

# Bessel filter (1-5 order)

**Collated and calculated by Zhou Yiliang (zhouyiliang0311@163.com) , 2023.2**

The filter implementation is base on the source on Wikipedia and python library `scipy.signal.bessel`.

## Bessel filter

The *Bessel filter* is given by the normalized transfer function

$$H(S) = \frac{c_{n,0}}{q_n(S)}$$

where n is the order of the filter and  $q_n$  are the Bessel polynomials

$$q_n(S) = \sum_{k=0}^n c_{n,k} S^k$$

with coefficients

$$c_{n,k} = \frac{(2n - k)!}{2^{n-k} k! (n - k)!}$$

The [transfer function](#) above is normalized (i.e., it is presented for the Bessel [low pass filter](#) with cutoff frequency 1). The Bessel low pass filter can be obtained from the transfer function above with the substitution  $S = s / \omega_c$ , where  $s = j\omega$ ,  $\omega_c$  is the cutoff frequency of the filter, and  $\omega$  is the angular frequency spanning the frequency spectrum between 0 and  $\pi$ . The substitution  $S = \omega_c / s$  produces the Bessel [high pass filter](#). The substitution  $S = (s^2 + \omega_c^2) / (B s)$  produces the Bessel [band pass filter](#), where  $\omega_c$  is the midpoint of the pass band and B is the width of the band. The substitution  $S = B s / (s^2 + \omega_c^2)$  produces the Bessel [band stop filter](#).

The Bessel filter is said to have an almost flat group delay (delay of the amplitude envelope for various frequencies). In other words, the Bessel filter has close to the same delay for all frequencies.

<https://www.recordingblogs.com/wiki/bessel-filter>

[https://en.wikipedia.org/wiki/Bessel\\_filter](https://en.wikipedia.org/wiki/Bessel_filter)

<https://docs.scipy.org/doc/scipy/reference/generated/scipy.signal.bessel.html>

## 1 st order

$$H(s) = \frac{1}{1 + s}$$

$$\text{Low: } s = \frac{s}{\omega_c} \quad \text{High: } s = \frac{\omega_c}{s}$$

$$\text{lowpass: } H(s) = \frac{1}{1 + \frac{s}{\omega_c}}$$

$$\text{Highpass: } H(s) = \frac{1}{1 + \frac{\omega_c}{s}}$$

Cutoff frequency  $\omega_c = 2\pi f_c/f_s$ .

Applying the [bilinear transformation](#):  $s=2*(z-1)/(z+1)$ , the transfer function in the z-domain can be expressed as:

lowpass:

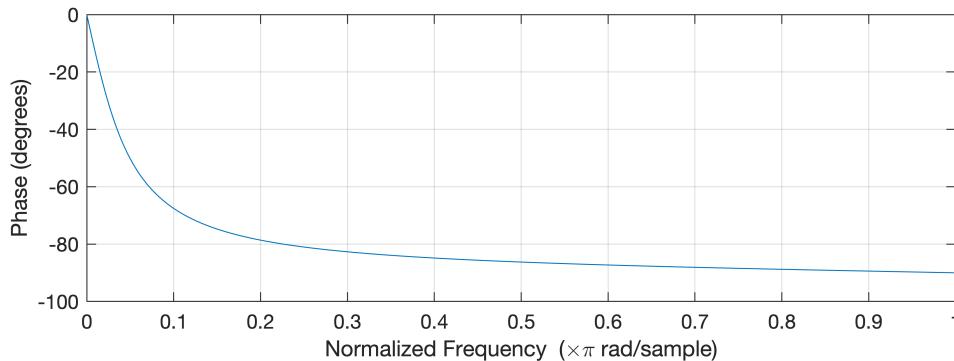
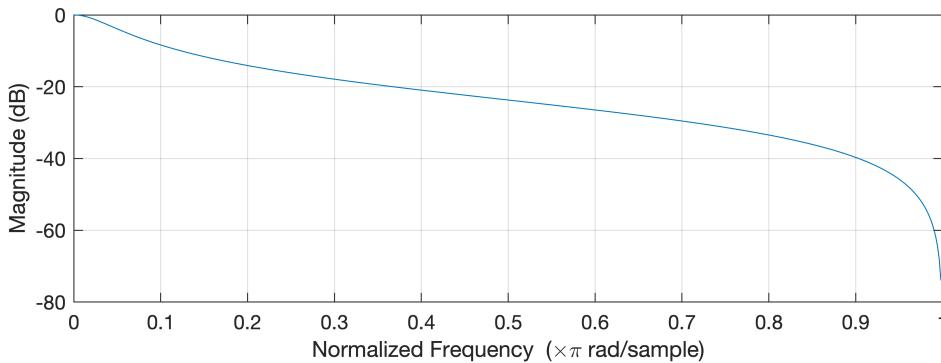
$$H(z) = \frac{w_c + w_c z^{-1}}{(w_c + 2) + (w_c - 2)z^{-1}}$$

Highpass

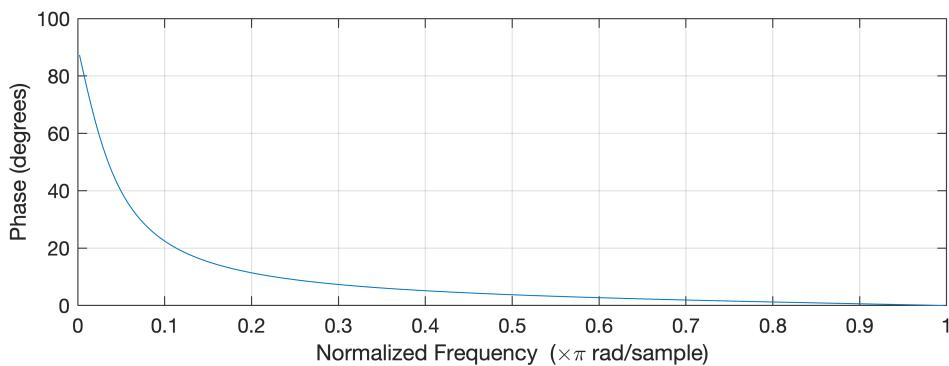
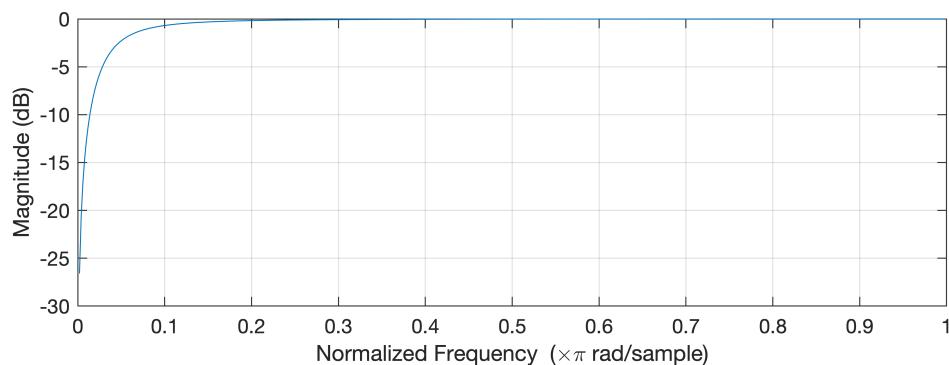
$$H(z) = \frac{2 - 2z^{-1}}{(w_c + 2) + (w_c - 2)z^{-1}}$$

