浙江大学 2013-2014 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号：21120520 开课学院： 计算机学院

考试试卷： ☑ A卷 ☐ B卷

考试形式： ☑ 闭卷 ☐ 开卷，允许带 ________入场

考试日期: 2014 年 1 月 15 日，考试时间： 120 分钟

诚信考试，沉着应考，杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2013
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true fill a ◯ otherwise a × in the bracket before the statement.**

   (a) (T) Language $\{a^m b^n c^j | m, n, j \in \mathbb{N} \text{ and } m + n + j \geq 2014\}$ is regular.

   (b) (T) Let L be a regular language, so is $\{ww^R | \ w \in \Sigma^* \text{ and } w \in L\}$.

   (c) (F) Let L_1 and L_2 be two languages. If L_1L_2 is regular, then either L_1 or L_2 is regular.

   (d) (T) Let L be a context-free language, then L^* is also context-free.

   (e) (   ) Language $\{w_1\#w_2\#\cdots\#w_n | n \in \mathbb{N}, \text{ for each } i, w_i \in \{a,b\}^* \text{ and for some } i, w_i$ is a palindrome$\}$ is context-free.

   ? (f) (T) Let L be a context-free language, then so is $H(L) = \{x | \exists y \in \Sigma^*, |x| = |y| \text{ and } xy \in L\}$.

   (g) (   ) Language $\{\text{“}M_1\text{”}\text{“}M_2\text{”} | M_1 \text{ and } M_2 \text{ are FA}, L(M_1) \subseteq L(M_2)\}$ is undecidable.

   (h) (   ) There's a function φ such that φ can be computed by some Turing machines, yet φ is not a primitive recursive function.

   (i) (   ) If L_1, L_2, and L_3 are all recursively enumerable, then $L_1 \cap (L_2 \cup L_3)$ must be recursively enumerable.

   (j) (   ) Let L_1 and L_2 be two recursively enumerable language. If $L_1 \cup L_2$ and $L_1 \cap L_2$ are recursive, then both L_1 and L_2 are recursive.

   (k) (   ) Let L be a recursively enumerable language and $L \leq_\tau \overline{H}$, then L is recursive, where $H = \{\text{“}M\text{”}\text{“}w\text{”} | \text{ Turing machine } M \text{ halts on } w\}$.

   (l) (T) The set of undecidable languages is uncountable.

2. **(20%)** Decide whether the following languages are regular or not and provide a formal proof for your answer.

   (a) $L_1 = \{uvu^R | u, v \in \{a, b\}^+\}$

   (b) $L_2 = \{uvu | u, v \in \{a, b\}^+\}$

   where $L^+ = LL^*$.

(a) Yes.

$L_1 = a\{a,b\}^+a \cup b\{a,b\}^+b$

(b) No. pump it!

3. **(20%)** Let $L_3 = \{ab^m c^n a^{m+2n} c | m, n \in \mathbb{N}\}$.

   (a) Give a context-free grammar for the language L_3.

   (b) Design a PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ accepting the language L_3.

   Solution: (a)

$S \to aXc$

$X \to bXa \mid Y$

$Y \to cYaa \mid e$

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

$K = \{\_\_\_\_\_\_\_\_\}$

$\Sigma = \{a, b, c\}$

$\Gamma = \{\_\_\_\_\_\_\_\_\}$

$s = \_\_\_\_\_\_\_\_$

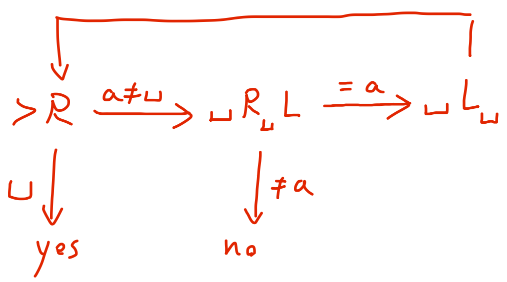
$F = \{\_\_\_\_\_\_\_\_\}$

(q, σ, β)	(p, γ)

4. **(12%)** Try to construct a Turing Machine to decide the following language

$$L = \{ww^R | w \in \{0,1\}^*\}.$$

Where w^R is the inverse of w. Always assume that the Turing machines start computation from the configuration $\triangleright\sqcup w$. When describing the Turing machines, you can use the elementary Turing machines described in textbook.



5. **(12%)** Show that the function: $\varphi : \mathbb{N} \to \mathbb{N}$ given by

$$\varphi(x) = \begin{cases} x \mod 3, & \text{if } x \text{ is a composite number;} \\ x^2 + 1, & \text{otherwise.} \end{cases}$$

is primitive recursive.

rem (x , 3)
exp (x , 2)
plus (exp(x,2))

6. **(12%)** Consider the problem

   $L_{even} = \{\text{"}M\text{"} | M$ is a TM and $L(M)$ contains at least one string of even number of b's$\}$

   (a) Show that L_{even} is recursively enumerable.

   (b) Show that L_{even} is non-recursive.

Enjoy your Spring Festival!