CS 5100 – PS8

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Problem 1: Feature representation for approximate Q-learning

Explain why your choice of features and weights gives an exact representation. Additionally, using your features and weights, show that the two learning rules shown above are equivalent.

I chose the distance to the nearest ghost and the distance to the nearest food as features because they are functions from states to real numbers that capture important properties of the state.

Let's say the feature of the distance to the nearest ghost is f_g and the distance to the nearest food is f_p, then we would have $Q(s,a) = w_g\, f_g(s,a) + w_p f_p(s,a) \approx \hat{Q}(s,a;\, w)$

Problem 2: Viral testing

Which test returning positive is more indicative of someone really carrying the virus?

Since the virus is carried by 1% of all people, we could say P (virus) = 0.01. Test A is 95% effective at recognizing the virus when it is present which means that P(A | virus) = 0.95, however it has a 10% false positive rate which means that P (A | $\overline{\text{virus}}$) = 0.1. Test B is 90% effective at recognizing the virus which means that P (B | virus) = 0.9, but has a 5% false positive rate which means that P (B | $\overline{\text{virus}}$) = 0.05.

First, we need to calculate the probability of virus given by A P (virus | A) and the probability of virus given by B P (virus | B):

$P(A, virus) = P(A|virus)P(virus) = 0.95 * 0.01 = 0.0095$. Since $P(A) = P(A, virius) + P(A, \overline{virus}) = 0.0095 + 0.1 * (1 - 0.01) = 0.1085$, we have $P(virus \mid A) = \frac{P(A,virus)}{P(A)} = \frac{0.0095}{0.1085} = 0.0876$. Similarly, $P(B, virus) = P(B|virus)P(virus) = 0.9 * 0.01 = 0.009$. Since $P(B) = P(B, virius) + P(B, \overline{virus}) = 0.009 + 0.05 * (1 - 0.01) = 0.0585$, we have $P(virus \mid B) = \frac{P(B,virus)}{P(B)} = \frac{0.009}{0.0585} = 0.1538$.

$P(virus \mid A) = 0.0876, P(virus \mid B) = 0.1538$ therefore, the test B is more indicative.

Extra credit: Crime scene investigation

a) Given this information, is it possible to calculate the most-likely color for the taxi? Justify your answer.

Define that a taxi is real blue as B and a taxi looks like blue which means it might be not blue as b. If under the dim lighting conditions, discrimination between blue and green is

75% which means P (b | B) = 0.75. If a taxi was green, it appears green 75% which means P ($\bar{b}$ | $\bar{B}$) = 0.75.

If we want to know the hit-and-run accident taxi is blue, our goal is to calculate the probability of blue given by it looks like blue P (B | b)

$$P(B \mid b) = \frac{P(B, b)}{P(b)} = \frac{P(b \mid B)\, P(B)}{P(b)} = \frac{0.75\, P(B)}{P(b)}$$

$$P(\bar{B} \mid b) = \frac{P(\bar{B}, b)}{P(b)} = \frac{P(b \mid \bar{B})\, P(\bar{B})}{P(b)} = \frac{0.25(1 - P(\bar{B}))}{P(b)}$$

Since we don't know P (B) (we could know P (b) if we know P (B)), we cannot decide the probability.

b) How would your answer to part (a) change if you know that 9 out of 10 Athenian taxis are green? Justify your answer.

If 9 out of 10 Athenian taxis are green which means the P (B) = 0.1 and P (b) = P (b, B) + P (b, $\bar{B}$) = P (b | B) P (B) + P (b | $\bar{B}$) P ($\bar{B}$) = 0.75 * 0.1 + 0.25 * 0.9 = 0.075 + 0.225 = 0.3. Therefore, $P(B \mid b) = \frac{0.75\, P(B)}{P(b)} = \frac{0.75 * 0.1}{0.3} = 0.25$ and $P(\bar{B} \mid b) = \frac{0.25\,(1 - P(B))}{P(b)} = 0.75$