Tree Traversals

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Chap.1-Introduction

It's easy to prove that one can reconstruct a unique binary tree according to its in-order traversal and another traversal. The question is, if parts of the traversal sequences are missing, can you still reconstruct the unique binary tree?

In this project, the number of nodes will be given in the first line. Incomplete in-order, pre-order and post-order traversal will be given in next 3 lines, respectively.

It's your turn now to figure out whether you can reconstruct a unique tree according to them. If you can, print complete in-order, pre-order, post-order and level order traversal result in 4 lines. And if not (including 2 cases: A. Cannot reconstruct a tree with given traversal. B. There are more than one tree that correspond to the given incomplete traversal.), just print "Impossible" on screen.

To make it a little simpler, tree nodes are numbered from 1 to N , no repetition and no number is given out of the range.

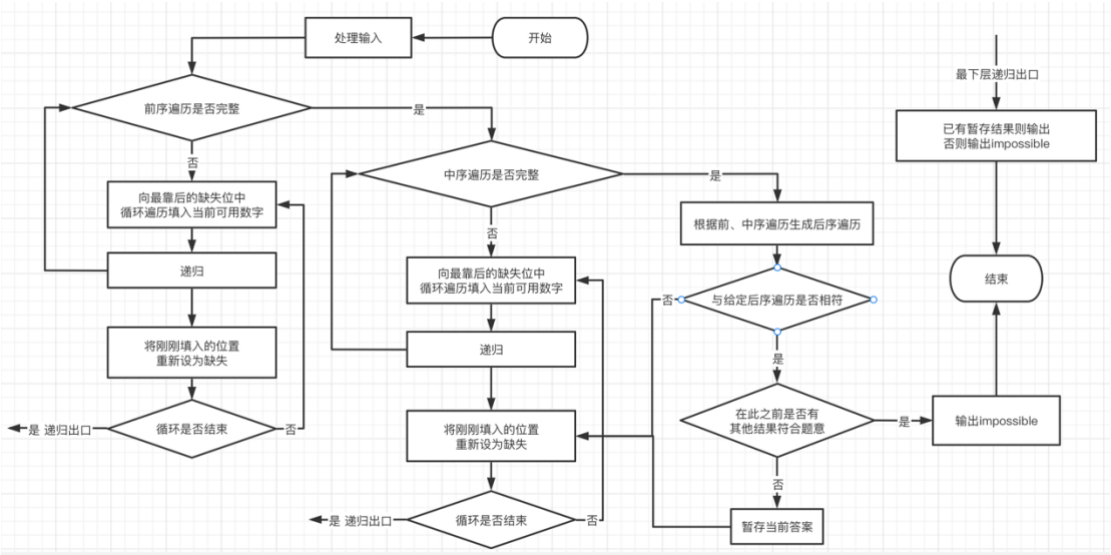
Chap.2-Algorithm Specification

As we all know that one can reconstruct a binary tree according to its in-order and another traversal. My algorithm consists of three parts.

Part 1: Recurse to find all possible complete arrangements of in-order and pre-order traversal. To do this, the missing number and the index of missing places will be saved at first.

Part 2: Try to reconstruct a tree by the traversal completed in Part 1. If feasible, compare the post-order traversal of it with the given post-order traversal. If not, just back to Part 1 and try another sequence.

Part 3: Assume that a tree has been successfully reconstructed and there is no conflict between this tree and the given post-order traversal. Save the tree and leave a flag which indicate that we have already own a result. Traversal in Part 1 cannot stop yet because we need to know whether the result is unique. The complete traversals will not print until tried all possible arrangements in Part 1 and 2.



