BIOSTAT 214

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For BIOSTAT 214 (Finite Population Sampling) assignments.

Chapter 2

HW1

Show that the mean of sample means over all possible samples drawn from a finite population using SRSWOR is equal to the mean of the finite population.

Proof:

Let N = number of units in the population, with the population mean

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} y_i.$$

Let n = size of a sample. When we draw the sample using SRSWOR, there are $\binom{N}{n}$ distinct samples. The number of samples of size n that will contain a given unit in a population of size N is

$$\binom{N}{n} \times \frac{n}{N} = \binom{N-1}{n-1}.$$

Let $S = \{s : s = \{i_1 < i_2 < ... < i_n\}, (i_1, ..., i_n) \subseteq (1, ..., N)\}$. We have $|S| = \binom{N}{n}$.

Let $\bar{y_s}$ = sample mean of $s \in S$. So,

$$\bar{y_s} = \frac{1}{n} \sum_{i \in s} y_i.$$

Then the mean of sample means over all possible samples is equal to

$$\frac{1}{\binom{N}{n}} \sum_{s \in S} \bar{y_s} = \frac{1}{\binom{N}{n}} \sum_{s \in S} (\frac{1}{n} \sum_{i \in s} y_i) \tag{2.1}$$

$$= \frac{1}{\binom{N}{n}} \frac{1}{n} \sum_{s \in S} \sum_{i \in s} y_i \tag{2.2}$$

$$= \frac{1}{\binom{N}{n}} \frac{1}{n} \sum_{s \in S} \sum_{s \ni i} y_i = (*) \tag{2.3}$$

Since for a given i, y_i will appear in exactly $\binom{N-1}{n-1}$ samples,

$$(*) = \frac{\binom{N-1}{n-1}}{n\binom{N}{n}} \sum_{i=1}^{N} y_i \tag{2.4}$$

$$= \frac{1}{N} \sum_{i=1}^{N} y_i = \bar{Y} \tag{2.5}$$

which concludes the proof.