Definition 0.0.1. Let $f:\ A \to B$ be a mapping. If every element of B is the second component of at most one order pair of f, then f is called a one-to-one mapping from A to B.

A function $f:\ A \to B$ is therefore one-to-one if an arbitrary element of B has either no preimage in A or exactly one preimage: any two preimages of $b \in B$ (if b has a preimage at all) must be equal. So the necessary and sufficient condition for a mapping: $f:\ A \to B$ to be one-to-one is

$$af = b \textit{ and } a_1 f = b \Longrightarrow a = a_1 \qquad (a, a_1 \in A, b \in B)$$

or, more shortly

$$af = a_1 f \Longrightarrow a = a_1 \qquad (a, a_1 \in A),$$

whose contrapositive reads

$$a \neq a_1 \Longrightarrow af \neq a_1 f \qquad (a, a_1 \in A)$$

A one-to-one mapping is a mapping by which different elements in the domain are matched with different elements in the range. Being a one-to-one function is the negation of being a "many-to-one" function, by which many elements in the domain are matched with one and the same element in the range.

Example 0.0.1. (a) $\{\,(x, y) : x^2 = y\,\} \subseteq \mathbb{R}$X $\mathbb{R}$is not a one-to-one function from $\mathbb{R}$into $\mathbb{R}$, for two distinct elements x and x (if $x \neq 0$) have the same image.

(b) Let $\mathbb{R}^+$denote the set of all positive real numbers. Then the mapping $\{\,(x, y) : x^2 = y\,\} \subseteq \mathbb{R}^+$X $\mathbb{R}^+$is a one-to-one function from $\mathbb{R}^+$into $\mathbb{R}^+$.

(c) The mapping $g : \{\,1, 2, 3\,\} \to \{\,a, b, c, d\,\}$, given by

$$1g = b, 2g = d, 3g = a,$$

is one-to-one.

(d) Let A be a nonempty set. Then $i_A : A \to A$ is one-to-one, for if $ai_A = bi_A$, then $a = b$ from the definition of $\imath_A$.