$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \end{vmatrix} = a_{11}C_{11} + ... + a_{1j}C_{1j} + ... + a_{1n}c_{1n}$$

$$(1) \qquad = \sum_{j=1}^{n} a_{1j}C_{1j}$$

where $C_{1j} = (-1)^{1+j} M_{1j} \quad (j = 1, ..., n)$ are cofactors so that the evaluation of D is reduced to the evaluation of determinants of order $n - 1$, which in turn are reduced to the evaluation of determinants of order $n - 2$, and so on. Thus

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}C_{11} + a_{12}C_{12} = a_{11}a_{22} - a_{12}a_{21}$$

The value given by 1 is the *Laplace expansion* of D with respect to the first row.

The same determinant D has Laplace expansions with respect to any other row or any column. It is proved in Linear Algebra that all these expansions have the same value, hence each one can be used for the evaluation of D.

Thus we have

$$(2) \qquad D = a_{i1}C_{i1} + a_{i2}C_{i2} + ... + a_{in}C_{in} = \sum_{j=1}^{n} a_{ij}C_{ij}$$

as Laplace expansion with respect to the ith row, and

$$(3) \qquad D = a_{1j}C_{1j} + a_{2j}C_{2j} + ... + a_{nj}C_{nj} = \sum_{i=1}^{n} a_{ij}C_{ij}$$

as Laplace expansion with respect to the jth column.

Example 0.1. Evaluate

$$\begin{vmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{vmatrix}$$

by expanding it with respect to the 3rd column, and 2nd row.