

3. Let \sim and \approx be two equivalence relations on a set A . We define \equiv by declaring $a \equiv b$ if and only if $a \sim b$ and $a \approx b$; and we define \cong by declaring $a \cong b$ if and only if $a \sim b$ or $a \approx b$. Determine whether \equiv and \cong are equivalence relations on A .
4. If a relation on A is symmetric and transitive, then it is also reflexive. Indeed, let \sim be the relation and let $a \in A$. Choose an element $b \in A$ such that $a \sim b$. Then $b \sim a$ by symmetry, and from $a \sim b$, $b \sim a$, it follows that $a \sim a$, by transitivity. So $a \sim a$ for any $a \in A$ and \sim is reflexive.

This argument is wrong. Why?