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A SUMMARY

2. 1. Operations with matrices:

A+B = $[a_{ij}]$ + $[b_{ij}]$ = $[a_{ij}$+$b_{ij}]$ = B+A
cA = c$[a_{ij}]$ = $[ca_{ij}]$
$A_{mxn}B_{nxp} = C_{mxp} = [c_{ij}], c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$

Definitions:

A^T = $[a_{ij}]^T$ (transpose of the matrix A)
Adj A = $[A_{ij}]^T$ = $[A_{ji}]^T$ (adjoint of A)
AA^{-1} = A^{-1}A = I (A^{-1} is the inverse of A)
A formula for A^{-1} is A^{-1} = Adj A/$|A|$
where $|A|$ = det A

Echelon matrix: Is a matrix $[a_{ij}]_{mxn}$ such that
a_{ij}= 0 (j = 1 , ... , k)
$a_{(i+1)j}$ = 0 (j = 1 , ... , k+1 at least)

An example of echelon matrix:

$$\begin{bmatrix} 0 & 3 & 1 & 0 & 5 & 0 & -7 & 8 & 3 \\ 0 & 0 & 0 & 2 & -4 & 0 & 6 & -3 & 4 \\ 0 & 0 & 0 & 0 & -2 & 0 & 1 & 12 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2. 2. Solution of a system of linear equans: $AX = B$

a) Square case:
By inverse matrix : $X = A^{-1}B$ when A is invertible
b) General case:
By GAUSS Method: Obtaining an echelon form of the augmeted matrix $|A : B|$ and solving step by step from bottom to top.

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MISCELLANEOUS EXERCISE

0.46. Let

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$$

Find a) AB b) A^3 + A^2 - 6A - 17I_3

0.47. Prove

$$a)\ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix} \qquad b)\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n & \frac{1}{2}n(n+1) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$$

0.48. Find a matrix B such that $B^{-1}AB$ is diagonal, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

0.49. List all 2x2 echelon matrices.

0.50. For an nxn matrix A, prove $|A||adjA| = |A|^n$

0.51. Obtain an echelon form of:

$$\begin{bmatrix} 2 & 1 & 0 & 3 & -2 \\ 2 & 4 & 1 & 3 & 2 \\ 4 & 2 & 1 & 3 & -2 \\ 2 & 1 & 1 & 0 & 5 \\ -4 & 1 & 1 & 3 & -2 \end{bmatrix}$$

0.52. If $ad - bc = 1$, then

$$\begin{bmatrix} ad & cd & -ab & -bc \\ -ac & -c^2 & a^2 & ac \\ bd & d^2 & -b^2 & -bd \\ -bc & -cd & ab & ad \end{bmatrix}^{-1} = \begin{bmatrix} ad & bd & -ac & -bc \\ -ab & -b^2 & a^2 & ab \\ cd & d^2 & -c^2 & -cd \\ -bc & -bd & ac & ad \end{bmatrix}$$