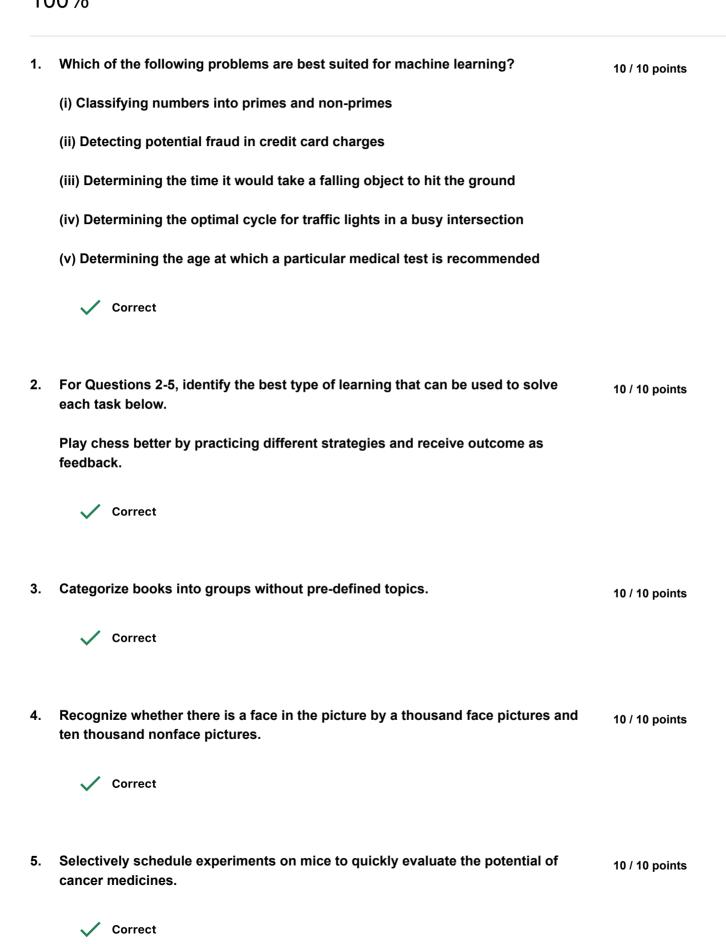
## 作業一

## **LATEST SUBMISSION GRADE**

## 100%



Let  $\mathcal{X} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N, \mathbf{x}_{N\!+\!1}, \dots, \mathbf{x}_{N\!+\!L}\}$  and  $\mathcal{Y} = \{-1, +1\}$  (binary classification). Here the set of training examples is  $\mathcal{D} = \left\{ (\mathbf{x}_n, y_n) \right\}_{n=1}^N$ , where  $y_n \in \mathcal{Y}$ , and the set of test inputs is  $\left\{ \mathbf{x}_{N\!+\!\ell} \right\}_{\ell=1}^L$ . The Off-Training-Set error (OTS) with respect to an underlying target f and a hypothesis g is

$$E_{OTS}(g,f) = \frac{1}{L} \sum_{\ell=1}^{L} \left[ \left[ g(\mathbf{x}_{NH\ell}) \neq f(\mathbf{x}_{NH\ell}) \right] \right].$$

Consider  $f(\mathbf{x}) = +1$  for all  $\mathbf{x}$  and

$$g(\mathbf{x}) = \begin{cases} +1, & \text{for } \mathbf{x} = \mathbf{x}_k \text{ and } k \text{ is odd and } 1 \le k \le N + L \\ -1, & \text{otherwise} \end{cases}.$$

 $E_{OTS}(g,f)$  =? (Please note the difference between floor and ceiling functions in the choices)



7. We say that a target function f can "generate"  $\mathcal{D}$  in a noiseless setting if  $f(\mathbf{x}_n) = y_n$  for all  $(\mathbf{x}_n, y_n) \in \mathcal{D}$ .

10 / 10 points

For all possible  $f \colon \mathcal{X} \to \mathcal{Y}$ , how many of them can generate  $\mathcal{D}$  in a noiseless setting?

Note that we call two functions  $f_1$  and  $f_2$  the same if  $f_1(\mathbf{x}) = f_2(\mathbf{x})$  for all  $\mathbf{x} \in \mathcal{X}$ .



8. A determistic algorithm  $\mathcal A$  is defined as a procedure that takes  $\mathcal D$  as an input, and outputs a hypothesis g. For any two deterministic algorithms  $\mathcal A_1$  and  $\mathcal A_2$ , if all those f that can "generate"  $\mathcal D$  in a noiseless setting are equally likely in probability,



9. For Questions 9-12, consider the bin model introduced in class. Consider a bin with infinitely many marbles, and let  $\mu$  be the fraction of orange marbles in the bin, and  $\nu$  is the fraction of orange marbles in a sample of 10 marbles. If  $\mu=0.5$ , what is the probability of  $\nu=\mu$ ? Please choose the closest number.

