

- (1) Justify each step in the proof of *Lemma 8.11*.
- (2) Let  $G$  be a group and  $a, b, c \in G$ . Suppose  $ab = ba$ . Prove that  $(a^m b^n c^r)^{-1} = c^{-r} a^{-m} b^{-n}$  for all  $m, n, r \in \mathbb{Z}$ , justifying each detail.
- (3) Let  $G$  be a nonempty set with an associative multiplication on it and let  $a_1, a_2, \dots, a_n$  be pairwise commuting elements of  $G$ . Show that

$$(a_1 a_2 \dots a_n)^m = a_1^m a_2^m \dots a_n^m$$

for all  $m \in \mathbb{N}$

- (4) Show that, if  $G$  is an additive commutative group, then

$$-(a_1 + a_2 + \dots + a_n) = (-a_1) + (-a_2) + \dots + (-a_n)$$

for all  $a_1, a_2, \dots, a_n$  in  $G$