The b and a coefficients can be obtained by comparing the numerator and denominator of the transfer function to the standard form of a digital filter:

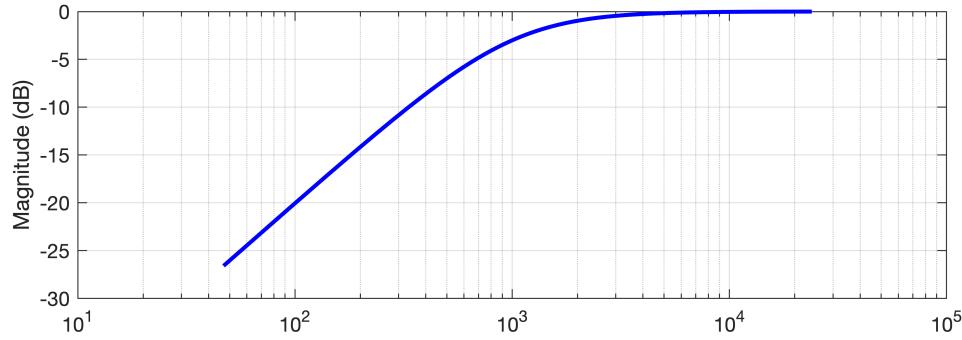
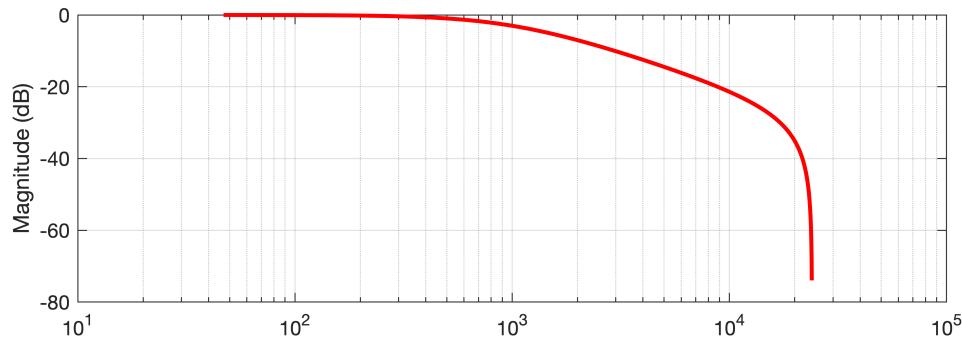
```
clear;clc;
fs = 48e3;
fc = 1000; %cutoff frequency
w_c = 2*pi*fc/fs; %normalized cutoff frequency of the filter
b_l1 = [w_c w_c];
a_l1 = [w_c+2 w_c-2];
% freqz(b_d, a_d)
freqz(b_l1, a_l1);
```



```
[H_low_1, w] = freqz(b_l1,a_l1);
b_h1 = [2 -2];
a_h1 = [w_c+2 w_c-2];
freqz(b_h1, a_h1);
```



```
[H_high_1, w] = freqz(b_h1,a_h1);
figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_1)), '-r', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_1)), '-b', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
```



## 2 st order

$$H(s) = \frac{3}{s^2 + 3s + 3}$$

$$\text{Low: } s = \frac{s}{w_c} \quad \text{High: } s = \frac{w_c}{s}$$

$$\text{lowpass: } H(s) = \frac{3}{\left(\frac{s}{w_c}\right)^2 + 3\frac{s}{w_c} + 3}$$

$$\text{Highpass: } H(s) = \frac{3}{\left(\frac{w_c}{s}\right)^2 + 3\frac{w_c}{s} + 3}$$

Cutoff frequency  $w_c = 2\pi f_c/f_s$ .

Applying the [bilinear transformation](#):  $s=2*(z-1)/(z+1)$ , the transfer function in the z-domain can be expressed as:

lowpass:

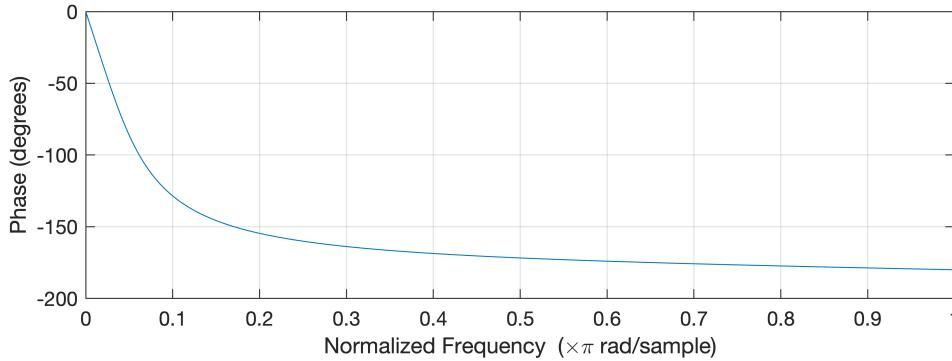
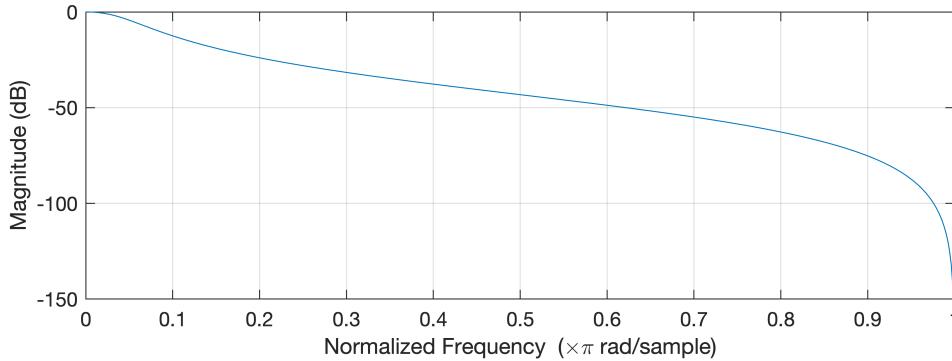
$$H(z) = \frac{3w_c^2(1 + 2z^{-1} + z^{-2})}{(w_c + 2) + (w_c - 2)z^{-1}}$$

