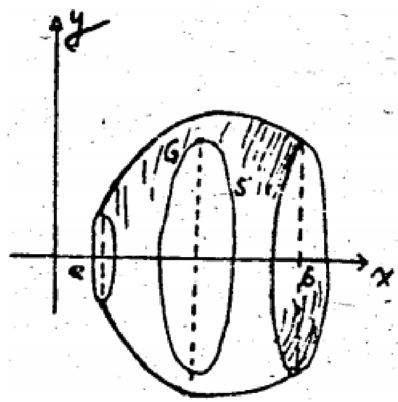
roid of the generating arc (region). They are extremely useful for finding the centroid when surface area (volume) is known, and for finding the latter when the centroid is known.

THEOREM 8.1. *The area of a surface of revolution generated by revolving an arc about a line in its plane not cutting the arc, is equal to the product of the length of arc and the circumference of the circle described by the centroid of the arc:*
$S_{0x} = s.2\pi\overline{y}$.



PROOF. Let the arc of the curve

$y = f(x) \in D\big(a, b\big), \qquad y \geq 0$

be revolved about the x-axis. We have

$S_{0x} = 2\pi \int_a^b y ds$

as the area of the surface, and

$s\overline{y} = M_{0x} = \int_a^b y ds \qquad (\delta = 1)$

$S_{0x} = 2\pi . s\overline{y} = s.2\pi\overline{y}$

$\square$

THEOREM 8.2. *The volume of a solid of revolution generated by revolving a region about a line in its plane not cutting the region, is equal to the product of the area of the region and the circumference of the circle described by the centroid of the region.*

PROOF. Let the region be

$R_{xy} = \Bigg(a, b;\ y_1(x),\ y_2(x)\Bigg)$

be revolved about the x-axis. We have

$V_{0x} = \pi \int_a^b (y_2^2 - y_1^2) dx$

as the volume of the solid, and

$\square$