Optimizing the Design of a Single-Phase Transformer Using Finite Element Analysis

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***Abstract*—This research paper presents an optimization methodology for the design of a single-phase 4 kVA 110/ 250 V, 60 Hz transformer. The proposed methodology utilizes ANSYS Maxwell and a multi-objective differential evolution algorithm to optimize the mass and loss of the transformer while maintaining the required voltage specifications. The design process involves the creation of a detailed 2D model of the transformer, followed by a simulation of its electromagnetic performance using ANSYS Maxwell. The outcomes of the simulation are utilized to enhance the design parameters of the transformer by utilizing a differential evolution algorithm that targets multiple objectives. The findings demonstrate that this approach can considerably boost the efficiency and cost-effectiveness of single-phase transformers.**

Keywords—transformer, ansys, single phase, optimization, FEA, differential evolution

I. Introduction

Transformers are fundamental components of the power distribution network, enabling the efficient conversion of electrical energy from one voltage level to another. The design of a single-phase transformer typically involves a magnetic core and two winding coils, one for the primary and one for the secondary [1]. The optimization of transformer design parameters is essential for improving their efficiency, reducing their weight and size, and enhancing their overall performance [2].

In recent years, the development of computer-aided design (CAD) tools has revolutionized the design process, enabling engineers to create highly accurate and sophisticated models of transformers. One such tool is ANSYS Maxwell, a powerful 2D electromagnetic simulation software that can accurately predict the performance of transformers [3].

The main objective of this study is to enhance the design of a single-phase transformer by utilizing ANSYS Maxwell software and a multi-objective differential evolution algorithm. The proposed methodology will aim to minimize the mass and loss of the transformer while maintaining the required voltage specifications. The design process will involve the creation of a detailed 2D model of the transformer, followed by a simulation of its electromagnetic performance using ANSYS Maxwell. The simulation results will then be used to optimize the transformer's design parameters.

The research paper will begin by providing a detailed review of the relevant literature on transformer design and optimization. It will then describe the methodology used in this study, including the specifications of the transformer, the design process, and the simulation and optimization methods. The results of the simulation and optimization process will be presented and analysed, with a discussion of their implications for the design and optimization of single-phase transformers.

Overall, this research paper will contribute to the development of more efficient and cost-effective transformers, which will have significant implications for the performance and reliability of the power distribution network.

II. Related Work

The design optimization of single-phase transformers is a key research area that has been investigated by scholars for decades. It involves finding the optimal design parameters that minimize the transformer's losses and maximize its efficiency, while ensuring that it operates within the specified voltage and current limits. Various optimization techniques have been applied to this problem to obtain accurate and reliable transformer designs.

Sizing and material selection are the crucial initial steps in transformer design optimization. Khatri proposed an optimization methodology for the design of single-phase transformers, considering the core cross-sectional area, the number of turns, and the material used in the core and winding [4]. The proposed methodology incorporates genetic algorithms to minimize the transformer's cost while satisfying the performance requirements. The authors found that the optimal design can lead to significant savings in the manufacturing cost and enhanced efficiency.

Moreover, optimal sizing of the transformer windings is essential in transformer design optimization, and this has been the focus of several studies. Coelho proposed a methodology for optimal winding design in power transformers [5]. The methodology includes a multi-objective optimization technique that aims to minimize the copper and core losses while satisfying the constraints. The results showed that the proposed methodology can provide a robust and efficient method for optimal winding design in power transformers.

Additionally, several studies have investigated the use of numerical modelling and simulation techniques to improve the transformer's performance. Li proposed a numerical study of the impact of eddy currents on a single-phase transformer [6]. The study analyses the transformer's operating performance under different harmonic load levels and shows the critical effects of eddy currents on the transformer's performance.

III. Method

A. Modelling and parametrization

As it is indicated, the aim is to reach an optimum transformer in terms of efficiency and mass. To approach the aim, first, required values were determined such as wire gauge, core material. Secondly, the core type was chosen as shell type. The reason of choosing it is the core loss and the copper requirement is less in the shell type compared to the core type. Thirdly, the design was drawn to the Ansys 2D

Maxwell. Finally, the design was parametrized to make it ready for optimization.

