Notes
Classical Electrodynamics

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Contents

Caprice

PART

I

1 Spring 2019

1.1 The angular integral in (4.16')

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Show the following equation is true

$$\int \mathrm{d}\Omega\, \mathbf{n} \cos\gamma = \frac{4\pi\mathbf{n}'}{3}, \tag{1.1}$$

where $\mathbf{n} = \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta$, and $\cos\gamma = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')$.

Proof. First look at the x component of the integral:

$$\begin{aligned}\int \mathrm{d}\Omega \sin\theta\cos\phi\cos\gamma &= \int \mathrm{d}\cos\theta\,\mathrm{d}\phi \sin\theta\cos\phi[cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi-\phi')] \\ (\text{integrate over } \phi) &= \pi\int \mathrm{d}\cos\theta\sin^2\theta\sin\theta'\cos\phi' \\ (u=\cos\theta) &= \pi\sin\theta'\cos\phi'\int_{-1}^{1}\mathrm{d}u\,(1-u^2) \\ &= \frac{4\pi}{3}\sin\theta'\cos\phi'.\end{aligned}$$

Similarly we can complete the proof. □

1.2 One tricky point on partial derivative below (5.108)

Just below (5.108), it claims $\partial r/\partial z = \cos\theta$, this partial derivative treats x and y as constant. What is $\partial z/\partial r$? If we use expression $z = r\cos\theta$, then it is easy to go to result $\partial z/\partial r = \cos\theta$. But we have $\partial r/\partial z = \cos\theta$ already, how could $\partial z/\partial r = \partial r/\partial z$? To resolve this "paradox", we should notice that when we do the partial derivative, we always need to specify what variables we keep as constant. In the first calculation $\partial r/\partial z = \cos\theta$, we keep x, y as constant. However, in $\partial z/\partial r = \cos\theta$ we treat θ, ϕ as constant. That's the reason why we have such inconsistent results.

1.3 Derivation of (6.27) and (6.28)

$$
\begin{aligned}
\mathbf{J_l} &= \int \mathrm{d}^3x'\delta(\mathbf{x}-\mathbf{x}')\mathbf{J_l}\\
&= -\frac{1}{4\pi}\int \mathrm{d}^3x'\nabla'^2\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)\mathbf{J_l}\\
&= -\frac{1}{4\pi}\int \mathrm{d}^3x'\boldsymbol{\nabla}'\cdot\left[\boldsymbol{\nabla}'\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)\mathbf{J_l}\right] + \frac{1}{4\pi}\int \mathrm{d}^3x'\boldsymbol{\nabla}'\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)(\boldsymbol{\nabla}'\cdot\mathbf{J_l})\\
&= -\frac{1}{4\pi}\int \mathrm{d}^3x'\boldsymbol{\nabla}\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)(\boldsymbol{\nabla}'\cdot\mathbf{J_l})\\
&= -\frac{1}{4\pi}\boldsymbol{\nabla}\int \mathrm{d}^3x'\frac{\boldsymbol{\nabla}'\cdot\mathbf{J_l}}{|\mathbf{x}-\mathbf{x}'|}\\
&= -\frac{1}{4\pi}\boldsymbol{\nabla}\int \mathrm{d}^3x'\frac{\boldsymbol{\nabla}'\cdot(\mathbf{J_l}+\mathbf{J_t})}{|\mathbf{x}-\mathbf{x}'|}\\
&= -\frac{1}{4\pi}\boldsymbol{\nabla}\int \mathrm{d}^3x'\frac{\boldsymbol{\nabla}'\cdot\mathbf{J}}{|\mathbf{x}-\mathbf{x}'|}.
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{J_t} &= \int \mathrm{d}^3x'\delta(\mathbf{x}-\mathbf{x}')\mathbf{J_t}\\
&= -\frac{1}{4\pi}\int \mathrm{d}^3x'\nabla^2\left(\frac{1}{|\mathbf{x}-\mathbf{x}'|}\right)\mathbf{J_t}\\
&= -\frac{1}{4\pi}\int \mathrm{d}^3x'\left[-\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\left(\frac{\mathbf{J_t}}{|\mathbf{x}-\mathbf{x}'|}\right)\right)+\nabla^2\left(\frac{\mathbf{J_t}}{|\mathbf{x}-\mathbf{x}'|}\right)\right]\\
&= \frac{1}{4\pi}\int \mathrm{d}^3x'\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\left(\frac{\mathbf{J_t}}{|\mathbf{x}-\mathbf{x}'|}\right)\\
&= \frac{1}{4\pi}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\int \mathrm{d}^3x'\frac{\mathbf{J}}{|\mathbf{x}-\mathbf{x}'|}
\end{aligned}
$$