Mathematics and Economics

Analysis of stock investment: companies and investors, risk and return

Chen Yijia
National Junior College

Abstract

A portfolio investment is a passive investment of securities, such as stocks and bonds. (James Chen, 2018) This essay is going to look specially into investment of stocks, from the angles of both stock sellers and investors. Companies make selling stocks profitable by carefully observing the demand and price of the stocks. For investors, expected return and risk are commonly concerned. At the end of this essay, a bilateral game played by investor and stock market is revealed. All we can do is to predict how the market will behave next, while the most exciting thing about the stock market is its unpredictability. (100 words)

Essay

Part I: How do companies benefit from selling stocks

When a business grows to the point that it is ready to go public or needs more funds for expansion or improvement, it sells its stocks. (Thomas Metcalf) To make sales profitable, companies have to carefully control the supply of stocks.

Price, supply and demand

The relationship among demand, supply and price of a stock is shown in figure 1:

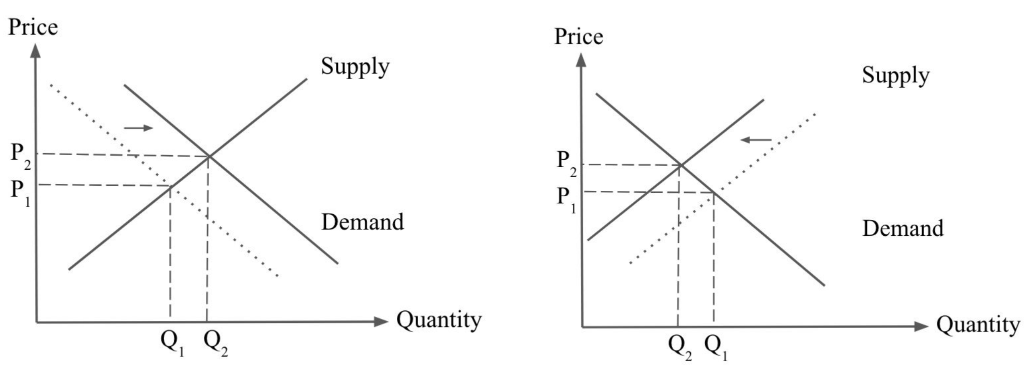

figure 1: how demand, supply and the price of a stock are related

As you can see, the three elements are closely related. Let us look at a simple example of recent data of Apple Inc (AAPL) in the US market:

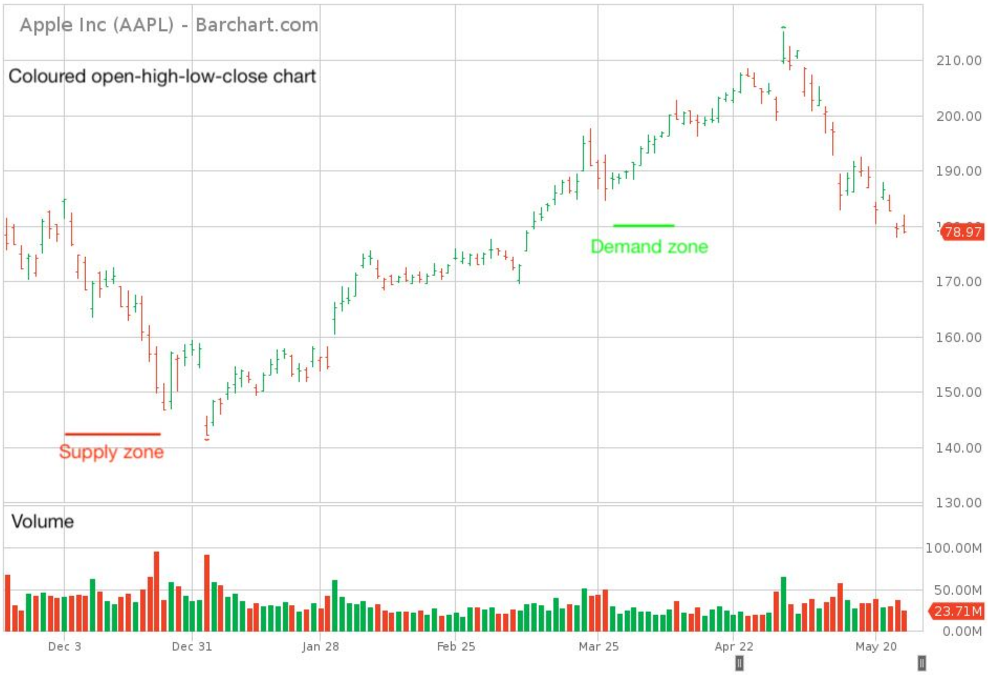

figure 2: coloured OHLC and volume charge of AAPL indice from 11/20/2018 to 24/5/2019, retrieved from Barchart.com on 25th May.

