a) to have max error of 10^{-2} in sum s, how many terms should be taken?

Solution.

a) Since $1 \geq \frac{1}{2} \geq \frac{1}{3} ... \geq \frac{1}{n} > ...$ and $\frac{1}{n} \to 0$, there is convergence.

b) $\frac{1}{n+1} < \frac{1}{10^2} \Rightarrow n+1 > 100 \Rightarrow n > 99$ (100 terms)

1. SERRIES OF ARBITRARY TERMS

If a series is one of positive terms, one applies a test given for such series, if the series is alternating one applies LEBNIZ'test (test for alternating serries). For an arbitrary series the following theorem holds:

Theorem A series

$$\sum_i a_n = a_1 + a_2 + ... + a_n + ...$$

is convergent if the series

$$\sum_i |a_n| = |a_1| + |a_2| + ... + |a_n| + ...$$

of absolute value is convergent.

Proof. Let s_n,S_n be the corresponding partial sums. The sum s_n contains non negative and negative terms and we write

$s_n = P_n - Q_n$, $S_n = P_n + Q_n$

where P_n and $-Q_n$ are sums of positive and negative terms respectively.

(P_n), (Q_n) are monotone increasing sequences bounded above by $S = \lim S_n$. Then $P_n \to P$, $Q_n \to Q$ and $S_n \to P + Q$. It follows that $s_n \to P - Q$