CHAPTER 2

2-1.*

a) $\overline{XYZ} = \bar{X} + \bar{Y} + \bar{Z}$

Verification of DeMorgan's Theorem

X	Y	Z	XYZ	$\overline{XYZ}$	$\bar{X}+\bar{Y}+\bar{Z}$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	1	1
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	1	1
1	1	1	1	0	0

b) $X + YZ = (X+Y)\cdot(X+Z)$

The Second Distributive Law

X	Y	Z	YZ	$X + YZ$	$X + Y$	$X + Z$	$(X + Y)(X + Z)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

c) $\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$

X	Y	Z	$\bar{X}Y$	$\bar{Y}Z$	$X\bar{Z}$	$\bar{X}Y+\bar{Y}Z+X\bar{Z}$	$X\bar{Y}$	$Y\bar{Z}$	$\bar{X}Z$	$X\bar{Y}+Y\bar{Z}+\bar{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

2-2.*

a) $\bar{X}\bar{Y} + \bar{X}Y + XY \quad = \quad \bar{X} + Y$

$$= (\bar{X}Y + \bar{X}\bar{Y}) + (\bar{X}Y + XY)$$
$$= \bar{X}(Y + \bar{Y}) + Y(X + \bar{X})$$
$$= \bar{X} + Y$$

b) $\bar{A}B + \bar{B}\bar{C} + AB + \bar{B}C \quad = \quad 1$

$= (\bar{A}B + AB) + (\bar{B}\bar{C} + \bar{B}C)$

$= B(A + \bar{A}) + \bar{B}(C + \bar{C})$

$B + B = 1$

c) $Y + \bar{X}Z + \bar{X}Y \quad = \quad X + Y + Z$

$= Y + X\bar{Y} + \bar{X}Z$

$= (Y + X)(Y + \bar{Y}) + \bar{X}Z$

$= Y + X + \bar{X}Z$

$= Y + (X + \bar{X})(X + Z)$

$= X + Y + Z$

d) $\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} \quad = \quad \bar{X}\bar{Y} + XZ + Y\bar{Z}$

$= \bar{X}\bar{Y} + \bar{Y}Z(X + \bar{X}) + XZ + XY + Y\bar{Z}$

$= \bar{X}\bar{Y} + X\bar{Y}Z + \bar{X}\bar{Y}Z + XZ + XY + Y\bar{Z}$

$= \bar{X}\bar{Y}(1 + Z) + X\bar{Y}Z + XZ + XY + Y\bar{Z}$

$= \bar{X}\bar{Y} + XZ(1 + \bar{Y}) + XY + Y\bar{Z}$

$= \bar{X}\bar{Y} + XZ + XY(Z + \bar{Z}) + Y\bar{Z}$

$= \bar{X}\bar{Y} + XZ + XYZ + Y\bar{Z}(1 + X)$

$= \bar{X}\bar{Y} + XZ(1 + Y) + Y\bar{Z}$

$= \bar{X}\bar{Y} + XZ + Y\bar{Z}$

2-3.+

a) $AB\bar{C} + B\bar{C}\bar{D} + BC + \bar{C}D \quad = \quad B + \bar{C}D$

$= AB\bar{C} + ABC + BC + B\bar{C}\bar{D} + B\bar{C}D + \bar{C}D$

$= AB(\bar{C} + C) + B\bar{C}(\bar{D} + D) + BC + \bar{C}D$

$= AB + B\bar{C} + BC + \bar{C}D$

$= B + AB + \bar{C}D$

$= B + \bar{C}D$

b) $WY + \bar{W}Y\bar{Z} + WXZ + \bar{W}X\bar{Y} \quad = \quad WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z$

$= (WY + W\bar{X}Y\bar{Z}) + (\bar{W}XY\bar{Z} + \bar{W}\bar{X}Y\bar{Z}) + (WXYZ + WX\bar{Y}Z) + (\bar{W}X\bar{Y}Z + \bar{W}X\bar{Y}\bar{Z})$

$= (WY + WXYZ) + (\bar{W}XY\bar{Z} + \bar{W}X\bar{Y}\bar{Z}) + (\bar{W}\bar{X}Y\bar{Z} + W\bar{X}Y\bar{Z}) + (WX\bar{Y}Z + \bar{W}X\bar{Y}Z)$

$= WY + \bar{W}X\bar{Z}(Y + \bar{Y}) + \bar{X}Y\bar{Z}(\bar{W} + W) + X\bar{Y}Z(W + \bar{W})$

$= WY + \bar{W}X\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}Z$

c) $A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C \quad = \quad (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)$

$= \overline{\overline{A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C}}$

$= \overline{(\bar{A} + D)(A + \bar{B})(C + \bar{D})(B + \bar{C})}$

$= \overline{(\bar{A}\bar{B} + AD + \bar{B}D)(BC + B\bar{D} + \bar{C}\bar{D})}$

$= \overline{\bar{A}\bar{B}\bar{C}\bar{D} + ABCD}$

$= (A + B + C + D)(\bar{A} + \bar{B} + \bar{C} + \bar{D}) = (\bar{A} + \bar{B} + \bar{C} + \bar{D})(A + B + C + D)$