Before starting to draw the transformer design, it would be better to determine the core material. Among various of materials, the M19 29G steel is determined as the best for our experiences. Because its B-H curve saturates at high point, and it has high permeability. These features help the transformer when operating at high temperatures. Besides, this steel reduces the eddy current effects. As it can be seen in the B-H curve of the M19 29G steel, the maximum B value before saturation is approximately 1.8T. (Fig. 1)

The transformer parameters are 4kVA, 110/250V, 60 Hz. From these values the primary and secondary currents can be calculated ass 36.36A and 16A respectively. By using the Table in the Power Stream, wire diameters were chosen, through the 10% higher of the calculated rated currents. For the primary winding wire gauge was chosen as 12 that is able to carry maximum of 41A. For the secondary winding, it is 17AWG and maximum amp it could handle is 19A. (Table I)

In the design, number of turns of primary winding (N_1) and width of the central leg (W) were chosen as input parameters of the system. Depending on the N_1, the windings width and height will change. However, it was considered that the input for the design should also include the number of layers in the primary winding (i.e., the number of turns in the horizontal direction), so that the optimal number of layers can be determined through the optimization process.

The number of turns vertically (floor), can be found by dividing the N_1 by layer count (4). The primary winding height ($W_{h,pri}$) and width ($W_{w,pri}$) can be found from the copper diameters ($d_{cu,pri}$ and $d_{cu,sec}$), and floor and layer numbers (1), (2). From the primary winding height, secondary winding measurements can be found with same logic. Between the core and winding, and windings there are 1mm gaps (window clearance). When calculating the window height (Wd_h) and width (Wd_w), these window clearances also considered (6), (7). By using the window height, width, and width of central leg of the core, the core height (C_h) and core width (C_w) were calculated (8), (9). After all these calculations, parametrization was applied to the 2D design in Ansys Maxwell (Fig. 2). By only changing the N_1, W, and $layer_{pri}$, the position and the lengths of the rectangles were being changing. Addition to the successful 2D parametrization, to get the results more accurate, the mesh of the design was created detailly (Fig. 3).

TABLE I.

COPPER PROPERTIES FOR THE WINGINGS

Coppers	Copper Properties		
	Rated Current (A)	Area* (mm²)	Conductor Diameter (mm)
Coil 1 Copper	36.36	3.31	2.05
Coil 2 Copper	16	1.04	1.15

*The cross-sectional area of the copper

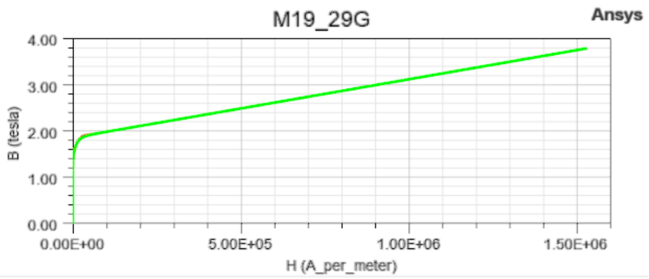

Fig. 1. The B-H curve of the M19_29G steel.

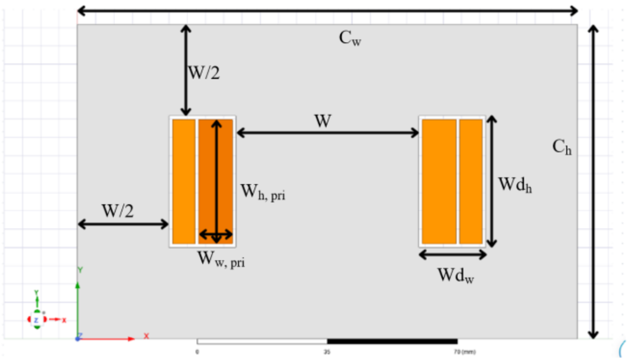

Fig. 2. The core design in ANSYS Maxwell and the measurement representation.