Highpass

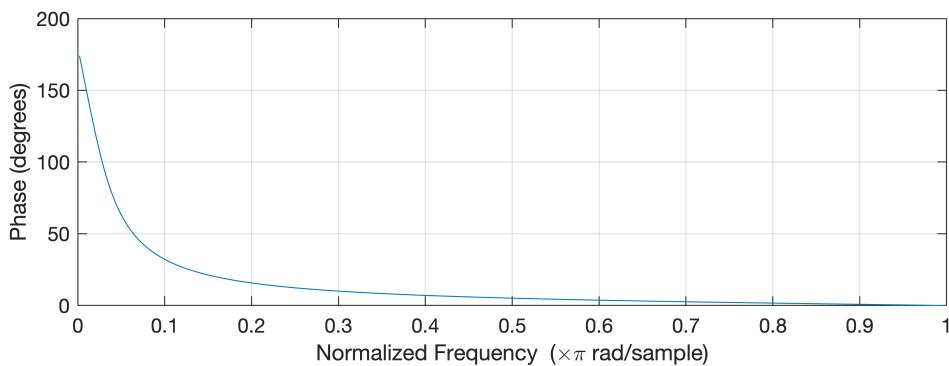
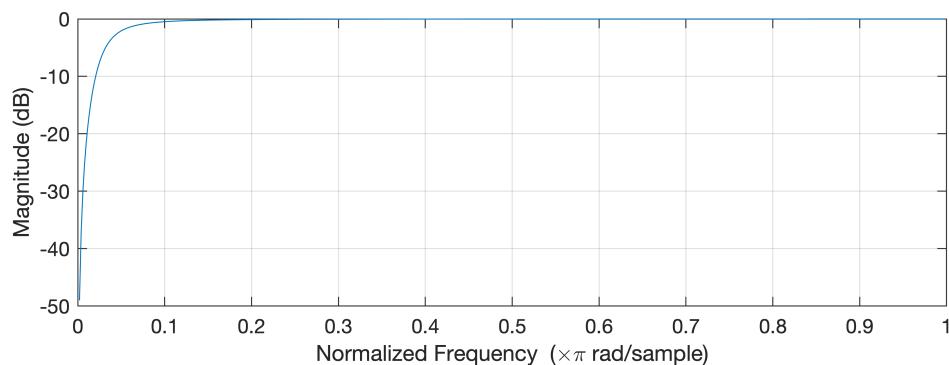
$$H(z) = \frac{12 - 24z^{-1} + 12z^{-2}}{(w_c^2 + 6w_c + 12) + (2w_c^2 - 24)z^{-1} + (w_c^2 - 6w_c + 12)z^{-2}}$$

The b and a coefficients can be obtained by comparing the numerator and denominator of the transfer function to the standard form of a digital filter:

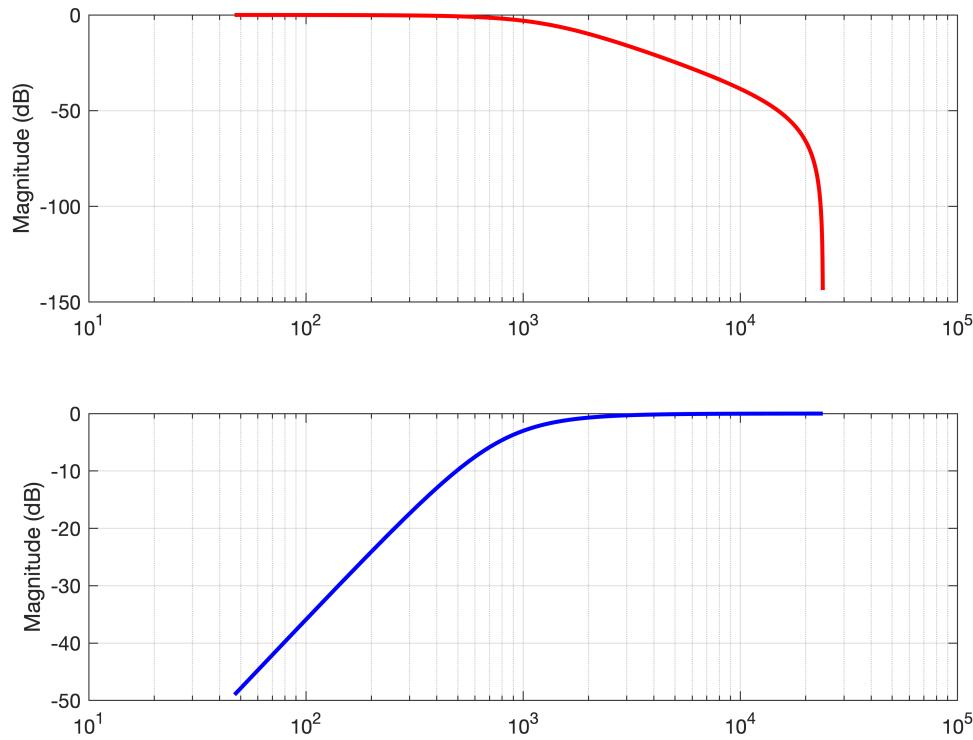
```
% fs = 48e3;
fc_l2 = fc/1.361654129; %3dB frequency normalization constant
w_c = 2*pi*fc_l2/fs; %normalized cutoff frequency of the filter
b_l2 = 3 * w_c^2 * [1, 2, 1];
a_l2 = [4+6*w_c+3*w_c^2, 6*w_c^2-8, 4-6*w_c+3*w_c^2];
% freqz(b_d, a_d)
freqz(b_l2, a_l2);
```



```
[H_low_2, w] = freqz(b_l2,a_l2);
fc_h2 = fc*1.361654129; %3dB frequency normalization constant
w_c = 2*pi*fc_h2/fs; %normalized cutoff frequency of the filter
b_h2 = [12, -24, 12];
a_h2 = [w_c^2+6*w_c+12, 2*w_c^2-24, w_c^2-6*w_c+12];
freqz(b_h2, a_h2);
```



```
[H_high_2, w] = freqz(b_h2,a_h2);
figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_2)), '-r', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_2)), '-b', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
```



### 3 rd order

lowpass:

$$H(s) = \frac{15w_c^3}{s^3 + 6w_c s^2 + 15w_c^2 s + 15w_c^3}$$

$$H(s) = \frac{15w_c^3 s^{-3}}{1 + 6w_c s^{-1} + 15w_c^2 s^{-2}}$$

Cutoff frequency  $w_c = 2\pi f_c/f_s$ .

First, we can simplify the denominator:

$$s^3 + 6 * w_c * s^2 + 15 * w_c^2 * s + 15 * w_c^3 = (2 * (z - 1)/(z + 1))^3 + 6 * w_c * (2 * (z - 1)/(z + 1))^2 + 15 * w_c^2 * (2 * (z - 1)/(z + 1)) + 15 * w_c^3$$

Applying the [bilinear transformation](#):  $s=2*(z-1)/(z+1)$ , the transfer function in the z-domain can be expressed as:

Highpass:

$$H(s) = \frac{15s^3}{\omega_c^3 + 6\omega_c^2 s + 15\omega_c s^2 + 15s^3} \quad H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}$$

## Example: High pass Bessel filter of the third order

Set  $n = 3$  and use the substitution  $S = \omega_c / s$ . The transfer function of the third order Bessel high pass filter is

$$H(s) = \frac{15s^3}{\omega_c^3 + 6\omega_c^2s + 15\omega_c s^2 + 15s^3}$$

The [bilinear transformation](#)  $s = 2(z - 1) / (z + 1)$  allows us to rewrite the transfer function using the [Z transform](#) as follows.

$$H(z) = \frac{a_0 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3}}{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}$$

$$a_0 = 120$$

$$a_1 = -360$$

$$a_2 = 360$$

$$a_3 = -120$$

$$b_0 = \omega_c^3 + 12\omega_c^2 + 60\omega_c + 120$$

$$b_1 = 3\omega_c^3 + 12\omega_c^2 - 60\omega_c - 360$$

$$b_2 = 3\omega_c^3 - 12\omega_c^2 - 60\omega_c + 360$$

$$b_3 = \omega_c^3 - 12\omega_c^2 + 60\omega_c - 120$$

Say that the cutoff frequency of the filter is  $\omega_c = 0.6$  (technically,  $\omega_c = 2 \arctan(0.6/2) \approx 0.583$ , because of the warping of the frequency domain by the bilinear transformation). The transfer function of this example Bessel high pass filter is

