THEOREM 0.1. [1]

PROOF. will be $-h$, h so that

$$A = \int_{-h}^{h} (\alpha x^2 + \beta x + \gamma)\,\mathrm{d}x = (\frac{\alpha}{3}x^3 + \frac{\beta}{2}x^2 + \gamma x)_{-h}^{h}$$
$$= \frac{2}{3}\alpha h^3 + 2\gamma h = \frac{h}{3}(2\alpha h^2 + 6\gamma)$$

Since,

$$\begin{aligned} y_0 &= \alpha h^2 - \beta h + \gamma \\ 4y_1 &= \qquad\qquad\quad 4\gamma \\ y_2 &= \alpha h^2 + \beta h + \gamma \end{aligned}$$

$$\overline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

$$y_0 + 4y_1 + y_2 = 2\alpha h^2 \qquad + 6\gamma$$

we have our result.

Now partitioning (a, b) regularly for an even number n and applying the above lemma for consecutive pairs of strips and adding the results of each pair, we have

$$\frac{h}{3}((y_0 + 4y_1 + y_2) + (y_2 + 4y_3 + y_4) + \cdots + (y_{n-2} + 4y_{n-1} + y_n))$$

and

$$\int_a^b f(x)\,\mathrm{d}x = \frac{h}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + \cdots + 2y_{n-2} + 4y_{n-1} + y_n)$$

where $h = (b-a)/n$ and n is an even number.

Observe that coefficients of y_1 are 1 for $i = 0$ and $i = n$; for others, 4 for odd i and 2 for even i. $\square$

EXAMPLE 0.1. Evaluate the definite integral

$$A = \int_1^3 \frac{\mathrm{d}x}{x}$$

approximately (numerically) using the three rules, taking $n = 6$.

Solution.

[2] We have $h = \frac{3-1}{6} = \frac{1}{3}$ and

x_i	1	$\frac{4}{3}$	$\frac{5}{3}$	2	$\frac{7}{3}$	$\frac{8}{3}$	3
y_i	1	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{3}{7}$	$\frac{3}{8}$	$\frac{1}{3}$

[1] proof of theorem continues from the previous page, 'THEOREM' and 'PROOF' words are not undesirable in my page.

[2] solution of the example continues to the next page. In order not to get errors while compiling, I closed my tags here.