EXAMPLE 0.1. [1]

(1) condition is jointly written with the rule.

(2)

$$(\frac{f}{g})(x) = \frac{f(x)}{g(x)} = \frac{|\,x\,|}{x^2\sqrt{1-x}}$$

As to the composition $g \circ f$ and $f \circ g$ we have.

(3) [2]

$$(g \circ f)(x) = g(f(x)) = g(\frac{|\,x\,|}{x}) = \frac{|\,x\,|}{x}\sqrt{1-\frac{|\,x\,|}{x}}$$

$$(f \circ g)(x) = f(g(x)) = f(x\sqrt{1-x}) = \frac{|\,x\sqrt{1-x}\,|}{x\sqrt{1-x}} = \frac{|\,x\,|\sqrt{1-x}}{x\sqrt{1-x}}$$

$$= \frac{|\,x\,|}{x} \quad (x \neq 1)$$

and

$$D_{g\circ f} = (-\infty, 1] - \{\,0\,\},\ D_{f\circ g} = (-\infty, 1] - \{\,0, 1\,\}$$
$$= (-\infty, 1) - \{\,0\,\} = (-\infty, 1)$$

EXAMPLE 0.2. Given the functions

$$f : \mathbb{R} \to \mathbb{R}, \quad f(x) = \frac{x}{x-2}; \quad g : \mathbb{R} \to \mathbb{R}, \quad g(x) = x^2 - x$$

find the rules for the composite functions $g \circ f$ and $f \circ g$, and then determine their domains.

Solution.

$$1.\ (g \circ f)(x) = g(f(x)) = f^2(x) - f(x) = \frac{x^2}{(x-2)^2} - \frac{x}{x-2}$$
$$= \frac{x^2 - x(x-2)}{(x-2)^2} = \frac{2x}{(x-2)^2}$$

$$2.\ (f \circ g)(x) = f(g(x)) = \frac{g(x)}{g(x)-2} = \frac{x(x-1)}{(x+1)(x-2)}$$

$$D_{g\circ f} = \mathbb{R} - \{\,2\,\}, \qquad D_{f\circ g} = \mathbb{R} - \{\,-1, 2\,\}$$

[1] example enumeration continues from the previous page.

[2] g(f(x) written for the first equation here in the original book, so I corrected them.