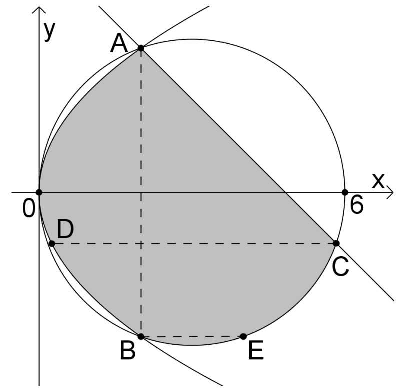
Observe that the shaded region is not normal. If it is split up into R_{xy} normal regions we get (at least) two such regions (AODB, ABEC). If it is split up into R_{yx} normal regions we get (at least) three such regions (AODC, DBEC, BFE).



It is reasonable to use the first splitting for this problem since the number of subregions is less than that of the other case. But in some problems such a selection may arise difficulty in integration.

Then our regions AODB, and ABEC are respectively:

$$R^1_{xy} = \{(x,y) : 0 \le x \le 2,\ -2\sqrt{x} < y < 2\sqrt{x}\} = (0,\ 2;\ -2\sqrt{x},\ 2\sqrt{x})$$

$$R^2_{xy} = \{(x,y) : 2 \le x \le 3+2\sqrt{2},\ -\sqrt{6x-x^2} \le y \le -x+2(1+\sqrt{2})\}$$
$$= (2,\ 3+2\sqrt{2};\ -\sqrt{6x-x^2},\ -x+2(1+\sqrt{2}))$$

$$|\,A\,| = \left|\,R^1_{xy}\,\right| + \left|\,R^2_{xy}\,\right|$$
$$= \int_0^2 (2\sqrt{x} - (-2\sqrt{x}))\,\mathrm{d}x + \int_2^{3+2\sqrt{2}} -x+2(1+\sqrt{2}) - (-\sqrt{6x-x^2}))\,\mathrm{d}x$$
$$= \frac{16}{3}\sqrt{2} + \frac{7}{2} + \int_2^{3+2\sqrt{2}} \sqrt{6x-x^2}\,\mathrm{d}x$$

Writing $6x - x^2 = 9 - (x-3)^2$ and setting $x - 3 = 3\sin t$ we have

$$\int \sqrt{6x-x^2}\,\mathrm{d}x = \int \sqrt{9-9\sin^2 t}\cdot 3\cos t\,\mathrm{d}t$$
$$= 9\int \cos^2 t\,\mathrm{d}t = \frac{9}{2}(t + \frac{\sin 2t}{2}) + c$$

$$\alpha = \int_2^{3+2\sqrt{2}} \sqrt{6x-x^2}\,\mathrm{d}x = \frac{9}{2}\arcsin\frac{2\sqrt{2}}{3} + \arcsin\frac{1}{3} + \frac{4\sqrt{2}}{9}$$
$$A = \frac{16}{3}\sqrt{2} + \frac{7}{2} + \alpha.$$