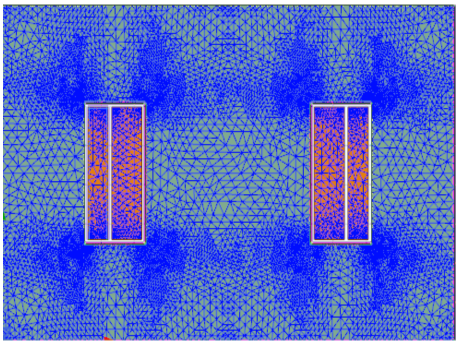

Fig. 3. Mesh representation of the design.

$$W_{h,pri} = floor_{pri} \cdot d_{cu,pri} \tag{1}$$

$$W_{w,pri} = layer_{pri} \cdot d_{cu,pri} \tag{2}$$

$$W_{h,sec} = W_{h,pri} \tag{3}$$

$$floor_{sec} = \frac{W_{h,pri}}{d_{cu,sec}} \, , \; layer_{sec} = \frac{N_2}{floor_{sec}} \tag{4}$$

$$W_{w,sec} = layer_{sec} \cdot d_{cu,sec} \tag{5}$$

$$\begin{gathered} \text{window clearance} = 1mm \\ Wd_w = W_{w,pri} + W_{w,sec} + 3 \cdot window\,clearance \end{gathered} \tag{6}$$

$$Wd_h = W_{h,pri} + 2 \cdot window\,clerence \tag{7}$$

$$C_h = Wd_h + 2 \cdot \frac{W}{2} \tag{8}$$

$$C_w = 2 \cdot Wd_w + W + 2 \cdot \frac{W}{2} \tag{9}$$

After the completion of the design, the resistance of the copper in each winding was calculated depending on the variables. For the resistance calculation the length of the wires should be driven. As it can be seen in figure 2, the wire length of a turn close to the central leg will be smaller than the wire length of a turn far from the central leg. Besides, the wires were winded as square. To make simpler the calculation of the wire length, measurement of one edge of inner square and outer square of winding was summed and divided by 2 to find the average. Then multiplied with 4 to get single turn wire length (10), (11), (12). Same process was applied to find the wire length of secondary winding. Then, by using the length of coppers and cross-sectional area of them, the resistances were calculated (14), (15).

$$One\ Edge_{outer\ square} = One\ Edge_{inner\ square} + 2W_{w,pri} \tag{10}$$

$$One\ Edge_{inner\ square} = W + 2 \cdot window\,clearance \tag{11}$$

$$L_{cu,pri,\text{single turn}} = 4 \cdot \frac{One\ Edge_{inner\ square} + One\ Edge_{outer\ square}}{2} \tag{12}$$

$$L_{cu,pri} = N_1 \cdot L_{cu,pri,\text{single turn}} \tag{13}$$

$$R_1 = \frac{\rho \cdot L}{A} = \frac{\rho \cdot L_{cu,pri}}{A_{cu,pri}} \tag{14}$$

$$R_2 = \frac{\rho \cdot L}{A} = \frac{\rho \cdot \text{Wire length}_{\text{sec}}}{A_{cu,\text{sec}}} \tag{15}$$

The resistance values of primary and secondary windings were calculated as 0.099Ω and 0.922Ω respectively.

B. Optimization design parameters

To optimize the provided transformer's parameters, various parametric sweep analyses were conducted. The objective was to find the optimal transformer design in terms of cost, which is determined by the transformer's mass, and efficiency. During this study, several core designs were tested, and the best design for the given power and voltage rates was selected based on the results, see in figure 2. An example of the multi-objective optimization results for the number of layers, the number of turns on the primary side, and the parameter "W" is illustrated in figure 5.

The optimization technique employed in this study, in conjunction with Finite Element Analysis (FEA), is the Multi-objective Differential Evolution Algorithm (MODE). [7]. As discussed earlier, evolutionary algorithms simulate the survival of the fittest concept in biological evolution. The optimization process using MODE begins with generating an initial design candidate, followed by analyzing it and producing a new set of candidate designs, referred to as "child vectors." These child vectors are then evaluated in ANSYS Maxwell, and compared with the initial design candidate to decide which designs should be selected for the next generation. This process is repeated until the optimal design is achieved.

