Example. Obtain the standard form of the equation

$$2x^2 + 3xy - 2y^2 + 8 = 0 \tag{1}$$

and compute H, D and T before and after the rotation.

Solution. The angle of the rotation is obtained from

$$tan\ 2\theta = \frac{B}{A-C} = \frac{3}{2+2} = \frac{3}{4}$$

which gives

$$cos\ 2\theta = \frac{1}{\sqrt{1+tan^2 2\theta}} = \frac{1}{\sqrt{1+9/16}} = 4/5$$

$$cos\ \theta = \sqrt{\frac{1+cos\ 2\theta}{2}} = \sqrt{\frac{1+4/5}{2}} = 3/\sqrt{10}$$

$$sin\ \theta = \sqrt{\frac{1-cos\ 2\theta}{2}} = \sqrt{\frac{1-4/5}{2}} = 1/\sqrt{10}$$

Then substituting

$$x = \frac{1}{\sqrt{10}}(3x' - y')$$

$$y = \frac{1}{\sqrt{10}}(x' + 3y')$$

into (1) we have

$$2(3x'-y')^2 + \frac{3}{10}(3x'-y')(x'+3y') - \frac{2}{10}(x'+3y')^2 + 8 = 0$$

or

$$\frac{2}{10}(3x'-y')^2 + 3(3x'-y')(x'+3y') - 2(x'+3y')^2 + 80 = 0$$

$$\Rightarrow (18+9-2)x'^2 + (2-9-18)y'^2 + 80 = 0$$

$$\Rightarrow 25x'^2 - 25y'^2 + 80 = 0 \Rightarrow \frac{y'^2}{80/25} - \frac{x'^2}{80/25} = 1 \Rightarrow a = b = \frac{4}{5}\sqrt{5}$$

Note that

$$H = A + C = 0, \qquad H' = A' + C' = 0$$

$$\delta = B^2 - 4AC = 25, \qquad \delta' = B'^2 - 4A'C' = -4(\frac{25}{10})(-\frac{25}{10}) = 25$$

(Since F = F'= 8 for a rotation)