Chap.3-Testing Result

Example 1 is an ordinary test case that contains no so much node and can reconstruct a tree. It also shows the format of input and output.

```
Input Example 1:
9                    //number of nodes
3 - 2 1 7 9 - 4 6    //in-order traversal
9 - 5 3 2 1 - 6 4    //pre-order traversal
3 1 - - 7 - 6 8 -    //post-order traversal
Output Example 1:
3 5 2 1 7 9 8 4 6    //complete in-order traversal
9 7 5 3 2 1 8 6 4    //complete pre-order traversal
3 1 2 5 7 4 6 8 9    //complete post-order traversal
9 7 8 5 6 3 2 4 1    //complete level order traversal
```

Example 2 is a test case that failed to reconstruct a tree for they are impossible traversal results.

```
Input Example 2:
9
1 - 3 5 9 8 - - 2
7 6 - 2 - 9 1 - -
```

```
3 5 8 1 9 - - 4 -
Output Example 2:
Impossible
```

Example 3 is a test case that failed to reconstruct a tree for the result is not unique.

```
Input Example 3:
10
1 2 3 4 5 6 7 - - -
10 - - 8 - - 7 - - -
- - - - - - - - - -
Output Example 3:
Impossible
```

Example 4 proves that you cannot reconstruct a tree by only pre-order and post-order traversal.

```
Input Example 4:
20
- - - - - - - - - - - - - - - - - - -
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
Output Example 4:
Impossible
```

Example 5, otherwise, proves that it's feasible to reconstruct a tree by its in-order and post-order traversal.

```
Input Example 5:
10
1 2 3 4 - 6 7 8 - 10
- - - - - - - - - -
10 - 8 7 6 5 - 3 2 -
Output Example 5:
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10
10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10
```

Example 6 is obviously right because I don't need post-order traversal to reconstruct my tree.

```
Input Example 6:
20
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 -
- 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
- - - - - - - - - - - - - - - - - - - -
Output Example 6:
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
```

```
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
```

Example 7 is an extreme test case with a very small size.

```
Input Example 7:
2
1 -
- -
2 1
Output Example 7:
1 2
1 2
2 1
1 2
```

Example 8 is also and extreme test case which is obviously impossible.

```
Input Example 8:
100
- - - - (else omitted)
- - - - (else omitted)
- - - - (else omitted)
Output Example 8:
Impossible
```

Chap.4-Analysis and Comment

There’re at most 2N Missing numbers in in-order and pre-order traversal altogether. It means I need at most 2N levels of recursion to complete these two sequences. The space for input and output is also of N complexity. So the space complexity is *O(N)*.

There’re at most N missing numbers in in-order traversal. Another N missing numbers in pre-order traversal as well. For each of them there’re N! cases to traverse all complete sequence. For each sensible in-order and pre-order traversal, I need a linear time complexity to reconstruct a tree and another linear time complexity to compare it with given post-order traversal. It also needs some time to handle input and output. So the time complexity of it is *O((N!)^2)*.

I know that it’s an algorithm of low efficiency. There’s a better way:

Treat incomplete traversals just like the complete. Pick out the first element of pre-order traversal or the last element of post-order traversal and locate it in in-order traversal. If you successfully locate it, continue to recursion. The only different is, if both first element of pre-order traversal and the last element of post-order traversal are missing or it absent in in-order traversal, iterate to try all possible numbers and positions. It seems not complicated but actually, it troublesome to code a recursion function like that.