The surviving designs are selected based on their ability to balance both cost and loss. The iterative process is repeated multiple times, with the goal of converging towards designs that achieve high efficiency while minimizing size. The model's design parameters that are targeted for optimization include the number of turns, the layer-count parameter that determines the copper rows' width and height ratio, the depth parameter, which is the thickness of the transformer limbs, and voltage regulation (VR).

rms(InducedVoltage(Secondary_Winding)) [V] Setup1 : LastAdaptive	rms(InducedVoltage(Primary_Winding)) [V] Setup1 : LastAdaptive
250.154094	109.991513

rms(InputCurrent(Secondary_Winding)) [mA] Setup1 : LastAdaptive	rms(Current(Primary_Winding)) [mA] Setup1 : LastAdaptive
16000.000000	36438.645359

Fig. 4. Simulation results for pf 1 and load 100 for voltage and current.

Evaluation	LayerCount	N_pri	W	Cost
1	3.375	50.833333	22.916667mm	8493.4
2	4.125	180.83333	46.25mm	119.29
3	4.875	115.83333	69.583333mm	91.179
4	5.625	245.83333	30.694444mm	164.06
5	6.375	83.333333	54.027778mm	24.946
6	7.125	213.33333	77.361111mm	467.24
7	7.875	148.33333	38.472222mm	90.171
8	8.625	278.33333	61.805556mm	610.01
9	9.375	67.083333	85.138889mm	46.406
10	10.125	197.08333	25.509259mm	575.44

Fig. 5. A part of optimization results.

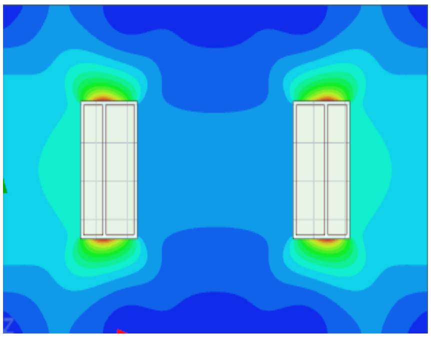

Fig. 6. Magnetic field contour of the core.

Maximum magnetic field came 1.62 Testa in Figure 5.

C. Efficiency and cost analysis

Transformer efficiency is a crucial parameter that must be taken into account during design, and it is calculated by dividing the power output in the secondary winding by the power input in the primary winding. To achieve high efficiency in transformers, it is necessary to minimize copper loss and core loss. In this research, two approaches were utilized to compute efficiency, as different power factor values necessitate different efficiency calculations. For power factor values equal to or greater than 1, the efficiency calculation method assumes a power factor value of 1. For power factor values less than 1, a different efficiency calculation method is employed. To calculate efficiency, the inductance value of the load is assumed to be zero in the case of a power factor value of 1. The output power for efficiency calculation is determined by measuring the power supplied to the load, while the input power is determined by multiplying the phase voltage of the transformer with the primary current.

$$Efficiency = \frac{P_{output}}{P_{input}} \cdot 100 \tag{16}$$

In this study, the calculation of output power for power factor values less than 1 involves multiplying the phase voltage (secondary), current (secondary), see (17) and power factor. The only difference between this method and the one used when the power factor is equal to 1 is that the resulting value is multiplied by the power factor.

$$P_{out} = V_2 \cdot I_2 \cdot (Power\ \ factor) \tag{17}$$

The main reason for using the power factor value in efficiency calculation is to account for the effect of apparent power, which arises when power is not multiplied by the power factor. The output power in transformers is the power consumed by the load, but the impedance value of any additional coil added to the load must be subtracted from the load resistance value in order to maintain a constant total

TABLE II.

DESIGN PARAMETERS FOR OPTIMIZING EFFICIENCY AND REDUCING MASS IN TRANSFORMERS

Optimum Design Parameters				
LayerCount	Limb Width (W) (mm)	Depth (W) (mm)	Number of Turn Primary/Secondary	V_R (V)
5	50.12	50.12	80/182	10.11

impedance value on the secondary winding. Apart from achieving high efficiency, minimizing cost is also a key objective. The cost of a transformer is closely related to its volume, as a larger volume typically results in higher production costs. In this research, the mass of the transformer was determined based on its volume. Transformer mass is calculated with copper mass and core mass. To determine the copper mass and core mass of the transformer, several factors are considered. The cross-sectional area of the copper wires is calculated based on the current rating, copper density, length of wire, winding number, and the core leg thickness. The cross-sectional area of the copper wires for the primary and secondary sides are calculated separately, as the current ratings differ for each. The mass of the core is calculated using the cross-sectional area of the core, its depth, and the density of the core material.

