within parentheses and separated by a comma. Thus (a, b) is an ordered pair. The adjective "ordered" is used to emphasize that the objects have a status of being first and being second. a is called the *first component* of the ordered pair (a, b), and b is called its *second component*. Two ordered pairs are declared equal if their first components are equal and their second components are equal. Thus (a, b) and (c, d) are equal if and only if $a = c$ and $b = d$, in which case we write $(a, b) = (c, d)$. Notice that we have $(a, b) \neq (b, a)$ unless $a = b$ (here $\neq$ means the negation of equality).

The set of all ordered pairs, whose first components are the elements of a set S and whose second components are the elements of a set T, is called the *cartesian product of* S *and* T, and is denoted by $S \times T$. Hence

$$S \times T = \{(a, b) : a \in S \text{ and } b \in T\}.$$

We can also define ordered triples (a, b, c), ordered quadruples (a, b, c, d), more generally ordered n-tuples $(a_1, a_2, \cdots, a_n)$. Equality of ordered n-tuples will mean the equaltiy of their corresponding components. The set of all ordered n-tuples, whose i-th components are the elements of a set S_i, is called the *cartesian product of* $S_1, S_2, \cdots, S_n$ and is denoted by $S_1 \times S_2 \times \cdots \times S_n$. Hence

$$S_1 \times S_2 \times \cdots \times S_n = \{(a_1, a_2, \cdots, a_n) : a_1 \in S_1, a_2 \in S_2, \cdots, a_n \in S_n\}.$$

It is possible to define the cartesian product of infinitely many sets, too. We do not give this definition, for we will not need it.

A set can have finitely many or infinitely many elements. The number of elements in a set S is called the *cardinality* or the *cardinal number of* S. The cardinality of S is denoted by $|S|$. The set S is said to be *finite* if $|S|$ is a finite number. S is said to be *infinite* if S is not finite. A rigorious definition of finite and infinite sets must be based on the notion of one-to-one correspondence between sets, which will be introduced in §3. However, we will not make any attempt to give a rigorous definition of finite and infinite sets. We shall be content with the suggestive description above.

Exercises.

(1) Show that, if R is a subset of S and S is a subset of T, then R is a subset of T.