$$u = 2 + 2\sqrt{3}i,\ v = \sqrt{3} - 1$$

find the polar form of their ratio u/v

a) by the property (2)

b) first finding the cartesian ratio, and then transforming to polar form.

Solution.

a) $\mid \frac{u}{v} \mid = \frac{|u|}{|v|} = \frac{4}{2} = 2,\ arg\frac{u}{v} = \arg u - \arg v = \frac{\pi}{3} - (-\frac{\pi}{6}) = \frac{\pi}{2}$

$\Rightarrow \frac{u}{v} = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$

b) $\frac{u}{v} = \frac{2+2\sqrt{3}i}{\sqrt{3}-i} = \frac{(22\sqrt{3})(\sqrt{3}+i)}{(\sqrt{3}-i)(\sqrt{3}+i)} = \frac{8i}{4} = 2i$

$\Rightarrow\mid \frac{u}{v} \mid = 2, \arg\frac{u}{v} = \frac{\pi}{2}$

$\Rightarrow \frac{u}{v} = 2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$.

Example. By use of De Moirve's formula, compute $\cos 5\theta$, $\sin 5\theta$ in terms of $\cos\theta$ and $\sin\theta$.

Solution.

$\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$
$= \cos^5\theta + 5\cos^4\theta(i\sin\theta) + 10\cos^3\theta(i\sin\theta)^2$
$+10\cos^2\theta(i\sin\theta)^3 + 5\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$
$= (\cos^5\theta - 10cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta)$
$+(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)i$

$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$
$\sin 5\theta = \sin^5\theta - 10\sin^3\theta\cos^2\theta + 5\sin\theta\cos^4\theta$

C. ROOTS OF NUMBERS

By an nth root of a complex number is meant a complex number whote nth power is equal to the given number. If