Example. find the inverses of the matrices

$a) A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 1 & 1 & 2 \end{bmatrix}$ $b) B = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$

Solution.

$$|A \;\; I| = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & -2 & -1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \downarrow \\ \leftarrow \end{array} \begin{array}{l} -2 \\ \end{array} \qquad \sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 0 & 1 & 0 \\ 0 & 5 & 3 & 1 & -2 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} \downarrow \\ \leftarrow \end{array} \begin{array}{l} -1 \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 3 & 1 & -2 & 0 \\ 0 & 3 & 3 & 0 & -1 & 1 \end{array}\right] \begin{array}{l} \\ \\ \frac{1}{3} \end{array} \qquad \sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 3 & 1 & -2 & 0 \\ 0 & 1 & 1 & 0 & -\frac{1}{3} & \frac{1}{3} \end{array}\right] \begin{array}{l} \\ \downarrow \\ \leftarrow \end{array} \begin{array}{l} \\ -\frac{1}{5} \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 5 & 3 & 1 & -2 & 0 \\ 0 & 0 & \frac{2}{5} & -\frac{1}{5} & \frac{1}{15} & \frac{1}{3} \end{array}\right] \begin{array}{l} \\ (1/5) \\ (5/2) \end{array} \qquad \sim \left[\begin{array}{ccc|ccc} 1 & -2 & -1 & 0 & 1 & 0 \\ 0 & 1 & \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right] \begin{array}{l} \leftarrow \\ \uparrow \\ \end{array} \begin{array}{l} \\ 2 \\ \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & & \frac{1}{5} & \frac{2}{5} & \frac{1}{5} & 0 \\ 0 & 1 & \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right] \begin{array}{l} \\ \\ -\frac{1}{5} \end{array} \qquad \sim \left[\begin{array}{ccc|ccc} 1 & & 0 & \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ 0 & 1 & \frac{3}{5} & \frac{1}{5} & -\frac{2}{5} & 0 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right] \begin{array}{l} \leftarrow \\ \uparrow \\ \end{array} \begin{array}{l} \\ -\frac{3}{5} \end{array}$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & & 0 & \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ 0 & 1 & 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right] \Rightarrow \quad A^{-1} = \left[\begin{array}{ccc} \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{6} & \frac{5}{6} \end{array}\right]$$