$$\text{Copper Mass} = A_{cu} \cdot density_{cu} \cdot L_{cu} \tag{18}$$

$$\text{Core Mass} = A_{core} \cdot W \cdot density_{core} \tag{19}$$

$$\text{Total Mass} = \text{Copper Mass} + \text{Core Mass} \tag{20}$$

During the optimization process of the designed system, the objective is to achieve the highest efficiency while minimizing mass. Therefore, it is not enough that the efficiency is too high, or the mass is too low. The study aims to find the optimal point between the two factors rather than favoring one over the other. This is achieved by assigning a 60% weight to efficiency and a 40% weight to mass while searching for the optimal point. This approach indicates that efficiency is given a slightly higher priority than mass. It should be noted that the relative importance of efficiency and mass can vary depending on the specific application area of the design.

According to the optimization results, the parameters in the Table II is the best option for the 4kVA 110/250V shell type transformer. Because the efficiency is wanted to be high and the copper length is the huge contributor to the losses, the number of turns in primary side resulted as 80 which is low.

D. *Equivalent circuit parameters*

To find the equivalent circuit parameters, open circuit and short circuit tests were applied to the transformer.

1) *Open circuit test*

To be able to apply open circuit test, the current value was assigned to 0 in secondary side. The current and voltage signals were plotted in time domain. Through the frequency, which is 60Hz and time delay, which is 4ms, phase difference was calculated as 86.4 degree. (21).

$$\varphi = 360° \cdot f \cdot \Delta t \tag{21}$$

In the simulation the primary voltage and primary current values came out as 112.51V and 2.26A respectively. With the phase value, which is 86.4, the R_c and X_m can be calculated as 792.21Ω and 49.85Ω (23), (26).

$$P = V_1 \cdot I_1 \cdot \cos(\varphi) = \frac{V_1^2}{R_c} \tag{22}$$

$$P = \frac{V_1^2}{R_c} \rightarrow R_c = V_1^2 / P \tag{23}$$

$$I_c = V_1 / R_c \tag{24}$$

$$I_m = \sqrt{I_1^2 - I_c^2} \tag{25}$$

$$X_m = V_1 / I_m \tag{26}$$

2) *Short circuit test*

In the short circuit tests, the secondary current assigned as 16A. In the calculations of the R_{eq} and X_{eq}, the primary voltage was taken as 10% of the V_1 which is 11.25V. The primary current was found as 24.87A from Ansys. R_{eq} and X_{eq} were found as 0.278Ω and 0.357Ω by using resistance of primary and secondary windings, calculated in (27), (30), primary voltage and current.

$$R_{eq} = R_1 + R_2' = R_1 + R_2 \cdot a^2 \tag{27}$$

$$V_R = R_{eq} \cdot I_1 \tag{28}$$

$$V_x = \sqrt{V_1^2 - V_R^2} \tag{29}$$

$$X_{eq} = V_x / I_1 \tag{30}$$

IV. RESULT

The optimization process was carried out using Optimetrics in ANSYS, enabling the identification of the optimal transformer design out of hundreds of candidates based on mass and efficiency criteria. An exemplary outcome of this optimization process is presented in Figure 5, while Table III shows the design that achieved high efficiency and low core and copper size. The selected design has a total mass of 5.09 kg for the copper and core and achieves 91.45% efficiency at the rated load. The equivalent circuit parameters for each phase were determined through short-circuit and open-circuit tests, as shown in Fig. 7. As transformers have no moving parts, Eddy Current analysis was conducted without requiring a graphical representation, and the results are presented in Table III.

There is an equivalent circuit in Figure 6. It was found from the open and short circuit test.

