A Quantum Dot Interacting With a Nano-mechanical Resonator

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Single Atom Laser

- Conventional laser: pump an ensemble of atoms into the excited state to achieve population inversion
- Early experiments on single atom laser:
  e.g. a Rb-85 atom couples to a microwave photon cavity

> One-Atom Maser
>
> D. Meschede, H. Walther, and G. Müller
> Phys. Rev. Lett. **54**, 551 – Published 11 February 1985

- Recent attempts using quantum dots as artificial atoms

> Semiconductor double quantum dot micromaser
>
> Y.-Y. Liu[1], J. Stehlik[1], C. Eichler[1], M. J. Gullans[2], J. M. Taylor[2,3], J. R. Petta[1,4,*]
> + See all authors and affiliations
>
> *Science* 16 Jan 2015:
> Vol. 347, Issue 6219, pp. 285-287
> DOI: 10.1126/science.aaa2501

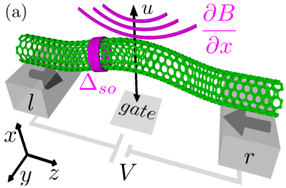
- Phonon lasing: quantum dots with mechanical resonators

> Resonant and Inelastic Andreev Tunneling Observed on a Carbon Nanotube Quantum Dot
>
> J. Gramich, A. Baumgartner, and C. Schönenberger
> Phys. Rev. Lett. **115**, 216801 – Published 16 November 2015

Quantum dot with carbon-nanotube

Control of vibrational states by spin-polarized transport in a carbon nanotube resonator

P. Stadler, W. Belzig, and G. Rastelli
Phys. Rev. B **91**, 085432 – Published 27 February 2015

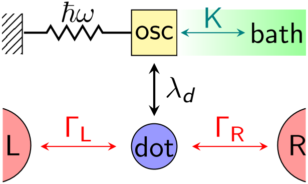


- A carbon nanotube quantum dot suspended between two ferromagnetic leads
- The dot spin couples to the vibration mode of the nanotube because of the nanotube spin-orbit interaction and/or a magnetic field gradient
- Lasing has been shown theoretically with collinearlly polarised leads [Mantovani *et al.*, 2019]

Our Goal:

Non-collinearlly polarised leads?

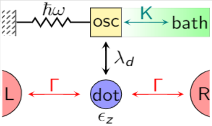
The System



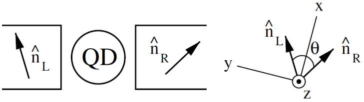
Dot State $\lvert\alpha\rangle$	$\lvert 0\rangle$	$\lvert\downarrow\rangle$	$\lvert\uparrow\rangle$	$\lvert\uparrow\downarrow\rangle$
Dot Energy E_α	0	$\epsilon - \frac{\epsilon_z}{2}$	$\epsilon + \frac{\epsilon_z}{2}$	$2\epsilon + U$

- Basis: $\lvert\alpha, n\rangle = \lvert\alpha\rangle \otimes \lvert n\rangle$; $n = 0, 1, 2, \cdots$ oscillator occupation number
- $E_{\downarrow,n+1} = \epsilon - \frac{\epsilon_z}{2} + (n+1)\hbar\omega$; $E_{\uparrow,n} = \epsilon + \frac{\epsilon_z}{2} + n\hbar\omega$
- When $\hbar\omega \approx \epsilon_z$, $E_{\downarrow,n+1} \approx E_{\uparrow,n}$: the dot can exchange energy with the oscillator by flipping its spin.
- Electrons tunnel between the dot and the leads.

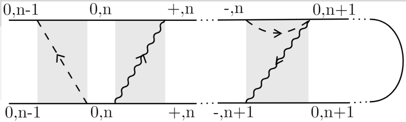
Assumptions



- Rotation Wave Approximation
  - the interaction strength between the dot and the oscillator is weak: $|\lambda_d| < \hbar\omega \approx \epsilon_z$
  - four eigenstates $|\chi, n\rangle$ of the dot and the oscillator:
    $|0, n\rangle$, $|\uparrow\downarrow, n\rangle$
    $|+, n\rangle = \sin\theta_n |\uparrow, n\rangle - \cos\theta_n |\downarrow, n+1\rangle$
    $|-, n\rangle = \cos\theta_n |\uparrow, n\rangle + \sin\theta_n |\downarrow, n+1\rangle$
- Unidirectional Transport (from left to right)
  - infinitely large voltage on the leads
  - Fermi functions: $f_L = 1$ and $f_R = 0$
- 0 Temperature
  - the bath cools down the oscillator
  - n=0 when the oscillator is in equilibrium
- Spin Flips in x Axis
  - $\hat{e}_x = \hat{n}_L + \hat{n}_R$
  - $\hat{e}_y = \hat{n}_L - \hat{n}_R$
  - $\hat{e}_z = \hat{n}_L \times \hat{n}_R$



Analysis

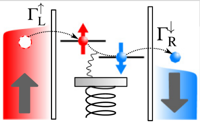


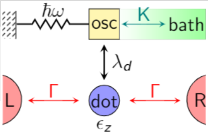
- Reduced Density Matrix of the Dot and the Oscillator
  - $\hat{\rho} = \sum_{\chi_1, n_1; \chi_2, n_2} P^{\chi_1, n_1}_{\chi_2, n_2} \left| \chi_2, n_2 \right\rangle \left\langle \chi_1, n_1 \right|$
  - diagonal terms: $P_{\chi,n} = P^{\chi,n}_{\chi,n}$ and $\sum_{\chi,n} P_{\chi,n} = 1$
- Master Equation at Steady State
  - $0 = \dot{P}^{\chi_1,n_1}_{\chi_2,n_2} = -\frac{i}{\hbar}(E_{\chi_1,n_1} - E_{\chi_2,n_2}) P^{\chi_1,n_1}_{\chi_2,n_2} + \sum_{\chi_1',n_1';\chi_2',n_2'} W^{\chi_1,n_1;\chi_1',n_1'}_{\chi_2,n_2;\chi_2',n_2'} P^{\chi_1',n_1'}_{\chi_2',n_2'}$
  - general transition rate: $W^{\chi_1,n_1;\chi_1',n_1'}_{\chi_2,n_2;\chi_2',n_2'} \propto \Gamma$ or K
- Keldysh Formalism with Diagrammatic Rule
  - $\left| \chi, n \right\rangle$ propagates on the Keldysh contour
  - leads electron tunnelling line & bath phonon tunnelling line

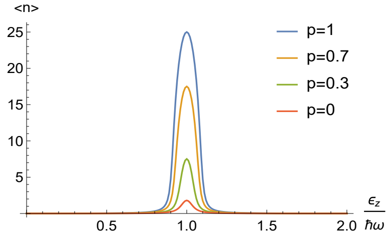
Numerical Simulation in First Order Expansion ($n_{\max} = 100$)

- off-diagonal elements reduce to zero
- diagrammatic results should agree with Fermi Golden Rule ones

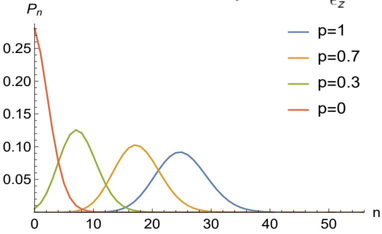
Anti-parallel Polarisation



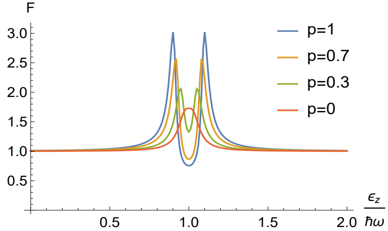




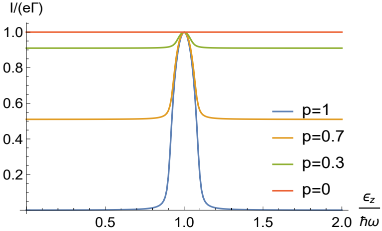
(a) average occupation number



(b) probability distribution at resonance

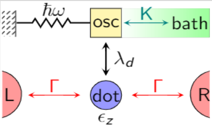


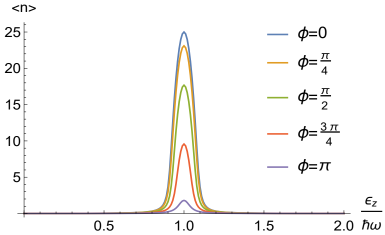
(c) Fano factor $(F) = \frac{\langle(\Delta n)^2\rangle}{\langle n\rangle}$



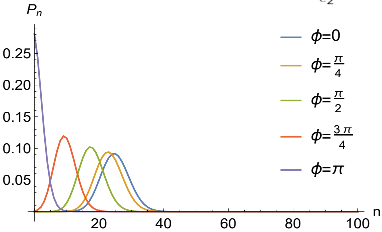
(d) the current in $e\Gamma$ units

Non-collinear Polarisation (1st order, p=1)

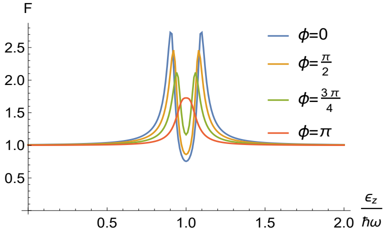




(a) average occupation number



(b) probability distribution at resonance



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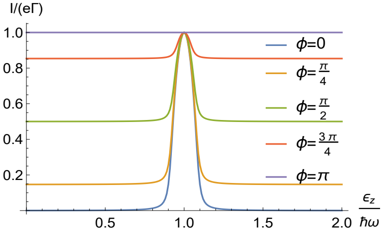


(d) the current in $e\Gamma$ units

Conclusion

Carbon-nanotube quantum dot with ferromagnetic leads

- Reproduced collinear polarisation results
  - Demonstrated the single atom lasing when the polarisation is sufficiently large
- Extended to non-collinear case
  - Developed diagrammatic rules
  - Demonstrated the single atom lasing when the angle between two polarisation axes is sufficiently small with full polarisation strength

