To prove $T = T'$, multiplying the first row by -h and the second by -k and adding to the last row, we have

$$T' = \begin{vmatrix} 2A & B & 2Ah + Bk + D \\ B & 2C & Bh + 2Ch + E \\ D & E & 2f(h,k) - Dh - E'k \end{vmatrix}$$

Now multiplying the first column by -h and the second by -k and adding the last column, we get

$$T' = \begin{vmatrix} 2A & B & D \\ B & 2C & E \\ D & E & 2f(h,k) - D'h - E'k - Dh - Ek \end{vmatrix}$$

where the element a_{33} is simplified to 2F. Hence $T' = T$.

Recalling that the coefficients of the transformed equation under a rotation are

$$\begin{array}{ll} A' = Ac^2 + Bcs + Cs^2, & D' = Dc + Es, \\ B' = B(c^2 - s^2) - 2(A - C)cs, & E' = -Ds + Ec, \\ C' = As^2 - Bcs + Cc^2, & F' = F, \end{array}$$

where c, s stand for $\cos\theta$, $\sin\theta$, we have

$$H' = A' + C' = A(c^2 + s^2) + C(s^2 + c^2) = A + C = H$$

To prove the invariance of Δ, we write A', B', C' in terms of $\cos 2\theta$, $\sin 2\theta$ by the use of $2\cos^2\theta = 1 + \cos 2\theta$, $2\sin^2\theta = 1 - \cos 2\theta$:

$$\begin{aligned} 2A' &= B\sin 2\theta + (A - C)\cos 2\theta + A + C \\ B' &= B\cos 2\theta + (A - C)\sin 2\theta \\ 2C' &= B\sin 2\theta + (A - C)\cos 2\theta + A + C, \end{aligned}$$

we have

$$\begin{aligned} \Delta' &= B'^2 - 4A'C' \\ &= [B\cos 2\theta - (A - C)\sin 2\theta]^2 - [(A + C)^2 - (B\sin 2\theta + (A - C)\cos 2\theta)^2] \\ &= B^2 + (A - C)^2 - (A + C)^2 = B^2 - 4AC^2 = \Delta \end{aligned}$$

The invariance of T is proved as follows: