i. $r_1 + r_2 \left(\frac{p_1}{q_1} + \frac{p_2}{q_2} = \frac{p_1 q_2 + p_2 q_1}{q_1 q_2} \right)$,

ii. $r_1 - r_2 \left(\frac{p_1}{q_1} - \frac{p_2}{q_2} = \frac{p_1 q_2 - p_2 q_1}{q_1 q_2} \right)$,

iii. $r_1 \cdot r_2 \left(\frac{p_1}{q_1} \cdot \frac{p_2}{q_2} = \frac{p_1 p_2}{q_1 q_2} \right)$,

iv. $r_1 : r_2 \left(\frac{p_1}{q_1} : \frac{p_2}{q_2} = \frac{p_1 q_2}{q_1 p_2} \right)$

are all rational.

Corollary 0.1. *Between any two distinct rational numbers there exists at least one rational number, hence infinitely many.*

Proof. Let the given rational numbers be r_1 and r_2 : $r_1 + r_2$ rational $\Longrightarrow \frac{1}{2}(r_1 + r_2)$ is rational. (why this arithmetic mean is between r_1 and r_2?) This process can be continued indefinitely.

0.1. Irrational numbers. A number which is not rational is called an *irrational number*. Since any cyclic decimal expansion is a rational number, then non cyclic ones represent irrational numbers:

$$0,81881888188881\cdots \text{ (Number of 8's increases by 1 in each step)}$$
$$4,303003000300003\cdots$$

The existence of irrational numbers may also be shown by the following theorem: