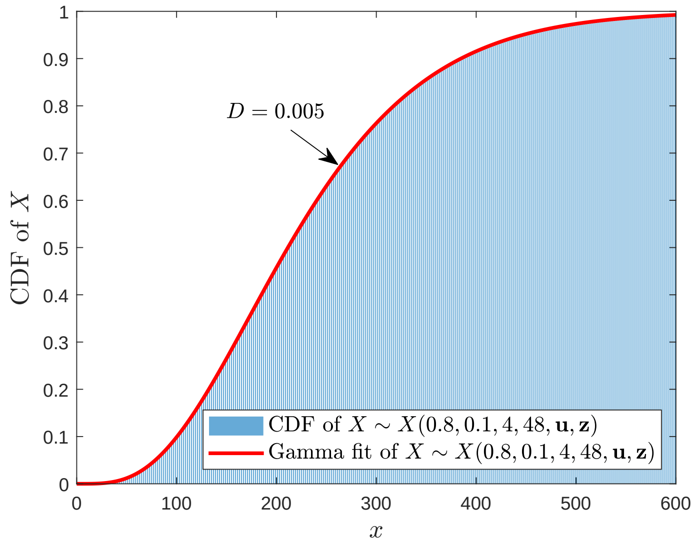

Fig. 1: KS test for Gamma distributions and real distributions of $X \sim X(0.8, 0.1, 4, 48, \mathbf{u}, \mathbf{z})$ via 10^5 Monte Carlo simulations.

Lemma 1. For any $n \times 1$ complex vector $\mathbf{u}$, $\beta \in [0, 1]$, $\beta \in \mathbb{R}^+$, two independent random variables $\hat{\mathbf{a}} \sim \mathcal{CN}_{m,1}(\sqrt{k/(1+k)}\bar{\mathbf{a}}, 1/(1+k)\mathbf{I}_m)$, and $\hat{\mathbf{C}} \sim \mathcal{CN}_{m,n}(\sqrt{k/(1+k)}\bar{\mathbf{C}}, 1/(1+k)\mathbf{I}_m \otimes \mathbf{I}_n)$, if we have random variable x as

$$x = |\beta \mathbf{a} + \mathbf{C}\mathbf{u} + \mathbf{z}|^2, \tag{1}$$

where $\mathbf{z} = \sqrt{k/(1+k)}\bar{\mathbf{a}} + \sqrt{k/(1+k)}\bar{\mathbf{C}}\mathbf{u}$, $\mathbf{a} \sim \mathcal{CN}_{m,1}(\mathbf{0}, \mathbf{I}_m)$ and $\mathbf{C} \sim \mathcal{CN}_{m,n}(\mathbf{0}, \mathbf{I}_m \otimes \mathbf{I}_n)$ are independent, the CDF of $X \sim X(\beta, k, m, n, \mathbf{u}, \mathbf{z})$ can be expressed as

$$F_X(x) = 1 - \frac{1}{\Gamma(\mu)}\Gamma(\mu, \frac{x}{\nu}), \tag{2}$$

where $\Gamma(x)$ is the Gamma function with respect to x, and $\Gamma(\epsilon, \eta)$ is the upper incomplete Gamma function defined as follows,

$$\Gamma(\epsilon, \eta) = \int_\eta^\infty \exp(-z) z^{\epsilon - 1} \mathbf{d}z, \tag{3}$$

$$\mu = m + \frac{|\mathbf{z}|^2}{\rho}, \quad \nu = \frac{\rho}{\kappa}, \tag{4}$$

$$\kappa = (1+k)^{-1} m(\beta^2 + |\mathbf{u}|^2) + |\mathbf{z}|^2, \tag{5}$$

$$\rho = (1+k)^{-2}(\beta^2 + |\mathbf{u}|^2)^2 m + 2(1+k)^{-1}|\mathbf{z}|^2(\beta^2 + |\mathbf{u}|^2). \tag{6}$$

