$$e^{ix} = 1 + \frac{ix}{1!} + \frac{(ix)^2}{2!} + ... + \frac{(ix)^n}{n!} + ...$$

$$= 1 + i\frac{x}{1!} - \frac{x^2}{2!} - i\frac{x^3}{3!} + ... + (i^n)\frac{x^n}{n!} + ...$$

$$= (1 - \frac{x^2}{2!} + \frac{x^4}{4} - ... + (-1)^n\frac{x^{2n}}{(2n!)} + ...)$$

$$+ i(x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + (-1)^n\frac{x^{n-1}}{(2n-1)!} + ...)$$

$$= cosx + isinx$$

$$e^{ix} = cosx + isinx \qquad (EULER)$$

Example 2. Obtain the McLAURIN series for $ln\sqrt{\frac{1+x}{1-x}}$.

Solution. Writing

$$ln\sqrt{\frac{1+x}{1-x}} = \frac{1}{2}ln\frac{1+x}{1-x} = \frac{1}{2}[ln(1+x) - ln(1-x)],$$

from

$$ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - ... + \frac{x^{2n}}{2n} - \frac{x^{2n-1}}{2n-1} + ... \qquad (-1,1)$$

$$ln(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - ... - \frac{x^{2n}}{2n} - \frac{x^{2n-1}}{2n-1} - ... \qquad (-1,1)$$

we get

$$ln\sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{5} + ... + \frac{x^{2n-1}}{2n-1} + ... \qquad (-1,1)$$

Example 3. Obtain the McLAURIN series for

a) x^3cosx b) $[ln(1+x)]sinx$

Solution.

a)

$$x^3cosx = x^3\sum_0 (-1)^n\frac{x^{2n}}{(2n)!} = \sum_0 (-1)^n\frac{x^{2n+3}}{(2n)!}$$

b) We have

$$[ln(1+x)]sinx = (x - \frac{x^2}{2} + \frac{x^3}{3} - ... + (-1)^{n-1}\frac{x^n}{n} + ...)$$

$$(x - \frac{x^3}{3!} + \frac{x^5}{5!} - ... + (-1)^n\frac{x^{2n+1}}{(2n+1)!} - ...)$$