- **Thank You** ☺ ☕

The Hamiltonian

$$\hat{H}_{\text{leads}} = \sum_{\eta,\sigma,k} \epsilon_{k,\sigma} \hat{c}^{\dagger}_{\eta,\sigma,k} \hat{c}_{\eta,\sigma,k},$$

$$\hat{H}_{\text{tun}} = \sum_{\eta,\sigma,k} \left(V_{\eta,\sigma} \hat{c}^{\dagger}_{\eta,\sigma,k} \hat{d}_{\sigma} + V^{*}_{\eta,\sigma} \hat{d}^{\dagger}_{\sigma} \hat{c}_{\eta,\sigma,k} \right)$$

$$\hat{H}_{\text{dot}} = \sum_{\sigma} \epsilon_{\sigma} \hat{d}^{\dagger}_{\sigma} \hat{d}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}$$

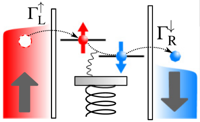
$$\hat{H}_{\text{intdot}} = -\lambda_d \left(\hat{d}^{\dagger}_{\downarrow} \hat{d}_{\uparrow} + \hat{d}^{\dagger}_{\uparrow} \hat{d}_{\downarrow} \right) \left(\hat{b}^{\dagger} + \hat{b} \right)$$

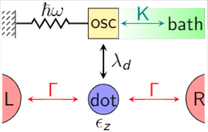
$$\hat{H}_{\text{osc}} = \hbar\omega \hat{b}^{\dagger} \hat{b},$$

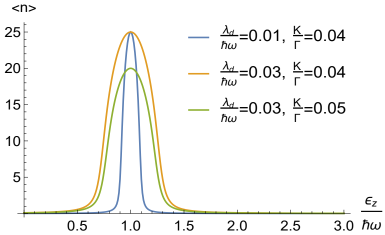
$$\hat{H}_{\text{intbath}} = \lambda_b \sum_{q} \left(a^{\dagger}_{q} b + b^{\dagger} a_{q} \right)$$

$$\hat{H}_{\text{bath}} = \sum_{q} \hbar\omega_q a^{\dagger}_{q} a_{q}$$

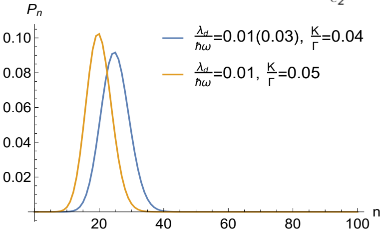
Anti-parallel Polarisation (p=1)







(a) average occupation number



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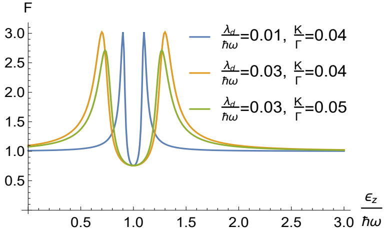
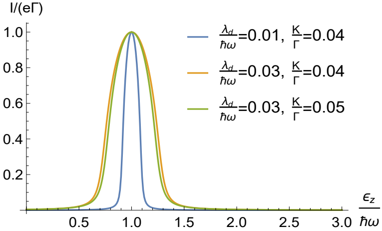
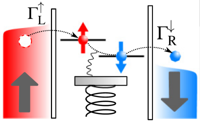


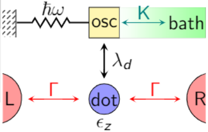
(c) Fano factor $(F) = \frac{\langle(\Delta n)^2\rangle}{\langle n\rangle}$



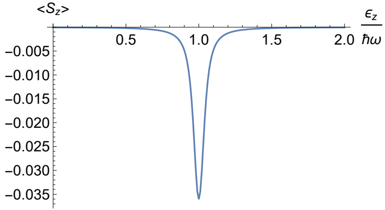
(d) the current in $e\Gamma$ units

p=0 vs p=1



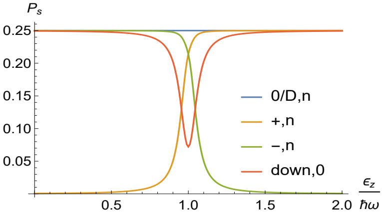


p=0



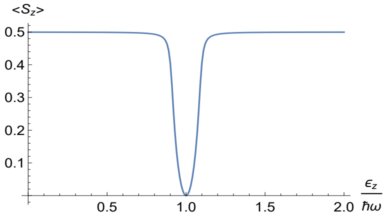
(a) average spin in the dot

p=0



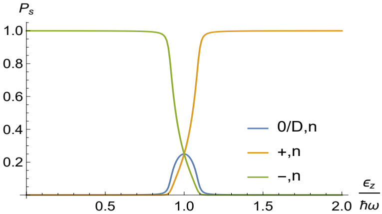
(b) probability of each state

p=1



(c) average spin in the dot

p=1



(d) probability of each state