Proof: See in Appendix A. ■

Theorem 1 (Expression of secrecy outage probability)**.** The secrecy outage probability of R_s, i.e., P_{out}, is expressed as

$$P_{\text{out}} = 1 - F_X(\phi_1) = \frac{1}{\Gamma(\mu_e)}\Gamma\left(\mu_e, \frac{\phi_1}{\nu_e}\right), \tag{7}$$

where $\phi_1 = \sigma_e^2(2^{C_m - R_s} - 1)/P$, $F_X(x)$ is defined in Eq. (2), $|(\beta\mathbf{H}_e + \mathbf{G}_e\mathbf{\Phi}\mathbf{H})\mathbf{b}|^2 \sim X(\beta, k, N_e, N_s, \mathbf{\Phi}\mathbf{H}\mathbf{b}, \mathbf{z}_e)$ represents a random variable, $\mathbf{b} = \mathbf{w}/\sqrt{P}$, P is the actual transmission power with $P \leq \rho$,

$$\mathbf{z}_e = \sqrt{k/(1+k)}\beta\bar{\mathbf{H}}_e\mathbf{b} + \sqrt{k/(1+k)}\bar{\mathbf{G}}_e\mathbf{\Phi}\mathbf{H}\mathbf{b}, \tag{8}$$

$$\mu_e = N_e + \frac{|\mathbf{z}_e|^2}{\rho}, \quad \nu_e = \frac{\rho}{\kappa}, \tag{9}$$

$$\kappa = (1+k)^{-1}N_e(\beta^2 + |\mathbf{\Phi}\mathbf{H}\mathbf{b}|^2) + |\mathbf{z}_e|^2, \tag{10}$$

$$\rho = (1+k)^{-2}N_e(\beta^2 + |\mathbf{\Phi}\mathbf{H}\mathbf{b}|^2)^2 + 2(1+k)^{-1}|\mathbf{z}_e|^2(\beta^2 + |\mathbf{\Phi}\mathbf{H}\mathbf{b}|^2). \tag{11}$$

Appendix

A. *Proof of Lemma 1*

Recalling the random variable $x = |\beta\mathbf{a} + \mathbf{C}\mathbf{u} + \mathbf{z}|^2$. We begin to calculate the mean and variance of $x \sim X(\beta, m, n, \mathbf{u}, \mathbf{z})$ as follows. At first, the mean of x is expressed as

$$\begin{aligned}
&\mathbb{E}(|\beta\mathbf{a} + \mathbf{C}\mathbf{u} + \mathbf{z}|^2) \\
&= \mathbb{E}(\mathbf{z}^{\mathrm{H}}\mathbf{z} + \beta\mathbf{z}^{\mathrm{H}}\mathbf{a} + \mathbf{z}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \beta\mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{a} + \mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{z} + \beta^2\mathbf{a}^{\mathrm{H}}\mathbf{a} + \beta\mathbf{a}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \beta\mathbf{a}^{\mathrm{H}}\mathbf{z}) \\
&= (1+k)^{-1}m(\beta^2 + |\mathbf{u}|^2) + |\mathbf{z}|^2 = \kappa. \qquad (12)
\end{aligned}$$

Then, we will deduce the variance of x, i.e., $\text{Var}(x)$, which is given as

$$\text{Var}(x) = \mathbb{E}(|x|^2) - |\mathbb{E}(x)|^2, \tag{13}$$

where $\mathbb{E}(|x|^2)$ can be transformed as

$$\begin{aligned}
&\mathbb{E}(|x|^2) \\
&= \mathbb{E}(\big||\beta\mathbf{a} + \mathbf{C}\mathbf{u}|^2\big|^2) \\
&= \mathbb{E}\big(\big|\mathbf{z}^{\mathrm{H}}\mathbf{z} + \beta\mathbf{z}^{\mathrm{H}}\mathbf{a} + \mathbf{z}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \beta\mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{a} + \mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \mathbf{u}^{\mathrm{H}}\mathbf{C}^{\mathrm{H}}\mathbf{z} + \beta^2\mathbf{a}^{\mathrm{H}}\mathbf{a} + \beta\mathbf{a}^{\mathrm{H}}\mathbf{C}\mathbf{u} + \beta\mathbf{a}^{\mathrm{H}}\mathbf{z}\big|^2\big) \\
&= \mathbb{E}\big(\big||\beta\mathbf{a}|^2\big|^2\big) + \mathbb{E}(\big||\mathbf{C}\mathbf{u}|^2\big|^2) + |\mathbf{z}|^4 \\
&+ 2\mathbb{E}(|\mathbf{z}|^2|\mathbf{C}\mathbf{u}|^2) + 2\mathbb{E}(\beta^2|\mathbf{a}|^2|\mathbf{C}\mathbf{u}|^2) + 2\mathbb{E}(\beta^2|\mathbf{a}|^2|\mathbf{z}|^2) \\
&+ 2\mathbb{E}(\beta^2|\mathbf{a}^{\mathrm{H}}\mathbf{z}|^2) + 2\mathbb{E}(|\mathbf{z}^{\mathrm{H}}\mathbf{C}\mathbf{u}|^2) + 2\mathbb{E}(\beta^2|\mathbf{a}^{\mathrm{H}}\mathbf{C}\mathbf{u}|^2), \qquad (14)
\end{aligned}$$

