with a $\leq$ x $\leq$ b, $\alpha \leq$ t $\leq \beta$, a, b const. and if f(x, t), f_2(x, t) ε C(D_f); then

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = \int_a^b \frac{\partial'}{\partial t} f(x,t)dx$$

Proof.

$$\begin{aligned}
\Delta F(t) &= F(t+\Delta t) - F(t) \\
&= \int_a^b (f(x,t+\Delta t) - f(x,t))dx \\
&= \int_a^b \Delta t f_2(x,\tau)dx \qquad\qquad \tau \ \varepsilon \ (\overline{t, t+\Delta t}) \\
\frac{\Delta F}{\Delta t} &= \int_a^b f_2(x,\tau)dx \\
\frac{\mathrm{d}F}{\mathrm{d}t} &= \lim_{\Delta t \to 0} \int_a^b f_2(x,\tau)dx = \int_a^b f_2(x,t)dx
\end{aligned}$$

Corollary. If in

$$F(t) = \int_{a(t)}^{b(t)} f(x,t)dx$$

a(t), b(t) are differantiable and f(x,t) f(x,t) are continious, then

$$\frac{\mathrm{d}F(t)}{\mathrm{d}t} = \int_{a(t)}^{b(t)} f_t(x,t)dt + f(b,t)b' - f(a,t)a'$$

Proof. Let

$$F(t) = G(a(t), b(t), t) = \int_{a(t)}^{b(t)} f(x,t)dx$$

Then

$$\begin{aligned}
\frac{\mathrm{d}F(t)}{\mathrm{d}t} &= \frac{\partial G}{\partial a}\frac{\partial a}{\partial t} + \frac{\partial G}{\partial b}\frac{\partial b}{\partial t} + \frac{\partial G}{\partial t}.1 \\
&= -f(a,t)a' + f(b,t)b' + \int_a^b f_t dx
\end{aligned}$$