Appendix-Source Code

```
#include <iostream>
#include <vector>
#include <string>
using namespace std;

#define MISSING (-1)

//only used to print level order traversal
struct tree {
	int element;
	struct tree* left;
	struct tree* right;
};

int N, traversalCount, postOrderCount;
vector<int> inOrder, preOrder, postOrder, tempPostOrder;
//to store the result
vector<int> inResult, preResult, postResult;
//numbers that not being used in each traversal, for node is numbered from 1 to N
vector<int> inMissingNumber, preMissingNumber, postMissingNumber;
//index that '-' is at in each traversal
vector<int> inMissingIndex, preMissingIndex, postMissingIndex;

//read a non-negative number from buffer separated by any character other than
'0'~'9'
//return #define MISSING when encounter '-' with no figure in front of it
//exit(1) if no more figure 'till EOF
int ReadNumber()
{
	char temp, flag=0, num=0;
```

```
	while(true){
		temp=getchar();//get a new char
		if(temp>='0' && temp<='9'){//encounter a figure
			num*=10;
			num+=temp-'0';
			flag=1;
		}else if(!flag && temp=='-'){//encounter a '-' with no figure before
			return MISSING;
		}else if(flag){//encounter a separator
			return num;
		}else if(!flag && temp==EOF){//no more figure
			printf("NO MORE FIGURE!");
			exit(1);
		}
	}
}

//load given traversal into corresponding vector
void Load()
{
	for(int i=0; i<N; i++) inOrder.push_back(ReadNumber());
	for(int i=0; i<N; i++) preOrder.push_back(ReadNumber());
	for(int i=0; i<N; i++) postOrder.push_back(ReadNumber());
}

//find out which number is absent in three traversal vector
//store result into vector<int> xxxMissing
void GetMissingNumber()
{
	int inFlag, preFlag, postFlag;

	for(int i=1; i<=N; i++){
		inFlag=0; preFlag=0; postFlag=0;
		for(int j=0; j<N; j++){//traverse from 1 to N, for node is numbered from 1 to
N
			if(inOrder[j]==i && !inFlag) inFlag=1;
			if(preOrder[j]==i && !preFlag) preFlag=1;
			if(postOrder[j]==i && !postFlag) postFlag=1;
		}
		//if not present, push i
		if(!inFlag) inMissingNumber.push_back(i);
		if(!preFlag) preMissingNumber.push_back(i);
		if(!postFlag) postMissingNumber.push_back(i);
	}
```

```
}

//store the index of element MISSING of each traversal into vector<int>
xxxMissingIndex
void GetMissingIndex()
{
	for(int i=0; i<N; i++){
		if(inOrder[i]==MISSING) inMissingIndex.push_back(i);
		if(preOrder[i]==MISSING) preMissingIndex.push_back(i);
		if(postOrder[i]==MISSING) postMissingIndex.push_back(i);
	}
}

//check if tempPostOrder correspond with postOrder. if so, traversalCount++ and
the traversal will be stored in vector<int> xxxResult
//once traversalCount>1, print "Impossible" and exit(1) because it means the result
is not unique
void CheckPostOrder()
{
	for(int i=0; i<N; i++){//check if correspond
		if(postOrder[i]!=MISSING && postOrder[i]!=tempPostOrder[i]) return;
	}

	if(traversalCount){//not unique, Impossible
		printf("Impossible");
		exit(1);
	}else{//available result, save it
		traversalCount++;
		inResult.assign(inOrder.begin(), inOrder.end());
		preResult.assign(preOrder.begin(), preOrder.end());
		postResult.assign(tempPostOrder.begin(), tempPostOrder.end());
	}
}

//try to restore the post-order traversal according to in-order and pre-order traversal
if it's possible
//temporary result will be stored in tempPostOrder
int GeneratePostOrder(int in_begin, int pre_begin, int post_begin, int length)
{
	int rootIndex, i, flag=1, leftCnt, rightCnt;
	for(i=0; i<length; i++){//preOrder[pre_begin] must be the root element
		if(inOrder[i+in_begin]==preOrder[pre_begin]){
			rootIndex=i+in_begin;
			break;
```

```
            }
        }
        if(i>=length){ //cannot find root element in inOrder traversal, cannot be a tree
            return 0;
        }else{ //root element is available in inOrder traversal
            postOrderCount++;
            tempPostOrder[post_begin+length-1]=preOrder[pre_begin];//save the
result

            //try restore left sub-tree
            leftCnt=rootIndex-in_begin;//node number of left sub-tree
            if( leftCnt > 0) flag=GeneratePostOrder(in_begin, pre_begin+1, post_begin,
leftCnt);

            //try restore right sub-tree
            rightCnt=in_begin+length-rootIndex-1;//node number of right sub-tree
            if(flag && rightCnt > 0) flag=GeneratePostOrder(rootIndex + 1, pre_begin +
leftCnt + 1,post_begin + leftCnt, rightCnt);

            if(postOrderCount==N) {//have traversed all node and get a possible result
