and these are verified easily. Hence $(e, a, b, \circ)$ is a group. There is a group of order 3. Any two groups of order 3 have essentialy the same Cayley table, namely the table in Figure 3. This statement will be made precise in §20.

The Cayley tables of ($\mathbb{N}$,+) and $(e, a, b, \circ)$ are symmetric about the principal diagonal (that joins the upper-left and lower-right cells). What does this signify? The symmetry of the Cayley table of a group $(G, \circ)$means that the cell where the i-th and j-th row column meet, and this for all $i, j = 1, 2, ..., |G|$.Assuming the i-th row is the row of $a \in G$and the j-th column is the column of $b \in G$ (and assuming we index the rows and columns by the elements of G in the same order), this means: $a \circ b = b \circ a$for all $a, b \in G$. So the group is commutative in the following sense.

Definition 0.1. A group $(G, \circ)$ is called a *commutative* group or an *abelian* group, if, in addition to the group axioms (i)-(iv) a fifth axiom
(v)$a \circ b = b \circ a$ for all $a, b \in G$.. holds.

A binary operation on a set G is called *commutative* when $a \circ b = b \circ a$ for all $a, b \in G$.. So a commutative group is one where the operation is commutative. The term "abelian" is used in honor of N. H. Abel, a Norwegian mathematician (1802-1829). We close this paragraph with some comments on the group axioms. The reader might ask why we should study the structures $(G, \circ)$ where $\circ$ satisfies the axioms (i),(ii),(iii),(iv).Why do we not study structures $(G, \circ)$ where $\circ$ satisfies the axioms (i),(ii),(iii),(iv) to some other combination of (i),(ii),(iii),(iv)? There is of course no reason why other combinations ought to be excluded from study. As a matter of fact, all combinations have a proper name and there are theories about them. However, they are very far from having the same importance as the combination (i),(ii),(iii),(iv).