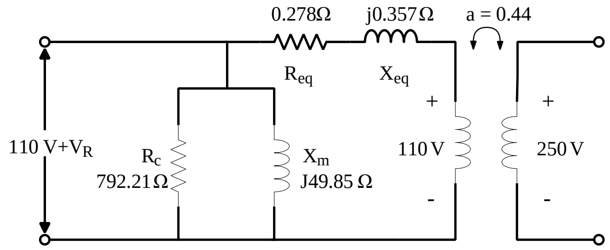

Fig. 7. Representation of equivalent circuit parameters.

TABLE III.
RESULTS OF TRANSFORMER ANALYSIS UNDER VARIOUS LOAD AND POWER FACTOR CONDITIONS

	pf = 1				pf = 0.9				pf = 0.8			
Load (%)	100	75	50	25	100	75	50	25	100	75	50	25
Efficiency	91.45	93.35	95.105	95.81	82.29	83.98	85.58	86.229	73.16	74.654	76.076	76.648
V_1 (V)	120.11	117.58	115.055	112.527	120.15	117.59	115.055	112.526	120.11	117.58	115.055	112.527
$V_{1(induced)}$ (V)	109.99	109.99	109.98	110	109.97	109.99	109.98	110	109.99	109.99	109.98	231.20
V_2 (V)	250.153	250.147	250.14	250.15	250.2	250.147	250.14	250.146	250.153	250.147	250.14	207.85
I_1 (A)	36.43	27.35	18.289	9.28	36.44	27.357	18.289	9.280	36.438	27.357	18.289	9.280
I_2 (A)	16	12	8	4	16	12	8	4	16	12	8	4
V_R (V)	10.11	7.58	5.055	2.527	10.18	7.59	5.054	2.526	10.11	7.58	5.055	2.527
R (ohm)	15.634	20.84	31.268	62.536	14.074	18.76	28.148	56.28	12.507	16.67	25.014	50.03
P_{in} (W)	4376.65	3215.81	2104.24	1044.25	4378.266	3216.71	2104.35	1044.340	4376.65	3216.71	2104.35	1044.34
P_{out} (W)	4002.468	3001.764	2001.155	1000.586	3602.88	2701.58	1801.039	900.527	3201.97	2401.410	1600.92	800.468

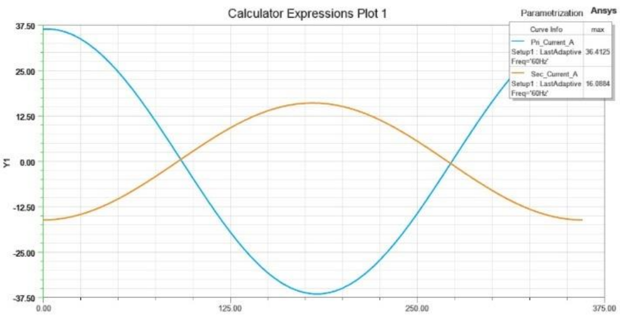

Fig. 8. Primary and secondary current graphs for pf=1 and 100% load.

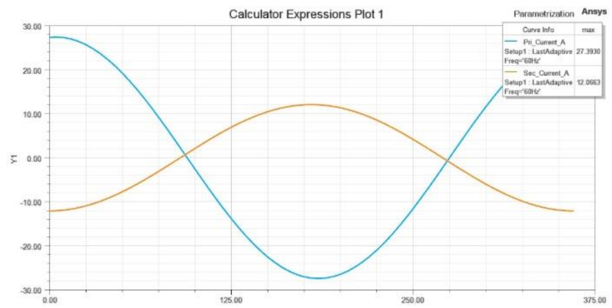

Fig. 9. Primary and secondary current graphs for pf=1 and 75% load.

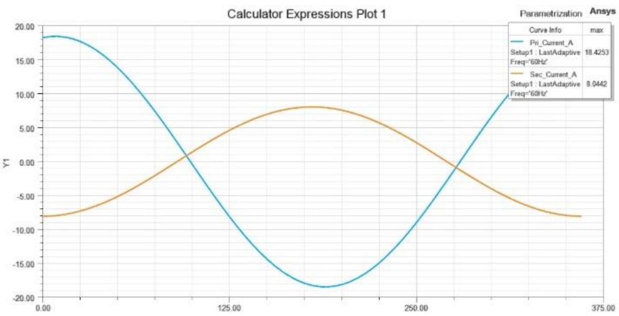

Fig. 10. Primary and secondary current graphs for pf=1 and 50% load.

