and

$$y = x^2 \Rightarrow \quad \sqrt{\frac{1}{2}}(x' + y') = \frac{1}{2}(x' - y')^2$$
$$\Rightarrow \quad x'^2 - 2x'y' + y'^2 - 2x' - 2y' = 0$$

3. $\tan 2\theta = \frac{-3}{1-1} = \infty \Rightarrow 0 = \pi/4$, and $x^2 - 3xy + y^2 - y = 0$

$$\Rightarrow \quad \frac{1}{2}(x' - y')^2 - \frac{3}{2}(x' - y')(x' + y') + \frac{1}{2}(x' + y')^2 - \frac{1}{2}(x' + y') = 0$$
$$\Rightarrow \quad x'^2 - 2x'y' + y'^2 - 3(x'^2 - y'^2) + (x'^2 + 2x'y' + y'^2) - 2(x' + y') = 0$$
$$\Rightarrow \quad -x'^2 + 5y'^2 - \sqrt{2}x' - \sqrt{2}y' = 0$$

Example 0.1. Translate the coordinate axes to eliminate the linear terms in (1) and compute Δ, H, T before and after the translation.

$$f(x, y) = x^2 + 4xy + 2y^2 + x - y - 5 = 0 \tag{0.1}$$

Solution.
Setting $x = x' + h$, $y = y' + k$ in (1) we have

$$x'^2 + 4x'y' + 2y'^2 + (2h + 4k + 1)x' + (4h + 4k - 1)y' + f(h, k) = 0$$

$$2h + 4k = -1, \qquad 4h + 4k = 1 \qquad h = 1, \qquad k = -3/4$$

Then (1) becomes

$$x'^2 + 4x'y' + 2y'^2 - \frac{33}{8} = 0$$

Now

$$H = A + C = 3, \qquad\qquad H' = A' + C' = 3$$
$$\Delta = B^2 - 4AC = 8 \qquad\qquad \Delta' = B'^2 - 4A'C' = 8$$

$$\text{T} = \begin{vmatrix} 2 & 4 & 1 \\ 4 & 4 & -1 \\ 1 & -1 & -10 \end{vmatrix} = 66, \qquad\qquad \text{T'} = \begin{vmatrix} 2 & 4 & 0 \\ 4 & 4 & 0 \\ 0 & 0 & \frac{-33}{4} \end{vmatrix} = 66$$

The equalities $H = H'$, $\Delta = \Delta'$ and $T = T'$ observed above are general as proved in the second of the following two theorem