2-4.+

Given: $A \cdot B = 0, A + B = 1$

Prove: $(A+C)(\bar{A}+B)(B+C) = BC$

$$= (AB + \bar{A}C + BC)(B+C)$$
$$= AB + \bar{A}C + BC$$
$$= 0 + C(\bar{A}+B)$$
$$= C(\bar{A}+B)(0)$$
$$= C(\bar{A}+B)(A+B)$$
$$= C(AB + \bar{A}B + B$$
$$= BC$$

2-5.+

Step 1: Define all elements of the algebra as four bit vectors such as *A*, *B* and *C*:

$A = (A_3, A_2, A_1, A_0)$

$B = (B_3, B_2, B_1, B_0)$

$C = (C_3, C_2, C_1, C_0)$

Step 2: Define OR_1, AND_1 and NOT_1 so that they conform to the definitions of AND, OR and NOT presented in Table 2-1.

a) $A + B = C$ is defined such that for all i, $i = 0, \dots ,3$, C_i equals the OR_1 of A_i and B_i.

b) $A\ B = C$ is defined such that for all i, $i = 0, \dots ,3$, C_i equals the AND_1 of A_i and B_i.

c) The element 0 is defined such that for A = "0", for all i, $i = 0, \dots ,3$, A_i equals logical 0.

d) The element 1 is defined such that for A = "1", for all i, $i = 0, \dots ,3$, A_i equals logical 1.

e) For any element A, $\bar{A}$ is defined such that for all i, $i = 0, \dots ,3$, $\bar{A}_i$ equals the NOT_1 of A_i.

2-6.

a) $\bar{A}\bar{C} + \bar{A}BC + \bar{B}C = \bar{A}\bar{C} + \bar{A}BC + (\bar{A}\bar{B}C + \bar{B}C)$

$$= \bar{A}\bar{C} + (\bar{A}BC + \bar{A}\bar{B}C \ + \bar{B}C$$
$$= (\bar{A}\bar{C} + \bar{A}C) + \bar{B}C = \bar{A} + \bar{B}C$$

b) $\overline{(A+B+C)}\overline{(ABC)}$

$$= \bar{A}\bar{A}B\bar{C} + \bar{A}\bar{B}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}\bar{C}$$
$$= (\bar{A}\bar{A})\bar{B}\bar{C} + \bar{A}(\bar{B}\bar{B})\bar{C} + \bar{A}\bar{B}(\bar{C}\bar{C})$$
$$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B}\bar{C}$$

c) $AB\bar{C} + AC = A(B\bar{C} + C) = A(B + C)$

d) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$

$$= (\bar{A}\bar{B} + B + \bar{A}\bar{C})D$$
$$= (\bar{A} + \bar{A}\bar{C} + B)D$$
$$= (\bar{A} + B)D$$

e) $(A+B)(A+C)(A\overline{B}C)$

$$= AAA\overline{B}C + ACA\overline{B}C + BAA\overline{B}C + BCA\overline{B}C$$
$$= A\overline{B}C$$

2-7.*

a) $\bar{X}\bar{Y} + XYZ + \bar{X}Y = \bar{X} + XYZ = (\bar{X} + XY)(\bar{X} + Z) = (\bar{X} + X)(\bar{X} + Y)(\bar{X} + Z)$

$= (\bar{X} + Y)(\bar{X} + Z) = \bar{X} + YZ$

b) $X + Y(Z + \overline{X + Z}) = X + Y(Z + \bar{X}\bar{Z}) = X + Y(Z + \bar{X})(Z + \bar{Z}) = X + YZ + \bar{X}Y$

$= (X + \bar{X})(X + Y) + YZ = X + Y + YZ = X + Y$

c) $\bar{W}X(\bar{Z} + \bar{Y}Z) + X(W + \bar{W}YZ) = \bar{W}X\bar{Z} + \bar{W}X\bar{Y}Z + WX + \bar{W}XYZ$

$= \bar{W}X\bar{Z} + \bar{W}XZ + WX = \bar{W}X + WX = X$

d) $(AB + \bar{A}\bar{B})(\bar{C}\bar{D} + CD) + \overline{AC} = ABC\bar{C}\bar{D} + ABCD + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A} + \bar{C}$

$= ABCD + \bar{A} + \bar{C} = \bar{A} + \bar{C} + A(BCD) = \bar{A} + \bar{C} + C(BD) = \bar{A} + \bar{C} + BD$

2-8.

a) $F = A\bar{B}C + \bar{A}\bar{C} + AB$

$= \overline{(\bar{A} + B + \bar{C})} + \overline{(A + C)} + \overline{(\bar{A} + \bar{B})}$

b) $\bar{\bar{F}} = \overline{\overline{A\bar{B}C + \bar{A}\bar{C} + AB}}$

$= \overline{\overline{(A\bar{B}C)}\,\overline{(\bar{A}\bar{C})}\,\overline{(AB)}}$

c) Same as part b.

