Notes

Analysis

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This is my note for analysis. The textbook I use are Stein's classic four volumes on analysis.

1.1 Cauchy criterion

Definition 1.1. If (a_n) is a sequence, then $\lim_{n\to\infty} a_n = L$ means that for every $\varepsilon > 0$ there exists a corresponding $N \in \mathbb{N}^+$ such that if $n \geq N$, then $|a_n - L| < \varepsilon$. If this limit exists, then we say that the sequence (a_n) converges, and if this limit doesn't exists then we say the sequence (a_n) diverges.

Proposition 1.1. A converging sequence of $\mathbb{R}$ has a unique limit.

Proof. Suppose both L_1 and L_2 are limits of (a_n) and $L_1 \neq L_2$, which means for any $\varepsilon > 0$, there exists $N_1 \in \mathbb{N}^+$ such that

$$|a_n - L_1|, \quad \text{for every} \quad n \geq N_1; \tag{1.1}$$

and for any $\varepsilon > 0$, there exists $N_2 \in \mathbb{N}^+$ such that

$$|a_n - L_2|, \quad \text{for every} \quad n \geq N_2. \tag{1.2}$$

Since $L_1 \neq L_2$, we can choose $\epsilon = |L_1 - L_2|/4$, $N = \max\{N_1, N_2\}$, then

$$\begin{aligned} |L_1 - L_2| &= |(a_n - L_2) + (L_1 - a_n)| \\ &\leq |a_n - L_1| + |a_n - L_2| \\ &\leq 2\varepsilon = |L_1 - L_2|/2, \quad \text{for every} \quad n \geq N, \end{aligned}$$

which can be true only if $L_1 = L_2$, and this contradicts to the assumption $L_1 \neq L_2$. Therefore we prove $L_1 = L_2$ by contradiction. □