Singapore Smart City: Predictive Maintenance of Public Housing using Markoc Chain Algorithms

Chen Yijia, Gao Yihe, Sun Yingzi
National Junior College

Abstract

The Singapore government aims to realise predictive maintenance in public housing by the end of 2025 - Artificial Intelligence technology is key to success. In this paper, a Markov Chain based algorithm for predictive maintenance of public housing is introduced and explained in detail with fabricated data. Concepts related to Markov Chain such as transition probability, transition (probability) matrix, eigenvalues and eigenvectors are applied. In the end, a more complicated Markov chain Monte Carlo algorithm will be briefly mentioned to address some limitations of our proposed algorithm.

Predictive maintenance and smart city plan

Singapore is making great strides forward to transform the country into a Smart City, which is largely driven by the use of Artificial Intelligence (AI) technology. With 78.7% of its residents living in public housing, the government is working on the maintenance of public housing to enhance citizen well-being. By 2025, sensors and AI algorithms are to be potentially used for predicting maintenance requirements of public housing. (Shamini Priya, 2019) This would decrease infrastructure and service downtime, as well as the number of major repairs. As a result, predictive maintenance ensures better conservation and increased life expectancy of assets. Residents' welfare is therefore improved.

Sensors and AI algorithms are necessary in order to achieve the goal of predictive maintenance. In real life, the future conditions, or states, of utilities in a building are only dependent on the present states but independent of previous states. A Markov chain can thus be applied, and Markov-chain based algorithms facilitate the prediction of potential defects in public housing.

Markov chain in discrete time

A *Markov process* is a random process indexed by time with the property that the future is only dependent on the present. The complexity of Markov processes depends greatly on whether the time space is discrete continuous, and whether the state space is discrete or a more general topological space. When the state space is discrete, Markov processes are known as *Markov chains*. (N.A., N.D.)

Modelling with Markov chain

Consider the states of utilities in a building in discrete time and space. The current condition of a utility can be determined by its extent of wear and tear, which is further divided into three *states*: mild (State A), medium (State B) and severe (State C) wear and tear. To classify the working condition of a utility into three discrete states, sensors are needed to obtain parameter values including surface corrosion of the object, power efficiency of internal electric circuit or vibration frequency in an ambient test etc.

State	Wear and Tear	Description of characteristics
A	Mild	The utility is almost new and working generally well.
B	Medium	The utility still functions but it may become faulty and need repairment or replacement soon
C	Severe	There is high risk of utility breakdown in the building

Table 1. Classification of three states in Markov process

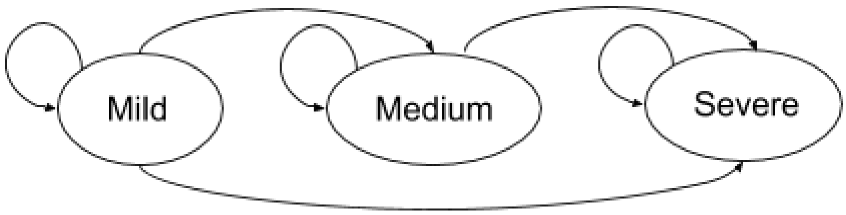

Figure 1. Transition between three states of utilities in public housing

The transitions between states are illustrated in Figure 1. Arrows in the figure indicate the transitions from one state to another. For example, a utility (such as a staircase) in mild wear-and-tear condition is largely working well. Its next state is very likely to be the same as the current state. It may also go into the medium wear-and-tear state but less likely. Rarely, the utility that is functioning well will suddenly become severely impaired in the next moment. Therefore, each transition from one state to another carries different *transition probabilities*.

Transition probabilities and transition matrices

In our predictive maintenance model, the *initial state matrix* shows the current possibility of a building falling into different states, which is determined by the data we gathered from sensors. It can be represented by a 1x3 matrix S_0 as shown below:

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ S_0 = & \begin{bmatrix} P_A & P_B & P_C \end{bmatrix} \end{array}$$

The *transition matrix* representing the possibility of transition from one state to another is shown by a 3x3 matrix *T*. The labels on the right of the matrix represent the current state of the building and the labels on top represent the next state which the building is possibly transiting to.

$$\begin{array}{ccc} & \begin{array}{ccc} A & B & C \end{array} & \\ T = & \begin{bmatrix} P_{AA} & P_{AB} & P_{AC} \\ P_{BA} & P_{BB} & P_{BC} \\ P_{CA} & P_{CB} & P_{CC} \end{bmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

For example, the number P_{AB} on the first row and second column is the possibility of a mild wear and tear (State A) building transiting to medium wear and tear (State B). The number P_{cc} on the third row and third column represents the possibility of a building staying in the severe wear and tear condition.

