

Another continuous case is the distribution of a mass m over a plane region R with density δ (mass per unit area) so that

$$dm = \delta dA$$

If a curve (or a region) is given with corresponding densities, the total mass is obtained by integrating δds or δdA , but in the second case the evaluation is possible when δ is function of x (or y) alone, which in the first case may be given as $\delta(x, y)$ and reducible to $\delta(x)$ or to $\delta(y)$ since x, y are related by equation of the curve.

Mass of a wire : Let the wire be in the shape of the curve
 $y=f(x) \in D(a, b)$ or $x=g(y) \in D(c, d)$

Then

$$m = \begin{cases} \int_a^b \delta(x) \sqrt{1 + f^2(x)} dx \\ \int_c^d \delta(y) \sqrt{1 + g^2(y)} dy \end{cases}$$

Mass of a plate : Let the plate be in the shape of the region

$$R_{xy} = (a, b; y_1(x) \text{ or } y_2(x)) \text{ or } R_{yx} = (c, d; x_1(y) \text{ or } x_2(y))$$

Then

$$m = \begin{cases} \int_a^b \delta(x)(y_2(x) - y_1(x)) dx \\ \int_c^d \delta(y)(x_2(y) - x_1(y)) dy \end{cases}$$

Example .Find the total mass of a wire bent to form the semicircle $x^2 + y^2 = a^2, y \geq 0$ with $\delta(y) = 2y$.