浙江大学 2014-2015 学年 秋冬 学期

《计算理论》课程期末考试试卷

课程号：21120520 开课学院：计算机学院

考试试卷：☑ A卷 ☐ B卷

考试形式：☑ 闭卷 ☐ 开卷，允许带 ________入场

考试日期: 2015 年 1 月 27 日，考试时间：120 分钟

诚信考试, 沉着应考, 杜绝违纪

考生姓名________________ 学号________________ 所属院系________________

题序	1	2	3	4	5	6	总分
得分							
评卷人							

Zhejiang University
Theory of Computation, Fall-Winter 2014
Final Exam

1. **(24%) Determine whether the following statements are true or false. If it is true fill a ◯ otherwise a × in the bracket before the statement.**

   (a) ( ) If L is any language, then the language LL^R must be equal to $\{ww^R \mid w \in L\}$.

   (b) ( ) Language $\{a^ib^jc^k | i,j,k \in \mathbb{N} \text{ and } i+j \not\equiv k (\bmod 3)\}$ is not regular.

   (c) ( ) If A is non-regular and both of B and $A \cap B$ are regular, then $A \cup B$ is non-regular.

   (d) ( ) For all languages L_1, L_2 and L_3, if $L_1 \subseteq L_2 \subseteq L_3$ and both L_1 and L_3 are regular, then L_2 is also regular.

   (e) ( ) Language $\{xcy | x, y \in \{a,b\}^* \text{ and } |x| \le |y| \le 2|x|\}$ is context-free.

   (f) ( ) Let L_1 be a regular language and L_2 be a context-free language, then $\{uv | u \in L_1, v \in L_2 \text{ and } |u| = |v|\}$ is also context-free.

   (g) ( ) Let $\mathbf{D}_{\text{DFA}} = \{\text{“}M\text{”} |$ DFA M rejects “M”, where “M” is the encoding of DFA M, just as Turing Machine$\}$, then $\mathbf{D}_{\text{DFA}}$ is recursively enumerable but not regular.

   (h) ( ) Let L be a language and there is a Turing machine M halts on x for every $x \in L$, then L is decidable.

   (i) ( ) Every countably infinite language is recursively enumerable.

   (j) ( ) A language is recursively enumerable if and only if it is Turing enumerable.

   (k) ( ) Let A be a recursively enumerable language and $A \le_\tau \overline{A}$, then A is recursive.

   (l) ( ) There are countably many Turing machines, and uncountably many languages, so most languages are not recursively enumerable.

2. **(20%)** Decide whether the following languages are regular or not and provide a formal proof for your answer. Let $\#_a(u)$ and $\#_a(v)$ be the number of a in string u and v, respectively.

   (a) $L_1 = \{ucv \mid u, v \in \{a,b\}^*, \#_a(u) = 2 \cdot \#_a(v)\}$

   Not Regular

   $\exists\ ucv \in L_1$, assume the pump length is $|u|$.

   let $u = xa$ or $u = xb$.

   for both case , $\forall i \neq 1,\ i \in N$,

   $xa^icv \notin L_1$, $xb^icv \notin L_1$

   (b) $L_2 = \{uv \mid u, v \in \{a,b\}^*, \#_a(u) = 2 \cdot \#_a(v)\}$

   Regular. $uv = (b^*ab^*ab^*ab^*)^*$

3. **(24%) On PDA and Context-Free Languages**
   Let $L_3 = \{xcy \mid x, y \in \{a,b\}^*, |x| = |y|, \text{ and } x \neq y^R\ \}$.

   (a) Construct a context-free grammar that generates the language L_3.

   (b) Construct a pushdown automata that accepts L_3.

Solution:

分类讨论.
- 外已不同：随意，可结束，
- 外无不同：第一个不同.

(a)

$$S \rightarrow aSa \mid bSb \mid aAb \mid bAa$$
$$A \rightarrow aAa \mid bAb \mid aAb \mid bAb \mid e$$

(b) The PDA $M = (K, \Sigma, \Gamma, \Delta, s, F)$ is defined below:

$K =$ ______

$\Sigma = \{a, b, c\}$

$\Gamma =$ ______

$s =$ ______

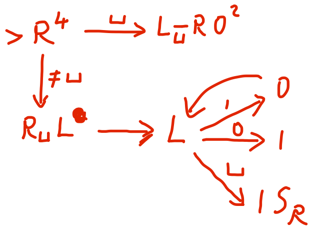
$F =$ ______

(q, σ, β)	(p, γ)

4. **(20%)** The function $\varphi : \mathbb{N} \to \mathbb{N}$ given by

$$\varphi(x) = \begin{cases} 4x, & \text{if } x < 8 \\ x + 2, & \text{if } x \geq 8 \end{cases}$$

(a) Try to construct a Turing Machine to compute the function $\varphi(x)$. When describing the Turing machines, you can use the elementary Turing machines described in textbook. Always assume that the Turing machines start computation from the configuration $\triangleright\underline{\sqcup}x$ where x is represented by binary string, i.e. $x \in \{0, 1\}^*$.



(b) Show that the function $\varphi(x)$ is primitive recursive.

$4x = mult(x, 4)$
$x + 2 = plus(x, 2)$
$iszero(x + 1 \dot{-} 8)$
p.r.

5. **(12%)** Consider the language

$$\textbf{Non-Empty} = \{\text{“}M\text{”} \mid \text{Turing machine } M \text{ halts some strings}\}$$

Show that **Non-Empty** is recursively enumerable. Justify your answer, and an informal description suffices.

Enjoy Your Spring Festival!