2-9.*

a) $\bar{F} = (\bar{A} + B)(A + \bar{B})$

b) $\bar{F} = ((V + \bar{W})\bar{X} + \bar{Y})Z$

c) $\bar{F} = [\bar{W} + \bar{X} + (Y + \bar{Z})(\bar{Y} + Z)][W + X + Y\bar{Z} + \bar{Y}Z]$

d) $\bar{F} = \bar{A}B\bar{C} + (A + B)\bar{C} + \bar{A}(B + C)$

2-10.*

Truth Tables a, b, c

X	Y	Z	a
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

A	B	C	b
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

W	X	Y	Z	c
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

a) Sum of Minterms: $\bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$

Product of Maxterms: $(X+Y+Z)(X+Y+\bar{Z})(X+\bar{Y}+Z)(\bar{X}+Y+Z)$

b) Sum of Minterms: $\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + ABC$

Product of Maxterms: $(A+\bar{B}+C)(\bar{A}+B+C)(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+C)$

c) Sum of Minterms: $\bar{W}\bar{X}Y\bar{Z} + \bar{W}XY\bar{Z} + W\bar{X}Y\bar{Z} + WX\bar{Y}\bar{Z} + WX\bar{Y}Z + WXY\bar{Z} + WXYZ$

Product of Maxterms: $(W+X+Y+Z)(W+X+Y+\bar{Z})(W+X+\bar{Y}+\bar{Z})$

$(W+\bar{X}+Y+Z)(W+\bar{X}+Y+\bar{Z})(W+\bar{X}+\bar{Y}+\bar{Z})$

$(\bar{W}+X+Y+Z)(\bar{W}+X+Y+\bar{Z})(\bar{W}+X+\bar{Y}+\bar{Z})$

2-11.

a) $E = \Sigma m(1,\ 2,\ 4,\ 6) = \Pi M(0,\ 3,\ 5,\ 7)$, $F = \Sigma m(0,\ 2,\ 4,\ 7) = \Pi M(1,\ 3,\ 5,\ 6)$

b) $\bar{E} = \Sigma m(0,\ 3,\ 5,\ 7)$, $\bar{F} = \Sigma m(1,\ 3,\ 5,\ 6)$

c) $E + F = \Sigma m(0,\ 1,\ 2,\ 4,\ 6,\ 7)$, $E \cdot F = \Sigma m(2,\ 4)$

d) $E = \bar{X}\bar{Y}Z + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XY\bar{Z}$, $F = \bar{X}\bar{Y}\bar{Z} + \bar{X}Y\bar{Z} + X\bar{Y}\bar{Z} + XYZ$

e) $E = \bar{Z}(X+Y) + \bar{X}\bar{Y}Z$, $F = \bar{Z}(\bar{X}+\bar{Y}) + XYZ$

2-12.*

a) $(AB+C)(B+\bar{C}D) = AB + AB\bar{C}D + BC = AB + BC$ s.o.p.

$= B(A+C)$ p.o.s.

b) $\bar{X} + X(X+\bar{Y})(Y+\bar{Z}) = (\bar{X}+X)(\bar{X}+(X+\bar{Y})(Y+\bar{Z}))$

$= (\bar{X}+X+\bar{Y})(\bar{X}+Y+\bar{Z})$ p.o.s.

$= (1+\bar{Y})(\bar{X}+Y+\bar{Z}) = \bar{X}+Y+\bar{Z}$ s.o.p.

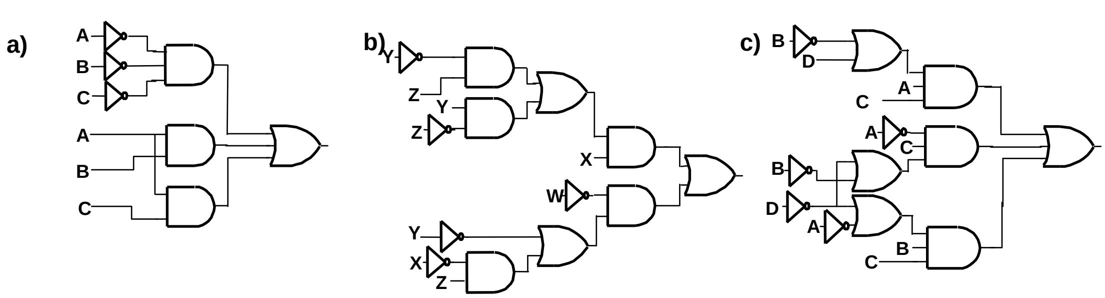
c) $(A+B\bar{C}+CD)(\bar{B}+EF) = (A+B+C)(A+B+D)(A+\bar{C}+D)(\bar{B}+EF)$

$= (A+B+C)(A+B+D)(A+\bar{C}+D)(\bar{B}+E)(\bar{B}+F)$ p.o.s.

$(A+B\bar{C}+CD)(\bar{B}+EF) = A(\bar{B}+EF) + B\bar{C}(\bar{B}+EF) + CD(\bar{B}+EF)$

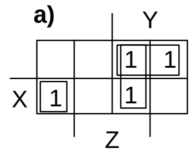
$= A\bar{B} + AEF + B\bar{C}EF + \bar{B}CD + CDEF$ s.o.p.

2-13.



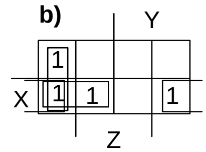
2-14.

a)



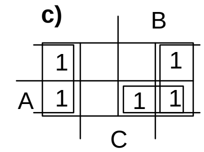
$\bar{X}Y + YZ + X\bar{Y}\bar{Z}$

b)



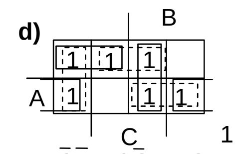
$X\bar{Y} + X\bar{Z} + \bar{Y}\bar{Z}$

c)



$\bar{C} + AB$

d)

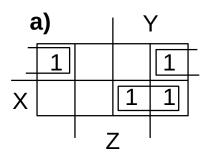


□ $\bar{A}\bar{B} + A\bar{C} + BC$

or ⬚ $\bar{B}\bar{C} + AB + \bar{A}C$

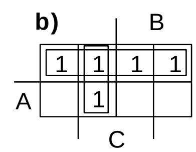
2-15.*

a)



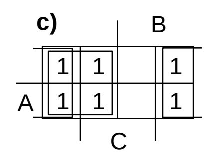
$\bar{X}\bar{Z} + XY$

b)



$\bar{A} + C\bar{B}$

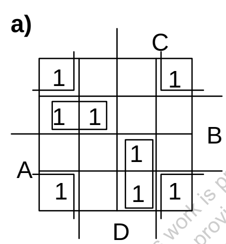
c)



$\bar{B} + \bar{C}$

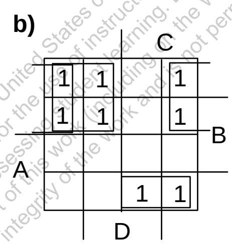
2-16.

a)



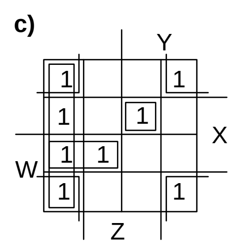
$\bar{B}\bar{D} + \bar{A}B\bar{C} + ACD$

b)



$\bar{A}\bar{C} + \bar{A}\bar{D} + A\bar{B}C$

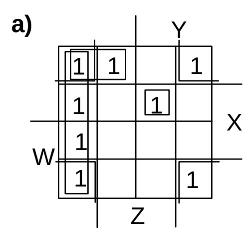
c)



$\bar{X}\bar{Z} + \bar{Y}\bar{Z} + WX\bar{Y} + \bar{W}XYZ$

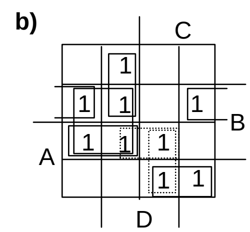
2-17.

a)



$F = \bar{X}\bar{Z} + \bar{Y}\bar{Z} + \bar{W}\bar{X}\bar{Y} + \bar{W}XYZ$

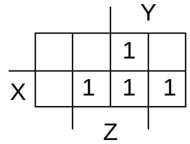
b)



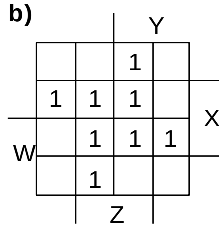
$F = B\bar{C} + \bar{A}\bar{C}D + \bar{A}B\bar{D} + A\bar{B}C + (ABD \text{ or } ACD)$

2-18.*

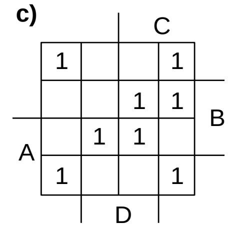
a)



b)



c)



$\Sigma m(3,\ 5,\ 6,\ 7)$ $\Sigma m(3,\ 4,\ 5,\ 7,\ 9,\ 13,\ 14,\ 15)$ $\Sigma m(0,\ 2,\ 6,\ 7,\ 8,\ 10,\ 13,\ 15)$

2-19.*

a) $Prime = XZ,\ WX,\ \bar{X}\bar{Z},\ W\bar{Z}$
$Essential = XZ,\ \bar{X}\bar{Z}$

b) $Prime = CD,\ AC,\ \bar{B}\bar{D},\ \bar{A}BD,\ \bar{B}C$
$Essential = AC,\ \bar{B}\bar{D},\ \bar{A}BD$

c) $Prime = AB,\ AC,\ AD,\ B\bar{C},\ \bar{B}D,\ \bar{C}D$
$Essential = AC,\ B\bar{C},\ \bar{B}D$

2-20.

a) $Prime = BD, \bar{A}\bar{C}D, \bar{A}BC, AB\bar{C}, ACD$

$Essential = \bar{A}\bar{C}D, \bar{A}BC, AB\bar{C}, ACD$

$Redundant = BD$

$F = \bar{A}\bar{C}D + \bar{A}BC + AB\bar{C} + ACD$

b) $Prime = \bar{W}\bar{Y}, \bar{X}Y, WXZ, \bar{W}\bar{X}, X\bar{Y}Z, WYZ$

$Essential = \bar{W}\bar{Y}, \bar{X}Y$

$Redundant = \bar{W}\bar{X}, X\bar{Y}Z, WYZ$

$F = \bar{W}\bar{Y} + \bar{X}Y + WXZ$

c) $Prime = W\bar{Z}, \bar{X}\bar{Z}, \bar{W}\bar{Y}Z, XYZ, \bar{W}\bar{X}\bar{Y}, \bar{W}XZ, WXY$

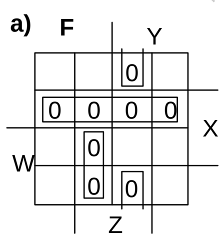
$Essential = W\bar{Z}, \bar{X}\bar{Z}$

$Redundant = \bar{W}\bar{X}\bar{Y}, \bar{W}XZ, WXY$

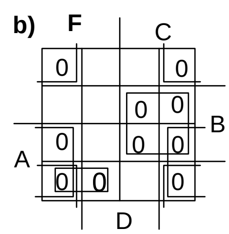
$F = W\bar{Z} + \bar{X}\bar{Z} + \bar{W}\bar{Y}Z + XYZ$

2-21.

a)



b)



$\bar{F} = \Sigma m(3,4,5,6,7,9,11,13)$

$F = \overline{\bar{W}X + W\bar{Y}Z + \bar{X}YZ}$

$F = (W + \bar{X})(\bar{W} + Y + \bar{Z})(X + \bar{Y} + \bar{Z})$

$\bar{F} = \Sigma m(0,\ 2,\ 6,\ 7,\ 8,\ 9,\ 10,\ 12,\ 14,\ 15)$

$F = \overline{\bar{B}\bar{D} + BC + A\bar{B}\bar{C} + A\bar{D}}$

$F = (B + D)(\bar{B} + \bar{C})(\bar{A} + B + C)(\bar{A} + D)$

2-22.*

a) s.o.p. $CD + A\bar{C} + \bar{B}D$
p.o.s. $(\bar{C} + D)(A + D)(A + \bar{B} + C)$

b) s.o.p. $\bar{A}\bar{C} + \bar{B}\bar{D} + A\bar{D}$
p.o.s. $(\bar{C} + \bar{D})(\bar{A} + \bar{D})(A + \bar{B} + \bar{C})$

c) s.o.p. $\bar{B}\bar{D} + \bar{A}BD + (\bar{A}BC\ or\ \bar{A}C\bar{D})$
p.o.s. $(\bar{A} + \bar{B})(B + \bar{D})(\bar{B} + C + D)$

2-23.

a) s.o.p. $A\bar{B}\bar{D} + AB\bar{C} + \bar{A}BD + \bar{A}\bar{B}C$

or $A\bar{C}\bar{D} + B\bar{C}D + \bar{A}CD + \bar{B}C\bar{D}$

p.o.s. $(\bar{A}+B+\bar{D})(\bar{A}+\bar{B}+\bar{C})(A+\bar{B}+D)(A+B+C)$

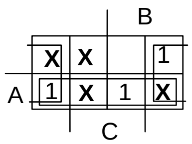
or $(A+C+D)(B+C+\bar{D})(\bar{A}+\bar{C}+\bar{D})(\bar{B}+\bar{C}+D)$

b) s.o.p. $\bar{X} + YZ + \bar{W}\bar{Z}$

p.o.s. $(\bar{X}+Y+\bar{Z})(\bar{W}+\bar{X}+Z)$

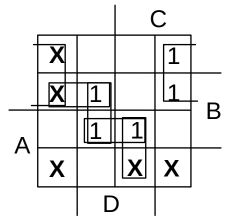
2-24.

a)



$F = A + \bar{C}$

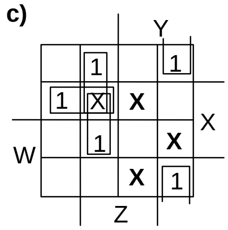
b)



$F = \bar{A}\bar{D} + (ABD + B\bar{C}D)$ or

$(ACD + B\bar{C}D)$ or $(ABD + \bar{A}B\bar{C})$

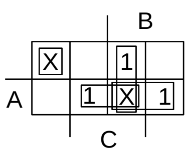
c)



$F = \bar{X}Y\bar{Z} + \bar{W}X\bar{Y} + \bar{W}\bar{Y}Z + XY\bar{Z}$

2-25.*

a)

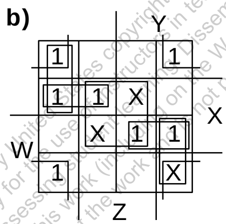


Primes $= AB,\ AC,\ BC,\ \bar{A}\bar{B}\bar{C}$

Essential $= AB,\ AC,\ BC$

$F = AB + AC + BC$

b)

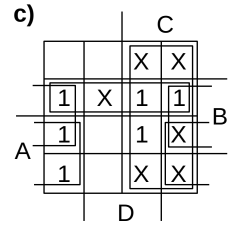


Primes $= \bar{X}\bar{Z},\ XZ,\ \bar{W}X\bar{Y},\ WXY,\ \bar{W}\bar{Y}\bar{Z},\ WY\bar{Z}$

Essential $= \bar{X}\bar{Z}$

$F = \bar{X}\bar{Z} + \bar{W}X\bar{Y} + WXY$

c)



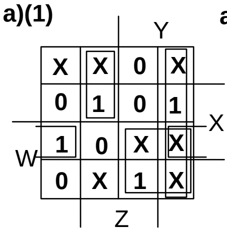
Primes $= \bar{A}B,\ C,\ A\bar{D},\ B\bar{D}$

Essential $= C,\ A\bar{D}$

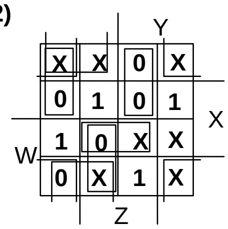
$F = C + A\bar{D}(B\bar{D}$ or $\bar{A}B)$

2-26.

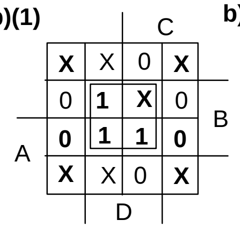
a)(1)



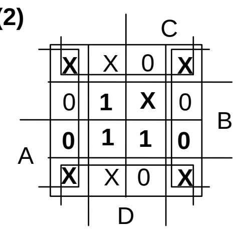
a)(2)



b)(1)



b)(2)



$F = WY + Y\bar{Z} + \bar{W}\bar{Y}Z + WX\bar{Z}$

$\bar{F} = (\bar{X}\bar{Y}$ or $\bar{X}\bar{Z}) + \bar{W}YZ + \bar{W}\bar{Y}\bar{Z} + (WXZ$ or $W\bar{Y}Z)$

$F = ((X+Y)$ or $(X+Z))(W+\bar{Y}+\bar{Z})(W+Y+Z)$

$((\bar{W}+\bar{X}+\bar{Z})$ or $(\bar{W}+Y+\bar{Z}))$

$F = BD$

$\bar{F} = \bar{B} + \bar{D}$

$F = BD$

2-27.*

$$X \oplus Y = X\bar{Y} + \bar{X}Y$$
$$\text{Dual}(X \oplus Y) = \text{Dual}(X\bar{Y} + \bar{X}Y)$$
$$= (X + \bar{Y})(\bar{X} + Y)$$
$$= \overline{\bar{X}Y + X\bar{Y}}$$
$$= \overline{X\bar{Y} + \bar{X}Y}$$
$$= \overline{X \oplus Y}$$

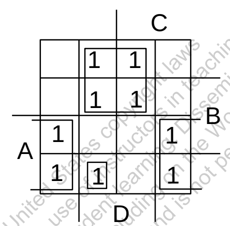
2-28.

$$AB\bar{C}D + A\bar{D} + \bar{A}D = AB\bar{C}D + (A \oplus D)$$

Note that $X + Y = (X \oplus Y) + XY$

Letting $X = AB\bar{C}D$ and $Y = A \oplus D$,

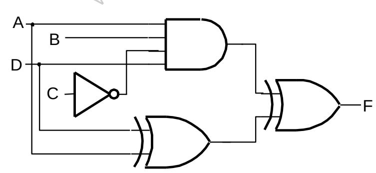
We can observe from the map below or determine algebraically that XY is equal to 0.



For this situation,

$$X + Y = (X \oplus Y) + XY$$
$$= (X \oplus Y) + 0$$
$$= X \oplus Y$$

So, we can write $F(A, B, C, D) = X \oplus Y = AB\bar{C}D \oplus (A \oplus D)$

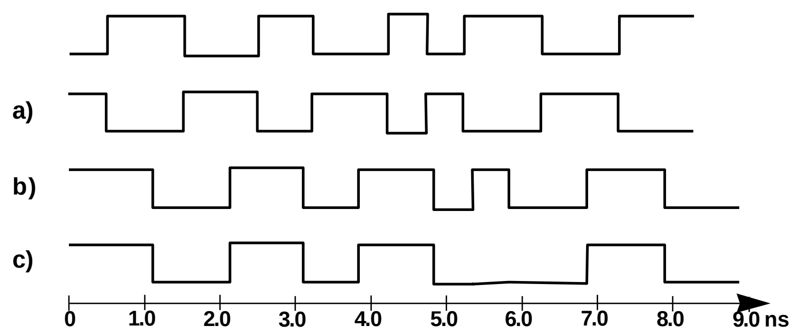


2-29.*

The longest path is from input C or $\bar{D}$.

0.073 ns + 0.073 ns + 0.048 ns + 0.073 ns = 0.267 ns

2-30.



2-31.

a) $t_{PHL\text{-C, D to F}} = 2t_{PLH} + 2\ t_{PHL} = 2(0.36) + 2(0.20) = 1.12$ ns

$t_{PLH\text{-C, D to F}} = 2t_{PHL} + 2t_{PLH} = 2(0.20) + 2(0.36) = 1.12$ ns

$t_{pd} = 1.12$ ns

$t_{PHL\text{-}\bar{B}\text{ to F}} = 2t_{PHL} + t_{PLH} = 2(0.20) + (0.36) = 0.76$ ns

$t_{PLH\text{-}\bar{B}\text{ to F}} = 2t_{PHL} + t_{PLH} = 2(0.36) + (0.20) = 0.92$ ns

