$$\text{Adj A} = \begin{bmatrix} -3 & -1 & 1 \\ -3 & 3 & 3 \\ 3 & -1 & -5 \end{bmatrix} = [A_{ji}]$$

b) $\text{Adj B} = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$

THEOREM 0.1. *$A^{-1} = \frac{1}{|A|}$ Adj $A = \frac{[A_{ji}]}{|A|}$ if $|A| \neq 0$, i. e., if $[A_{ij}]$ is invertible.*

PROOF. We need to show that

$$\text{A } \frac{Adj A}{|A|} = \text{I or A Adj A} = |A| \text{ I}$$

Indeed,

$$\text{A Adj A} = \begin{bmatrix} \cdots & \cdots & \cdots \\ a_{il} & \cdots & a_{in} \\ \cdots & \cdots & \cdots \end{bmatrix} \begin{bmatrix} \vdots & A_{lj} & \vdots \\ \vdots & \vdots & \vdots \\ \vdots & A_{nj} & \vdots \end{bmatrix} /|A|$$

$$= [\sum_k a_{ik} A_{kj}]/|A| = [\delta_{ij} |A|]/|A| = [\delta_{ij}] = I$$

by Theorem 6 on determinant. (Book I) □

EXAMPLE 0.1. . Find the inverses of the matrices A and B in Example 1, if any.

Solution.

a) The classical adjoint of A was obtained as the matrix

$$\begin{bmatrix} -3 & -3 & 3 \\ -1 & 3 & -1 \\ 1 & 3 & -5 \end{bmatrix}$$

and the inverse is obtained by dividing this matrix by $|A|$= -6