1. If a sequence is convergent, then every subsequence of it is convergent,

2. If a subsequence is divergent, the original sequence is divergent,

3. If two subsequences converge to distinct limits, the original sequence is divergent.

Example 0.1.

1. 3, $\sqrt{10}$, ..., $\sqrt{n}$, ... diverges to ∞,
2. $((1+\frac{1}{n})^n)_2$ converges to e,
3. 1, -1, 1, -1 ... , $(-1)^{n-1}$, ... diverges since it has the subsequences (1) and (-1) having distinct limits 1 and -1.

Theorem 0.1. *If* $(a_n) \to a$*,* $(b_n) \to b$*, and* $c \in R$*, then*

$$a)(c\ a_n) \to ca \qquad b)(a_n + b_n) \to a + b$$
$$c)(a_n b_n) \to ab \qquad d)(\frac{a_n}{b_n}) \to \frac{a}{b} \quad (if\ b_n \neq 0,\ b \neq 0)$$
$$d)(|a_n|) \to |a|$$

Proof. We prove c) only. Those of the others are similar. The proof runs in the same way as that for functions with continous variable.

$\square$

Let $a_n \to a$, $b_n \to b$. Then given $\epsilon > 0$ there exists $N > 0$ such that