We have obtained K when the plane curve Γ is given in vector or cartesian form. Now, for a parametric curve

$$\Gamma : x = x(t), y = y(t),$$

setting

$$y' = \dot{y}/\dot{x},\ y'' = \frac{d}{dx}\ \frac{\ddot{y}\dot{x}-\dot{y}\ddot{x}}{\dot{x}^2}\ \frac{1}{\dot{x}},$$

in $K = y''/(1+{y'}^2)^{3/2}$, we have

$$K = \frac{\dot{x}\ddot{y}-\ddot{x}\dot{y}}{(\dot{x}^2+\dot{y}^2)^{3/2}}$$

When Γ is given in polar form:

$$\Gamma\text{: } r = f(\theta) \text{ or } x = r\cos\theta,\ y = r\sin\theta,$$

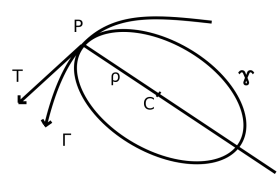
computing $\dot{x}, \dot{y}, \ddot{x}, \ddot{y}$ and setting in above formulas one gets

$$K = \frac{r^2+2r'^2-rr''}{(r^2+r'^2)^{3/2}}$$

which can also be obtained from $K = dx/ds$ where $\alpha = \psi + \theta$ and $\psi = \arctan\frac{r}{r'}$

Circle of curvature:

The circle of curvature of a curve Γ at a point P of it, is the limiting circle of the circle passing through P and two nearby points Q, R when $Q \to P$, $R \to P$.



It can be shown that the circle center at $C = P + \varrho N$ and radius ϱ is the circle of curvature γ at P (in 2- or 3- space). γ lies in the oscuating plane at P since it is tangent to Γ. (or the tangent vector T) and center is on principal normal.