If in a square matrix all entries below (above) the diagonal are zero, the matrix is called an upper (lower) triangular matrix.
The following are"triangular matrices:

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 3 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 3 & 4 & -5 \end{bmatrix}$$

An upper triangular matrix   A lower triangular matrix

$$(a_{ij} = 0 \quad for \quad i > j) \qquad (a_{ij} = 0 \quad for \quad i < j)$$

The above definition may be extended to any matrix, with

$$\begin{bmatrix} 1 & 0 & 8 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 0 \end{bmatrix}$$

Transpose of matrix:

The transpose of a matrix $A = [a_{ij}]_{(mxn)}$ is the matrix $A^T = [a_{ji}]_{(nxm)}$. According to this definition the transpose is obtained by changing rows into columns and column into rows. Why $(A^T)^T = A$?

Example: Write the transpose of the following matrices:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \quad , \qquad B = \begin{bmatrix} 3 & 5 \\ 0 & -2 \\ 1 & 4 \end{bmatrix} \quad , \qquad C = \begin{bmatrix} 3 & 0 & 7 \end{bmatrix}$$

and give reasons for unalteration of the diagonal elements.

Answer.

$$A^T = \begin{bmatrix} a & d & g \\ d & e & h \\ g & f & i \end{bmatrix} \quad , \qquad B^T = \begin{bmatrix} 3 & 0 & 1 \\ 5 & -2 & 4 \end{bmatrix} \quad , \qquad C^T = \begin{bmatrix} 3 \\ 0 \\ 7 \end{bmatrix}$$