SUPPLEMENTAL FILE FOR "SECRECY OUTAGE PROBABILITY FAIRNESS FOR INTELLIGENT REFLECTING SURFACE-ASSISTED UPLINK CHANNEL: ALTERNATING OPTIMIZATION VS. LEARNING"

Dear Reader:

A reviewer suggested replacing the SDR algorithm with an alternative method based on the smoothing technique and FISTA algorithm. Based on the literature provided by the reviewer, we have designed the corresponding algorithm as follows.

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Here, we provide a detailed design based on the smoothing technique and FISTA algorithm you suggested. The smoothing technique in [R1] is an effective method for smoothing non-smooth functions. We use it to transform P2 into PR1 as follows,

$$\text{P2:} \min_{\mathbf{\Phi},\mathbf{W}} \max_{k} -z_k, \tag{1a}$$

$$\text{s.t.} \ |\exp(j\theta_n)|^2 = 1, \quad n = 1, ..., N_s, \tag{1b}$$

$$\mathbf{w}_k^{\mathrm{H}}\mathbf{w}_k = 1, \quad k = 1, ..., K. \tag{1c}$$

The log-sum-exp function is utilized to smooth the non-smoothed objective function $f(\mathbf{\Phi}, \mathbf{W}) \triangleq \max_k -z_k$ in P2 as follows,

$$f(\mathbf{\Phi}, \mathbf{W}) \approx f(\mathbf{\Phi}, \mathbf{W}, \beta) = \frac{1}{\beta} \log \left(\sum_{k=1}^{K} e^{-\beta z_k} \right), \tag{2}$$

where $\beta > 0$ is the smoothness parameter. According to Eq. (2), P2 can be transformed as follows,

$$\text{PR1:} \min_{\mathbf{\Phi},\mathbf{W}} \quad f(\mathbf{\Phi}, \mathbf{W}, \beta), \tag{3a}$$

$$\text{s.t. Eqs. } (1b) \text{ and } (1c). \tag{3b}$$

As suggested by the reviewers, we attempted to solve PR1 by using FISTA [R2]. FISTA is a gradient descent algorithm, and we applied it to solve PR1, as shown in Algorithm 1.

Algorithm 1: FISTA Algorithm for PR1

1 Initialize $\mathbf{\Phi}_0, \mathbf{W}_0, t = 0$;
2 **while** $t < \mathrm{Iter}_{\max}$ **do**
3   $\mathbf{\Phi}_{t+1} \leftarrow \mathrm{Proj}_{\Phi}\big(\mathbf{\Phi}_t - \alpha \nabla_{\Phi} f(\mathbf{\Phi}, \mathbf{W}, \beta)\big)$;
4   $\mathbf{w}_{k,(t+1)} \leftarrow \mathrm{eigvec}_{\lambda_{max}}(\mathbf{B}_k^{-1}\mathbf{A}_k), \forall k = 1, ..., K$;
5   **if** $\|f(\mathbf{\Phi}_{t+1}, \mathbf{W}_{t+1}, \beta) - f(\mathbf{\Phi}_t, \mathbf{W}_t, \beta)\| < \epsilon$ **then**
6   **break**;
7   **end**
8   $t \leftarrow t + 1$;
9 **end**
10 Return $\mathbf{\Phi} \leftarrow \mathbf{\Phi}_{t+1}, \mathbf{W} \leftarrow \mathbf{W}_{t+1}$.