$t_{pd\text{-}\bar{B}\text{ to F}} = 0.76 + 0.92 = 0.84$ ns

$t_{PHL\text{-A, B, }\bar{C}\text{ to F}} = t_{PLH} + t_{PHL} = 0.36 + 0.20 = 0.56$ ns

$t_{PLH\text{-A, B, }\bar{C}\text{ to F}} = t_{PHL} + t_{PLH} = 0.20 + 0.36 = 0.56$ ns

$t_{pd\text{-A, B, }\bar{C}\text{ to F}} = 0.56$ ns

b) $t_{pd\text{-C, D to F}} = 4\ t_{pd} = 4(0.28) = 1.12$ ns

$t_{pd\text{-}\bar{B}\text{ to F}} = 3\ t_{pd} = 3(0.28) = 0.78$ ns

$t_{pd\text{-A, B, C}}$ to $F = 2\ t_{pd} = 2(0.28) = 0.56$ ns

c) For paths through an odd number of inverting gates with unequal gate t_{PHL} and t_{PLH}, path t_{PHL}, t_{PLH}, and t_{pd} are different. For paths through an even number of inverting gates, path t_{PHL}, t_{PLH}, and t_{pd} are equal.

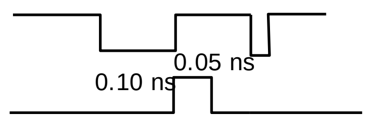
2-32.

If the rejection time for inertial delays is greater than the propagation delay, then an output change can occur before it can be predicted whether or not it is to occur due to the rejection time.
For example, with a delay of 2 ns and a rejection time of 3 ns, for a 2.5 ns pulse, the initial edge will have already appeared at the output before the 3 ns has elapsed at which whether to reject or not is to be determined.

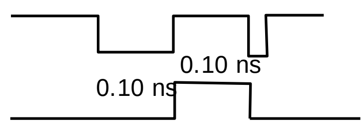
2-33.+

a) The propagation delay is $t_{pd} = \max(t_{PHL} = 0.05,\ t_{PLH} = 0.10) = 0.10$ ns.

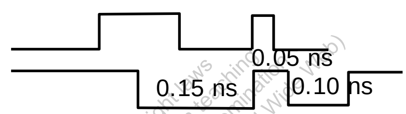
Assuming that the gate is an inverter, for a positive output pulse, the following actually occurs:



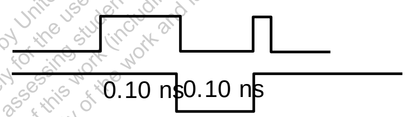
If the input pulse is narrower than 0.05 ns, no output pulse occurs so the rejection time is 0.05 ns. The resulting model predicts the following results, which differ from the actual delay behavior, but models the rejection behavior: :



b) For a negative output pulse, the following actually occurs:

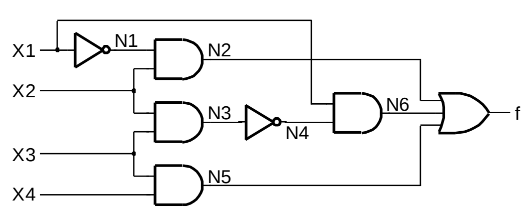


The model predicts the following results, which differs from the actual delay behavior and from the actual rejection behavior:



Overall, the model is inaccurate for both cases a and b, and provides a faulty rejection model for case b. Using an average of t_{PHL} and t_{PLH} for t_{pd} would improve the delay accuracy of the model for circuit applications, but the rejection model still fails.

2-34.*

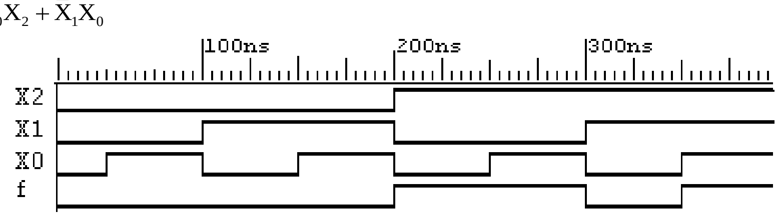


2-35.

```
-- Figure 4-40: Structural VHDL Description
library ieee;
use ieee.std_logic_1164.all;
entity nand2 is
  port(in1, in2: in std_logic;
       out1 : out std_logic);
end nand2;
```

```
architecture concurrent of nand2 is
begin
  out1 <= not (in1 and in2);
end architecture;

library ieee;
use ieee.std_logic_1164.all;
entity nand3 is
    port(in1, in2, in3 : in std_logic;
        out1 : out std_logic);
end nand3;

architecture concurrent of nand3 is
begin
    out1 <= not (in1 and in2 and in3);
end concurrent;

library ieee;
use ieee.std_logic_1164.all;
entity nand4 is
    port(in1, in2, in3, in4: in std_logic;
        out1 : out std_logic);
end nand4;
-- The code above this point could be eliminated by using the library, func_prims.

library ieee;
use ieee.std_logic_1164.all;
entity fig440 is
  port(X: in std_logic_vector(2 to 0);
      f: out std_logic);
end fig440;
architecture structural_2 of fig440 is

component NAND2
  port(in1, in2: in std_logic;
      out1: out std_logic);
end component;

component NAND3
  port(in1, in2, in3: in std_logic;
      out1: out std_logic);
end component;

signal T: std_logic_vector(0 to 4);
begin
      g0: NAND2 port map (X(2),X(1),T(0));
      g1: NAND2 port map (X(2),T(0),T(1));
      g2: NAND2 port map (X(1),T(0),T(2));
      g3: NAND3 port map (X(1),T(1),T(2),T(3));
      g4: NAND2 port map (X(1),T(2),T(4));
      g5: NAND2 port map (T(3),T(4),f);
end structural_2;
```

