HW2.

problem 1 (a)

$$\hat{\pi}\hat{\lambda}_{0,1:D}\hat{\lambda}_{1,1:D} = \arg\max \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} (\ln p(\lambda_{0}d) + \ln p(\lambda_{1,}d)$$
$$+ \sum_{i=1}^{n} \ln p(x_i, d|\lambda_{y_i}d)$$

From above $L = \sum_{i=1}^{n} \ln p(y_i|\pi) + \sum_{d=1}^{D} (\ln p(\lambda_{0,}d) + \ln p(\lambda_{1}d) + \sim)$.

$$= \sum_{i=1}^{n} \ln \pi^{y_i}(1-\pi)^{1-y_i} + \sum_{d=1}^{D} \left(\ln \frac{\lambda_{0d} e^{-\lambda_{0d}}}{\Gamma(2)} + \ln \frac{\lambda_{0d} e^{-\lambda_{1d}}}{\Gamma(2)} + \sum_{i=1}^{n} \ln \frac{\lambda_{y_id}^{x_i} e^{-\lambda_{y_id}}}{x_i!}\right)$$

$$\Rightarrow (\sum y_i) \ln \pi + (n - \sum y_i) \ln(1-\pi).$$

$$\frac{\partial f}{\partial \pi} = (\sum y_i)\frac{1}{\pi} + (n - \sum y_i)\frac{-1}{1-\pi} = 0$$

$$= \frac{\sum y_i}{\pi} - \frac{n - \sum y_i}{1-\pi} = 0$$

$$\Rightarrow (\sum y_i)(1-\pi) = \pi(n - \sum y_i)$$

$$\Rightarrow \hat{\pi} = \frac{\sum y_i}{n}$$

(b) for $\hat{\lambda}_{y,d} = \sum_{i=1}^{n} \ln \pi^{y_i}(1-\pi)^{1-y} + \sum_{d=1}^{P} \left(\ln \frac{\lambda_{0d} e^{-\lambda_{0d}}}{\Gamma(2)} + \ln \frac{\lambda_{0d} e^{-\lambda_{1d}}}{\Gamma(2)} + \sum_{i=1}^{n} \ln \frac{\lambda_{y_id}^{x_i} e^{-\lambda_{yd}}}{x_i}\right)$

denote: $\lambda_{y_id} = y_i\lambda_{1,d} + (1-y_i)\lambda_{0d}$.

$$(\ln(\lambda_{0,d}) - \lambda_{0,d} + \ln(\lambda_{1,d}) - \lambda_{1d} + \sum_{i=1}^{n} (\overbrace{x_{id} \ln(y_i\lambda_{1,d} + (1-y_i)\lambda_{0}d)}^{\lambda_{y_id}^{x_i}}$$
$$- \underbrace{(y_i\lambda_{1d} + (1-y_i)\lambda_{0}d)}_{\ln e^{-\lambda_{yd}}} - \ln(x_{i}d!))$$

$$\frac{\partial}{\partial \lambda_{0,d}} = \frac{1}{\lambda_{0,d}} - 1 + \sum_{i=1}^{n} \left(\frac{1-y_i}{y_i \lambda_{1,d} + (1-y_i)\lambda_{0,d}} \cdot x_{i,d} - (1-y_i) \right) = 0$$

$y_i \{0, 1\}$ $\quad \frac{1}{\lambda_{0,d}} - 1 + \sum_{i=1}^{n} \left(\frac{x_{i,d}}{\lambda_{0,d}} \cdot -1 \right) = 0$ $\qquad \frac{1}{\lambda_{1,d}} - 1 + \sum_{i \mid y=1} \left(\frac{x_{i,d}}{\lambda_{1,d}} - 1 \right) = 0$

$$\Downarrow \qquad\qquad\qquad\qquad \Downarrow$$

$$\hat{\lambda}_{0,d} = \frac{1 + \sum_{i \mid y_i = 0} x_{i,d}}{1 + \sum_{i \mid y_i = 0} 1} \qquad\qquad \hat{\lambda}_{1,d} = \frac{1 + \sum_{i \mid y_i = 1} x_{i,d}}{1 + \sum_{i \mid y_i = 1} 1}$$

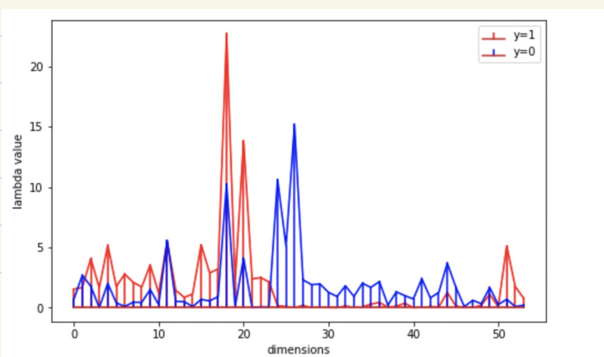
y

$$\hat{\lambda}_{y,d} = \frac{1 + \sum_{i \mid y_i = y} x_{i,d}}{1 + \sum_{i \mid y_i = y} 1}$$

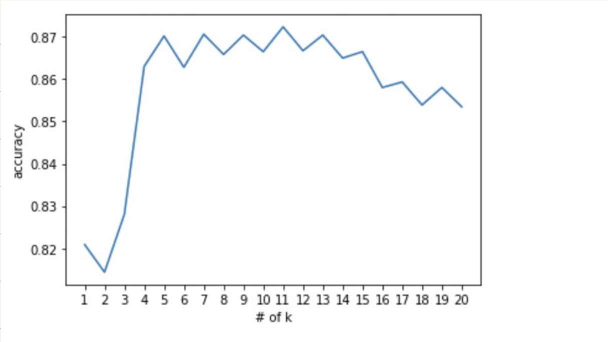
problem 2. (a)

True positive	False positive
1702	560
False negative	True negative
111	2227

(b)



(c)



Problem 3. (a).

(b)

(c). The drawback is Gaussian doesn't have a penalization term as RR in first homework thus it may overfit

(d)