In Step 3, $\mathbf{\Phi}$ is updated based on the gradient. The gradient can be calculated as follows,

$$\nabla_{\mathbf{\Phi}} f(\mathbf{\Phi}, \mathbf{W}, \beta) = -\sum_{k=1}^{K} \frac{e^{-\beta z_k}}{\sum_{l=1, l\neq k}^{K} e^{-\beta z_l}} \nabla_{\mathbf{\Phi}} z_k, \tag{4}$$

where

$$\nabla_{\mathbf{\Phi}} z_k = c_{1k} \cdot \frac{L_k}{(\sum_{i=1, i\neq k}^{K} \rho_i |\mathbf{w}_k^{\mathrm{H}}(\mathbf{h}_i + \mathbf{G\Phi f}_i)|^2 + \sigma_b^2)^2}, \tag{5}$$

$$L_k = (\sum_{i=1, i\neq k}^{K} \rho_i |\mathbf{w}_k^{\mathrm{H}}(\mathbf{h}_i + \mathbf{G\Phi f}_i)|^2 + \sigma_b^2)(2\mathbf{w}_k^{\mathrm{H}}(\mathbf{h}_k + \mathbf{G\Phi f}_k)(\mathbf{G}\mathrm{diag}(\mathbf{f}_k))^{\mathrm{H}}\mathbf{w}_k) \tag{6}$$

$$-|\mathbf{w}_k^{\mathrm{H}}(\mathbf{h}_k + \mathbf{G\Phi f}_k)|^2(2\sum_{i=1, i\neq k}^{K} \rho_i \mathbf{w}_k^{\mathrm{H}}(\mathbf{h}_i + \mathbf{G\Phi f}_i)(\mathbf{G}\mathrm{diag}(\mathbf{f}_i))^{\mathrm{H}}\mathbf{w}_k), \tag{7}$$

c_{1k} is defined in Eq. (16) of the paper, and $\mathrm{Proj}_{\Phi}(\mathbf{\Phi})$ is a projection operation as follows,

$$\mathbf{\Phi}_{i,i} = \frac{\mathbf{\Phi}_{i,i}}{|\mathbf{\Phi}_{i,i}|}. \tag{8}$$

In Step 4, $\mathbf{w}_k$ is effectively updated based on the generalized Rayleigh quotient, which has the closed-form expression in Eq. (22) of the paper.

The numerical simulation results are shown in Fig. 1. Notice that the AO scheme based on the smoothing technique and FISTA algorithm (AO-Grad) has the lowest performance compared to the proposed AO and MSB schemes. The step-size α is one, the smoothness parameter β is set to 10, the maximum iteration $\mathrm{Iter}_{\max}$ is 50, and the convergence threshold ϵ is 10^{-4}.

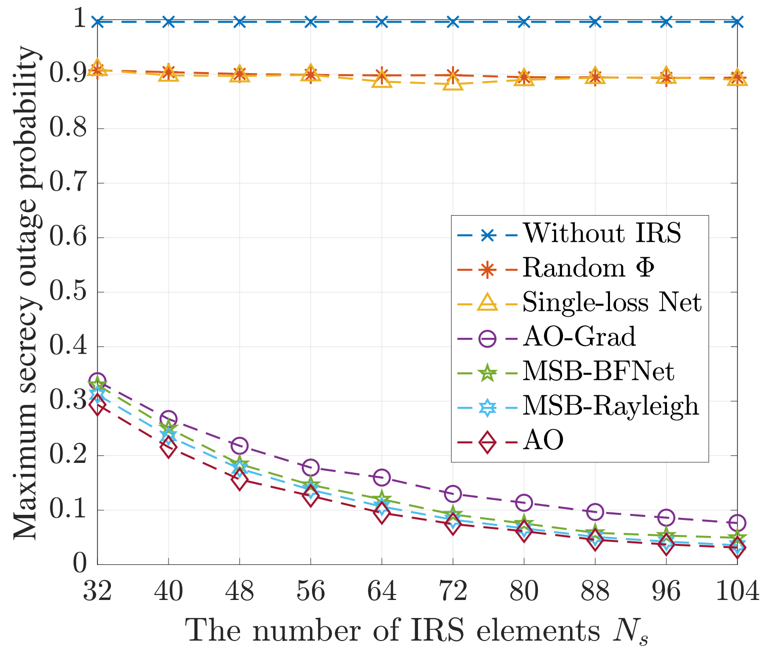

Fig. 1: Performance comparison of methods with an increasing number of IRS elements.