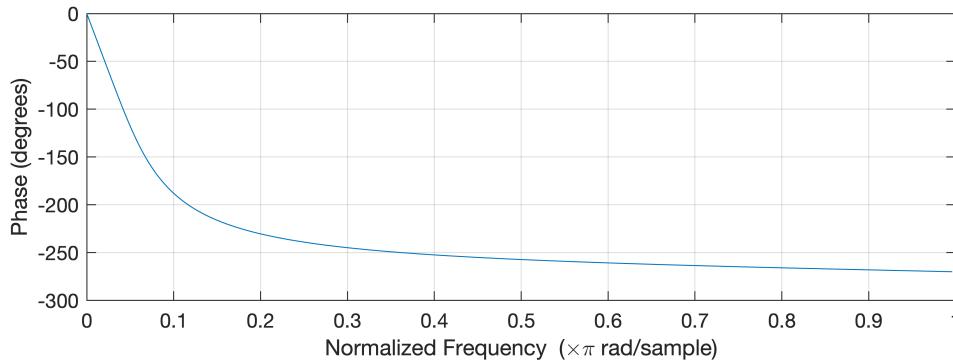
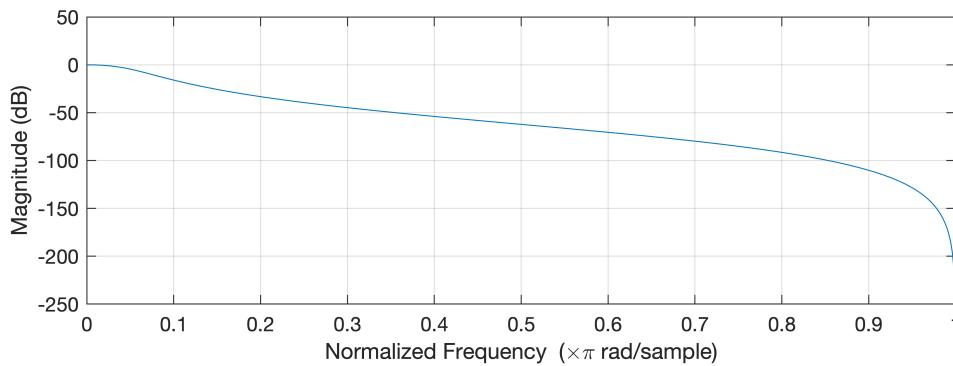
$$H(z) = \frac{0.747496 - 2.242488z^{-1} + 2.242488z^{-2} - 0.747496z^{-3}}{1 - 2.435790z^{-1} + 1.995366z^{-2} - 0.548811z^{-3}}$$

and the filter itself is

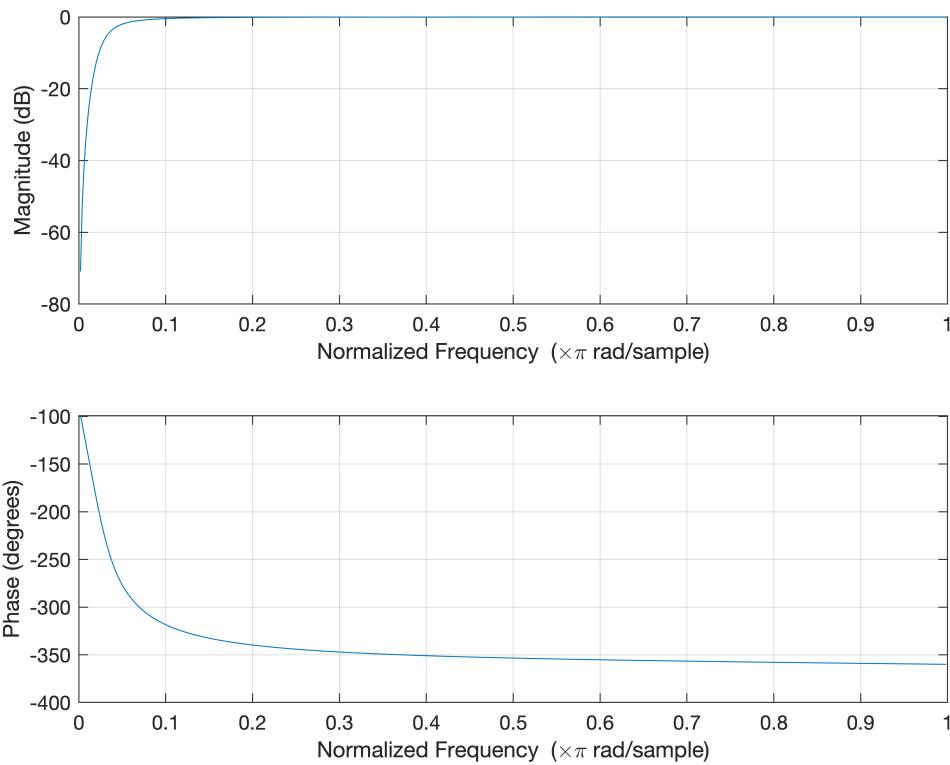
$$\begin{aligned} y(k) = & 0.747496x(k) - 2.242488x(k-1) + 2.242488x(k-2) - 0.747496x(k-3) \\ & + 2.435790y(k-1) - 1.995366y(k-2) + 0.548811y(k-3) \end{aligned}$$

Suppose that the [sampling frequency](#) is 2000 Hz. The cutoff frequency then is  $\omega_c = (0.6 * 2000) / (2\pi) = 191$  Hz. The [magnitude response](#) of the filter is shown in the graph below.

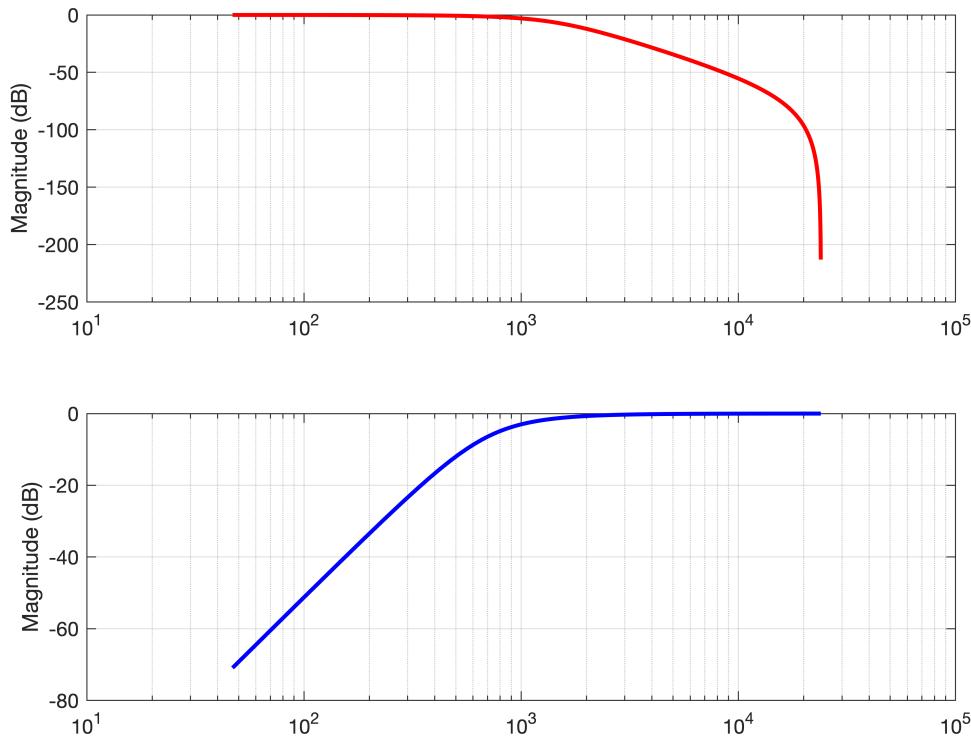
```
% fs = 48e3;
fc_l3 = fc/1.755672389; %3dB frequency normalization constant
w_c = 2*pi*fc_l3/fs; %normalized cutoff frequency of the filter
b_l3 = 15*w_c^3*[1, 3, 3, 1];
a_l3 = [(8 + 24*w_c + 30*w_c^2+15*w_c^3), (-24 - 24*w_c + 30*w_c^2 + 45*w_c^3),(24 - 2*
```



```
[H_low_3, w] = freqz(b_l3,a_l3);
fc_h3 = fc*1.755672389; %3dB frequency normalization constant
w_c = 2*pi*fc_h3/fs; %normalized cutoff frequency of the filter
b_h3 = [120 -360 360 -120];
a0=w_c^3+12 * w_c^2+60*w_c+120;
a1= 3*w_c^3 + 12*w_c^2 - 60*w_c - 360;
a2= 3*w_c^3 - 12*w_c^2 - 60*w_c + 360;
a3= w_c^3 - 12*w_c^2 + 60*w_c - 120;
a_h3 = [a0 a1 a2 a3];
% [b_d, a_d] = bilinear(b, a, fs);
% freqz(b_d, a_d)
freqz(b_h3, a_h3)
```



```
[H_high_3, w] = freqz(b_h3,a_h3);
figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_3)), '-r', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_3)), '-b', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
```



