C. Some Important Convergent Sequences

1. a) $(\sqrt[n]{a})_1 : a, \sqrt{a}, \sqrt[3]{a}, \cdots, \sqrt[n]{a}, \cdots$ converges to 1 (for $a > 0$);

   b) $(\sqrt[n]{n})_1 : 1, \sqrt{2}, \sqrt[3]{3}, \cdots, \sqrt[n]{n}, \cdots$ converges to 1

2. $((1 + \frac{\lambda}{n})^n) \to e^\lambda$ or $(\frac{n+\lambda}{n})^n \to e^\lambda$

3. a) $(\frac{\ln^p n}{n}) \to 0$ for any constant p.

   b) $\frac{n^p}{e^n} \to 0$ for any constant p.

4. If $(a_n)_1 : a_1, a_2, \cdots, a_n, \cdots$ is a sequence with positive terms converging to the limit a, then

   a) $A_n = \frac{a_1, \cdots, a_n}{n} \to a$

   b) $G_n = \sqrt[n]{a_1 \cdots a_n} \to a$

   where A_n, G_n are called the arithmetic mean and geometric mean of the positive numbers $a_1, a_2, \cdots, a_n$.

5. a) $(\frac{n}{\sqrt[n]{n!}}) \to e$, b)$(\frac{e^n}{n!}) \to 0$

The proofs of 1 to 3 are obtained by limit process considering the functions $a^{1/x}, x^{1/x}, (1 + \frac{\lambda}{x})^x, x^p/e^x$ of continuous variable x.

Proof. Since $(a_n) \to a$, then given $\varepsilon > 0$ there is $\mathbb{N} > 0$ such that $\square$