$$\begin{aligned}
I &= cos^3 x sinx + 3\int cos^2 x sin^2 x dx \\
&= cos^3 x sin + 3\int cos^2 x dx - 3\int cos^4 x dx \\
&= cos^3 x sinx + 3\int cos^2 x dx - 3\int cos^4 x dx \\
4I &= cos^3 x sinx + \frac{3}{2}\int (1 + cos2x)dx \\
&= cos^3 x sinx + \frac{3}{2}(x + \frac{sin2x}{2}) + c_1 \\
I &= \frac{1}{4}cos^3 x sinx + \frac{1}{16}sin2x + \frac{3}{8}x + c
\end{aligned}$$

Properties: (Indefinite integrals of even, odd functions]
Let $e_i(x)$ be even, odd functions respectively. Then recalling properties $De_1(x) = \omega_1(x)$, $D\omega_2(x) = e_2(x)$ (§2.1, Exercise 20) we may have the converse. Indeed the following properties hold:

$$1. \int e_1 dx = \omega_1(x) + c, \qquad 2. \int \omega_2(x)dx = e_2(x) + c$$

Proof:

1. Let $F(x) = \int e_1(x)dx$ without constant of integration. Then
$$F(-x) = \int e_1(x)d(-x) = -\int e_1(-x)dx = -\int e_1 dx = -F(x),$$
Showing that $F(x)$ is an odd function, namely $\omega_1(x)$
2. Proved similarly. $\square$

Discuss periodicity of the integral of a periodic function.

EXERCISES (5.1)

1. Simplify the following
   a) $\int df(x)$ b) $d\int f(x)dx$ c) $\frac{d}{dx}\int arccosxdx$
   d) $\int \frac{d}{df}arccosxdx$ e) $\int d(x^7 + x + 7)^7$ f) $\frac{d}{dx}\int \frac{d}{dx}arcsecx$
2. If $F_1(x)$, $F_2(x)$ are two primitives of $f(x)$, then show that $c_1F_1(x) + c_2F_2(x)$ is a primitive of $f(x)$ when c_1, c_2