## 4 th order

The minimum phase transfer function is

$$H(s) = \frac{105}{s^4 + 10s^3 + 45s^2 + 105s + 105}.$$

$$\text{Low: } s = \frac{s}{w_c} \quad \text{High: } s = \frac{w_c}{s}$$

Applying the [bilinear transformation](#)  $s=2^*(z-1)/(z+1)$

$$\text{lowpass: } H(s) = \frac{105w_c^4}{s^4 + 10s^3w_c + 45s^2w_c^2 + 105sw_c^3 + 105w_c^4}$$

$$H(z) = \frac{105w_c^4(z+1)^4}{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4}}$$

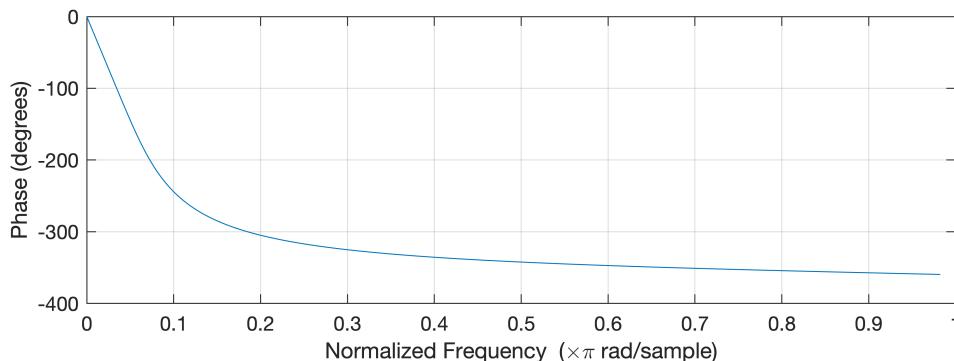
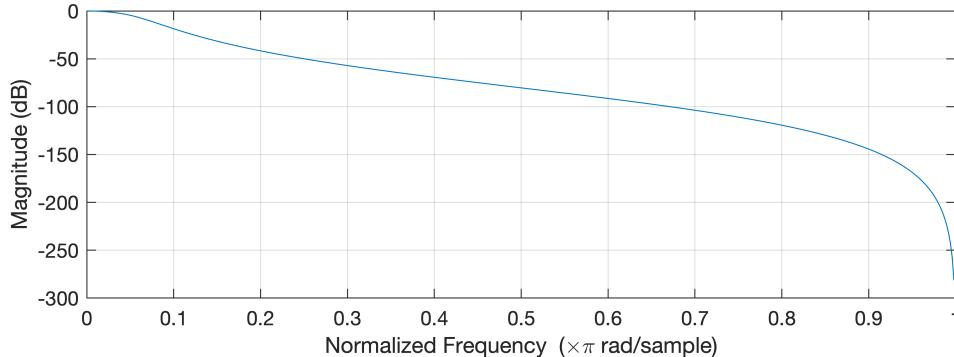
$$\text{highpass: } H(s) = \frac{105s^4}{w_c^4 + 10w_c^3s + 45w_c^2s^2 + 105w_c s^3 + 105s^4}$$

```
% fs = 48e3;
fc_l4 = fc/2.113917675; %3dB frequency normalization constant
w_c = 2*pi*fc_l4/fs; %normalized cutoff frequency of the filter
b_l4 = 105*w_c^4*[1 4 6 4 1];
a0= 105*w_c^4 +210*w_c^3 + 180*w_c^2 + 80*w_c + 16; %z^4
```

```

a1= 420*w_c^4 + 420*w_c^3 + 0*w_c^2 - 160*w_c -64; %z^3
a2= 630*w_c^4 + 0*w_c^3 + (-360)*w_c^2 + 0*w_c + 96; %z^2
a3= 420*w_c^4 - 420*w_c^3 + 0*w_c^2 + 160*w_c - 64; %z^1
a4= 105*w_c^4 - 210*w_c^3 + 180*w_c^2 - 80*w_c + 16; %z^0
a_l4 = [a0 a1 a2 a3 a4];
freqz(b_l4, a_l4)

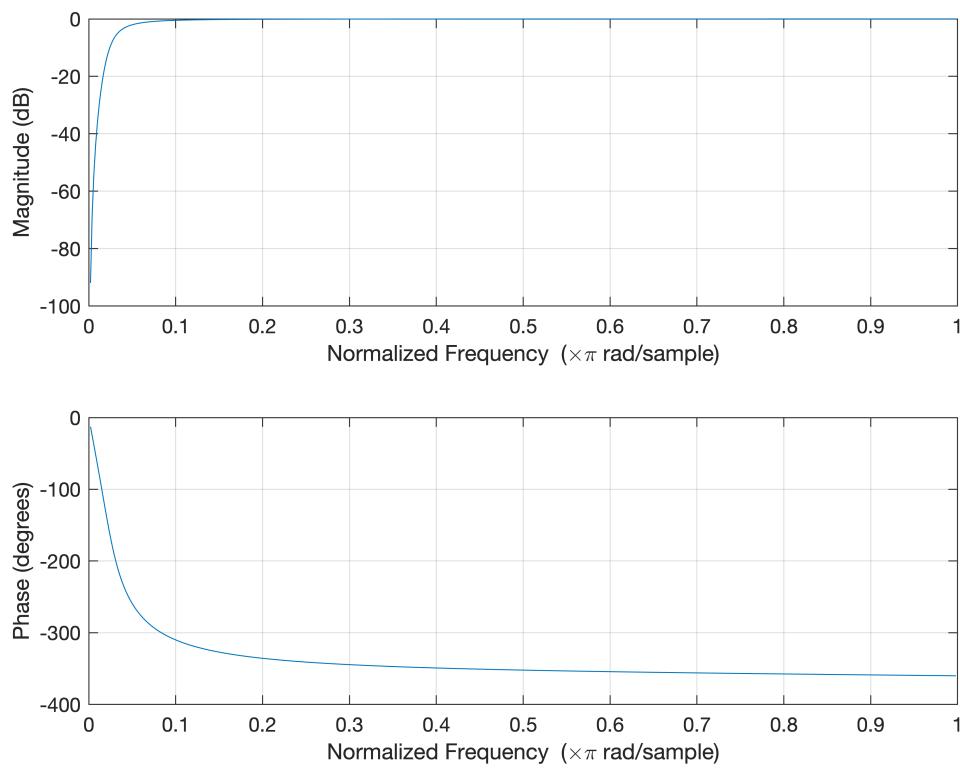
```



