hypothesis of (1) is satisfied when we replace a by a^{-1} and b by b^{-1}. Using (1) with a^{-1}, b^{-1} in place of a, b, respectively, we obtain

$$(ab)^{-n} = ((ab)^{-1})^n = (a^{-1}b^{-1})^n = (a^{-1})^n(b^{-1})^n = a^{-n}b^{-n}$$

for $n \in \mathbb{N}$. Thus $(ab)^n = a^n b^n$ is valid also when $n \leq -1$. So $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$.

LEMMA 0.1. *Let G be a commutative group. Then $(ab)^n = a^n b^n$ for all $a, b \in G$ and for all $n \in \mathbb{Z}$*

PROOF. This follows immediately from Lemma 8.14. □

So far, we dealt with multiplicative groups. For additive groups, there are some modifications. In the case of an additive group, the unique element in $P_n(a_1, a_2, ..., a_n)$ of Lemma 8.3 is called the sum of $a_1, a_2, ..., a_n$ and is denoted by $a_1 + a_2 + ... + a_n$ or by $\sum_{i=1}^{n} a_i$. We write na for $a_1 + a_2 + ... + a_n$ in case $n \in \mathbb{N}$ and $a_1, a_2, ..., a_n$ are all equal to $a \in G$. Also, we define $0a = 0$(the first 0 is the integer 0, the second 0 is the identity element of G) and $(-m)a = -(ma)$ for $m \in \mathbb{N}$. Thus we defined na for all $n \in \mathbb{Z}, a \in G$.

LEMMA 0.2. *Let G be a additively written commutative group. Then*

(1) $ma + na = (m + n)a$
(2) $(-m)a = m(-a)$
(3) $n(ma) = (nm)a$
(4) $n(a + b) = na + nb$

for all $m, n \in \mathbb{Z}, a, b \in G$

PROOF. (1),(2),(3) foolow from Lemma 8.7 and (4) from Lemma 8.15. □

Notice that commutativity is essential for (4).

Exercises

(1) Let G be a group such that $a^2 = 1$ for all $a \in G$. Prove that G is commutative.