Subsequences:

If every term of an $(infinite)$ sequence $(b_n)_l$ is also a term of a sequence $(a_n)_l$ then $(b_n)_l$ is said to be a subsequence of $(a_n)_l$.

Clearly, every sequence is a subsequence of itself, Among other subsequences of $(a_n)_l$ we mention the following:

$$(a_{2n})_5, (a_{n+3})_l, (a_{n^2})_7, (a_{n!})_2, (a_n)_N$$

A notation for arbitrary subsequence of $(a_n)_l$ is $(a_{n_k})_{n=1}$ where (n_k) is a sequence of integers.

Some subsequences of $((-1)^n)_2$ are

$$1, 1, 1, \cdots, 1, \cdots$$

$$-1, -1, -1, \cdots, -1, \cdots$$

$$1, \underbrace{-1}_{1}, .1, \underbrace{-1, -1}_{2}, \cdots, 1, \underbrace{-1, \cdots, -1}_{\text{n}}, \cdots$$

0.1. BEHAVIOR OF A SEQUENCE.

0.1.1. *Monotonocity.* :

A sequence $(a_n)_l$ is called monotone if

$$a1 \leqslant a2 \leqslant \cdots \leqslant a_n \leqslant \cdots$$

or else

$$a1 \geqslant a2 \geqslant \cdots^{1} \geqslant a_n \geqslant \cdots$$

[1]it was "..:" but i thought that it should be "..."

In the former (latter) case the sequence is said to be