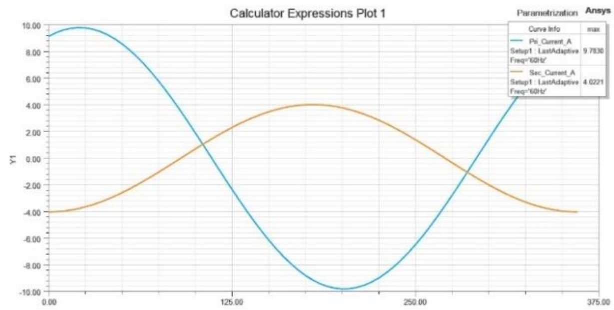

Fig. 11. Primary and secondary current graphs for pf=1 and 25% load.

The current values can be seen from figures 7, 8, 9 and 10 for all different load conditions. In addition, the same results apply to pf 0.9 and pf 0.8 since the power factor values has no change on the magnitude of the currents. On the other hand, when the load conditions change the current values changes in direct proportion.

It is observed that voltage remains constant as load decreases, while the current values decrease, leading to an increase in resistance values as shown in the table. As a result, current values decrease in both the primary and secondary windings. Notably, when the load is 25% and the power factor value is equal to 1, the highest efficiency of 95.81% is achieved. This is attributed to the decrease in winding losses with the decrease in current values in windings as load reduces.

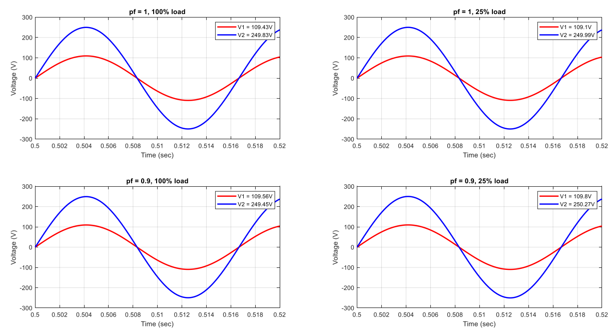

Fig. 12. Induced voltages at primary and secondary winding for power factor values of 1 and 0.9 and for 100% and 25% loading conditions.

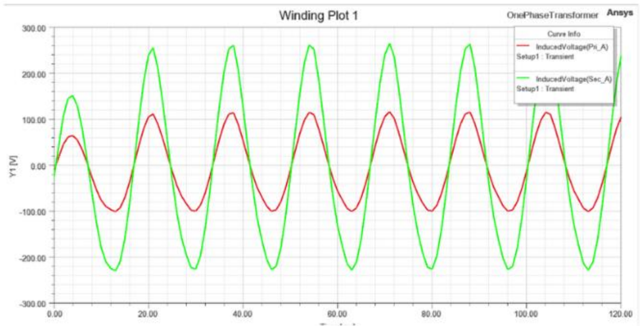

Fig. 13 Primary and secondary induced voltage graphs for pf=1 and 100% load

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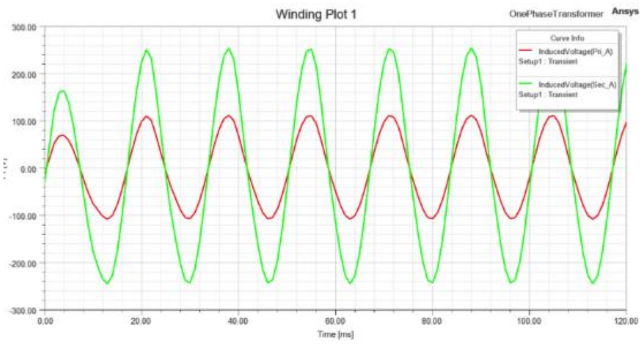

