

**Solution.** For a horizontal tangent line we have  $\Psi = \Pi - \Theta$ ,  
(or  $\Psi = \Theta$ ). Then

$$\frac{r}{r'} = \tan \Psi = -\tan \Theta$$

$$\Rightarrow -\tan \Theta = \frac{r}{r'} = \frac{r^2}{rr'} = \frac{9 \cos 2\Theta}{-9 \sin 2\Theta} = -\cot 2\Theta$$

$$\Rightarrow \tan \Theta \tan 2\Theta = 1 \Rightarrow \tan \Theta = \pm 1/\sqrt{3} \Rightarrow \Theta = \pm \Pi/6 + \Pi$$

$$\Rightarrow \Theta_{1,2} = \pm \Pi/6, \quad \Theta_{3,4} = \pm \Pi/6 + \Pi$$

and

$$\Psi_{1,2} = \Pi - \Theta_{1,2} = \Pi \pm \frac{\Pi}{6}, \quad \Psi_{3,4} = \pm \frac{\Pi}{6}.$$

**EXAMPLE 0.1.** Show that the circles  $r = 2 \sin \Theta$  and  $r = 4 \cos \Theta$  intersect orthogonally.

**Solution.**

$$2 \sin \Theta = 4 \cos \Theta \Rightarrow \tan \Theta = 2 \Rightarrow \Theta_0 = \arctan 2.$$

$$\tan \Psi_1 = \frac{2 \sin \Theta}{2 \cos \Theta} = \tan \Theta = 2, \quad \tan \Psi_2 = \frac{4 \cos \Theta}{-4 \sin \Theta} = -\cot \Theta = -1/2$$

Then

$$\mu_1 \cdot \mu_2 = -1$$

## C. SURFACE AREA OF A SURFACE OF REVOLUTION.

A surface of revolution is a surface generated when a curve is revolved about a (straight) line. This line is called the symmetry axis of the surface.

Sphere is a familiar example of a surface of revolution.

Let  $y = f(x)$  be a function with continuous derivative on  $(a, b)$ . When the curve is rotated about the x-axis (or y-axis) it generates a surface of revolution of area  $S_{ox}$  (or  $S_{oy}$ ).

Consider an element of arc  $ds$ . After revolution it generates an element of surface in the shape of a slice of a