10 / 10 points



- 10. If  $\mu = 0.9$ , what is the probability of  $\nu = \mu$ ? Please choose the closest number. 10 / 10 points Correct 11. If  $\mu = 0.9$ , what is the actual probability of  $\nu \le 0.1$ ? 10 / 10 points Correct 12. If  $\mu = 0.9$ , what is the bound given by Hoeffding's Inequality for the probability of 10 / 10 points  $\nu \le 0.1$ ? Correct 13. Questions 13-14 illustrate what happens with multiple bins using dice to indicate 6 10 / 10 points bins. Please note that the dice is not meant to be thrown for random experiments in this problem. They are just used to bind the six faces together. The probability below only refers to drawing from the bag. Consider four kinds of dice in a bag, with the same (super large) quantity for each kind. A: all even numbers are colored orange, all odd numbers are colored green B: all even numbers are colored green, all odd numbers are colored orange C: all small (1~3) are colored orange, all large numbers (4~6) are colored green D: all small (1~3) are colored green, all large numbers (4~6) are colored orange If we pick 5 dice from the bag, what is the probability that we get 5 orange 1's? Correct
- 14. If we pick 5 dice from the bag, what is the probability that we get "some number"  $$_{10\,/\,10~points}$$  that is purely orange?

Correct

15. For Questions 15-20, you will play with PLA and pocket algorithm. First, we use an artificial data set to study PLA. The data set is in

Each line of the data set contains one  $(\mathbf{x}_n, y_n)$  with  $\mathbf{x}_n \in \mathbb{R}^4$ . The first 4 numbers of the line contains the components of  $\mathbf{x}_n$  orderly, the last number is  $y_n$ .

Please initialize your algorithm with  $\mathbf{w}=0$  and take  $\mathrm{sign}(0)$  as -1. Please always remember to add  $x_0=1$  to each  $\mathbf{x}_n$ .

Implement a version of PLA by visiting examples in the naive cycle using the order of examples in the data set. Run the algorithm on the data set. What is the number of updates before the algorithm halts?



- 16. Implement a version of PLA by visiting examples in fixed, pre-determined random cycles throughout the algorithm. Run the algorithm on the data set. Please repeat your experiment for 2000 times, each with a different random seed. What is the average number of updates before the algorithm halts?
- 10 / 10 points

17. Implement a version of PLA by visiting examples in fixed, pre-determined random cycles throughout the algorithm, while changing the update rule to be

$$\mathbf{w}_{t+1} \leftarrow \mathbf{w}_t + \eta y_{n(t)} \mathbf{x}_{n(t)}$$

with  $\eta=0.5$ . Note that your PLA in the previous Question corresponds to  $\eta=1$ . Please repeat your experiment for 2000 times, each with a different random seed. What is the average number of updates before the algorithm halts?



18. Next, we play with the pocket algorithm. Modify your PLA in Question 16 to visit examples purely randomly, and then add the "pocket" steps to the algorithm. We will use

https://www.csie.ntu.edu.tw/~htlin/mooc/datasets/mlfound math/hw1 18 train.dat

as the training data set  $\mathcal{D}$ , and

https://www.csie.ntu.edu.tw/~htlin/mooc/datasets/mlfound math/hw1 18 test.dat

as the test set for "verifying" the g returned by your algorithm (see lecture 4 about verifying). The sets are of the same format as the previous one. Run the pocket algorithm with a total of 50 updates on  $\mathcal{D}$ , and verify the performance of  $\mathbf{w}_{POCKET}$  using the test set. Please repeat your experiment for 2000 times, each with a different random seed. What is the average error rate on the test set?



19. Modify your algorithm in Question 18 to return  $\mathbf{w}_{50}$  (the PLA vector after 50updates) instead of  $\hat{\mathbf{w}}$  (the pocket vector) after 50 updates.

10 / 10 points

Run the modified algorithm on  $\mathcal{D}$ , and verify the performance using the test set.

Please repeat your experiment for 2000 times, each with a different random seed. What is the average error rate on the test set?



20. Modify your algorithm in Question 18 to run for 100 updates instead of 50, and verify the performance of  $\mathbf{W}_{POCKET}$  using the test set. Please repeat your experiment for 2000 times, each with a different random seed. What is the average error rate on the test set?

10 / 10 points



Correct