a) $\frac{df(A)}{ds} = ye^{x-1}s\frac{1}{\sqrt{5}} + e^{x-1}\frac{2}{\sqrt{5s}}|_A = \frac{4}{\sqrt{5}} + \frac{2}{\sqrt{5}} = \frac{6}{\sqrt{5}}$

b) $\frac{df(A)}{ds} = ye^{x-1}s(\frac{1}{\sqrt{-5}}) + e^{x-1}(\frac{-2}{\sqrt{5s}})_A = -\frac{6}{\sqrt{5}}$

Example 2. Given u =xy $\ln z$,find its directional derivative along the curve r: x $\ln t$ $y = t^2/2$, z =2t at $A(0, 1/2, 2)$ int he positive sense.

Solution. A tangent vector of r A(in the positive sense of r) being

$$(\dot{x}, \dot{y}, \dot{z})_A = (1/t, t, 2)_A = (1, 1, 2)$$

the unit tangent vector T has components

$$cos\alpha = \frac{1}{\sqrt{6}}, cos\beta = \frac{1}{\sqrt{6}}, cos\gamma = \frac{2}{\sqrt{6}}$$

and

$$\frac{du(A)}{ds} = (y\ ln\ z)\ \frac{1}{\sqrt{6}} + (x\ ln\ z)\frac{1}{\sqrt{6}} + (\frac{xy}{2})\ \frac{2}{\sqrt{6}}|_A = \frac{1}{2}\ (lnz)\ \frac{1}{\sqrt{6}}$$

Example 3. If the temperature distribution in a room $(x\epsilon[0, 4], y\epsilon[0, 12], z\epsilon[0, 3])$ is given by

$$\text{T} = \frac{z}{(x+1)(y+2)} \qquad \text{(in degrees)}$$

find the rate of change of T along the diagonal [OB] of the room in the sense from 0 to B, at the center C.

Solution.

a = (4, 12, 3), a/$|a|$ = (4, 12, 3)/13, C(2, 6, 3/2)

$$\frac{dT}{ds} = T_a = T_x\ \frac{4}{13} + T_y\ \frac{12}{13} + T_z\ \frac{3}{13}|_C$$

where

$$T_x(C) = \frac{z}{(x+1)(y+2)}|_C = -1/48$$

$$T_x(C) = -\frac{z}{(x+1)(y+2)}2|_C = -1/128$$