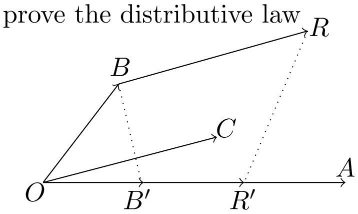
Properties:

1. A . B = B . A (com. law)
2. (λA). B = A.(λB) = λ(A.B)
3. A. (B+C) = A.B + A.C (dist. law)

Proof: The first two properties are direct consequences of the definition. To prove the distributive law



$$A.(B + C) = A.B + A.C \tag{0.1}$$

consider $\vec{BR} = \vec{OC}$ (see fig.), and projections B', R' of B, R on OA. Then,

A.(B+C) = A.R

=A.R' (Geom. interp. 1)

=A. (B'+C')

=A.B' + A.C' (collinearity of vectors)

=A.B + A.C (Geom. interp. 1)

Now we derive the analytic expression

$$A.B = a_1b1 + a_2b_2 + a_3b_3 for A = (a_1, a_2, a_3), B = (b_1, b_2, b_3). \tag{0.2}$$

Expanding

$$A.B = (a_1i + a_2j + a_3k).(b_1i + b_2j + b_3k) \tag{0.3}$$

by distributive law, we get nine terms, six of which are zero by properties

$$i.j = 0, j.k = 0, k.i = 0 \tag{0.4}$$

for orthogonal vectors i,j,k and the remaining terms are

$$a_1b_1, a_2b_2, a_3b_3 \tag{0.5}$$

by the properties i.i = 1, j.j = 1, k.k = 1 for unit vectors i,j,k