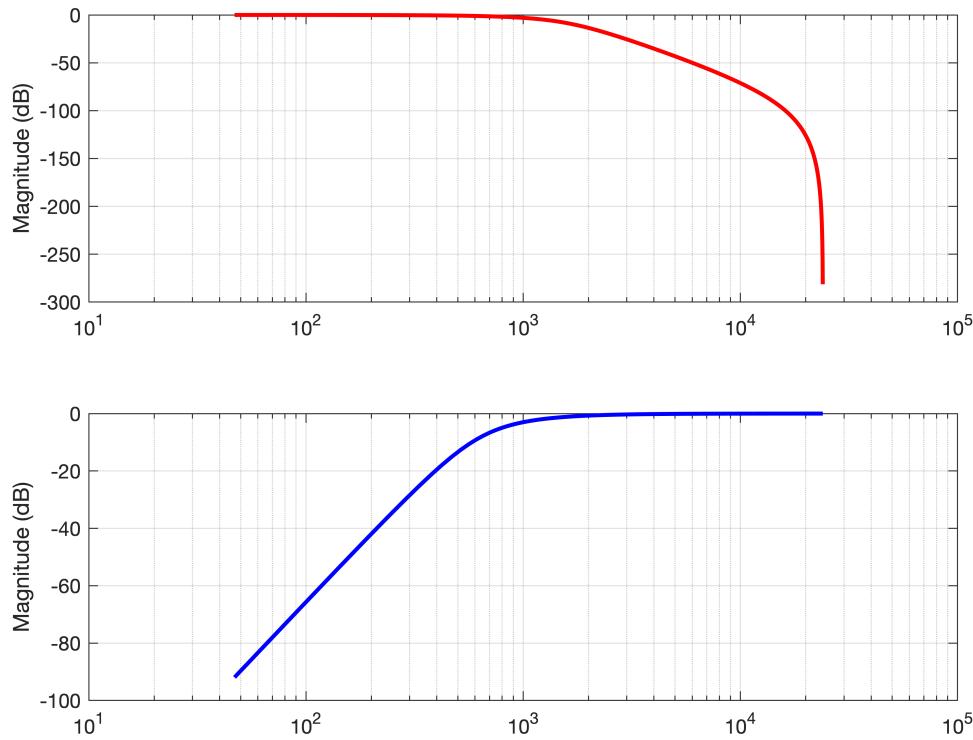
```

[H_low_4, w] = freqz(b_l4,a_l4);
fc_h4 = fc*2.113917675; %3dB frequency normalization constant
w_c = 2*pi*fc_h4/fs; %normalized cutoff frequency of the filter
b_h4 = 105*16*[1 -4 +6 -4 1];
a0= 1*w_c^4 +20*w_c^3 + 180*w_c^2 + 840*w_c + 1680; %z^4
a1= 4*w_c^4 + 40*w_c^3 + 0*w_c^2 - 1680*w_c -6720; %z^3
a2= 6*w_c^4 - 0*w_c^3 + (-360)*w_c^2 + 0*w_c + 10080; %z^2
a3= 4*w_c^4 - 40*w_c^3 + 0*w_c^2 + 1680*w_c - 6720; %z^1
a4= 1*w_c^4 - 20*w_c^3 + 180*w_c^2 - 840*w_c + 1680; %z^0
a_h4 = [a0 a1 a2 a3 a4];
% [b_d, a_d] = bilinear(b, a, fs);
% freqz(b_d, a_d)
freqz(b_h4, a_h4)

```



```
[H_high_4, w] = freqz(b_h4,a_h4);
figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_4)), '-r', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_4)), '-b', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
```



## 5 th order

The minimum phase transfer function is

$$H(s) = \frac{945}{s^5 + 15s^4 + 105s^3 + 420s^2 + 945s + 945}.$$

$$\text{Low: } s = \frac{s}{w_c} \quad \text{High: } s = \frac{w_c}{s}$$

Applying the [bilinear transformation](#)  $s=2*(z-1)/(z+1)$

$$\text{lowpass: } H(s) = \frac{945w_c^5}{s^5 + 15s^4w_c + 105s^3w_c^2 + 420s^2w_c^3 + 945sw_c^4 + 945w_c^5}$$

$$H(z) = \frac{945w_c^5(z+1)^5}{a_0 + a_1z^{-1} + a_2z^{-2} + a_3z^{-3} + a_4z^{-4} + a_5z^{-5}}$$

