CHAPTER 2
Groups

1. Basic Definitions

Before giving the formal definition of a group, we would rather present some concrete examples.

1.1. Examples.

(a) Consider the addition of integers. From the numerous properties of this binary operation, we single out the following ones.
   (i) $+$ is a binary operation on Z, so, for any $a, b \in Z$, we have $a + b \in Z$.
   (ii) For all $a, b, c \in Z$, we have $(a + b) + c = a + (b + c)$.
   (iii) There is an integer, namely $0 \in Z$, which has the property $a + 0 = a$ for all $a \in Z$.
   (iv) For all $a \in Z$, there is an integer, namely $-a$, such that $a + (-a) = 0$.

(b) Consider the multiplication of positive real numbers. Let $\mathbb{R}^+$ be the set of positive real numbers. Here the multiplication enjoys properties apalogous to the ones above.
   (i) $\cdot$ is a binary operation on $\mathbb{R}^+$, so, for any $a, b \in \mathbb{R}^+$; we have $a \cdot b \in \mathbb{R}^+$.
   (ii) For all $a, b, c \in \mathbb{R}^+$, we have $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
   (iii) There is a positive real number, namely $1 \in \mathbb{R}^+$, which has the property $a \cdot 1 = a$ for all $a \in \mathbb{R}^+$.