Example. Evaluate

$$a)\frac{d}{dt}\int_{\frac{\pi}{2}}^{\pi}\frac{\cos xt}{x}\,dx \qquad\qquad b)\frac{d}{dt}\int_{0}^{x^2}\frac{\arctan x}{y^2}\,dx$$

Solution.

$$\begin{aligned}
a)\frac{d}{dt}\int_{\frac{\pi}{2}}^{\pi}\frac{\cos xt}{x}\,dx &= \int_{\frac{\pi}{2}}^{\pi}\frac{\partial}{\partial x}\frac{\cos xt}{x}\,dx\\
&= \int_{\frac{\pi}{2}}^{\pi}-\frac{x\,\sin xt}{x}\,dx = \left.\frac{\cos xt}{t}\right|_{x=\frac{\pi}{2}}^{\pi}\\
&= \frac{\cos \pi t-\cos\frac{\pi}{2}t}{t}\\
b)\frac{d}{d\,y}\int_{0}^{y^2}\arctan\frac{x}{y^2}\,dx&\\
&= (\arctan\frac{y^2}{y^2})\,2y-(\arctan 0)0+\int_{0}^{y^2}\frac{\partial}{\partial y}\arctan\frac{x}{y^2}\,dx\\
&= \frac{\pi}{2}\,y+\int_{0}^{y^2}-\frac{-2xy}{x^2+y^4}\,dx-y\,\ln 2
\end{aligned}$$

$$EXERCISES(4,2)$$

16. Evaluate

$$a)\ \left.\frac{\partial}{\partial x}\frac{x-y+1}{x+y-1}\right|_{(0,0)} \qquad\qquad b)\ \left.\frac{\partial}{\partial y}\frac{x-y+1}{x+y-1}\right|_{(0,0)}$$

17. If $u(x,y,z)=x^2y+y^2z+z^2x$, verify that

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=(x+y+z)^2$$

18.Verify the equality $f_{xy}=f_{yx}$
for

$$\begin{aligned}
a)f(x,y)&=\cos xy^2 & b)f(x,y)&=\sin^2 x\cos y\\
c)f(x,y)&=\frac{y}{x} & d)f(x,y)&=\sqrt{x^2+y^2}
\end{aligned}$$