(ii) $(a, b) \approx (c, d)$ then $ad = bc$, then $da = bc$, then $cb = da$, so $(c, d) \approx (a, b)$

(iii) (a,b) $\approx (c, d)$ and (c,d) = (e,f), then

ad=bc and cf = de

adf = cbf and bcf = bde

adf = bde

d(af-be) =0

af-be = 0 (since $d \neq 0$)

af = be

$$(a, b) \approx (e, f) \tag{0.1}$$

Thus, $\approx$ is an equivalence relation on S.

(g) Let T be the set of all triangles in the Euclidean plane. Congruence of triangles is an equivalence relation on T.

(h) Let S be the set of all continuous functions defined on the closed interval [0,1]. For any two functions f,g in S, let us write $f \stackrel{\mathrm{A}}{=} g$ if

$$\int_0^1 f(x)dx = \int_0^1 g(x)dx$$

Then, $\stackrel{\mathrm{A}}{=}$ is an equivalence relation on S.
An equivalence relation is a weak form of equality. Suppose we have various objects, which are similar in one respect and dissimilar in certain other respects. We may wish to ignore their dissimilarity and focus our attention on their similar behaviour. Then there is no need to distinguish between our various objects that behave in the same way. We may regard them as equal or identical. Of course, "equal" or "identical" are poor words to employ here, for the objects are not absolutely identical, they are equal only in one respect that we wish to investigate more closely. So we employ the word "equivalent". That a and b

are equivalent means, then, a and b are equal, not in every respect, but rather as far as a particular property is concerned. An equivalence relation is a formal tool for disregarding differences between various objects and treating them as equals. Let us examine our examples under this light. In example 2.3(b) the points P are R may be different, but the lines they determine with the origin are equal. In Example 2.3(c), the lines may be different, but their directions are equal. In Example 2.3(d), the integers may be different, but their parities are equal. In Example 2.3(e), the integers