

a) $\frac{f(x)}{g(x)}$

b) $\frac{g(x)}{f(x)}$

14. Let f be a differentiable function. Show that the tangent line to f and $1/f$ at the point x_0 intersect the x -axis at points which are symmetrically placed with respect to x_0 .

15. If $u = u(x), v = v(x)$ are functions differentiable up to any order, there is a formula for the n th derivative $(uv)^n$ established by Leibniz. For $n = 2$ and 3 the formulas are

a) $(uv)'' = u''v + 2u'v' + uv''$

b) $(uv)''' = u'''v + 3u''v' + 3u'v'' + uv'''$

Prove these and write the formula for $n = 4$.

c) $(uv)^{(4)} = \dots$

16. Apply the linear operator $1 + D + 2D^2 - D^3$ to

a) $x^4 - 3x^2$

b) $8x\sqrt{x}$

17. Prove by induction

a) $Dx^n = nx^{n-1} \quad n \in \mathbb{Z}$.

b) $D^m x^n = n \dots (n - m + 1)x^{n-m}, \quad m \in \mathbb{Z}, \quad (m \leq n), n \text{ is fixed.}$

18. Find the derivatives of the following functions

a) $(x+2)(x^3 + 7x^{5/2})$

b) $(5x^2 + x)(\frac{2}{x} - \frac{3}{x^2})$

c) $\sqrt{5x^3 - x}$

d) $\sqrt[3]{7x + x^2}$

19. Find $f'(x)$ if $f(x) = |3x^2 - 4x + 5| + \frac{1}{x}$

20. Prove the following:

a) Derivative of a periodic function is a periodic function,