$$= (x_1 - x_2)(x_1 + x_2) \begin{cases} > 0 \text{ when} & x_1, x_2 \in \mathbb{R}_0^- \\ < 0 \text{ when} & x_1, x_2 \in \mathbb{R}_0^+ \end{cases}$$

If f is an increasing (or decreasing) function on an interval $I \subseteq D$, then f is said to be a monotonic increasing (or monotonic decreasing) function in the interval I.

The function given in the above example, is the monotonic increasing in $\mathbb{R}_0^-$ and monotonic decreasing in $\mathbb{R}_0^+$.

A monotonic increasing (or decreasing) function f in an interval expressed usually by saying that f is one-to-one (or simply 1-1) in I to mean that to distinct numbers x_1, x_2 in I correspond distinct images $f(x_1), f(x_2)$.

D. Inverse of a function

A function

$$f : D \to \mathbb{R}, y = f(x) \text{ or } f = (x, y) : x \in D, y = f(x) \tag{0.0.1}$$

with D as the domain and $\mathbb{R}$ as the range, being a relation from $D \to \mathbb{R}$, its inverse

$$f^{-1} = \{ \, (\, x, y \,) : x \in \mathbb{R}, x = f(y) \, \} \tag{0.0.2}$$

is a relation from $\mathbb{R}$ to D. If the relation f^{-1} is a function we call f^{-1} the inverse function of f, and f is said to be an invertible on the set D.

Since f is a function it maps an x in D into a image y in $\mathbb{R}$, and since f^{-1} is a function from $\mathbb{R}$ to D it maps y backward to the single image x in D. This means that f is an one-to-one function and consequently f^{-1} is one-to-one function.

The graphs of f and f^{-1} are symmetric with respect