$F = X_0X_2 + \bar{X}_1X_0$

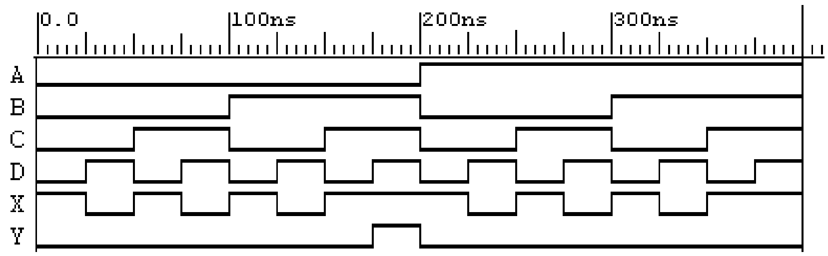


2-36.

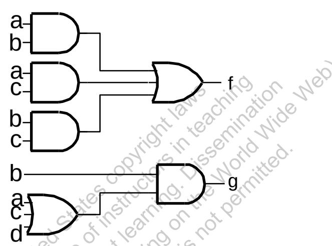
```
begin
    g0: NOT_1 port map (D, x1);
    g1: AND_2 port map (B, C, x2);
    g2: NOR_2 port map (A, x1, x3);
    g3: NAND_2 port map (x1, x3, x4);
    g4: OR_2 port map (x1, x2, x5);
    g5: AND_2 port map (x4, x5, X);
    g6: AND_2 port map (x3, x5, Y);
end structural_1;
```

$X = \bar{D} + BC$

$Y = \bar{A}BCD$



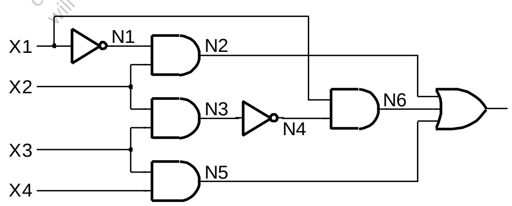
2-37.



2-38.*

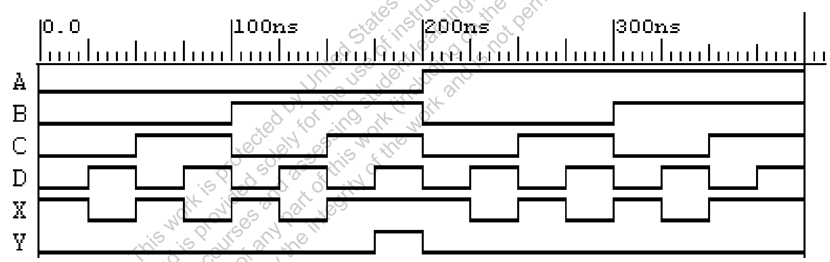
```
begin
    F <= (X and Z) or ((not Y) and Z);
end;
```

2-39.*



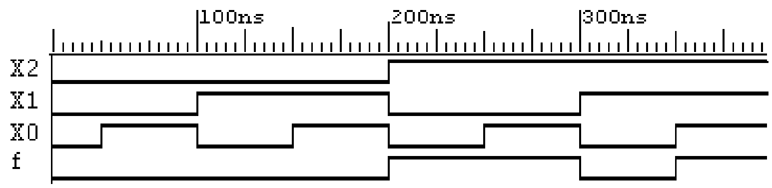
2-40.

```
module circuit_4_50(A, B, C, D, X, Y);
  input A, B, C, D;
  output X, Y;

  wire n1, n2, n3, n4, n5;

  not
    go(n1, D);

  nand
    g1(n4, n1, n3);

  and
    g2(n2, B, C),
    g3(X, n4, n5),
    g4(Y, n3, n5);

  or
    g5(n5, n1, n2);
  nor
    g6(n3, n1, A);

  endmodule
```

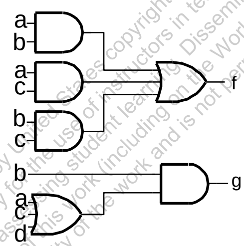


2-41.

```
module circuit_4_51(X, F);
   input [2:0] X;
   output F;
   wire [0:4] T;
  nand
   g0(T[0],X[0],X[1]),
   g1(T[1],X[0],T[0]),
   g2(T[2],X[1],T[0]),
   g3(T[3],X[2],T[1],T[2]),
   g4(T[4],X[2],T[2]),
   g5(F,T[3],T[4]);
endmodule
```



2-42.



2-43.*

```
module circuit_4_53(X, Y, Z, F);
   input X, Y, Z;
   output F;
   assign F = (X & Z) | (Z & ~Y);
endmodule
```