

MAT2041: Linear Algebra and Applications  
Autumn 2022  
Tutorial 1

Student ID:

Student Name:

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**Exercise**

1. Prove the following properties of scalar product:

(i.) If  $a$  is a vector and  $\beta$  and  $\gamma$  are scalars, we have

$$(\beta\gamma)a = \beta(\gamma a)$$

(ii.) If  $a$  is a vector and  $\beta, \gamma$  are scalars, then

$$(\beta + \gamma)a = \beta a + \gamma a$$

(iii.) When scalar multiplication is written with the scalar on the right, we have the right-distributive property:

$$a(\beta + \gamma) = a\beta + a\gamma$$

(iv.) Scalar-vector multiplication also satisfies another version of the right-distributive property:

$$\beta(a + b) = \beta a + \beta b$$

for any scalar  $\beta$  and any  $n$ -vectors  $a$  and  $b$ . In this equation, both of the  $+$  symbols refer to the addition of  $n$ -vectors.

2. *Total score from course record.* The record for each student in a class is given as a 10 vector  $r$ , where  $r_1, \dots, r_8$  are the grades for the 8 homework assignments, each on a 0–10 scale,  $r_9$  is the midterm exam grade on a 0 – 120 scale, and  $r_{10}$  is final exam score on a 0 – 160 scale. The student's total course score  $s$ , on a 0 – 100 scale, is based 25% on the homework, 35% on the midterm exam, and 40% on the final exam. Express  $s$  in the form  $s = w^\top r$ . (That is, determine the 10-vector  $w$ .)
3. **(Optional)** *Word count and word count histogram vectors.* A vector of length  $n$  can represent the number of times each word in a dictionary of  $n$  words appears in a document. For example,  $(25, 2, 0)$  means that the first dictionary word appears 25 times, the second one twice, and the third one not at all. (Typical dictionaries used for document word counts have many more than 3 elements.)

Suppose the  $n$ -vector  $w$  is the word count vector associated with a document and a dictionary of  $n$  words. For simplicity we will assume that all words in the document appear in the dictionary.

- (i.) What is  $\mathbf{1}^\top w$  ?
- (ii.) What does  $w_{282} = 0$  mean?
- (iii.) Let  $h$  be the  $n$ -vector that gives the histogram of the word counts, i.e.,  $h_i$  is the fraction of the words in the document that are word  $i$ . Use vector notation to express  $h$  in terms of  $w$ . (You can assume that the document contains at least one word.)