

Solution.

$$\begin{aligned}
 \text{a) } \frac{x-t}{-1} &= \frac{y-t^2}{2} = \frac{t}{3} \\
 \Rightarrow 2x-2t &= -y+t^2, \quad 3y-3t^2=2z \\
 \Rightarrow 6x-6t &= -3y+3t^2, \quad 3t^2=3y-2z \\
 \Rightarrow 6x-6t &= -3y+3y-2z \Rightarrow t = \frac{x-z}{3} \\
 \Rightarrow 3y-3\left(\frac{x-z}{3}\right)^2 &= 2z \\
 \text{b) Taking } Q(x-t, y+2t, z+3t) \text{ on } r, \text{ one has} \\
 (x-t)^2 &= 2(z+3t), \quad (y+2t)-(z+3t)=1 \\
 \Rightarrow (x-t)^2 &= 2(z+3t), \quad t=y-z-1 \\
 \Rightarrow (x-y+z+1)^2 &= 2(z+3y-3z-3) \\
 \Rightarrow x^2+y^2+z^2-2xy+2xz-2yz+2x-8y+6z+7 &= 0
 \end{aligned}$$

0.1. Cones. A surface S generated by a variable line l passing through a fixed point P_0 and subject to another condition such as intersecting a curve r (or remaining tangent to a given surface Σ) is called a cone.

The line l is the generatrix, r the directrix, and P_0 the vertex of the cone S , and we say that S is defined by P_0 and r .

Equation of a cone

The equation of the cones defined by $P_0(x_0, y_0, z_0)$ and

$$\begin{aligned}
 \text{a) } r : x &= f(t), y = g(t), z = h(t), \\
 \text{b) } r : F(x, y, z) &= 0, G(x, y, z) = 0
 \end{aligned}$$