Stock volume is the total amount of trade happening on a particular day. A red volume bar means that the stock closed lower on that day compared to the previous day's close. In other words, it means that more stock were sold by the company than bought by investors. Fourteenth December to 24th December is clearly a *supply zone* and 27th March to 8th April is a typical *demand zone*. More buying than selling of stocks drove the price of the stock up.

Part II: What are the considerations investors need to take

<u>Expected return</u>

Expected return is the profit or loss an investor anticipates on an investment. Expected return is usually represented by a *probability density function*, and the numerical value of expected return is represented by the area bounded by the graph and x-axis, obtaining via *integration*.

The distribution of stock returns is described as *log-normal shape*. The formula of calculating a log normal function is given below:

$$f(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2}\left(\frac{ln(x)-\mu}{\sigma}\right)^2},$$

where μ is the mean of all data and σ is the standard deviation.

Here is an example of such distribution (positive return rates only):

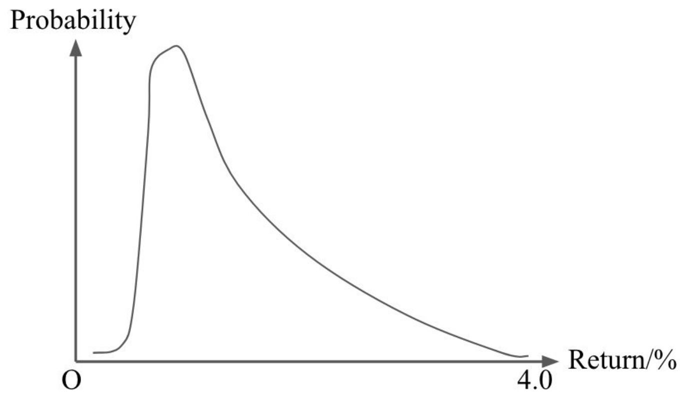

figure 3: example of probability density function in a right-skewed log-normal distribution shape

We can calculate the *mean* and *standard deviation* of returns within a certain period and apply the log-normal function formula. This will give us a graph of similar shape as *figure 3.* However, a log-normal distribution cannot show negative returns (negative x-axis values) as log function is only defined for positive x. So what is the case when the average of returns is negative? Let us use Straits Times Index (STI.) as an example and try to apply the method above:

Date	Open	High	Low	Close*
May 24, 2019	3,152.08	3,172.72	3,149.97	3,169.89
May 01, 2019	3,389.52	3,397.18	3,149.97	3,169.89
Apr 01, 2019	3,229.11	3,415.18	3,227.62	3,400.20
Mar 01, 2019	3,210.84	3,251.72	3,156.79	3,212.88
Feb 01, 2019	3,194.22	3,286.08	3,174.00	3,212.69
Jan 01, 2019	3,072.99	3,250.27	2,993.42	3,190.17
Dec 01, 2018	3,154.22	3,192.88	3,000.45	3,068.76
Nov 01, 2018	3,045.68	3,132.42	3,007.31	3,117.61
Oct 01, 2018	3,262.43	3,272.88	2,955.68	3,018.80
Sep 01, 2018	3,209.97	3,265.01	3,102.73	3,257.05
Aug 01, 2018	3,331.05	3,347.98	3,187.83	3,213.48
Jul 01, 2018	3,277.43	3,341.42	3,176.26	3,319.85
Jun 01, 2018	3,423.50	3,492.34	3,237.77	3,268.70

figure 4: OHLC table of Straits Times Index from June 2018 to May 2019, retrieved from Yahoo Finance on 25th May.

By applying the following relationship, Return = (Ending price - Starting price) / Starting price * 100%, we get:

Date	Return
Jun 01, 2018	(3,268.70-3,423.50)/3,423.50*100% = -4.52%
Jul 01, 2018	(3,319.85-3,277.43)/3,277.43*100% = +1.29%
Aug 01, 2018	-3.53%
Sep 01,2018	+1.47%
Oct 01, 2018	-7.47%
Nov 01, 2018	+2.36%
Dec 01,2018	-2.71%
Jan 01, 2019	+3.81%
Feb 01,2019	+5.78%
Mar 01,2019	+0.06%
Apr 01,2019	+5.30%
May 01,2019	-6.48%

Table 1: calculation of returns of STI. in each month from June 2018 to May 2019

The *mean* and *standard deviation* are:

Mean = (-5.42+1.29-3.53+1.47-7.47+2.36-2.71+3.81+5.78+0.06+5.30-6.48)/12 = -0.462

Standard deviation = 4.38.

The final graph looks as follows:

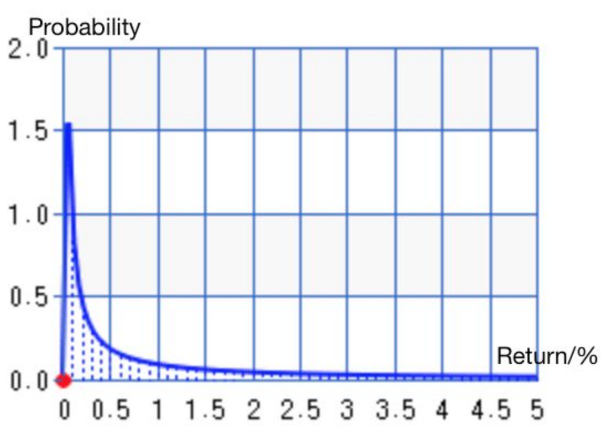

figure 5: probability distribution of returns of STI.

