DIVERGENCE AND CURL OF A VECTOR FUNCTION

If $f(x,y,z) = (P,Q,R)$ is a vector funtion, then

$$div\ F = \bigtriangledown .F = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$curl\ F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} =$$

which are scalar and vector function respectively.

Example. Given the vector functions

$$F(x,y,z) = (x^2, 2xy, xyz), \quad G(x,y,z) = (yz, z^2, xy),$$

find

a)$div\ F$, b)$curl\ G$, c)$div\ curl\ G$.

Solution.

a) $div\ F = 2x + 2y + xy$

b) $curl\ G = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (x - 2z, 0, -z)$

c) $div(curl\ G) = 0$.

Properties. For a scalar function f and vector functions F, G:

1.$\bigtriangledown .fF = (\bigtriangledown f).F + f\ \bigtriangledown .F$

2. $\bigtriangledown \times fF = (\bigtriangledown f) \times F + f\ \bigtriangledown \times F$

3. $\bigtriangledown .(F \times G) = (\bigtriangledown \times F).G - F.(\bigtriangledown \times G)$

4. $\bigtriangledown . \bigtriangledown f = (\bigtriangledown . \bigtriangledown) f$

5. $\bigtriangledown \times \bigtriangledown f = 0$

6. $\bigtriangledown . \bigtriangledown \times F = 0$, $\bigtriangledown .r = 3, \bigtriangledown \times r = 0, F. \bigtriangledown r$

where r is the position vector.