statement p_n is true for all $n \in N$. We formulate the axiom as an operational procedure.

0.1. Principle of mathematical induction: Let p_n be a statement involving a natural number n. We can prove the proposition

$$\text{for all } n \in N,\ p_n$$

by establishing that

I. p_1 is true,

II. for all $k \in N$ if p_k is true, then p_{k+1} is true.

Proofs by the principle of mathematical induction consist of two steps. In the first step, we show that p_1 is true. In practise, this is often quite easy, but we should not neglect it. In the second step, we assume that p_k is true. This assumption is the inductive hypothesis. Using this hypothesis, we prove that p_{k+1} is true. A proof by induction will not be complete (and valid) if we carry out the first step but not the second, or if we carry out the second step but not the first.

0.2. Examples:(a). Prove that $1 + 2 + ... + n = \frac{n(n+1)}{2}$ for all $n \in N$. We use the principle of mathematical induction.

I. $1 = \frac{1(1+1)}{2}$, so the formula is true for $n = 1$.

II. Make the inductive hypothesis that $1 + 2 + ... + k = \frac{k(k+1)}{2}$.

We want to establish $1 + 2 + ... + k + (k + 1) = \frac{(k+1)(k+2)+1}{2}$. We have

$$\begin{aligned} 1 + 2 + ... + k + (k + 1) &= \tfrac{k(k+1)}{2} + (k + 1) \text{ (by inductive hyp.)} \\ &= (\tfrac{k}{2} + 1)(k + 1) \\ &= \tfrac{(k+1)(k+2)}{2}, \end{aligned}$$

so the formula is true for $n = k + 1$ it is true for $n = k$. Hence

$$1 + 2 + ... + n = \frac{n(n+1)}{2} \text{ for all } n \in N.$$