CS 5100 – PS9

Yijing Xiao

Problem 1: Bayesian Networks for Astronomy

a) **Based on the problem description, what independence / conditional independence assumptions should hold between the five variables?**

According to the description, we could know that we could have M1 by given F1 and N, and could have M2 by given F2 and N. The number of stars is totally not influenced by telescope focus and the probability of small measurement error. Moreover, if the telescope is out focus, the measurements would be different. Therefore:

F1 is independence of N, but F1 is not conditional independence of N given M1.

F2 is independence of N, but F2 is not conditional independence of N given M2.

M1 and M2 both are not independent.

b) **Using your answer to part (a), which of the three Bayesian networks shown in Figure 1 above is best for modeling this problem? Justify your answer.**

Figure (i) is not correct because the figure has F1 conditional independence of N given M1.

Figure (ii) and (iii) both are correct, but figure (ii) is the best one since it represents the causal structure correctly and it is not more complicated.

c) **Suppose M1 = 1 and M2 = 3. What are the possible numbers of stars if you assume no prior constraint on the values of N?**

The two measurement results are different which means that one of the telescopes may have out of focus. In this case, the scientist will undercount by three or more stars. If we assume that F2 is true, there still has some small probability e of error by up to one star. Therefore, we have different situations here:

i. <u>Case 1: F1 and F2 both are true and M1 see one fewer star and M2 see one more star: N = 2</u>
ii. <u>Case 2: F1 is false and F2 is true and M2 see one fewer star and M1 undercount by three: N = 4</u>
iii. <u>Case 3: F1 and F2 both are false and both M1 and M2 are undercount (by three or bigger): N ≥ 6</u>

Hence, N could be 2, 4 or bigger than 6.

d) **What is the most likely number of stars, given these observations? Explain how to compute this mathematically, and what additional information is needed (if any).**

If I don't know the distribution of P(N), I don't think I could calculate mathematically; the only way for now is inference by using observation. According to our analysis on question (c), we could know that the probability e of error and the probability of telescope out of focus f would influence values of N. Since the telescope can with a much smaller probability f which means $f \gg e$, The probability of case 2 and 3 would be smaller than the probability of case 1, in other words F1 and F2 both are true and both M1 and M2 has probability of error. Therefore, N = 2 is the most likely number of stars.

e) **For the remainder of the problem, consider the network from Figure 1(ii), and suppose N ∈ {1, 2, 3}, M1, M2 ∈ {0, 1, 2, 3, 4}. Suppose each value of N is equally likely. Write out the conditional probability tables (CPTs) for all nodes in the network: P (F), P (N), P (M | N, F).**

If N is less than 3, if the telescope is out of focus, then we fail to detect any stars at all. Assume that the probability of telescope is out of focus is f, and the probability of telescope is not out of focus is (1-f). Since the astronomer may see an additional star with probability e and may see one fewer star with probability e, the probability of no error is (1-2e).

The conditional probability tables (CPTs) in the network:

	N = 1		N = 2		N = 3	
	F1	$\overline{F1}$	F1	$\overline{F1}$	F1	$\overline{F1}$
M1 = 0	f	e(1-f)	f	0	f	0
M1 = 1	0	(1-2e)(1-f)	0	e(1-f)	0	0
M1 = 2	0	e(1-f)	0	(1-2e)(1-f)	0	e(1-f)
M1 = 3	0	0	0	e(1-f)	0	(1-2e)(1-f)
M1 = 4	0	0	0	0	0	e(1-f)

F1, M1, N and F2, M2, N has identical table, and both are same.

f) Using variable elimination and the CPTs computed in the previous part, calculate the probability distribution P (N |M1 = 2, M2 = 3).

P (N = 1 | M1 = 2, M2 = 3) = e(1-f) * 0 = 0

P (N = 2 | M1 = 2, M2 = 3) = (1-2e)(1-f) * e(1-f)

P (N = 3 | M1 = 2, M2 = 3) = e(1-f) * (1-2e)(1-f)

P(N) = P (N = 1 | M1 = 2, M2 = 3) + P (N = 2 | M1 = 2, M2 = 3) + P (N = 3 | M1 = 2, M2 = 3) = 0 + (1-2e)(1-f) * e(1-f) + e(1-f) * (1-2e)(1-f) = $2e(1\text{-}f)^2(1\text{-}2e)$