

is a Vector Space  $M_{m \times n}(+, \mathbb{R})$  where the inner product (dot product) is defined as

$$\begin{aligned} \langle A, B \rangle = A \cdot B &= a_{11}b_{11} + \dots + a_{1n}b_{1n} \\ &+ a_{21}b_{21} + \dots + a_{2n}b_{2n} \\ &+ a_{n1}b_{n1} + \dots + a_{nn}b_{nn} \end{aligned}$$

and the norm of A, by  $\|A\| = \sqrt{A \cdot A}$

The vector space  $R^3(+, \mathbb{R})$  has the natural generalization  
 $R^n(+, \cdot, R)$

where

$$R^n = \{(x_1, \dots, x_n) : x_i \in R\}$$

is the set of all ordered n-tuples or vectors in n-space, in which the operation of addition, multiplication by scalars and inner product are defined as:

$$\begin{aligned} (x_1, \dots, x_n) + (y_1, \dots, y_n) &= (x_1 + y_1, \dots, x_n + y_n) \\ \lambda(x_1, \dots, x_n) &= (\lambda x_1, \dots, \lambda x_n) \\ (x_1, \dots, x_n) \cdot (y_1, \dots, y_n) &= x_1 y_1 + \dots + x_n y_n \end{aligned}$$

The vectors

$$e_1 = (1, 0, \dots, 0), e_2 = (0, 1, 0, \dots, 0), \dots, e_n = (0, \dots, 0, 1)$$

in this space are unit vectors and pairwise orthogonal as seen by application inner product:

$$e_i \cdot e_j = \begin{cases} 1 & \text{when } i = j \\ 0 & \text{when } i \neq j \end{cases}$$

They are said to lie on  $n(> 1)$  mutually orthogonal axes  $0_{x_1}, \dots, 0_{x_n}$   
 sketch of which cannot be realized when  $n > 3$ .