

## C. ROTATION OF COORDINATE AXES AND APPLICATION:

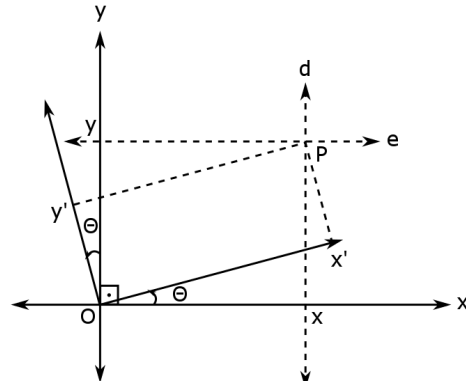
A transformation which rotates (turns) all points of a figure through the same angle  $\Theta$  about a given point  $O$  is called a *rotation*.

The point  $O$  is the *center of rotation* and  $\Theta$  the *angle of rotation*.  $\Theta$  is considered positive (negative) when measured counterclockwise (clockwise).

A rotation with center at the origin and with angle  $\Theta$  rotates the coordinate system  $Oxy$  into a new coordinate system  $Ox'y'$ .

To obtain the transforming formulas for coordinates  $x, y$  and  $x', y'$  of the same point  $P$  in the systems  $Oxy$  and  $Ox'y'$ , consider the lines  $d$  and  $e$  through  $P$  and perpendicular to  $Ox$  and  $Oy$  respectively.

The normal equations of  $d$  and  $e$  in  $Oxy$  system are



$$\begin{cases} d : x' \cos(-\Theta) + y' \sin(-\Theta) - x = 0 \\ e : x' \cos(\frac{\pi}{2} - \Theta) + y' \sin(\frac{\pi}{2} - \Theta) - y = 0 \end{cases}$$

or

$$\begin{cases} x' \cos \Theta - y' \sin \Theta - x = 0 \\ x' \sin \Theta + y' \cos \Theta - y = 0 \end{cases}$$

from which we have the transforming formulas:

<u>From new to old</u>	<u>From old to new</u>
$x = x' \cos \Theta - y' \sin \Theta$	$x' = x \cos \Theta + y \sin \Theta$
$y = x' \sin \Theta + y' \cos \Theta$	$y' = -x \sin \Theta + y \cos \Theta$

Application to SDE:

Let  $f(x, y) = 0$  be an equation. Then  $f(x' \cos \Theta - y' \sin \Theta$