§2
Equivalence Relations

In mathematics, we often investigate relationships between certain objects (numbers, functions, sets, figures, etc.). If an element a of a set$A$ is related to an element b of a set B, we might write

$$a \text{ is related to } b$$

or shortly

$$a \text{ related } b$$

or even more shortly

$$a \; R \; b$$

The essential point is that we have two objects; a and b, that are related in some way. Also, we say "a is related to b", not "b is related to a",so the *order* of a and b is important. In other words, the ordered pair(a, b) is distinguished by the relation. This observation suggests the following formal definition of a relation.

DEFINITION 1. Let A and B two sets. A *relation R from A into B* is a subset of the cartesian product A $\times$B.

If A and B happen to be equal, we speak of a relation on A instead of using the longer phrase "a relation from$A$ into A"

Equivalence relations constitute a very important type of relations on a set.

DEFINITION 2. Let A be a nonempty set. A relation R on A (that is, a subset R of $R \times R$) is called and *equivalence relation on A* if the following holds:

(i) $(a, a) \in R$ for all $a \in A$,
(ii) if $(a, b) \in R$, then $(b, a) \in R$ (for all $a, b \in$ A).
(iii) if $(a, b) \in R$ and $(b, c) \in$ R, then $(a, c) \in R$
(for all $a, b, c \in R$)