Application of the model

Consider a public toilet in a building with initial state matrix S_0 and transition matrix T

$$S_0 = \begin{bmatrix} 0.60 & 0.35 & 0.05 \end{bmatrix}$$

$$T = \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{bmatrix}$$

We can see that the toilet is 60% likely to be in mild condition at the initial state. Multiply S_0 and T, we get

$$S_1 = S_0 T = [0.60\ 0.35\ 0.05] \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{bmatrix} = [0.42\ 0.38\ 0.20]$$

The toilet is still most likely to be in mild condition, but only with a probability of 42%, much lower than the first time. Since we multiply the same transition matrix again and again, to make the calculation process easier, we can diagonalise matrix T by calculating its eigenvalues λ_1, λ_2, λ_3 and corresponding eigenvectors $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$.

Let $T = QDQ^{-1}$, where

$$Q = \begin{bmatrix} v_1 \mid v_2 \mid v_3 \end{bmatrix}, \text{ where } v_1, v_2, v_3 \text{ are all } 3 \times 1 \text{ matrices}$$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

Thus,

$$\begin{aligned} S_n &= S_0 T^n \\ &= \begin{bmatrix} 0.60 & 0.35 & 0.05 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.70 & 0 \\ 0 & 0 & 0.75 \end{bmatrix}^n \begin{bmatrix} 1 & 1 & 4 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 1 & 0.6 & 2.75 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.70^n & 0 \\ 0 & 0 & 0.75^n \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & -4 & 3 \\ 0 & 1 & -1 \end{bmatrix} \end{aligned}$$

Below shows values of S_n for n = 1, 2, 3, …

n	Probability of falling in State A	Probability of falling in State B	Probability of falling in State C
1	0.42	0.38	0.20
2	0.29	0.37	0.34
3	0.21	0.34	0.46
4	0.14	0.29	0.56
5	0.10	0.25	0.65
6	0.07	0.21	0.72
7	0.05	0.17	0.78
8	0.03	0.14	0.83
	… …		
n	0	0	1

Table 2. Probabilities of the utility in different states

From Table 2, we can deduce that at least one maintenance is needed when n = 2, at which time the utility is mostly likely to be in State B with medium wear and tear. Without repair or replacement of certain components of toilet, it is likely to become not usable in 4 years as the possibility of state C increases to the highest (46%) among all 3 states. In 5 years and later, it is very likely that the toilet will breakdown as the possibility of toilet in state C (56%) not only is the highest but also exceeds 50%.

Also, when n approaches infinity, S_n approaches [0 0 1]. It also shows an example of *absorbing Markov process*, in which every state can reach an absorbing state. An *absorbing state* is a state that, once entered, cannot be left**.** In this model, State C is the absorbing state since a faulty utility can by no means go back to State A or B.

Based on the results of the possibility of the toilet in different states, the government can make a plan of the maintenance of the toilet ahead to reduce the downtime of it and also possibly the number of repairs and cost needed. This would increase the welfare of residents and ensure their safety.

Conclusion

In conclusion, a Markov chain related algorithm can be applied to achieve predictive maintenance in public housing. However, before the creation of such an algorithm, a big data base is needed to derive the transition matrix, and the transition matrix may not be fixed in real life.

In view of the limitations, a Markov Chain Monte Carlo algorithm can be applied Similar to Markov chain in discrete time. In the paper named ‘*Bayesian model updating of a coupled-slab system using field test data utilizing an enhanced Markov chain Monte Carlo simulation algorithm*’ (Lam et. al, 2015), a more sophisticated Markov Chain Monte Carlo algorithm is presented. It is based on Bayes’ theorem and posterior probability density function (PDF) in the form of

$$p(\mathbf{x}|\mathbf{D}) = \frac{p(\mathbf{D}|\mathbf{x})\, p(\mathbf{x})}{p(\mathbf{D})}$$

where $p(\mathbf{x})$ is the prior PDF, and $1/p(\mathbf{D})$ is the normalizing constant such that the integration of the posterior PDF over the parameter space always equals to unity. Details of derivations formulae and construction of algorithm is elaborated in the paper.

Bibliography

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