Marker Code Trace Reconstruction

Wan Yiliang (A0250640R)

1 Forward-Backward Algorithm

1.1 Forward Messages

$$\begin{aligned} f^M(i,j) &= P(s_1...s_j, \pi^M(i,j)|\mathbf{t}) \\ f^I\ (i,j) &= P(s_1...s_j, \pi^I\ (i,j)|\mathbf{t}) \\ f^D\ (i,j) &= P(s_1...s_j, \pi^D\ (i,j)|\mathbf{t}) \end{aligned} \tag{1}$$

$$\begin{aligned} f^M(i,j) &= P(\pi^M(i,j), s_1...s_j|\mathbf{t}) = \sum\nolimits_{\pi_{prev}} P(s_1...s_j, \pi^M(i,j), \pi_{prev}|\mathbf{t}) \\ &= P(s_1...s_j, \pi^M(i,j), \pi^M(i-1,j-1)|\mathbf{t}) + P(\pi^I(i-1,j-1), \pi^M(i,j), s_1...s_j|\mathbf{t}) \\ &\quad + P(s_1...s_j, \pi^M(i,j), \pi^D(i-1,j-1)|\mathbf{t}) \end{aligned} \tag{2}$$

First, we examine the first term of (2)

$$\begin{aligned} &P(\pi^M(i-1,j-1), \pi^M(i,j), s_1...s_j|\mathbf{t}) \\ &= P(s_1...s_{j-1}|s_j, \pi^M(i-1,j-1), \pi^M(i,j), \mathbf{t}) \times P(s_j|\pi^M(i-1,j-1), \pi^M(i,j), \mathbf{t}) \\ &\quad \times P(\pi^M(i,j)|\pi^M(i-1,j-1), \mathbf{t}) \times P(\pi^M(i-1,j-1)|\mathbf{t}) \\ &= P(s_1...s_{j-1}|\pi^M(i-1,j-1)\mathbf{t}) \times P(s_j|\pi^M(i,j), \mathbf{t}) \times P(\pi^M(i,j)|\pi^M(i-1,j-1), \mathbf{t}) \\ &\quad \times P(\pi^M(i-1,j-1)|\mathbf{t}) \\ &= P(s_1...s_{j-1}|\pi^M(i-1,j-1)\mathbf{t}) \times P(\pi^M(i-1,j-1)|\mathbf{t}) \times P(\pi^M(i,j)|\pi^M(i-1,j-1), \mathbf{t}) \\ &\quad \times P(s_j|\pi^M(i,j), \mathbf{t}) \\ &= P(s_1...s_{j-1}, \pi^M(i-1,j-1)|\mathbf{t}) \times P(\pi^M(i,j)|\pi^M(i-1,j-1)) \times\ P(s_j|\pi^M(i,j), \mathbf{t}) \\ &= f^M(i-1,j-1) \times T_{MM} \times Memission[i](s_j) \end{aligned} \tag{3}$$

Similar procedures can be applied to the second and the third term of (2)

$$P(\pi^I(i-1,j-1), \pi^M(i,j), s_1...s_j|\mathbf{t}) = f^I(i-1,j-1) \times T_{IM} \times Memission[i](s_j) \tag{4}$$

$$P(\pi^D(i-1,j), \pi^M(i,j), s_1...s_j|\mathbf{t}) = f^D(i-1,j-1) \times T_{DM} \times Memission[i](s_j) \tag{5}$$

Put (3) (4) and (5) into (2), and we can obtain the recursion of f^M

$$f^M(i,j) = (f^M(i-1,j-1) \times T_{MM} + f^I(i-1,j-1) \times T_{IM} + f^D(i-1,j-1) \times T_{DM}) \times Memission[i](s_j) \tag{6}$$

Similarly, we can obtain the recursion formula of f^I and f^D

$$\begin{aligned} f^I(i,j) &= \sum\nolimits_{\pi_{prev}} P(\pi_{prev}, \pi^I(i,j), s_1...s_j|\mathbf{t}) \\ &= (f^M(i,j-1) \times T_{MI} + f^I(i,j-1) \times T_{II} + f^D(i,j-1) \times T_{DI}) \times Iemission(s_j) \end{aligned} \tag{7}$$

$$\begin{aligned} f^D(i,j) &= \sum\nolimits_{\pi_{prev}} P(\pi_{prev}, \pi^D(i,j), s_1...s_j|\mathbf{t}) \\ &= f^M(i-1,j) \times T_{MD} + f^I(i-1,j) \times T_{ID} + f^D(i-1,j) \times T_{DD} \end{aligned} \tag{8}$$

