

## B. EVALUATION

When S has the equation  $\phi(x, y, z) = 0$  defining  $x = x(x, z)$ ,  $y = y(z, x)$ ,  $z = z(x, y)$  the surface integral (1) is the sum in

$$\begin{aligned} & \int \int_{S_{yz}} P(x(y, z), y, z) \overline{dydz} \\ & + \int \int_{S_{zx}} P(x, y(z, x), z) \overline{dzdx} \\ & + \int \int_{S_{xy}} P(x, y, z(x, y)) \overline{dxdy} \quad (2) \end{aligned}$$

of three double integrals, where  $S_{yz}$  for instance is the projection of S onto yz-plane.

When  $\phi(x, y, z) = 0$  defines z, for instance, as a function of x, y not uniquely, say  $z_1$  and  $z_2$ , one evaluates surface integral for both surfaces (lower and upper surface).

When the equation of S is given parametrically as

$$x = x(u, v), y = y(u, v), z = z(u, v)$$

, then by the usual transformations (change of variables) from yz-, zx-, xy-planes to uv-plane, (1) becomes

$$\begin{aligned} & \int \int_{S_1} P(x(u, v), y(u, v), z(u, v)) \left| \frac{\partial(y, z)}{\partial(u, v)} \right| \overline{dudv} \\ & + \int \int_{S_2} Q(x(u, v), y(u, v), z(u, v)) \left| \frac{\partial(z, x)}{\partial(u, v)} \right| \overline{dudv} \\ & + \int \int_{S_3} R(x(u, v), y(u, v), z(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \overline{dudv} \end{aligned}$$

where  $S_1, S_2, S_3$  are the images of  $S_{yz}, S_{zx}, S_{xy}$  under the transformation.