1. METHODS OF INTEGRATION

In calculus there are essentialy two methods of integration : "change of variables" and "by parts".

1.1. Integration by change of variable (substitution).

Let the indefinite integral

$$I = \int f(x)\ dx$$

to be evaluated.

One makes (tries) the substitution

$$x = u(t) \quad \text{or} \quad t = u^{-1}(x) = v(x)$$

$$I = \int f(x)\ dx = \int f(u(t)).u'(t)\ dt = \int g(t)\ dt$$

If the substitution is properly selected the new integral is more easily integrable than the original one, getting G(t) + c and replacing t by v(x), one has

$$G(t) + c = G(v(x)) + c = F(x) + c$$

EXAMPLE 1.1. Evaluate

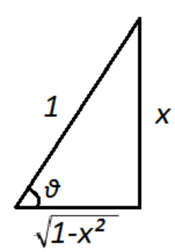
$$I = \int \frac{dx}{(1-x^2)^{3/2}}$$

SOLUTION 1.1. Since square root is not involved, $1 - x^2 > 0$ follows and the substitution $x = sin\theta$ may work

$$I = \int \frac{cos\theta\ d\theta}{(1-sin^2\theta)^{3/2}} = \int \frac{cos\theta\ d\theta}{cos^3\theta} = \int sec^2\theta\ d\theta = tan\theta + c$$

The result is to be written in terms of x. Using the relation $x = sin\theta$ we have

$$I = tan\theta + c = \frac{x}{\sqrt{1-x^2}} + c$$



sin Triangle[1]

1.1.

EXAMPLE 1.2. Evaluate

$$I = \int (x^3 - 2x + 3)^{15}(3x^2 - 2)\ dx$$

SOLUTION 1.2. Observing that, $D(x^3-2x+3) = (3x^2-2)$ the proper substitution is

$$u = (x^3 - 2x + 3)\ , \quad du = (3x^2 - 2)\ dx$$