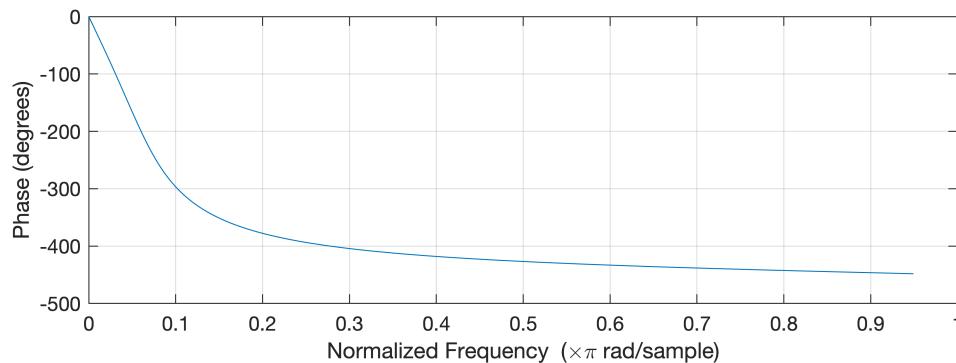
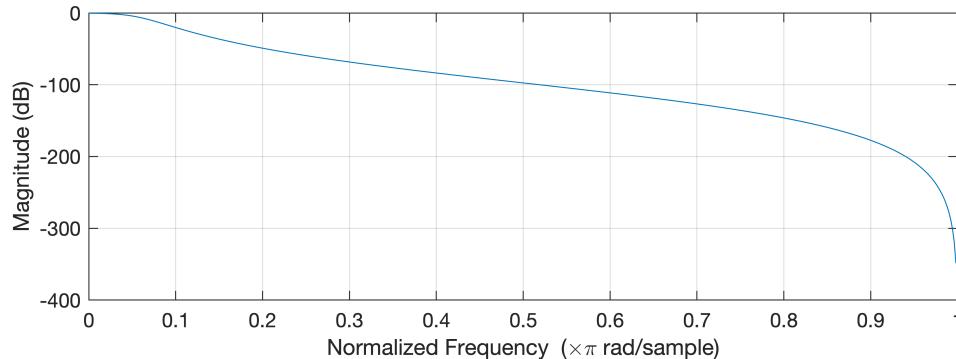
$$\text{highpass: } H(s) = \frac{945s^5}{w_c^5 + 15w_c^4s + 105w_c^3s^2 + 420w_c^2s^3 + 945w_c s^4 + 945s^5}$$

```
% fs = 48e3;
fc_l5 = fc/2.427410702; %3dB frequency normalization constant
w_c = 2*pi*fc_l5/fs; %normalized cutoff frequency of the filter
b_l5 = 945*w_c^5*[1 5 10 10 5 1];
a0= 945*w_c^5 + 1890*w_c^4 + 1680*w_c^3 + 840*w_c^2 + 240*w_c + 32; %z^5
```

```

a1= 4725*w_c^5 + 5760*w_c^4 +1680*w_c^3 + (-840)*w_c^2 + (-720)*w_c + (-160); %z^4
a2= 9450*w_c^5 + 3780*w_c^4 +(-3360)*w_c^3 + (-1680)*w_c^2 + 480*w_c + 320; %z^3
a3= 9450*w_c^5 + (-3780)*w_c^4 +(-3360)*w_c^3 + 1680*w_c^2 + 480*w_c + (-320); %z^2
a4= 4725*w_c^5 - 5670*w_c^4 +1680*w_c^3 + 840*w_c^2 + (-720)*w_c + 160; %z^1
a5= 945*w_c^5 - 1890*w_c^4 +1680*w_c^3 - 840*w_c^2 + 240*w_c - 32; %z^0
a_l5 = [a0 a1 a2 a3 a4 a5];
freqz(b_l5, a_l5)

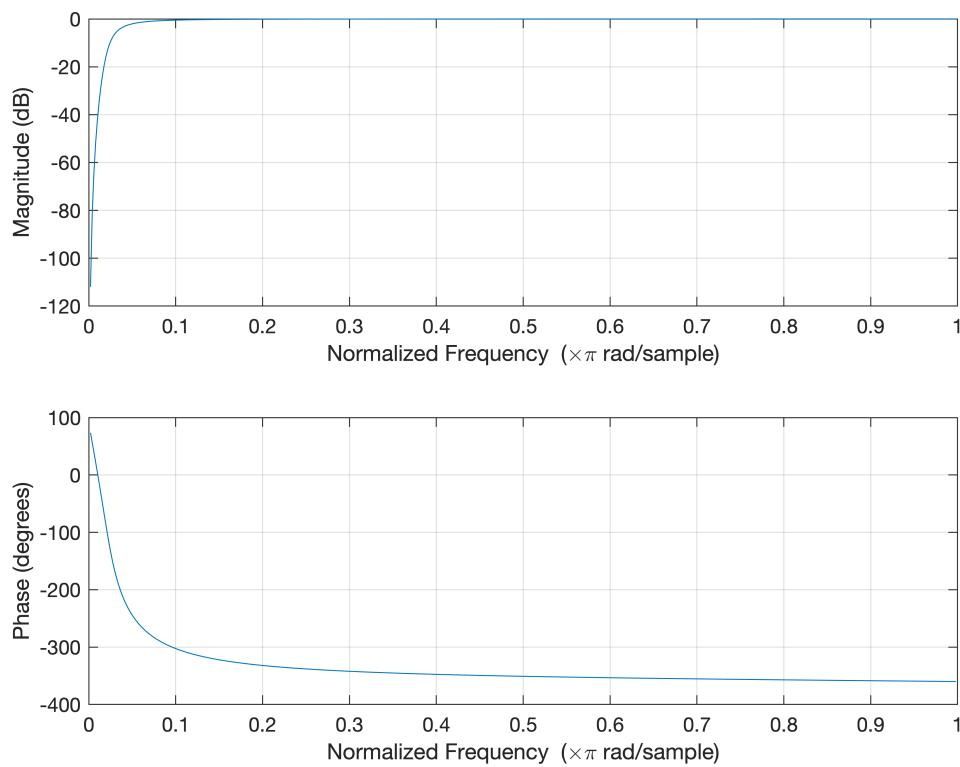
```



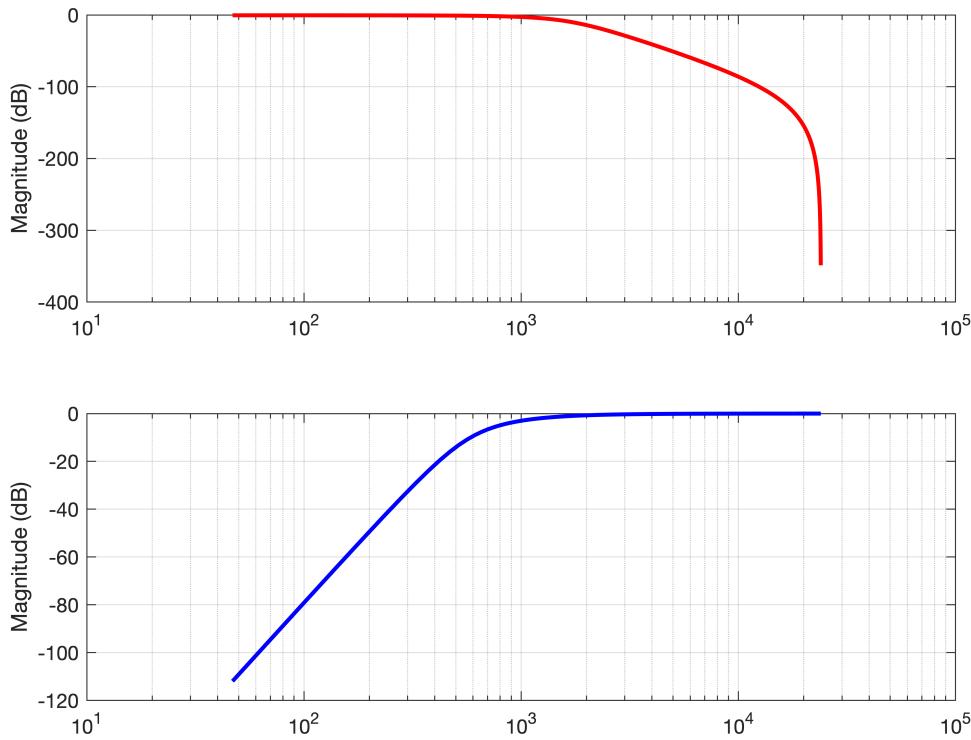
```

[H_low_5, w] = freqz(b_l5,a_l5);
fc_h5 = fc*2.427410702; %3dB frequency normalization constant
w_c = 2*pi*fc_h5/fs; %normalized cutoff frequency of the filter
b_h5 = 945*32*[1 -5 10 -10 5 -1];
a0= 1*w_c^5 + 30*w_c^4 +420*w_c^3 + 3360*w_c^2 + 15120*w_c + 30240; %z^5
a1= 5*w_c^5 + 90*w_c^4 +420*w_c^3 + (-3360)*w_c^2 + (-45360)*w_c + (-151200); %z^4
a2= 10*w_c^5 + 60*w_c^4 +(-840)*w_c^3 + (-6720)*w_c^2 + 30240*w_c + 302400; %z^3
a3= 10*w_c^5 + (-60)*w_c^4 +(-840)*w_c^3 + 6720*w_c^2 + 30240*w_c + (-302400); %z^2
a4= 5*w_c^5 - 90*w_c^4 + 420*w_c^3 + 3360*w_c^2 + (-45360)*w_c + 151200; %z^1
a5= 1*w_c^5 - 30*w_c^4 + 420*w_c^3 - 3360*w_c^2 + 15120*w_c - 30240; %z^0
a_h5 = [a0 a1 a2 a3 a4 a5];
% [b_d, a_d] = bilinear(b, a, fs);
% freqz(b_d, a_d)
freqz(b_h5, a_h5)

