yxw190015

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hub and authority:

authority centrality $= x_i$

hub centrality $= y_i$

$x_i = \alpha \sum_j A_{ij} y_j$ similarly $\rightarrow$ $y_i = \beta \sum_j A_{ji} x_j$

$\alpha \beta$ are constant

In matrix:

$x = \alpha A y$ $y = \beta A^T x$

combine

$AA^T x = \lambda x$ $A^T A y = \lambda y$

$\lambda = (\alpha\beta)^{-1}$

authority and hub centrality are given by eigenvectors of AA^T A^TA with same eigenvalue !

$$AA^Tx = \lambda x$$

then

$$A^TA(A^Tx) = \lambda(A^Tx)$$

$$y = A^Tx$$

Page Rank $\alpha \neq 1$?

$$Av = \lambda v$$

Katz : $x = \alpha Ax + \beta 1$

$$x = (I - \alpha A)^{-1} \cdot 1 =$$

$$AA^T - \alpha A$$

$$\det(A - \alpha^{-1} I) = 0$$

$$A - \lambda I = 0$$

Page :

$$x = (I - \alpha AD^{-1})^{-1} 1 = D(D - \alpha A)^{-1} 1$$

$$\det[I - \alpha AD^{-1}]^{-1} = 0$$

let $AD^{-1} = S$

$$\det(I - \alpha S) = 0$$

$$\det(S - \frac{1}{a} I) = 0$$

eigenvector $AD^{-1} = [k_1, k_2, k \cdots, k_n]$

$$AD^{-1} w = \lambda w$$

$$(AD^{-1} - \frac{1}{a} I) w = 0$$

if $a = 1$

$$(AD^{-1} - \lambda I) w = 0$$

$$AD^{-1} = \lambda I w$$

The network will not converge

we get a singular matrix

we need to avoid .