

Theorem.

- a) In a convergent series, the general term tends to zero,
- b) If, in a series, the general terms does not tend to zero, the series is divergent.

We call Part b test for divergence by general-term test.

Proof.

- a) Let $\sum_1 a_n = s$. Then $a_n = s_{n+1} - s_n \rightarrow s - s = 0$
- b) Let $a_n \rightarrow k \neq 0$, but suppose the series is convergent.

Then by a), $a_n \rightarrow 0$ contradicting $k \neq 0$.

Remark. As the harmonic series shows, the approach of a_n to zero does not imply the convergence of the series, that is, the series in which $a_n \rightarrow 0$ may converge or diverge. Test for convergence and divergence are generally given for series of positive terms.

B.SERIES OF POSITIVE TERMS

A series having almost every term (all terms except perhaps finitely many ones) positive is called a series of positive terms.

Thus the series

$$\sum_1 \frac{1}{n}, \sum_0 |a_n|, \sum_1 \frac{n^2-2n-3}{n^2} = -4 - \frac{3}{4} - 0 + \frac{5}{6} + \dots + \frac{n^2-2n-3}{n^2} + \dots$$

are series of positive terms, while $\sum_0 \left(\frac{-1}{2}\right)^n$ is not.

The tests that are given here are comparison tests by which a given series of positive terms is compared either with a positive improper integral or with another series of positive terms. There are also intrinsic tests which are applied directly to a given series itself.