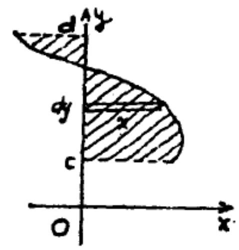
$$|R| = \int_c^d |f(y)| \, \mathrm{d}y.$$

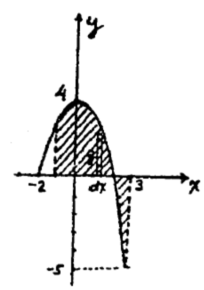
In evaluation area it will be useful to sketch the region in the first step and also draw an elementary area as a horizontal or vertical strip of width $\mathrm{d}y$ or $\mathrm{d}x$ respectively.



Example 0.0.1. Find the area of the plane region bounded by the parabola $y = 4 - x^2$, y-axis and vertical lines $x = -1, x = 3$.

Solution.
The parabola intersects x-axis at $x = 2$ and $x = -2$, and y-axis at $y = 4$. The region is then the shaded one. Hence

$$\begin{aligned}
|R| &= \int_{-1}^{3} \left| 4 - x^2 \right| \mathrm{d}x \\
&= \int_{-1}^{2} (4 - x^2) \, \mathrm{d}x + \int_{2}^{3} (x^2 - 4) \, \mathrm{d}x \\
&= \left(4x - \frac{x^3}{3} \right)_{-1}^{2} + \left(\frac{x^3}{3} - 4x \right)_{2}^{3} \\
&= \left(8 - \frac{8}{3} \right) - \left(-4 + \frac{1}{3} \right) + (9 - 12) - \left(\frac{8}{3} - 8 \right) \\
&= 17 - \frac{16}{3} - \frac{1}{3} = \frac{34}{3}.
\end{aligned}$$



Example 0.0.2. Find the area of a quarter an ellipse with semi major axis a and semi major axis b.

Solution.
The standard equation of the ellipse (center at the origin) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Taking a vertical strip as elementary area (or differential of the area)