                CheckPostOrder();
                postOrderCount=0;//don't check a same result multiple times in
different levels of recursion
            }
            return flag;
        }
}

//use recursion to find all possible pre-order traversal sequence
void FillPreOrder()
{
        if(preMissingNumber.empty()){//have filled all MISSING places
            postOrderCount=0;
            GeneratePostOrder(0, 0, 0, N);
        }else{
            int tempNumber, tempIndex;
            vector<int>::iterator iter;
            for(iter=preMissingNumber.begin(); iter<preMissingNumber.end(); iter++){
                //preparation of recursion
                tempNumber=*iter;
                preMissingNumber.erase(iter);
                tempIndex=*(--preMissingIndex.end());
                preMissingIndex.pop_back();
                //fill preOrder traversal sequence
```

```
                preOrder[tempIndex]=tempNumber;
                //recursion
                FillPreOrder();
                //restore variables
                preMissingNumber.insert(iter, tempNumber);
                preMissingIndex.push_back(tempIndex);
            }
        }
}

//use recursion to find all possible in-order traversal sequence
void FillInOrder()
{
        if(inMissingNumber.empty()){//have filled all MISSING placed
            FillPreOrder();
        }else{
            int tempNumber, tempIndex;
            vector<int>::iterator iter;
            for(iter=inMissingNumber.begin(); iter<inMissingNumber.end(); iter++){
                //preparation of recursion
                tempNumber=*iter;
                inMissingNumber.erase(iter);
                tempIndex=*(--inMissingIndex.end());
                inMissingIndex.pop_back();
                //fill inOrder traversal sequence
                inOrder[tempIndex]=tempNumber;
                //recursion
                FillInOrder();
                //restore
                inMissingNumber.insert(iter, tempNumber);
                inMissingIndex.push_back(tempIndex);
            }
        }
}

//creat a binary tree according to inResult and postResult
struct tree* CreatTree(int begin, int end)
{
        static int offset=0;

        //exit of recursion
        if(begin > end) return 0;

        //postOrder[end] is the root of current tree
```

```
	//find the position of root in inorder array
	int rootElement, rootPosition;
	rootElement=postResult[end];
	for(rootPosition= begin + offset; rootPosition <= end + offset; rootPosition++)
		if(inResult[rootPosition]==rootElement) break;

	//creat new tree by recursion
	struct tree *root;
	root=new tree;
	root->element=rootElement;

	//rootPosition refer to the index of inOrder
	//begin and end refer to the index of postOrder
	//take care of this! What's the begin and end next time?
	offset++;
	root->right=CreatTree(rootPosition-offset+1, end - 1);
	offset--;
	root->left=CreatTree(begin, rootPosition - offset - 1);

	return root;
}

//print level order traversal according to inResult and postResult
void PrintLevelOrder()
{
	struct tree* queue[N];
	struct tree* t= CreatTree(0, N-1);//creat a tree
	int begin=0, end=0, tempend;
	queue[end++]=t;
	while(begin<end){
		tempend=end;
		for(int i=begin; i<tempend && i<N; i++){
			//print the element in a same level
			printf("%d ", queue[i]->element);
			//push the child node into queue[]
			//for queue[begin]~queue[end] must be in a same level, the child
node must be in a same level as well
			if(queue[i]->left) queue[end++]=queue[i]->left;
			if(queue[i]->right) queue[end++]=queue[i]->right;
			begin=tempend;
		}
	}
	printf("\n");
}
```

```
void PrintResult()
{
	//print in-order traversal
	for(int i=0; i<N; i++) printf("%d ", inResult[i]);
	printf("\n");
	//print pre-order traversal
	for(int i=0; i<N; i++) printf("%d ", preResult[i]);
	printf("\n");
	//print post-order traversal
	for(int i=0; i<N; i++) printf("%d ", postResult[i]);
	printf("\n");
	//print level order traversal
	PrintLevelOrder();
}

int main()
{
	//preparation
	scanf("%d", &N);
	tempPostOrder.resize(N);
	Load();
	GetMissingNumber();
	GetMissingIndex();
	//start to traverse all possible cases
	FillInOrder();

	if(traversalCount){
		PrintResult();
	}else{
		printf("Impossible\n");
	}

	return 0;
}
```

Declaration

I hereby declare that all the work done in this project titled "Tree Traversals" is of my independent effort.