooked. | It converges faster when f(x) is a straight line. Also, when the magnitude of the slope of the tangent line is larger, it converges faster. | It converges faster when the magnitude of the slope of the secant line is larger. |

1. How to compare ACCURACY for IVP?

We compare the accuracy of numerical methods by comparing the truncation error (TE), which is in term of O(h). Higher power of h will lead to higher accuracy.

Euler’s Method - O(h2)

Heun’s Method - O(h3) (more accurate than Euler’s Method)

Taylor Series Method of Order p - O(hp+1) (More accurate than Heun’s Method)

Runge-Kulta Method of Order 4 - O(h5)

Adam’s Method - O(h5) (Similar with RK4)

1. Shows Steps to approximate solution for IVP using Heun’s Method.

We obtain the f(x,y) = y’ and step size h given in the question.

First, we apply the predictor formula,

Then, we apply the corrector formula,

1. Find Approximate Solutions for sets of linear equations up to certain degree of accuracy (4 decimal places).

(Eg. Jacobi Method, Gauss-Seidel Method, SOR)

To check the CONDITION:

Strictly Row Diagonally Dominant, which means for each row, the diagonal element must greater than the sum of the absolute values of the rest of the elements of that row. (Eg. Write down: 4>1+1(row1)).

If not fulfilling the CONDITION:

Interchange the equations to make it follows “Strictly Row Diagonally Dominant” rule. Make sure they fulfil the CONDITION at last.

In this question, I use Gauss-Seidel Method.

Build formula:

Left hand side(x,y,z), Right hand side (algebra…)

Use k-1 is the previous one, k is the recent one. It will follow the updated one.

To stop iteration:

When for all unknowns, the accuracy smaller than 0.00005 (up to 4 decimal places).