Notes

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PART

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This is my note for some non-trivial but not systematic problems which involves some interesting physics or maths.

1.1 Walkway equilibrium

Suppose the mass of the objects attached to each end of the rope are m_1 and m_2, The angles between each segment of the rope, bended by the central object which has mass M, with the horizontal plane are θ and ϕ. The distance between two pulleys is L, and what we want to know is the vertical displacement d of the central object. Thus we can obtaind the equations for d when the system is at equilibrium.

$$L = d(\cot\theta + \cot\phi), \tag{1.1}$$
$$m_1 g\cos\theta = m_2\cos\phi, \tag{1.2}$$
$$m_1 g\sin\theta + m_2 g\sin\phi = Mg, \tag{1.3}$$

From (1.2), we have $\cos\phi = \frac{m_1}{m_2}\cos\theta$, thus (1.3) can be written as

$$m_1\sin\theta + m_2\sqrt{1 - \tfrac{m_1^2}{m_2^2}(1-\sin^2\theta)} = M, \tag{1.4}$$

such that we can solve for $\sin\theta$ and $\cos\theta$

$$\sin\theta = \frac{M^2 + m_1^2 - m_2^2}{2Mm_1}, \tag{1.5}$$
$$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{1}{2Mm_1}\sqrt{[(m_1+m_2)^2 - M^2][M^2 - (m_1-m_2)^2]}, \tag{1.6}$$
$$\cot\theta = \frac{\sqrt{[(m_1+m_2)^2 - M^2][M^2 - (m_1-m_2)^2]}}{M^2 + m_1^2 - m_2^2}, \tag{1.7}$$

together with $\sin\phi$ and $\cos\phi$

$$\cos\phi = \frac{m_1}{m_2}\cos\theta = \frac{1}{2Mm_2}\sqrt{[(m_1+m_2)^2 - M^2][M^2 - (m_1-m_2)^2]}, \tag{1.8}$$
$$\sin\phi = \sqrt{1-\cos^2\phi} = \frac{M^2 - m_1^2 + m_2^2}{2Mm_2}, \tag{1.9}$$
$$\cot\phi = \frac{\sqrt{[(m_1+m_2)^2 - M^2][M^2 - (m_1-m_2)^2]}}{M^2 - m_1^2 + m_2^2}. \tag{1.10}$$

Therefore we can plug into (1.1) and obtain the expression of d as follows

$$d = \frac{L[M^4 - (m_1^2 - m_2^2)^2]}{2M^2\sqrt{[(m_1+m_2)^2 - M^2][M^2 - (m_1-m_2)^2]}}. \tag{1.11}$$

The equilibrium condition in this case is

$$|m_1 - m_2| < M < (m_1 + m_2). \tag{1.12}$$

such that the argument under the square root is positive. Also we can easily check that if $m_1 = m_2 = m$ then this result reduces to our former result

$$d = \frac{LM}{2\sqrt{4m^2 - M^2}}. \tag{1.13}$$