

$$4A \frac{dA}{dt} = x(y^2 + z^2) \frac{dx}{dt} + y(z^2 + x^2) \frac{dy}{dt} + z(x^2 + y^2) \frac{dz}{dt}$$

When P is at  $P_O(6, 0, 0)$ , then Q is at  $Q_O(0, 9, 0)$  and R is at  $R_O(0, 0, 12)$  with  $|P_O Q_O R_O|_2 = 9\sqrt{61}$ . Then

$$4.9\sqrt{61} \frac{dA}{dt} = 6(225)2 + 9(680)3 + 12(117)4$$

$$9\sqrt{61} \frac{dA}{dt} = 675 + 27.45 + 12.117$$

$$\sqrt{61} \frac{dA}{dt} = 75 + 135 + 156 = 366$$

$$\frac{dA}{dt} = \frac{766}{\sqrt{61}} = 6\sqrt{61} \text{ unit}^2 / \text{sec}$$

### 0.1. TAYLOR'S FORMULA AND SERIES.

THEOREM 0.1. *If  $f(x, y)$  has continuous partial derivatives up to order  $n+1$  in a neighborhood of  $(a, b)$  ev  $f$ , then*

$$f(x, y) = f(a, b) + \sum_{k=1}^n \frac{1}{k!} \left( (x-a) \frac{a}{ax} + (y-b) \frac{a}{ay} \right)^k f(x, y)|_{(a,b)} + R_{n+1}$$

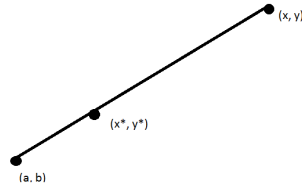
where the remainder is given by

$$R_{n+1} = \frac{1}{(n+1)!} \left( (x-a) \frac{a}{ax} + (y-b) \frac{a}{ay} \right)^{n+1} f(x, y)_{(x^*, y^*)}$$

with  $(x^*, y^*)$  a point on the open segment  $(P_O P)$  joining  $P_O(a, b)$  to  $P(x, y)$

PROOF. Since every point of the line segment  $[P_O P]$  can be represented parametrically as

$$x = a + ht, \quad y = b + kt \quad 0 \leq t \leq 1,$$



The end points of the segment correspond to  $t=0$  and  $t=1$  (observe that  $h, k$  are direction numbers of the line segment) Substituting (2) in  $f(x, y)$  gives the function

□