Fig. 14. Primary and secondary induced voltage graphs for pf=1 and 25% load

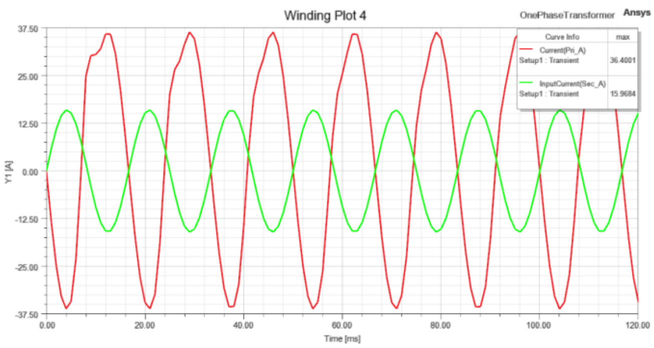

Fig. 15. Primary and secondary current graphs for pf=1 and 100% load

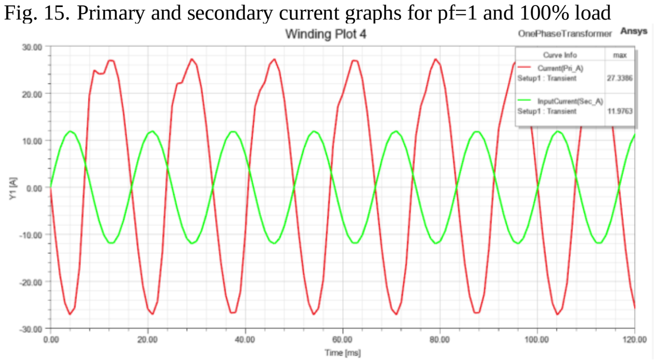

Fig. 16. Primary and secondary current graphs for pf=1 and 75% load

Figure 12 shows the primary and secondary induced voltage graphs for p.f 1 and p.f 0.9. As can be seen, there was no effect on the voltage values of the power factor. In the same way, load conditions also had no effect. In addition, the first cycles are distorted because the analysis is performed transient. Therefore, the cycle after 0.5 seconds is shown so that the results can be explained and examined properly. In this way, voltage graphs are obtained, crystal clearly.

Figure 13 and figure 14 show the induced voltage graphs over ansys. Figures 15 and 16 also show the transient current graph over ANSYS. As you can see, there is some deterioration in the first cycles. The reason is because it is run as a transient. In subsequent cycles, the waveform recovers and returns to the sinusoidal.

V. Discussion

This research has some limitations and challenges that need to be addressed to obtain better results. One of the limitations is that the core material used in the transformer was not laminated, on the other hand, the results suggest that Eddy current losses are not significant. Another limitation, such as the significant difference between the primary winding voltage and input voltage due to core and copper losses. To address this limitation, the study employed a voltage regulation (VR) parameter. Moreover, the optimization algorithm prioritizes decreasing mass over total loss, but this can be improved by implementing a better control structure for selecting candidate designs. To improve the transformer model, parameters that manipulate the thickness of the transformer in different regions of the core can be defined to allow the optimization algorithm to give better results. In the transformer model, the thickness and depth are assumed to be W at the limb, while W/2 is for the rest of the transformer. In the optimization, in addition to the Multiobjective genetic algorithm, a parametric solution is also used to solve it in the desired value ranges. Modifying the range of values to prioritize higher efficiency and lower cost for the transformer model has the potential to enhance its performance. Lastly, running the optimization algorithm repeatedly on a high-performance computer can help converge to higher efficiency and lower mass values, as computer properties play an important role in transformer analysis and optimization.

VI. Conclusion

This research paper has presented an optimization methodology for the design of a single-phase 4 kVA 110/ 250 V, 60 Hz transformer. The proposed methodology utilizes ANSYS Maxwell and a multi-objective differential evolution algorithm to optimize the mass and loss of the transformer while maintaining the required voltage specifications. The design process involves the creation of a detailed 2D model of the transformer, followed by a simulation of its electromagnetic performance using ANSYS Maxwell. The simulation results are then used to optimize the transformer's design parameters using a multi-objective differential evolution algorithm.

The results of the simulation and optimization process show that the proposed methodology can significantly improve the efficiency and cost-effectiveness of single-phase transformers. The optimized design parameters of the transformer can be determined, and the equivalent circuit parameters of the transformer can be calculated.

In conclusion, this research paper has contributed to the development of more efficient and cost-effective transformers, which will have significant implications for the performance and reliability of the power distribution network.

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