Since

$$l : x + \sqrt{3}y - 2a = 0$$

the square of the distance of (x, y) from l is

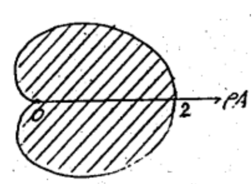
$$d^2 = x + \sqrt{3}y - 2a^{2}/4.$$

$I_l =$ [1] $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} \frac{1}{4}(x+\sqrt{3}y-2a)^2 ky0.05cmdydx$
$= \frac{1}{4}\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2 + 2\sqrt{3}xy + 3y^2 - 4ax - 4a\sqrt{3}y + 4a^2)kydydx$
$= \frac{1}{4}\Big(I_{oy} + 0 + 3I_{ox} - \underbrace{4aM_{oy}}_{0} - 4a\sqrt{3}M_{ox} + 4a^2m\Big)$
$= \frac{k}{4}\Big(\frac{2}{15}a^5 + 0 + 3\frac{4a^5}{15} - 0 - 4a\sqrt{3}\frac{\pi}{8}a^4 + 4a^2\frac{2}{3}a^3\Big)$
$= \frac{k}{4}a^5\Big(\frac{2}{15} + \frac{12}{15} - \frac{\sqrt{3}}{2}\pi + \frac{5}{3}\Big)$
$= \frac{k}{40}(36 - 5\sqrt{3}\pi)a^5.$

If R is a polar region, by usual transformations one can obtain m, N_{PA}, M_{CPA}, I_{PA}, I_{CPA}, I_0 and the coordinates $\bar{\theta}, \bar{r}$ of G can be computed by

$$\bar{\theta} = \arctan\frac{\bar{y}}{\bar{x}}, \quad \bar{r} = \sqrt{\bar{x}^2 + \bar{y}^2}.$$

Example. Find the centroid, and moment of inertia I_0 of a homogeneous plate in the shape of the cardioid $r = a(1+\cos\theta)$.
[2]



Solution.
$m = \int_{-\pi}^{\pi}\int_{0}^{a(1+\cos\theta)} \delta r dr d\theta$
$= \delta\int_{-\pi}^{\pi} {}^{3}\frac{r^2}{2}\Big|_{\theta}^{a(1+\cos\theta)} d\theta$
(since δ is const)

[1]There are 2 equal operator.
[2]wrapfig package is used
[3]The lower bound of the integral must be $-\pi$