page=b2p2/266

A function of two variables may also be defined implicitly as $F(x, y, z) = 0$ (by a restrictor). Unless otherwise stated it is considered that z is the dependent variable. However $F(x, y, z) = 0$ may define x (or y) as a function of the other two variables.

Example 0.1. Which ones of the following relations are functions. If not, write a restriction to be a function.

a) $x^2 + y^2 + z^2 = 16$

b) $x^2 + y - z^2 = 0$

c) $-2z = 2x^2 + y^2$

d) $z^3 = x$

Solution.

a) This relation is not a function, since $P(x, y)$ has more than one image. A restriction for this to be a function is $z > 0$. Another restriction is, for instance, the point $(0, \sqrt{7}, -3)$ lies on the surface.
b) Same as in (a), A restriction is $z > 5$. Observe that y is a function of x and z.
c) This is a function, since to each pair (x, y) there is assigned a single image, namely $z = -x^2 -^2 /2$.
d) It is a function: $z = f(x, y) = \sqrt[3]{x}$.

Example 0.2. Find and sketch the domains of the following functions:

a) $z = e^{xy}$

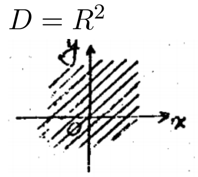
b) $z = \ln xy$

c) $z = \sqrt{1 - x^2 - y^2}$

d) $z = \frac{e^x}{1-y}$

Solution.

a) $D = R^2$



b) $D = (x, y) : xy > 0$