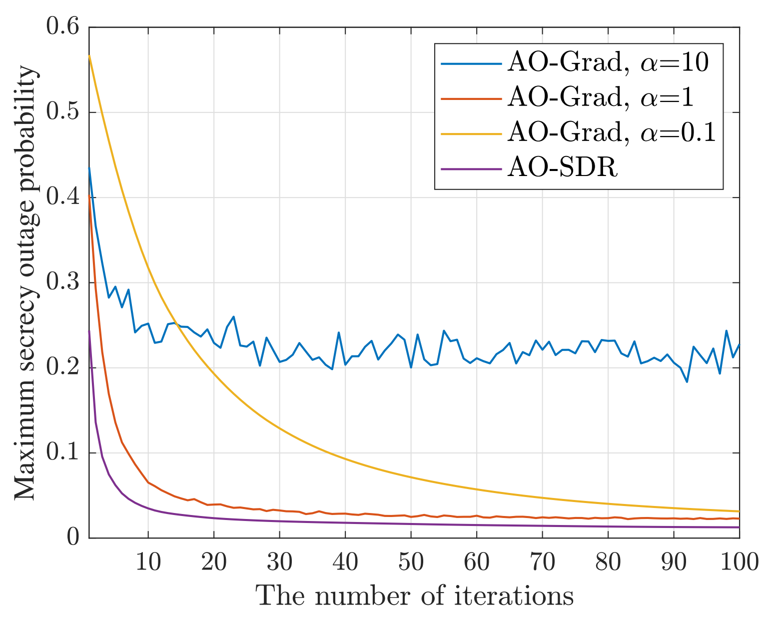

Fig. 2: Maximum secrecy outage probability under different step-sizes α.

The maximum secrecy outage probability under different step sizes is illustrated in Fig. 2. For the selection of the step size α, we conducted experiments within the range of 10, 1, and 0.1, ultimately determining $\alpha = 1$ to be the optimal step-size due to its superior security and convergence performance. The value $\alpha = 10$ exhibits significant oscillations, indicating that the step size is too large. In contrast, $\alpha = 0.1$ converges more slowly, suggesting that the step size is too small.

Here, we provide a simple analysis of why AO-Grad exhibits lower performance. Consider the smoothed objective function $f(\mathbf{\Phi}, \mathbf{W}, \beta) = \frac{1}{\beta} \log \left(\sum_{k=1}^{K} e^{-\beta z_k} \right)$, where z_k is defined as

$$z_k = c_{1k} \left(\frac{|\mathbf{w}_k^{\mathrm{H}} (\mathbf{h}_k + \mathbf{G}\mathbf{\Phi}\mathbf{f}_k)|^2}{\sum_{i=1, i \neq k}^{K} \rho_i |\mathbf{w}_k^{\mathrm{H}} (\mathbf{h}_i + \mathbf{G}\mathbf{\Phi}\mathbf{f}_i)|^2 + \sigma_b^2} \right) + c_{2k}. \tag{9}$$

The variable $\mathbf{\Phi}$ affects both the numerator and denominator in Eq. (9), resulting in a non-convex fractional programming problem. Although $f(\mathbf{\Phi}, \mathbf{W}, \beta)$ is a differentiable function, gradient descent-based algorithms (e.g., FISTA) is not appropriate to find the optimal result straightforwardly because $f(\mathbf{\Phi}, \mathbf{W}, \beta)$ remains non-convex.

In conclusion, we adopt the AO scheme based on the SDR and the generalized Dinkelbach's algorithm after carefully considering both security performance and support for the subsequent DL schemes.

[R1] Y. Nesterov, "Smooth minimization of non-smooth functions," Math. Program., Ser. A, vol. 103, pp. 127–152, 2005.

[R2] A. Beck and M. Teboulle, "A fast iterative shrinkage-thresholding algorithm for linear inverse problems," SIAM J. Imag. Sci., vol. 2, no. 1, pp. 183–202, 2009.