Example 1. Transform the cartesian equation to polar one:

$$a)ax + by + c = 0(line)b)x^2 + y^2 - 2ax = 0(circle)$$

Solution. Setting $x = rcos\theta$, $y = rsin\theta$, we have

a) $ar\ cos\theta + br\ \sin\theta + c = 0 \Rightarrow r = \frac{c}{a\cos\theta + b\sin\theta}$
b) $r^2 - 2arcos\theta \Rightarrow r(r - 2acos\theta) = 0 \Rightarrow r = 0$ (pole) or $r = 2acos\theta$. But $r = 0$ is contained in the second for $\theta = \pi/2$. Hence transformed equation is $r = 2acos\theta$.

Example 2. Transform the polar equation to cartesian:

a) $r = a(1 + cos\theta)$ (cardioid) b) $r^2 = a^2\cos 2\theta$ (lemniscate)

Solution.

a) We express first $cos\theta$ in terms of x and r and then replace r^2 by $x^2 + y^2$:

$$r = a(1 + cos\theta) \Rightarrow r = a(1 + \frac{x}{r}) \Rightarrow r^2 = a(r + x)$$
$$x^2 + y^2 = ar + ax \Rightarrow (x^2 + y^2 - ax)^2 \Rightarrow a^2(x^2 + y^2).$$

b)

$$x^2 = a^2cos2\theta \Rightarrow x^2 + y^2 = a^2(cos2\theta - sin2\theta)$$
$$x^2 + y^2 = a^2(\frac{x^2}{r^2} - \frac{y^2}{r^2}) \Rightarrow (x^2 + y^2)^2 = a^2(x^2 - y^2)$$

Example 3. Write two representations of points A, B, C, D, E, F in the given figure, where OAB, OCD are equilateral triangles, and OEFA is, a square ($OA = 2$, $OD = 3$)

Solution. Since $OA = OB = 2$, $OC = OD = 3$, we have form Figure,
$A(0, 2) = A(\pi, -2)$
$B(\frac{\pi}{3}, 2) = B(\frac{\pi}{3} + \pi, -2)$
$C(2\frac{\pi}{3}, 3) = C(-\frac{\pi}{3}, -3)$
$D(\pi, 3) = D(0, -3)$