and factorize it completely.

3.37. If a, b, c are distinct real numbers, show

$$\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0 \qquad abc = 1$$

3.38. Prove that

$$\begin{vmatrix} x^3 & x^2 & x & 1 \\ 3x^2 & 2x & 1 & 0 \\ y^3 & y^2 & y & 1 \\ 3y^2 & 2y & 1 & 0 \end{vmatrix} = (x-y)^4$$

3.39. Show that

$$\begin{vmatrix} 1 & A & B & AB \\ 1 & a & B & aB \\ 1 & A & b & Ab \\ 1 & a & b & ab \end{vmatrix} = (A-a)^2(B-b)^2$$

3.40. Show that

$$\begin{vmatrix} \cos(x+y) & \sin(x+y) & -\cos(x+y) \\ \sin(x-y) & \cos(x-y) & \sin(x-y) \\ \sin 2x & 0 & \sin 2y \end{vmatrix} = \sin 2(x+y)$$

3.41. Prove $D_n = a_n D_{n-1} + D_{n-2}$, where

$$D_n = \begin{vmatrix} a_1 & 1 & 0 & \cdots & 0 \\ -1 & a_2 & 1 & \ddots & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & -1 & a_n \end{vmatrix}$$

(Hint: Expand by the last row or column)

3.42. If D_n denotes the determinant