$\mathbb{E}(\beta^2|\mathbf{a}^{\mathrm{H}}\mathbf{z}|^2) = (1+k)^{-1}\beta^2 m|\mathbf{z}|^2$, $\mathbb{E}(\beta^2|\mathbf{a}^{\mathrm{H}}\mathbf{C}\mathbf{u}|^2) = (1+k)^{-1}\beta^2 m|\mathbf{u}|^2$, $\mathbb{E}(|\mathbf{z}^{\mathrm{H}}\mathbf{C}\mathbf{u}|^2) = (1+k)^{-1}|\mathbf{z}|^2|\mathbf{u}|^2$, $\mathbb{E}(|\mathbf{z}|^2|\mathbf{C}\mathbf{u}|^2) = (1+k)^{-1}m|\mathbf{z}|^2|\mathbf{u}|^2$, $\mathbb{E}(\beta^2|\mathbf{a}|^2|\mathbf{C}\mathbf{u}|^2) = (1+k)^{-2}\beta^2 m^2|\mathbf{u}|^2$, $\mathbb{E}(\beta^2|\mathbf{a}|^2|\mathbf{z}|^2) = (1+k)^{-1}\beta^2 m|\mathbf{z}|^2$. According to the property of noncentral chi-square distribution, the mean and variance of $|\beta\mathbf{a}|^2$ is $(1+k)^{-2}\beta^2 m$ and $(1+k)^{-2}\beta^4 m$, respectively. Hence, $\mathbb{E}(\left||\beta\mathbf{a}|^2\right|^2)$ can be expressed as

$$\mathbb{E}(\left||\beta\mathbf{a}|^2\right|^2) = \mathrm{Var}(|\beta\mathbf{a}|^2) + [\mathbb{E}(|\beta\mathbf{a}|^2)]^2 = (1+k)^{-2}\beta^4(m+m^2). \tag{15}$$

We introduce an auxiliary random variable $\mathbf{z}_1 = \mathbf{C}\mathbf{u}/|\mathbf{u}|$ such that $\mathbf{z}_1 \sim \mathcal{CN}_{m,1}(\mathbf{0}, 1/(1+k)\mathbf{I}_m)$. We change the form of $\mathbb{E}(\left||\mathbf{C}\mathbf{u}|^2\right|^2)$ as

$$\mathbb{E}(\left||\mathbf{C}\mathbf{u}|^2\right|^2) = \mathbb{E}(|\mathbf{u}|^4|\mathbf{z}_1|^4) = (1+k)^{-2}(m^2+m)|\mathbf{u}|^4. \tag{16}$$

Following that, we have

$$\mathbb{E}(|x|^2) = (1+k)^{-2}(\beta^2+|\mathbf{u}|^2)^2(m^2+m) + 2(1+k)^{-1}|\mathbf{z}|^2(\beta^2+|\mathbf{u}|^2)(1+m) + |\mathbf{z}|^4. \tag{17}$$

Then, according to Eq. (13), Var(x) can be expressed as

$$\mathrm{Var}(x) = (1+k)^{-2}(\beta^2+|\mathbf{u}|^2)^2 m + 2(1+k)^{-1}|\mathbf{z}|^2(\beta^2+|\mathbf{u}|^2) = \rho. \tag{18}$$

Hence, the shape and scale of the Gamma distribution can be expressed as

$$\mu = \frac{[\mathbb{E}(x)]^2}{\mathrm{Var}(x)} = m + \frac{|\mathbf{z}|^2}{\rho}, \quad \nu = \frac{\mathrm{Var}(x)}{\mathbb{E}(x)} = \frac{\rho}{\kappa}. \tag{19}$$

At last, on the basis of the definition of the Gamma distribution, we get the CDF of X as follows.

$$F_X(x) = 1 - \frac{1}{\Gamma(\mu)}\Gamma(\mu, \frac{x}{\nu}), \tag{20}$$

where $\Gamma(x)$ is the Gamma function of variable x, and $\Gamma(\epsilon,\eta)$ is the upper incomplete Gamma function defined in Eq. (8). The proof is completed. ■