1.2 Backward Messages

$$\begin{aligned} b^M(i,j) &= P(s_{j+1}...s_N|\pi^M(i,j), \mathbf{t}) \\ b^I\ (i,j) &= P(s_{j+1}...s_N|\pi^I\ (i,j), \mathbf{t}) \\ b^D\ (i,j) &= P(s_{j+1}...s_N|\pi^D\ (i,j), \mathbf{t}) \end{aligned} \tag{9}$$

$$\begin{aligned} b^M(i,j) &= P(s_{j+1}...s_N|\pi^M(i,j), \mathbf{t}) = \sum\nolimits_{\pi_{next}} P(s_{j+1}...s_N, \pi_{next}|\pi^M(i,j), \mathbf{t}) \\ &= P(s_{j+1}...s_N, \pi^M(i+1,j+1)|\pi^M(i,j), \mathbf{t}) + P(s_{j+1}...s_N, \pi^I(i,j+1)|\pi^M(i,j), \mathbf{t}) \\ &\quad + P(s_{j+1}...s_N, \pi^D(i+1,j))|\pi^M(i,j), \mathbf{t}) \end{aligned} \tag{10}$$

Again, $b^M(i,j)$ is the sum of three terms which correspond to three possible next states. Examine the first term of (10).

$$\begin{aligned}
&P(s_{j+1}...s_N, \pi^M(i+1,j+1)|\pi^M(i,j), \mathbf{t}) \\
&\quad = P(s_{j+2}...s_N|s_{j+1}, \pi^M(i+1,j+1), \pi^M(i,j), \mathbf{t}) \ \times P(s_{j+1}, \pi^M(i+1,j+1)|\pi^M(i,j), \mathbf{t}) \\
&\quad = P(s_{j+2}...s_N|s_{j+1}, \pi^M(i+1,j+1), \pi^M(i,j), \mathbf{t}) \ \times P(s_{j+1}|\pi^M(i+1,j+1)\pi^M(i,j), \mathbf{t}) \\
&\qquad \times P(\pi^M(i+1,j+1)|\pi^M(i,j), \mathbf{t}) \\
&\quad = P(s_{j+2}...s_N|\pi^M(i+1,j+1), \mathbf{t}) \ \times P(s_{j+1}|\pi^M(i+1,j+1), \mathbf{t}) \ \times P(\pi^M(i+1,j+1)|\pi^M(i,j)) \\
&\quad = b^M(i+1,j+1) \times Memission[i+1][s_{j+1}] \times T_{MM}
\end{aligned} \tag{11}$$

Similar procedures can be applied to the second and the third term of (10)

$$P(s_{j+1}...s_N, \pi^I(i,j+1)|\pi^M(i,j), \mathbf{t}) \ = b^I(i,j+1) \times Iemission[s_{j+1}] \times T_{MI} \tag{12}$$

$$P(s_{j+1}...s_N, \pi^D(i+1,j))|\pi^M(i,j), \mathbf{t}) \ = b^D(i+1,j) \times T_{MD} \tag{13}$$

Put (11) (12) and (13) into (10), and we can obtain the recursion of b^M

$$\begin{aligned}
b^M(i,j) \ &= b^M(i+1,j+1) \times Memission[i+1][s_{j+1}] \times T_{MM} \ + b^I(i,j+1) \times Iemission[s_{j+1}] \times T_{MI} \\
&\quad + b^D(i+1,j) \times T_{MD}
\end{aligned} \tag{14}$$

Similarly, we can obtain the recursion formula of b^I and b^D

$$\begin{aligned}
b^I(i,j) \ &= b^M(i+1,j+1) \times Memission[i+1][s_{j+1}] \times T_{IM} \ + b^I(i,j+1) \times Iemission[s_{j+1}] \times T_{II} \\
&\quad + b^D(i+1,j) \times T_{ID}
\end{aligned} \tag{15}$$

$$\begin{aligned}
b^D(i,j) \ &= b^M(i+1,j+1) \times Memission[i+1][s_{j+1}] \times T_{DM} \ + b^I(i,j+1) \times Iemission[s_{j+1}] \times T_{DI} \\
&\quad + b^D(i+1,j) \times T_{DD}
\end{aligned} \tag{16}$$