EXAMPLE 0.1. a) i.

$$a \cdot 1 = a \text{ for all } a \in \mathbb{R}^+.$$

ii. For all $a \in \mathbb{R}^+$, there is a positive real number, namely 1/a, such that

$$a \cdot \frac{1}{a} = 1.$$

b) Let n be a natural number and consider the addition in $\mathbb{Z}_n$, which we introduced in §6.

i. $+$ is a binary operation on $\mathbb{Z}_n$, so, for any $\bar{a}, \bar{b} \in \mathbb{Z}_n$, we have $\bar{a} + \bar{b} \in \mathbb{Z}_n$.

ii. For all $\bar{a}, \bar{b}, \bar{c} \in \mathbb{Z}_n$, we have $(\bar{a} + \bar{b}) + \bar{c} = \bar{a} + (\bar{b} + \bar{c})$.

iii. There is an integer mod n, namely $\bar{0} \in \mathbb{Z}_n$, which has the property

$$\bar{a} + \bar{0} = \bar{a} \text{ for all } \bar{a} \in \mathbb{Z}_n.$$

iv. For all $\bar{a} \in \mathbb{Z}_n$, there is an integer mod n, namely $\overline{-a}$, such that

$$\bar{a} + (\overline{-a}) = \bar{0}.$$

c) Let X be a nonempty set and let S_X be the set of all one-to-one mappings from X onto X. Consider the composition $\circ$ of mappings in S_X.

i. $\circ$ is a binary operation on S_X, for if σ and τ are one-to-one mappings from X onto X, so is $\sigma \circ \tau$ by Theorem 3.13.

ii. For all $\sigma, \tau, \mu \in S_X$, we have $(\sigma \circ \tau) \circ \mu = \sigma \circ (\tau \circ \mu)$ (Theorem 3.10).

iii. There is a mapping in S_X, namely $\iota_X \in S_X$, such that

$$\sigma \circ \iota_X = \sigma \text{ for all } \sigma \in S_X \quad \text{(Example 3.9(a))}$$

iv. For all $\sigma \in S_X$, there is a mapping in S_X, namely σ^{-1}, such that

$$\sigma \circ \sigma^{-1} = \iota_X.$$

(See Theorem 3.14 and Theorem 3.16. That $\sigma^{-1} \in S_X$ follows from Theorem 3.17(1).)

These are examples of groups. In each case, we have a nonempty set and a binary operation on that set which enjoys some special properties. A group will be defined as a nonempty set and a binary operation on that set having the same properties as in the examples above. A group will thus consist of two parts: a set and a binary operation. Formally, a