```



```
[H_high_5, w] = freqz(b_h5,a_h5);
figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_5)), '-r', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_5)), '-b', 'LineWidth', 2);
ylabel('Magnitude (dB)');
grid on;
```

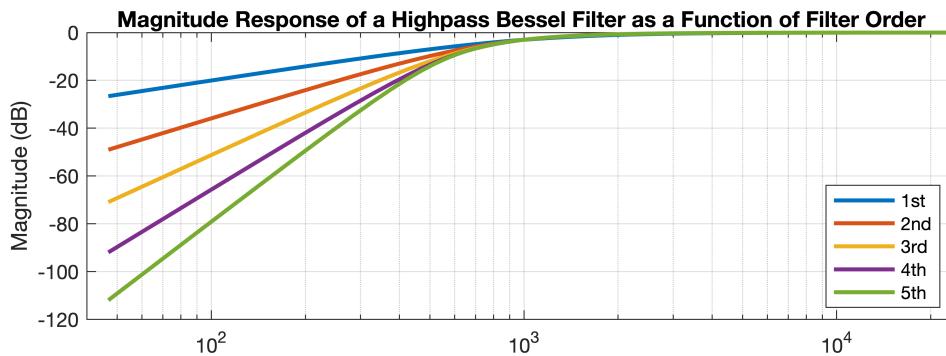
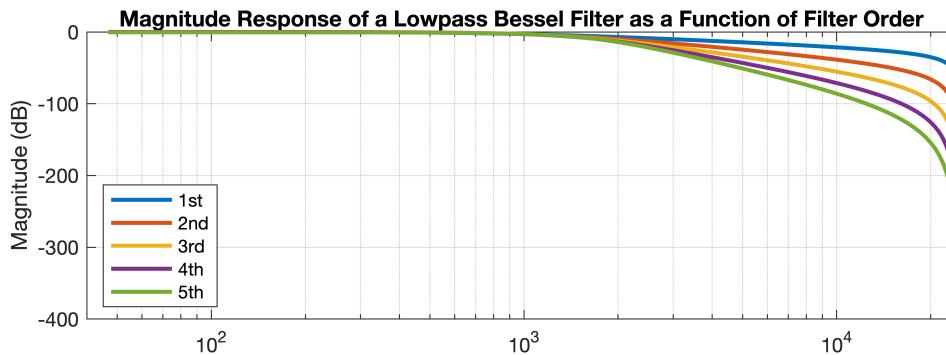


Plot together

```

figure
subplot(2,1,1)
semilogx(w/pi*fs/2, 20*log10(abs(H_low_1)), 'LineWidth', 2);
hold on
semilogx(w/pi*fs/2, 20*log10(abs(H_low_2)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_3)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_4)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_5)), 'LineWidth', 2);
legend({'1st', '2nd', '3rd', '4th', '5th'},'Location','southwest')
ylabel('Magnitude (dB)');
xlim([40 fs/2])
title('Magnitude Response of a Lowpass Bessel Filter as a Function of Filter Order')
grid on;
subplot(2,1,2)
semilogx(w/pi*fs/2, 20*log10(abs(H_high_1)), 'LineWidth', 2);
hold on
semilogx(w/pi*fs/2, 20*log10(abs(H_high_2)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_3)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_4)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_5)), 'LineWidth', 2);
legend({'1st', '2nd', '3rd', '4th', '5th'},'Location','southeast')
ylabel('Magnitude (dB)');
xlim([40 fs/2])
title('Magnitude Response of a Highpass Bessel Filter as a Function of Filter Order')
grid on;

```



```
% Here is the measured result
freq = [500, 750, 1000, 1500, 2000];
y_low = zeros(8, 5);
y_high = zeros(8, 5);
y_low(1,:) = [-1, -2, -3.1, -5.3, -7.2]; % -6dB
y_low(2,:) = [-0.7, -1.8, -3.1, -6.8, -10];% -12dB
y_low(3,:) = [-0.7, -1.7, -3.1, -7.5, -12.4];% -18dB
y_low(4,:) = [-0.7, -1.7, -3.1, -7.9, -13.8];% -24dB
y_low(5,:) = [-0.7, -1.7, -3.1, -7.9, -14.8];% -30dB
y_low(6,:) = [-0.7, -1.7, -3.1, -7.9, -14.8];% -36dB
y_low(7,:) = [-0.7, -1.7, -3.1, -7.8, -14.8];% -42dB
y_low(8,:) = [-0.7, -1.7, -3.1, -7.8, -14.3];% -48dB
y_high(1,:) = [-7.1, -4.8, -3.1, -1.6, -1];% -6dB
y_high(2,:) = [-10, -5.4, -3.1, -1.4, -0.8];% -12dB
y_high(3,:) = [-12.4, -6, -2.9, -1.05, -0.5];% -18dB
y_high(4,:) = [-13.9, -6, -3, -1.2, -0.6];% -24dB
y_high(5,:) = [-17.6, -7, -3, -1, -0.5];% -30dB
y_high(6,:) = [-18.4, -7, -3.5, -1.4, -0.6];% -36dB
y_high(7,:) = [-18.9, -6, -2.8, -1, -0.5];% -42dB
y_high(8,:) = [-19.8, -6.5, -3.1, -1.3, -0.6];% -48dB
%interested range
figure
subplot(2,1,1)
for i = 1:length(y_low)-3
    sz = 50;
    % c = 1:1:length(freq);
    scatter(freq, y_low(i,:), sz);
```

```

    hold on
end
semilogx(w/pi*fs/2, 20*log10(abs(H_low_1)), 'LineWidth', 2);
hold on
semilogx(w/pi*fs/2, 20*log10(abs(H_low_2)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_3)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_4)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_low_5)), 'LineWidth', 2);
legend({'6dB', '12dB', '18dB', '24dB', '30dB', '1st', '2nd', '3rd', '4th', '5th'}, 'Location', 'North');
ylabel('Magnitude (dB)');
xlim([500 2000])
title('Magnitude Response of a Lowpass Bessel Filter as a Function of Filter Order')
grid on;
subplot(2,1,2)
for i =1:1: length(y_low)-3
    sz = 50;
%    c = linspace(1,1,length(freq));
    scatter(freq, y_high(i,:), sz);
    hold on
end
semilogx(w/pi*fs/2, 20*log10(abs(H_high_1)), 'LineWidth', 2);
hold on
semilogx(w/pi*fs/2, 20*log10(abs(H_high_2)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_3)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_4)), 'LineWidth', 2);
semilogx(w/pi*fs/2, 20*log10(abs(H_high_5)), 'LineWidth', 2);
legend({'6dB', '12dB', '18dB', '24dB', '30dB', '1st', '2nd', '3rd', '4th', '5th'}, 'Location', 'North');
ylabel('Magnitude (dB)');
xlim([500 2000])
title('Magnitude Response of a Highpass Bessel Filter as a Function of Filter Order')
grid on;

```

