Answer.

$$M_{13} = \begin{vmatrix} 4 & 3 \\ 0 & 7 \end{vmatrix}, \qquad C_{13} = (-1)^{1+3} M_{13} = M_{13};$$

$$M_{21} = \begin{vmatrix} 2 & 5 \\ 7 & 2 \end{vmatrix}, \qquad C_{21} = (-1)^{2+1} M_{21} = -M_{21}$$

Transpose of a Determinant:

By the *transpose* of a determinant

$$\mathrm{D} = \begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = |a_{ij}|_n$$

is meant the determinant

$$\begin{vmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & & \vdots \\ a_{n1} & \cdots & a_{nn} \end{vmatrix} = |a_{ji}|_n$$

obtained from D by replacing each row by respective column. It is denoted by D^{T} (read: D transpose)

Thus the transpose of

$$\mathrm{D} = \begin{vmatrix} 1 & 2 & 3 \\ 5 & 5 & 5 \\ 0 & 7 & 4 \end{vmatrix} is \qquad \mathrm{D}^{\mathrm{T}} = \begin{vmatrix} 1 & 5 & 0 \\ 2 & 5 & 7 \\ 3 & 5 & 4 \end{vmatrix}$$

Why the transpose of a symmetric determinant is identical with itself, and that of a skew symmetric one is skew symmetric?

B. Evaluation of a Determinant:

The real determinant $|a_{11}|$ of order 1 is by definiton the real number a_{11} itself. Thus, $|-5| = -5, |\sqrt{2}| = \sqrt{2}$.

If the order is greater than 1, we define it by cofactors as follows: