$$|a_n - a| < \varepsilon,\ |b_n - b| < \varepsilon \text{ for all } n > N$$

To show $|a_n - a| < \varepsilon$ we from $a_n b_n - ab$ and get

$$\begin{aligned}
|a_n b_n - ab| &= |a_n b_n - ab_n - ab + ab_n| \\
&= |(a_n - a)b_n - a(b - b_n)| \\
&\leqslant |a_n - a||b_n| + |a||b - b_n| \\
&\leqslant |b_n|\varepsilon + |a|\varepsilon
\end{aligned}$$

Since for all $n > N$, b_n lies in the interval $(b - \varepsilon,\ b - \varepsilon)$ it follows that $b_n < K$ for some positive K, and one has

$$|a_n b_n - ab| < K\varepsilon + |a|\varepsilon = (K + |a|)\varepsilon$$

showing that $a_n b_n \to ab$.

Theorem 2

a) A monotone sequence is convergent,
b) A convergent sequence is bounded,
c) $(a_n) \to a$, $(b_n) \to n$ and $a_n \leqslant c_n \leqslant b_n$ for all $n > N \implies (c_n) \to c$ and $a \leqslant c \leqslant b$

Proof. Omitted