and ℓ is a lower bound. Any number larger than u is an upper bound, and any number smaller than ℓ is also a lower bound of the sequence.

If $(a_n)_\urcorner$ is bounded there exists, clearly, a positive number K such that

$$-K \leq a_n \leq K \text{ or } |\, a_n \,| \leq K \text{ for all } n \geq 1$$

Examples.

1. 1,2, . . . , n, . . . monotone and bounded below,
2. 1, 1/2, . . . , 1/n, . . . monotone and bounded,
3. 2, 2, . . . , 2, . . . monotone and bounded,
4. -1, 1, . . . , $(-1)^n$, . . . non monotone, but bounded.

Example. Show boundedness of

a) $(\frac{\sin n}{\sqrt{n}+8})_1$ b) [1] $(\frac{n+8}{n^{3/2}})_4$

Solution.

a) $\left|\, \frac{\sin n}{\sqrt{n}+8} \,\right| = \frac{|\sin n\,|}{\sqrt{n}+8} \leq \frac{1}{\sqrt{n}+8} \leq \frac{1}{\sqrt{1}+8} = \frac{1}{9} \quad (K = \frac{1}{9}),$

since max($\sin n$)=1 and min n = 1.

b) $\left|\, \frac{n+8}{n^{3/2}} \,\right| = \frac{n+8}{n\sqrt{n}} = \frac{1}{\sqrt{n}} + \frac{8}{n\sqrt{n}} \leq \frac{1}{\sqrt{4}} + \frac{8}{4\sqrt{4}} = \frac{1}{2} + 1 \quad (K = 3/2)$

since min n = 4.

[1] a package modifying and adding to the enumerate environment is used to enable inline enumerating.