This graph shows a completely different shape, and the value does not match. This is caused by a relatively large standard deviation and limitations of log-normal function. Thus a new strategy is needed.

Another property of the stock market is that it is operated autonomously and its state is unobservable, while only the output, which is namely the profit or loss, is visible. In this case, a *hidden Markov model* (HMM) can be used to predict the future behaviour of a stock based only on the previous state. A general function of HMM can be written as:

$$f(b) = \Sigma^{S}_{y1=1}\, \Sigma^{S}_{y2=1} \ldots \Sigma^{S}_{yT=1}\, f(y_1,\ldots,y_T)\, f(b;\, y_1,\ldots,y_T),$$

where $f(y_1,\ldots,y_T) = f(y_1)\mathbf{\Pi}^{T}_{t=2} f(y_t \mid y_{t-1})$ and $f(b;\, y_1,\ldots,y_T) = \mathbf{\Pi}^{T}_{t=1} f(b_t \mid y_t)$, y_t is the *discrete variable*, b_t is the *latent variable* at y_t and S is the *latent states* of each latent variable. (Luca De Angelis, Leonard J. Paas, 2010)

In the stock market context, let us firstly get a clearer idea of what are the different components in this Markov model. The *observable variables*, or the *discrete variables*, are previous returns of stocks. The *hidden variables*, or the *latent variables*, are the continuous hidden behaviour of the stock. (“Behaviour” is loosely used here, as it can refer to the fluctuations of the stock or the future prospects of the stock.) The behavior of the stock can be characterised by the *probability* of different values of returns. So the latent state (S) is equal to one as we only consider the probability of each return.

Let b_t denote the probability of a particular return at time t (t = 1,2,..., T) and y_t denote the discrete latent variables, or known returns. So $f(b)$ will be the function of expected return:

$$\begin{aligned} f(b) &= f(y_1,\ldots,y_T)\, f(b;\, y_1,\ldots,y_T) \\ &= f(y_1)\mathbf{\Pi}^{T}_{t=2} f(y_t \mid y_{t-1}) \cdot \mathbf{\Pi}^{T}_{t=1} f(b_t \mid y_t) \end{aligned}$$

Since we calculate return rates by taking the profit or loss over the just previous return, and probability of return is independent of the value of return, we get:

$$f(b) = f(y_t \mid y_{t-1}) \cdot f(b_t),$$

which is the expected return at point t.

This expression now looks very familiar and it actually shows the weighted average of different return values. So we can use the formula for *expected value* and get another expression that conveys the same idea:

$$f(b) = \Sigma^{T}_{n=1}\, f(y_n)\, f(b_n).$$

Applying this equation, it clearly gives a more accurate and reliable expected return than simply looking at a log-normal graph. However, to use this model, a large number of data is needed and it is usually done by computational work.

Risk assessment

Standard deviation is usually used as an indicator of risk. Besides, investment and commercial banks use *value-at-risk* (*VaR*) to determine the extent and occurrence ratio of potential losses in their institutional portfolios. It is done through *Monte Carlo analysis* in which computational algorithms will be run multiple trials using repeated random sampling.

Part III: How do companies and investors play the stock management game

A hypothetical stock management game is played by at least two sides: the investor and the stock market. The best strategy for both sides is revealed by game theory, which is prevalently used in portfolio management.

Nash Equilibrium

Nash equilibrium is namely the optimal outcome of a game in which no participant can gain by a unilateral change of strategy if the strategies of the others remain unchanged. In one game, two stocks, Dow Jones Industrial (DOW J) and AAPL, have actual return rates of 25.08% and 46.11% respectively in 2017, -5.63% and -6.79% respectively in 2018:

Investor	Return in 2017 /%	Return in 2018 /%
DOW J	25.08	-5.63
AAPL	46.11	-6.79

table 2: actual return rates of DOW J and AAPL in two consecutive years

Assuming that the market is predictable, the payoff of stock market is the opposite of the investor:

Investor \ Market	Return in 2017 /%	Return in 2018 /%
DOW J	25.08, -25.08	-5.63, 5.63
AAPL	46.11, -46.11	-6.79, 6.79

table 3: an investment game played by the investor and the stock market

As we can see, if the market chooses Return in 2017, the investor has better to choose AAPL as it gives a higher return. If the market chooses Return in 2018, the investor has better to choose DOW J instead.
Also, if the investor chooses DOW J, the market will choose Year 2017, and if the investor chooses AAPL, the market will choose Year 2018.

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